- Tangents and Secants to Circles

Tangents and Secants to Circles

A **tangent** to a circle is a line that touches the circle at exactly one point, called the point of contact. A **secant** is a line that intersects the circle at two distinct points.

The tangent line is always perpendicular to the radius drawn to the point of contact.

From an external point outside the circle, exactly two tangents can be drawn, and these tangents have equal lengths from the external point to the points of contact on the circle.

Formula Derivation: Length of Tangents from an External Point

Let the circle have center O and radius r. Let P be an external point from which tangents PA and PB are drawn touching the circle at points A and B respectively.

Since OA and OB are radii, and PA and PB are tangents, triangles OAP and OBP are right angled at A and B respectively.

Using the Pythagoras theorem in $\triangle OAP$:

$$OP^2 = OA^2 + AP^2$$

$$AP^2 = OP^2 - OA^2$$

$$AP = \sqrt{OP^2 - r^2}$$

Similarly, $PB = AP$. Thus, the lengths of tangents from an external point are equal.

Worked Illustration

Given a circle with center O and radius 5 cm, and an external point P such that $OP = 13$ cm, find the length of the tangent from P to the circle.

Solution:

Using the formula derived,

$$AP = \sqrt{OP^2 - r^2} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12 \text{ cm}$$

Therefore, the length of the tangent from P to the circle is 12 cm.

Solved Examples

Example 1: From a point P outside a circle with radius 7 cm, the length of the tangent is 24 cm. Find the distance from P to the center of the circle.

Solution:

Let $OP = d$. Using the tangent length formula,

$$AP = \sqrt{d^2 - r^2} = 24$$

$$24 = \sqrt{d^2 - 7^2} = \sqrt{d^2 - 49}$$

$$24^2 = d^2 - 49$$

$$576 = d^2 - 49$$

$$d^2 = 576 + 49 = 625$$

$$d = 25 \text{ cm}$$

Distance from P to the center is 25 cm.

Practice Set

- **Level 1 – Easy**
- Find the length of the tangent from a point 10 cm away from the center of a circle with radius 6 cm.
- How many tangents can be drawn from a point on the circle?
- **Level 2 – Moderate**
- From a point 15 cm away from the center of a circle with radius 9 cm, find the length of each tangent.
- Prove that the tangents drawn from an external point to a circle are equal in length.
- **Level 3 – Challenging**
- Two concentric circles have radii 4 cm and 7 cm. Find the length of the chord of the larger circle that is tangent to the smaller circle.
- From a point outside a circle, two tangents are drawn. If the distance from the point to the center is 13 cm and the radius is 5 cm, find the length of each tangent and the distance between the points of contact.

Answer Key

- Level 1:
  - Length of tangent = $\sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8$ cm
  - Exactly one tangent can be drawn from a point on the circle.
- Level 2:
  - Length of tangent = $\sqrt{15^2 - 9^2} = \sqrt{225 - 81} = \sqrt{144} = 12$ cm
  - Proof: Using right triangles formed by radius and tangent, lengths are equal by Pythagoras theorem.
- Level 3:
  - Length of chord tangent to smaller circle = $2 \times \sqrt{7^2 - 4^2} = 2 \times \sqrt{49 - 16} = 2 \times 5 = 10$ cm
  - Length of tangent = $\sqrt{13^2 - 5^2} = 12$ cm; distance between points of contact = 2 $\times$ length of tangent $= 24$ cm

Quick Reference

Concept	Formula	Notes
Length of tangent from external point	$\sqrt{OP^2 - r^2}$	OP = distance from center to external point, r = radius
Number of tangents from point	1 if on circle, 2 if outside, 0 if inside	
Tangent and radius	Perpendicular at point of contact	$\angle \text{between tangent and radius} = 90^\circ$

Glossary

- **Tangent:** A line touching a circle at exactly one point.
- **Secant:** A line intersecting a circle at two points.
- **Point of Contact:** The point where a tangent touches the circle.
- **Radius:** A line segment from the center of the circle to any point on the circle.
- **Chord:** A line segment joining two points on the circle.
- **External Point:** A point outside the circle from which tangents can be drawn.
- **Perpendicular:** Two lines meeting at a right angle (90 degrees).