Angle of Elevation and Depression

The **angle of elevation** is the angle formed between the horizontal line and the line of sight when an observer looks upward at an object. Conversely, the **angle of depression** is the angle formed between the horizontal line and the line of sight when the observer looks downward at an object.

Concept Explanation

To solve problems involving angles of elevation and depression, first draw a horizontal line from the observer's eye level. Then, draw the line of sight to the object. The angle between these two lines is the angle of elevation if the object is above the horizontal, or the angle of depression if below.

Formula Derivation

Consider a right triangle formed by the observer's position, the base of the object, and the top of the object. Let θ be the angle of elevation or depression, h be the height of the object, and d be the horizontal distance from the observer to the object.

Using trigonometry,

$$\tan\theta = \frac{h}{d}$$

Rearranging,

$$h = d \times \tan\theta$$

Worked Illustration

Given an observer at point O looking at the top of an object at point B, with horizontal distance AO and height AB, the angle of elevation $\angle AOB = \theta$. Using the formula above, the height AB can be calculated if AO and θ are known.

Solved Examples

Example 1: From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building, and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from point P. (Take $\sqrt{3} = 1.732$)

Solution:

Let the distance from P to the building be $AP = x$ meters.

In right-angled triangle $\triangle PBA$,

$$\tan 30^\circ = \frac{BA}{AP} = \frac{10}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{x} \implies x = 10\sqrt{3} = 17.32 \text{ m}$$

Let the length of the flagstaff be $BD = h$ meters.

Then, total height $AD = AB + BD = 10 + h$.

In right-angled triangle $\triangle PDA$,

$$\tan 45^\circ = \frac{AD}{AP} = \frac{10+h}{17.32}$$

$$1 = \frac{10+h}{17.32} \implies 10 + h = 17.32 \implies h = 7.32 \text{ m}$$

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Thus, the length of the flagstaff is 7.32 m, and the distance of the building from point P is 17.32 m.

Practice Set

- **Level 1 – Easy:** From a point 20 m away from a tree, the angle of elevation to the top of the tree is 30°. Find the height of the tree.
- **Level 2 – Moderate:** A man standing on the top of a 50 m tall building observes a car moving on the ground. The angle of depression of the car from the top of the building is 45°. Find the distance of the car from the base of the building.
- **Level 3 – Challenging:** The angles of elevation of the top of a tower from two points on the ground, which are 30 m apart on the same side of the tower, are 30° and 60°. Find the height of the tower.

Answer Key

- **Level 1:** Height = 20 $\times \tan 30^\circ = 20 \times \frac{1}{\sqrt{3}} = 11.55$ m
- **Level 2:** Distance = Height = 50 m (since $\tan 45^\circ = 1$)
- **Level 3:** Let the height be h. Using two points and tangent values, $h = 17.32$ m (detailed solution involves simultaneous equations)

Quick Reference

Quantity	Formula
Height of object	$h = d \times \tan\theta$
Distance from object	$d = \frac{h}{\tan\theta}$

Glossary

- **Angle of Elevation:** Angle between horizontal and line of sight looking upward.
- **Angle of Depression:** Angle between horizontal and line of sight looking downward.
- **Line of Sight:** Straight line from observer's eye to the object.
- **Horizontal:** A line parallel to the ground or x-axis.
- **Observer:** The point from which the object is viewed.

Applications of Trigonometry in Measuring Heights and Distances

Trigonometry is used to calculate unknown heights and distances by measuring angles of elevation and depression and applying trigonometric ratios.

Concept Explanation

By measuring the angle of elevation or depression and the horizontal distance, one can calculate the height of an object or the distance between objects using the tangent function.

Formula Derivation

In a right triangle,

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{h}{d}$$

where h is the height and d is the horizontal distance.

Solved Examples

Example 2: The angles of depression of the top and bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Solution:

Let the height of the multi-storeyed building be PC and the distance between buildings be BD. Given $AB = 8$ m.

Using alternate interior angles,

$$\angle PBD = 30^\circ, \quad \angle PAC = 45^\circ$$

In right triangle $\triangle PBD$,

$$\tan 30^\circ = \frac{PD}{BD} = \frac{1}{\sqrt{3}} \implies BD = \sqrt{3}PD \quad (1)$$

In right triangle $\triangle PAC$,

$$\tan 45^\circ = \frac{PC}{AC} = 1 \implies AC = PC$$

Since $PC = PD + DC$ and $DC = AB = 8$,

$$AC = PD + 8$$

From (1),

$$BD = \sqrt{3}PD = PD + 8$$

Rearranged,

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$$PD(\sqrt{3}-1) = 8 \implies PD = \frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{8(\sqrt{3}+1)}{2} = 4(\sqrt{3}+1)$$

Therefore,

$$PC = PD + 8 = 4(\sqrt{3}+1) + 8 = 4(\sqrt{3}+3) \text{ m}$$

Distance between buildings,

$$BD = 4(\sqrt{3}+3) \text{ m}$$

Prepzy

Practice Set

- **Level 1 – Easy:** Find the height of a tower if the angle of elevation from a point 50 m away is 45°.
- **Level 2 – Moderate:** The angle of depression of a boat from the top of a lighthouse 60 m high is 30°. Find the distance of the boat from the base of the lighthouse.
- **Level 3 – Challenging:** Two buildings are 40 m apart. From the top of the first building, the angle of depression of the base of the second building is 30°, and from the top of the second building, the angle of depression of the base of the first building is 45°. Find the heights of the two buildings.

Answer Key

- **Level 1:** Height = 50 m (since $\tan 45^\circ = 1$)

- **Level 2:** Distance = $60 \times \sqrt{3} = 103.92$ m
- **Level 3:** Heights can be found using simultaneous equations involving tangent of angles; detailed steps required.

Quick Reference

Quantity	Formula
Height of building	$h = d \times \tan\theta$
Distance from building	$d = \frac{h}{\tan\theta}$

Glossary

- **Angle of Depression:** Angle between horizontal and line of sight looking downward.
- **Alternate Interior Angles:** Angles formed when a transversal crosses parallel lines, equal in measure.
- **Right Triangle:** Triangle with one 90° angle.