

- Mean
- Median
- Mode
- Empirical Relationship Between Mean, Median and Mode

Mean

Concept Explanation: Mean is the average value of a set of numbers. It is calculated by summing all observations and dividing by the total number of observations.

Formula Derivation

For ungrouped data with observations x_1, x_2, \dots, x_n , mean \bar{x} is given by:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

For grouped data, where f_i is the frequency of the i^{th} class and x_i is the class mark (midpoint), mean is:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Class mark x_i is calculated as:

$$x_i = \frac{\text{lower limit} + \text{upper limit}}{2}$$

Assumed Mean Method (shortcut):

$$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$$

where A is the assumed mean and $d_i = x_i - A$ are deviations.

Worked Illustrations

Given class intervals and frequencies, calculate mean using both direct and assumed mean methods.

Solved Example

Marks obtained by 30 students are grouped as follows:

Class Interval	Frequency (f_i)	Class Mark (x_i)	Deviation ($d_i = x_i - 47.5$)	$f_i x_i$	$f_i d_i$
10-25	1	17.5	-30	17.5	-30
25-40	3	32.5	-15	97.5	-45
40-55	5	47.5	0	237.5	0
55-70	4	62.5	15	250	60
70-85	7	77.5	30	542.5	210
85-100	10	92.5	45	925	450
Total	30			2070	645

Calculate mean by:

- Direct Method: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2070}{30} = 69$
- Assumed Mean Method: $\bar{x} = 47.5 + \frac{645}{30} = 47.5 + 21.5 = 69$

Practice Set

- *Level 1:* Find mean of ungrouped data: 12, 15, 18, 20, 25.
- *Level 2:* Calculate mean using assumed mean method for grouped data: Class intervals 0-10, 10-20, 20-30 with frequencies 5, 8, 7.
- *Level 3:* Given grouped data with class intervals and frequencies, calculate mean using both methods and compare.

Answer Key

- Level 1: Mean = $\frac{12+15+18+20+25}{5} = 18$
- Level 2: Assume mean $A = 15$, calculate deviations and mean accordingly.
- Level 3: Detailed steps required for both methods.

Quick Reference

- Mean (ungrouped): $\bar{x} = \frac{\sum x_i}{n}$
- Mean (grouped direct): $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
- Mean (assumed): $\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$

Glossary

- **Class Mark:** Midpoint of class interval.
- **Frequency:** Number of observations in a class.
- **Deviation:** Difference between class mark and assumed mean.

Median

Concept Explanation: Median is the middle value of an ordered data set dividing it into two equal halves.

Formula Derivation

For ungrouped data:

- If number of observations n is odd, median is the $\left(\frac{n+1}{2}\right)^{th}$ value.
- If n is even, median is average of $\frac{n}{2}^{th}$ and $\left(\frac{n}{2} + 1\right)^{th}$ values.

For grouped data, median is calculated using:

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c. f.}{f} \right) \times h$$

where:

- l = lower limit of median class
- N = total frequency
- $c. f.$ = cumulative frequency before median class
- f = frequency of median class
- h = class width

Worked Illustrations

Identify median class by locating $\frac{N}{2}$ in cumulative frequency, then apply formula.

Solved Example

Given cumulative frequency data for heights of 51 girls:

Class Interval	Frequency (f)	Cumulative Frequency ($c. f.$)
Below 140	4	4
140-145	7	11
145-150	18	29
150-155	15	44
155-160	7	51

Total $N = 51$, so $\frac{N}{2} = 25.5$. Median class is 145-150.

Applying formula:

$$\text{Median} = 145 + \left(\frac{25.5 - 11}{18} \right) \times 5 = 145 + \frac{14.5}{18} \times 5 = 145 + 4.03 = 149.03$$

Practice Set

- *Level 1:* Find median of ungrouped data: 12, 15, 18, 20, 25.
- *Level 2:* Calculate median for grouped data with given frequencies and class intervals.
- *Level 3:* Given cumulative frequency table, find median and interpret results.

Answer Key

- Level 1: Median = 18 (middle value)

- Level 2: Identify median class and apply formula.
- Level 3: Detailed calculation as shown in example.

Quick Reference

- Median (ungrouped odd): middle value
- Median (ungrouped even): average of two middle values
- Median (grouped): $l + \left(\frac{\frac{N}{2} - c.f.}{f} \right) \times h$

Glossary

- **Median Class:** Class interval containing the median.
- **Cumulative Frequency:** Sum of frequencies up to a class.
- **Class Width:** Difference between upper and lower class limits.

Mode

Concept Explanation: Mode is the value that occurs most frequently in a data set.

For ungrouped data, mode is the observation with highest frequency.

For grouped data, mode is estimated using the modal class (class with highest frequency) and the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where:

- l = lower limit of modal class
- h = class width
- f_1 = frequency of modal class
- f_0 = frequency of class before modal class
- f_2 = frequency of class after modal class

Worked Illustrations

Identify modal class by highest frequency, then apply formula.

Solved Example

Family size data:

Class Interval	Frequency
1-3	7
3-5	8
5-7	2
7-9	3
9-11	0

Modal class is 3-5 with frequency $f_1 = 8$.

Applying formula:

$$\text{Mode} = 3 + \left(\frac{8 - 7}{2 \times 8 - 7 - 2} \right) \times 2 = 3 + \frac{1}{7} \times 2 = 3 + 0.286 = 3.286$$

Practice Set

- *Level 1:* Find mode of ungrouped data: 2, 3, 3, 5, 7.
- *Level 2:* Calculate mode for grouped data with given frequencies.
- *Level 3:* Given frequency distribution, find modal class and mode using formula.

Answer Key

- Level 1: Mode = 3
- Level 2: Identify modal class and apply formula.
- Level 3: Detailed calculation as shown in example.

Quick Reference

- Mode (ungrouped): most frequent value
- Mode (grouped): $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

Glossary

- **Modal Class:** Class with highest frequency.
- **Frequency:** Number of observations in a class.
- **Class Width:** Difference between class limits.

Empirical Relationship Between Mean, Median and Mode

Concept Explanation: The mean, median and mode of a moderately skewed distribution are related by empirical formulas.

Formula Derivation

The empirical relations are:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

$$\text{Median} = \frac{1}{3} \times \text{Mode} + \frac{2}{3} \times \text{Mean}$$

$$\text{Mean} = \frac{3}{2} \times \text{Median} - \frac{1}{2} \times \text{Mode}$$

Worked Illustrations

Given any two of mean, median, mode, calculate the third using these formulas.

Solved Example

If mean = 50 and median = 52, find mode:

$$\text{Mode} = 3 \times 52 - 2 \times 50 = 156 - 100 = 56$$

Practice Set

- *Level 1:* Given mean and median, find mode.
- *Level 2:* Given mode and mean, find median.
- *Level 3:* Given median and mode, find mean.

Answer Key

- Level 1: Use $\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$
- Level 2: Use $\text{Median} = \frac{1}{3} \times \text{Mode} + \frac{2}{3} \times \text{Mean}$
- Level 3: Use $\text{Mean} = \frac{3}{2} \times \text{Median} - \frac{1}{2} \times \text{Mode}$

Quick Reference

- $\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$
- $\text{Median} = \left(\frac{1}{3}\right) \text{ Mode} + \left(\frac{2}{3}\right) \text{ Mean}$
- $\text{Mean} = \left(\frac{3}{2}\right) \text{ Median} - \left(\frac{1}{2}\right) \text{ Mode}$

Glossary

- **Empirical Relation:** Approximate relationship between mean, median and mode.

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