- Properties of a Parallelogram
- Midpoint Theorem

Properties of a Parallelogram

A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Understanding its properties helps in solving various geometric problems.

Concept Explanation

- Opposite sides of a parallelogram are parallel and equal in length.
- Opposite angles of a parallelogram are equal.
- Consecutive angles are supplementary (sum to 180°).
- Diagonals bisect each other.
- A diagonal divides the parallelogram into two congruent triangles.
- If a quadrilateral has one pair of opposite sides equal and parallel, it is a parallelogram.

Formula Derivation

Let ABCD be a parallelogram with AB \parallel DC and AD \parallel BC.

Since AB \parallel DC and AD \parallel BC, by the properties of parallel lines and transversals, alternate interior angles are equal, and opposite sides are equal.

For diagonals bisecting each other, let diagonals AC and BD intersect at O.

In triangles AOB and COD,

- AB = DC (opposite sides equal)
- $\angle AOB = \angle COD$ (vertically opposite angles)
- OB = OD (since O is midpoint of BD)

By SAS criterion, $\triangle AOB \cong \triangle COD$, so AO = OC.

Worked Illustration

Given parallelogram ABCD with AB = 8 cm and BC = 5 cm, find CD and AD.

Since opposite sides are equal, CD = AB = 8 cm and AD = BC = 5 cm.

Solved Example

Example: Prove that the diagonals of parallelogram ABCD bisect each other.

Solution:

Let diagonals AC and BD intersect at O.

Consider $\triangle AOB$ and $\triangle COD$.

- AB = DC (opposite sides equal)
- $\angle AOB = \angle COD$ (vertically opposite angles)

- OB = OD (since O is midpoint of BD)

By SAS criterion, $\triangle AOB \cong \triangle COD$.

Therefore, AO = OC and BO = OD, proving diagonals bisect each other.

Practice Set

Level 1 – Easy

- In parallelogram PQRS, if PQ = 10 cm and QR = 6 cm, find PS and SR.
- State whether a quadrilateral with one pair of opposite sides equal and parallel is a parallelogram.

Level 2 – Moderate

- In parallelogram ABCD, diagonals AC and BD intersect at O. If AO = 7 cm, find OC and BO.
- Prove that opposite angles of a parallelogram are equal.

Level 3 – Challenging

- Prove that the diagonal of a parallelogram divides it into two congruent triangles.
- If the diagonals of a quadrilateral bisect each other, prove that it is a parallelogram.

Answer Key

Level 1

- PS = QR = 6 cm, SR = PQ = 10 cm.
- Yes, such a quadrilateral is a parallelogram.

Level 2

- OC = AO = 7 cm, BO = OD (equal halves of diagonal BD).
- Proof: In parallelogram ABCD, AB \parallel DC and AD \parallel BC. Using alternate interior angles and congruent triangles, opposite angles are equal.

Level 3

- Proof: Diagonal divides parallelogram into two triangles sharing a side and having equal opposite sides and angles, hence congruent by SAS.
- Proof: If diagonals bisect each other, triangles formed are congruent, implying opposite sides are equal and parallel, so the quadrilateral is a parallelogram.

Quick Reference

Property	Description
Opposite sides	Equal and parallel
Opposite angles	Equal
Consecutive angles	Supplementary (sum to 180°)
Diagonals	Bisect each other

Glossary

- **Parallelogram:** A quadrilateral with both pairs of opposite sides parallel.
- **Bisect:** To divide into two equal parts.
- **Congruent Triangles:** Triangles that are identical in shape and size.
- **SAS Criterion:** Two triangles are congruent if two sides and the included angle of one are equal to the corresponding parts of the other.

Midpoint Theorem

The Midpoint Theorem is a fundamental result in triangle geometry that relates the segment joining midpoints of two sides to the third side.

Concept Explanation

- The line segment joining the midpoints of any two sides of a triangle is parallel to the third side.
- This segment is equal to half the length of the third side.
- The converse states that a line drawn through the midpoint of one side parallel to another side bisects the third side.

Formula Derivation

Consider triangle ABC with M and N as midpoints of sides AB and AC respectively.

By definition,

- AM = MB = \frac{1}{2}AB
- AN = NC = \frac{1}{2}AC

Line segment MN is drawn.

By the Midpoint Theorem,

$$MN \parallel BC \quad \text{and} \quad MN = \frac{1}{2}BC$$

Worked Illustration

In triangle PQR, M and N are midpoints of PQ and PR respectively. If MN = 6 cm, find the length of side QR.

Using Midpoint Theorem,

$$MN = \frac{1}{2}QR \Rightarrow QR = 2 \times MN = 2 \times 6 = 12 \text{ cm}$$

Solved Example

Example: Prove the Midpoint Theorem.

Solution:

In triangle ABC, let M and N be midpoints of AB and AC.

Join MN.

Consider triangles MNB and CND.

Since M and N are midpoints,

- AM = MB and AN = NC
- By construction, MN is parallel to BC.

Therefore, $MN = \frac{1}{2} BC$ and $MN \parallel BC$.

Practice Set

Level 1 – Easy

- In triangle XYZ, M and N are midpoints of XY and XZ. If MN = 5 cm, find YZ.
- State the Midpoint Theorem.

Level 2 – Moderate

- In triangle ABC, prove that the segment joining midpoints of two sides is parallel to the third side.
- Find the length of the third side if the segment joining midpoints is 7 cm.

Level 3 – Challenging

- Prove the converse of the Midpoint Theorem.
- In triangle PQR, a line through midpoint M of PQ is drawn parallel to PR. Prove that it bisects QR.

Answer Key

Level 1

- $YZ = 2 \times MN = 2 \times 5 = 10$ cm.
- The line segment joining midpoints of two sides of a triangle is parallel to the third side and equal to half of it.

Level 2

- Proof involves showing congruent triangles formed by joining midpoints and using properties of parallel lines.
- Third side length = 2 \times 7 = 14 cm.

Level 3

- Converse proof uses properties of parallel lines and congruent triangles to show bisection.
- Line through midpoint M parallel to PR bisects QR by congruent triangles.

Quick Reference

Theorem	Statement
Midpoint Theorem	Segment joining midpoints of two sides is parallel to third side and half its length.
Converse	Line through midpoint parallel to one side bisects the third side.

Glossary

- **Midpoint:** The point dividing a line segment into two equal parts.
- **Parallel Lines:** Lines in the same plane that never intersect.
- **Bisection:** Division into two equal parts.
- **Congruent Triangles:** Triangles identical in size and shape.