

- Basic Properties of Circles
- Cyclic Quadrilaterals

Basic Properties of Circles

A circle is the set of all points in a plane that are at a fixed distance, called the radius, from a fixed point called the centre.

Points inside the circle are called interior points, and points outside are exterior points. The interior of the circle is the set of all interior points, and the exterior is the set of all exterior points.

Key properties include:

- Equal chords subtend equal angles at the centre.
- Chords corresponding to congruent arcs are equal.
- If two arcs are congruent, their chords are equal, and vice versa.
- Only one circle passes through three non-collinear points.
- Perpendicular bisectors of two chords intersect at the centre.
- The angle subtended by an arc at the centre is twice the angle subtended at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- An angle in a semicircle is a right angle.

Formula Derivation

Consider a circle with centre O and radius r .

The circumference (perimeter) of the circle is given by:

$$C = 2\pi r$$

The area of the circle is:

$$A = \pi r^2$$

For a semicircle, the area is half of the circle's area:

$$A_{semi} = \frac{1}{2}\pi r^2$$

The perimeter of a semicircle includes the diameter and the curved part:

$$P_{semi} = \pi r + 2r = (\pi + 2)r$$

Worked Illustration

Given two circles with radii 10 cm and 8 cm intersecting such that their common chord length is 12 cm, find the distance between their centres.

Solved Example

Step 1: Let O and O' be the centres of the two circles with radii 10 cm and 8 cm respectively. Let PQ be the common chord.

Step 2: Given OP = 10 cm, O'P = 8 cm, and PQ = 12 cm. Find OO'.

Step 3: The perpendicular from the centre to the chord bisects the chord, so:

$$PL = \frac{1}{2}PQ = 6 \text{ cm}$$

Step 4: Apply Pythagoras theorem in triangle OLP:

$$OP^2 = OL^2 + PL^2 \implies OL = \sqrt{OP^2 - PL^2} = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8 \text{ cm}$$

Step 5: Apply Pythagoras theorem in triangle O'LP:

$$O'P^2 = O'L^2 + PL^2 \implies O'L = \sqrt{8^2 - 6^2} = \sqrt{64 - 36} = \sqrt{28} = 5.29 \text{ cm}$$

Step 6: Distance between centres OO' is:

$$OO' = OL + O'L = 8 + 5.29 = 13.29 \text{ cm}$$

Practice Set

- **Level 1 – Easy:** Find the circumference and area of a circle with radius 7 cm.
- **Level 2 – Moderate:** Two chords of lengths 8 cm and 6 cm are equidistant from the centre of a circle. Find the radius if the distance from the centre to each chord is 4 cm.
- **Level 3 – Challenging:** Two circles intersect such that the length of their common chord is 10 cm. If the radii of the circles are 13 cm and 15 cm, find the distance between their centres.

Answer Key

- Level 1: Circumference = $2\pi \times 7 = 14\pi$ cm, Area = $\pi \times 7^2 = 49\pi$ cm².
- Level 2: Using Pythagoras theorem, radius $r = \sqrt{4^2 + 8^2} = \sqrt{16 + 64} = \sqrt{80} = 8.94$ cm.
- Level 3: Use the method of the solved example to find the distance between centres.

Quick Reference

Property	Formula/Statement
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Circumference	$2\pi r$
Area	πr^2
Semicircle Area	$\frac{1}{2}\pi r^2$
Semicircle Perimeter	$(\pi + 2)r$

Glossary

- **Radius:** Distance from the centre to any point on the circle.
- **Chord:** A line segment joining two points on the circle.
- **Arc:** A part of the circumference of a circle.
- **Segment:** The region bounded by a chord and the arc subtended by it.
- **Sector:** The region bounded by two radii and the arc between them.

Cyclic Quadrilaterals

A quadrilateral is cyclic if all its vertices lie on a single circle.

Key properties include:

- If a line segment joining two points subtends equal angles at two other points on the same side, the four points are concyclic.
- The sum of opposite angles of a cyclic quadrilateral is 180° .
- Any exterior angle of a cyclic quadrilateral equals the interior opposite angle.
- Concentric circles share the same centre.
- The degree measure of a semicircle is 180° , and a full circle is 360° .
- The degree measure of a major arc is $360^\circ - \theta$, where θ is the minor arc.

Formula Derivation

For a cyclic quadrilateral ABCD, the sum of opposite angles is 180° :

$$\angle A + \angle C = 180^\circ \quad \text{and} \quad \angle B + \angle D = 180^\circ$$

Solved Example

Prove that the quadrilateral formed by the internal angle bisectors of any quadrilateral is cyclic.

Step 1: Let PQRS be a quadrilateral with internal angle bisectors PB, QD, RD, and SB forming quadrilateral ABCD.

Step 2: We need to prove ABCD is cyclic, i.e., $\angle A + \angle C = 180^\circ$.

Step 3: Using angle properties and bisectors, we find:

$$\angle BAD = 180^\circ - \frac{1}{2}(\angle P + \angle Q) \quad \angle BCD = 180^\circ - \frac{1}{2}(\angle R + \angle S)$$

Step 4: Adding these,

$$\angle BAD + \angle BCD = 360^\circ - \frac{1}{2}(\angle P + \angle Q + \angle R + \angle S) = 360^\circ - \frac{1}{2} \times 360^\circ = 180^\circ$$

Hence, ABCD is cyclic.

Practice Set

- **Level 1 – Easy:** Verify if a rectangle is a cyclic quadrilateral.
- **Level 2 – Moderate:** Prove that the opposite angles of a cyclic quadrilateral sum to 180° using angle bisectors.
- **Level 3 – Challenging:** Given a quadrilateral, construct the quadrilateral formed by its internal angle bisectors and prove it is cyclic.

Answer Key

- Level 1: Yes, a rectangle is cyclic because all vertices lie on a circle and opposite angles sum to 180° .
- Level 2: Use angle bisector properties and angle sum in triangles to prove the sum of opposite angles is 180° .
- Level 3: Follow the steps in the solved example to construct and prove cyclicity.

Quick Reference

Property	Statement
Opposite Angles	Sum to 180° in cyclic quadrilateral
Exterior Angle	Equals interior opposite angle
Concentric Circles	Circles with common centre

Glossary

- **Cyclic Quadrilateral:** A quadrilateral whose vertices lie on a circle.
- **Concyclic Points:** Points lying on the same circle.
- **Angle Bisector:** A line dividing an angle into two equal parts.
- **Exterior Angle:** An angle formed outside a polygon by one side and the extension of an adjacent side.