

- Congruence of Triangles
- Properties of Triangles

Congruence of Triangles

Two triangles are said to be congruent if their corresponding sides and corresponding angles are equal. This means the two triangles are identical in shape and size.

Formula Derivation and Criteria

Congruence can be established using the following criteria:

- **SAS (Side-Angle-Side)**: Two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of another triangle.
- **ASA (Angle-Side-Angle)**: Two angles and the included side of one triangle are equal to the corresponding two angles and included side of another triangle.
- **AAS (Angle-Angle-Side)**: Two angles and any one corresponding side of one triangle are equal to the corresponding parts of another triangle.
- **SSS (Side-Side-Side)**: All three sides of one triangle are equal to the corresponding three sides of another triangle.
- **RHS (Right angle-Hypotenuse-Side)**: In right-angled triangles, if the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of another triangle, the triangles are congruent.

Worked Illustration

Consider two triangles $\triangle ABC$ and $\triangle PQR$ such that $AB = PQ$, $\angle B = \angle Q$, and $BC = QR$. By SAS rule, $\triangle ABC \cong \triangle PQR$.

Solved Example

Example 1: In an isosceles triangle $\triangle PQR$ with $PQ = PR$, points S and T lie on QR such that $QT = RS$. Show that $PS = PT$.

Solution:

Given $PQ = PR$ (isosceles triangle), and $QT = RS$.

From $QT = RS$, subtract ST from both sides:

$$QT - ST = RS - ST \implies QS = RT$$

Consider triangles $\triangle PQS$ and $\triangle PRT$:

- $PQ = PR$ (given)
- $\angle Q = \angle R$ (angles opposite equal sides in isosceles triangle)
- $QS = RT$ (from above)

By SAS rule, $\triangle PQS \cong \triangle PRT$.

Therefore, corresponding parts are equal, so $PS = PT$.

Practice Set

Level 1 – Easy

- Prove that two equilateral triangles are congruent.
- In $\triangle ABC$, if $AB = AC$, prove that $\angle B = \angle C$.

Level 2 – Moderate

- In $\triangle XYZ$, $XY = XZ$ and $YZ = YW$. Prove that $\triangle XYZ \cong \triangle XWZ$.
- Prove that the bisector of the vertex angle of an isosceles triangle bisects the base.

Level 3 – Challenging

- In right-angled triangles, prove the RHS congruence criterion.
- Given two triangles with two pairs of equal angles and one pair of equal sides not included between the angles, prove the triangles are congruent.

Answer Key

- Equilateral triangles have all sides equal, so by SSS rule, they are congruent.
- In isosceles triangle, angles opposite equal sides are equal.
- Use SAS congruence by showing two sides and included angle are equal.
- Use properties of isosceles triangle and angle bisector theorem.
- Use RHS congruence by showing hypotenuse and one side equal.
- Use AAS congruence criterion.

Quick Reference

Criterion	Condition
SAS	Two sides and included angle equal
ASA	Two angles and included side equal

AAS	Two angles and any side equal
SSS	All three sides equal
RHS	Right angle, hypotenuse and one side equal

Glossary

- **Congruent Triangles:** Triangles with equal corresponding sides and angles.
- **Isosceles Triangle:** Triangle with two equal sides.
- **Included Angle:** Angle between two given sides.
- **CPCT:** Corresponding Parts of Congruent Triangles.

Properties of Triangles

Triangles have several important properties, especially isosceles and equilateral triangles.

Key Properties

- In an isosceles triangle, angles opposite equal sides are equal.
- In an isosceles triangle, sides opposite equal angles are equal.
- The bisector of the vertex angle of an isosceles triangle bisects the base.
- The medians of an equilateral triangle are equal in length.
- A point equidistant from two intersecting lines lies on the bisector of the angle formed.

Solved Example

Example 2: AB is a line segment. Points C and D lie on opposite sides of AB such that each is equidistant from points A and B. Show that line CD is the perpendicular bisector of AB.

Solution:

Given $CA = CB$ and $DA = DB$.

In $\triangle CAD$ and $\triangle CBD$:

- $CA = CB$ (given)
- $AD = BD$ (given)
- $CD = CD$ (common side)

By SSS rule, $\triangle CAD \cong \triangle CBD$.

Therefore, $\angle ACD = \angle BCD$.

In $\triangle CAO$ and $\triangle CBO$:

- $CA = CB$
- $\angle ACO = \angle BCO$
- $CO = CO$ (common side)

By SAS rule, $\triangle CAO \cong \triangle CBO$.

Therefore, $AO = BO$ and $\angle AOC = \angle BOC$.

Since $\angle AOC + \angle BOC = 180^\circ$, we have:

$$2 \times \angle AOC = 180^\circ \implies \angle AOC = 90^\circ$$

Thus, CD is perpendicular to AB and bisects it.

Practice Set

Level 1 – Easy

- Prove that in an isosceles triangle, the angles opposite equal sides are equal.
- Show that the medians of an equilateral triangle are equal.

Level 2 – Moderate

- Prove that the bisector of the vertex angle of an isosceles triangle bisects the base.
- Show that a point equidistant from two intersecting lines lies on the angle bisector.

Level 3 – Challenging

- Prove that the medians of an equilateral triangle are concurrent and equal.
- Prove that the perpendicular bisector of a chord passes through the center of the circle.

Answer Key

- Use properties of isosceles triangle and congruence criteria.
- Use triangle congruence and properties of medians.
- Use angle bisector theorem and congruence.
- Use concurrency of medians and equilateral triangle properties.
- Use circle theorems and perpendicular bisector properties.

Quick Reference

Property	Description
Isosceles Triangle	Two equal sides; opposite angles equal

Equilateral Triangle	All sides and medians equal
Angle Bisector	Bisects opposite side in isosceles triangle
Perpendicular Bisector	Bisects side at 90° and passes through center (circle)

Glossary

- **Isosceles Triangle:** Triangle with two equal sides.
- **Median:** Line segment joining vertex to midpoint of opposite side.
- **Bisector:** A line dividing an angle or segment into two equal parts.
- **Perpendicular Bisector:** A line perpendicular to a segment at its midpoint.

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