Types of Angles

Understanding the types of angles is fundamental in geometry. An angle is formed by two rays originating from the same point called the vertex. The rays are called the arms of the angle.

Concept Explanation

- **Acute Angle:** An angle x such that $0^\circ < x < 90^\circ$.
- **Right Angle:** An angle $x = 90^\circ$.
- **Obtuse Angle:** An angle x such that $90^\circ < x < 180^\circ$.
- **Straight Angle:** An angle $x = 180^\circ$.
- **Reflex Angle:** An angle x such that $180^\circ < x < 360^\circ$.
- **Complete Angle:** An angle $x = 360^\circ$.

Formula Derivation

Angles are measured in degrees, representing the amount of rotation from one arm to the other around the vertex.

Worked Illustrations and Solved Examples

Refer to the images for visual understanding of each angle type.

Practice Set

- **Level 1 – Easy:** Identify the type of angle for $45^\circ, 90^\circ, 120^\circ$.
- **Level 2 – Moderate:** Draw angles of $30^\circ, 135^\circ, 270^\circ$ using a protractor.
- **Level 3 – Challenging:** Given an angle x, if its complement is 40°, find x. Also, find its supplement.

Answer Key

- Level 1: 45° - Acute, 90° - Right, 120° - Obtuse
- Level 3: $x + 40^\circ = 90^\circ \Rightarrow x = 50^\circ$; Supplement = $180^\circ - 50^\circ = 130^\circ$

Quick Reference

- Complementary angles sum to 90°.
- Supplementary angles sum to 180°.

Glossary

- **Vertex:** The common endpoint of two rays forming an angle.
- **Arms:** The two rays that form an angle.
- **Complementary Angles:** Two angles whose sum is 90°.
- **Supplementary Angles:** Two angles whose sum is 180°.

Revision Notes

Concept Explanation

Basic definitions and properties related to lines and angles.

- **Line:** A collection of points extending infinitely in both directions with length but no breadth or thickness.
- **Line Segment:** A part of a line bounded by two endpoints.
- **Ray:** A part of a line with one endpoint extending infinitely in one direction.
- **Angle:** Formed by two rays with a common endpoint called the vertex.
- **Complementary Angles:** Two angles whose sum is 90°.
- **Supplementary Angles:** Two angles whose sum is 180°.
- **Adjacent Angles:** Two angles with the same vertex and a common arm, and their non-common arms are on opposite sides of the common arm.
- **Vertically Opposite Angles:** When two lines intersect, the opposite angles formed are equal.
- **Linear Pair of Angles:** Two adjacent angles whose non-common arms form a straight line; their sum is 180°.

Formula Derivation

For linear pair of angles $\angle A + \angle B = 180^\circ$.

Vertically opposite angles are equal: $\angle A = \angle B$.

Worked Illustrations and Solved Examples

Example: Find x if two adjacent angles are $6x$ and $3x$ forming a linear pair.

Solution:

Since they form a linear pair,

$$6x + 3x = 180^\circ$$

$$9x = 180^\circ$$

$$x = 20^\circ$$

Practice Set

- **Level 1 – Easy:** Identify adjacent angles in given diagrams.
- **Level 2 – Moderate:** Calculate unknown angles using linear pair property.
- **Level 3 – Challenging:** Prove vertically opposite angles are equal using algebraic expressions.

Answer Key

- Level 2: Use $\angle A + \angle B = 180^\circ$ to find unknowns.
- Level 3: Set $\angle A = x, \angle B = y$, use linear pair and equality to prove $x = y$.

Quick Reference

- Linear pair angles sum to 180°.
- Vertically opposite angles are equal.

Glossary

- **Linear Pair:** Two adjacent angles whose non-common arms form a straight line.
- **Vertically Opposite Angles:** Angles opposite each other when two lines intersect.
- **Adjacent Angles:** Angles sharing a common arm and vertex.

Transversal Line

Concept Explanation

A transversal is a line that intersects two or more lines at distinct points. When the lines are parallel, the transversal creates several special angle pairs with specific properties.

Key Angle Types Formed by a Transversal

- **Corresponding Angles:** Angles in the same relative position at each intersection.
- **Alternate Interior Angles:** Angles on opposite sides of the transversal but inside the two lines.
- **Alternate Exterior Angles:** Angles on opposite sides of the transversal but outside the two lines.
- **Consecutive Interior Angles:** Angles on the same side of the transversal and inside the two lines.

Formula Derivation and Theorems

- If a transversal intersects two parallel lines, then:
  - Each pair of corresponding angles is equal.
  - Each pair of alternate interior angles is equal.
  - Each pair of interior angles on the same side of the transversal is supplementary (sum to 180°).
- Conversely, if any one of these angle relationships holds, the two lines are parallel.
- **Exterior Angle Theorem:** The exterior angle of a triangle equals the sum of the two opposite interior angles.

Proof of Alternate Interior Angles Theorem

Given two parallel lines $AB \parallel CD$ and transversal PQ, angles x and y are alternate interior angles.

Proof:

$$\angle PRB = y \quad (\text{Corresponding angles})$$

$$\angle PRB = x \quad (\text{Vertically opposite angles})$$

Therefore, $x = y$.

Worked Illustrations and Solved Examples

Example: Two parallel lines are cut by a transversal. If $\angle 1 : \angle 2 = 4 : 5$ and $\angle 1 + \angle 2 = 180^\circ$, find all eight angles formed.

Solution:

Let $\angle 1 = 4x$ and $\angle 2 = 5x$.

$$4x + 5x = 180^\circ \Rightarrow 9x = 180^\circ \Rightarrow x = 20^\circ$$

Thus, $\angle 1 = 80^\circ$, $\angle 2 = 100^\circ$.

Using properties of parallel lines and transversal, find other angles accordingly.

Practice Set

- **Level 1 – Easy:** Identify corresponding and alternate interior angles in diagrams.
- **Level 2 – Moderate:** Given some angles, find unknown angles using transversal properties.
- **Level 3 – Challenging:** Prove lines are parallel given angle conditions.

Answer Key

- Level 2: Use angle sum and equality properties to find unknowns.
- Level 3: Use converse theorems of parallel lines and transversal.

Quick Reference

- Corresponding angles are equal if lines are parallel.
- Alternate interior angles are equal if lines are parallel.
- Consecutive interior angles are supplementary if lines are parallel.

Glossary

- **Transversal:** A line that intersects two or more lines.
- **Corresponding Angles:** Angles in the same relative position at each intersection.
- **Alternate Interior Angles:** Angles on opposite sides of the transversal inside the two lines.
- **Consecutive Interior Angles:** Angles on the same side of the transversal inside the two lines.
- **Parallel Lines:** Lines in the same plane that never intersect.