Linear Equations in Two Variables

A linear equation in two variables is an equation of the form $ax + by + c = 0$, where a, b, c are real numbers and $a \neq 0, b \neq 0$. The variables x and y appear to the first power only.

Such an equation represents a straight line when graphed on the Cartesian plane.

Formula Derivation

Starting from the general form:

$$ax + by + c = 0$$

Express y in terms of x:

$$by = -ax - c$$

$$y = -\frac{a}{b}x - \frac{c}{b}$$

This is the slope-intercept form $y = mx + c'$ where slope $m = -\frac{a}{b}$ and intercept $c' = -\frac{c}{b}$.

Worked Illustrations

Consider the equation $2x + y = 7$.

To find solutions, assign arbitrary values to x and solve for y:

- For $x = 0$: $y = 7 - 2(0) = 7$ → solution $(0, 7)$
- For $x = 1$: $y = 7 - 2(1) = 5$ → solution $(1, 5)$
- For $x = 3$: $y = 7 - 2(3) = 1$ → solution $(3, 1)$
- For $y = 0$: $2x + 0 = 7 \rightarrow x = \frac{7}{2}$ → solution $(\frac{7}{2}, 0)$

Solved Examples

Example: Find four different solutions of $2x + y = 7$.

Solution:

1. Given equation: $2x + y = 7$
2. Put $x = 0$: $y = 7$ → solution $(0, 7)$
3. Put $y = 0$: $2x = 7 \rightarrow x = \frac{7}{2}$ → solution $(\frac{7}{2}, 0)$
4. Put $x = 1$: $y = 7 - 2(1) = 5$ → solution $(1, 5)$
5. Put $y = 1$: $2x + 1 = 7 \rightarrow 2x = 6 \rightarrow x = 3$ → solution $(3, 1)$

Thus, four solutions are $(0, 7), (\frac{7}{2}, 0), (1, 5), (3, 1)$.

Practice Set

Level 1 – Easy

- Find two solutions of $x + y = 5$.
- Find three solutions of $3x + 2y = 12$.
- Find one solution of $4x - y = 7$ when $x = 2$.

Level 2 – Moderate

- Find four solutions of $5x + 3y = 15$.
- Find three solutions of $2x - 4y = 8$.
- Find the value of y when $x = -1$ in $7x + y = 0$.

Level 3 – Challenging

- Find five solutions of $6x + 5y = 30$ where both x and y are integers.
- Find three solutions of $3x - 2y = 1$ such that y is positive.
- Verify if $(2, 3)$ is a solution of $4x + 5y = 23$.

Answer Key

Level 1

- $(0, 5), (5, 0)$
- $(0, 6), (4, 0), (2, 3)$
- $4(2) - y = 7 \Rightarrow 8 - y = 7 \Rightarrow y = 1 \rightarrow$ solution $(2, 1)$

Level 2

- $(0, 5), (3, 0), (1, 10/3), (6, -5)$
- $(0, -2), (4, 0), (8, 4)$
- $7(-1) + y = 0 \Rightarrow -7 + y = 0 \Rightarrow y = 7$

Level 3

- $(0, 6), (5, 0), (1, 4.8), (2, 3.6), (3, 2.4)$ (integers only: $(0, 6), (5, 0)$ and others approximate)
- $(1, 1), (3, 4), (5, 7)$
- Check: $4(2) + 5(3) = 8 + 15 = 23$ → Yes, $(2, 3)$ is a solution.

Quick Reference

Form	Equation	Slope	Intercept
General	$ax + by + c = 0$	$-\frac{a}{b}$	$-\frac{c}{b}$
Slope-Intercept	$y = mx + c$	m	c

Glossary

- **Linear Equation:** An equation where variables have power 1.
- **Solution:** An ordered pair (x, y) satisfying the equation.
- **Slope:** The rate of change of y with respect to x.
- **Intercept:** The point where the line crosses the y-axis.

Graphing Linear Equations in Two Variables

A linear equation in two variables can be represented graphically by plotting its solutions as points on the Cartesian plane and joining them to form a straight line.

Concept Explanation

Given $ax + by + c = 0$, express y in terms of x:

$$y = -\frac{a}{b}x - \frac{c}{b}$$

Choose arbitrary values of x, compute corresponding y, plot points (x, y), and join them with a straight line.

Worked Illustrations

Graph the equation $2x + y = 3$.

Express y:

$$y = 3 - 2x$$

Choose values:

- $x = 0, y = 3 \rightarrow (0,3)$
- $x = 1, y = 1 \rightarrow (1,1)$
- $x = -1, y = 5 \rightarrow (-1,5)$

Plot these points and join them to form the line.

Solved Examples

Example: Draw the graph of $2x + y = 3$.

Solution:

1. Express y: $y = 3 - 2x$
2. Calculate points:
   - $x = 0, y = 3 \rightarrow (0,3)$
   - $x = 1, y = 1 \rightarrow (1,1)$
   - $x = -1, y = 5 \rightarrow (-1,5)$
3. Plot points on graph paper.
4. Join points with a straight line.

Practice Set

Level 1 – Easy

- Graph $x + y = 4$ using three points.
- Graph $y = 2x$ using four points.
- Graph $y = 5$ and describe the line.

Level 2 – Moderate

- Graph $3x - y = 6$ using three points.
- Graph $2x + 3y = 12$ and find intercepts.
- Graph $x = 4$ and describe the line.

Level 3 – Challenging

- Graph $4x + 5y = 20$ and find slope.
- Graph $y = -\frac{1}{2}x + 3$ and find intercepts.
- Verify if points $(2, 3)$, $(4, 1)$, and $(0, 5)$ lie on the line $2x + y = 7$.

Answer Key

Level 1

- Points: (0,4), (1,3), (2,2)
- Points: (0,0), (1,2), (2,4), (3,6)
- Line is horizontal passing through y=5.

Level 2

- Points: (0,-6), (2,0), (4,6)
- Intercepts: x=4, y=4
- Line is vertical passing through x=4.

Level 3

- Slope $m = -\frac{4}{5}$
- Intercepts: y-intercept = 3, x-intercept = 6
- Points (2,3) and (4,1) satisfy equation; (0,5) does not.

Quick Reference

Step	Action
1	Express y in terms of x
2	Choose values of x
3	Calculate corresponding y
4	Plot points (x, y)
5	Join points with straight line

Glossary

- **Cartesian Plane:** A plane with x-axis and y-axis intersecting at origin.
- **Coordinate:** An ordered pair (x, y) representing a point.
- **Slope:** Ratio of vertical change to horizontal change.
- **Intercept:** Point where line crosses axis.