

- Surface Area and Volume of Sphere (Including Hemisphere)
- Surface Area and Volume of Right Circular Cone

Surface Area and Volume of Sphere (Including Hemisphere)

Concept Explanation: A sphere is a perfectly round three-dimensional object where every point on the surface is equidistant from the center. A hemisphere is exactly half of a sphere. The radius r is the distance from the center to any point on the surface.

Formula Derivation:

For a sphere of radius r :

$$\text{Surface area } A = 4\pi r^2$$

$$\text{Volume } V = \frac{4}{3}\pi r^3$$

For a hemisphere of radius r :

$$\text{Curved surface area } A_c = 2\pi r^2$$

$$\text{Total surface area } A_t = 3\pi r^2 \text{ (curved surface + base area)}$$

$$\text{Volume } V = \frac{2}{3}\pi r^3$$

Worked Illustrations:

Consider a sphere with radius $r = 7$ cm.

$$\text{Surface area} = 4\pi(7)^2 = 4\pi \times 49 = 196\pi \text{ cm}^2$$

$$\text{Volume} = \frac{4}{3}\pi(7)^3 = \frac{4}{3}\pi \times 343 = \frac{1372}{3}\pi \text{ cm}^3$$

For a hemisphere with the same radius:

$$\text{Curved surface area} = 2\pi(7)^2 = 2\pi \times 49 = 98\pi \text{ cm}^2$$

$$\text{Total surface area} = 3\pi(7)^2 = 3\pi \times 49 = 147\pi \text{ cm}^2$$

$$\text{Volume} = \frac{2}{3}\pi(7)^3 = \frac{2}{3}\pi \times 343 = \frac{686}{3}\pi \text{ cm}^3$$

Solved Examples:

Example: Find the volume of a sphere whose surface area is $616\pi \text{ cm}^2$.

Solution:

$$\text{Given surface area } A = 4\pi r^2 = 616\pi$$

Divide both sides by π :

$$4r^2 = 616$$

$$r^2 = \frac{616}{4} = 154$$

$$r = \sqrt{154} \text{ cm}$$

$$\text{Volume } V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\sqrt{154})^3 = \frac{4}{3}\pi(154 \times \sqrt{154})$$

$$\text{Volume} = \frac{4}{3}\pi \times 154 \times 12.4097 = \frac{4}{3}\pi \times 1909.9 = 2546.5\pi \text{ cm}^3 \text{ (approx.)}$$

Practice Set:

- **Level 1 – Easy:** Calculate the surface area and volume of a sphere with radius 5 cm.
- **Level 2 – Moderate:** A hemisphere has a volume of $\frac{500\pi}{3} \text{ cm}^3$. Find its radius and total surface area.
- **Level 3 – Challenging:** Two spheres have radii in the ratio 3:4. Find the ratio of their volumes and surface areas.

Answer Key:

- Level 1: Surface area = $4\pi(5)^2 = 100\pi \text{ cm}^2$, Volume = $\frac{4}{3}\pi(5)^3 = \frac{500}{3}\pi \text{ cm}^3$
- Level 2: Volume = $\frac{2}{3}\pi r^3 = \frac{500\pi}{3} \Rightarrow r^3 = 250 \Rightarrow r = \sqrt[3]{250} \approx 6.3 \text{ cm}$; Total surface area = $3\pi r^2 = 3\pi(6.3)^2 = 119\pi \text{ cm}^2$ approx.
- Level 3: Volume ratio = $(3^3) : (4^3) = 27 : 64$; Surface area ratio = $3^2 : 4^2 = 9 : 16$

Quick Reference:

Shape	Surface Area	Volume
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere	$3\pi r^2$ (total)	$\frac{2}{3}\pi r^3$

Glossary:

- **Radius (r):** Distance from the center to the surface of the sphere or hemisphere.
- **Surface Area:** Total area covered by the surface of the solid.
- **Volume:** Space occupied by the solid.
- **Curved Surface Area:** Area of the curved part of the solid, excluding base(s).

Surface Area and Volume of Right Circular Cone

Concept Explanation: A right circular cone is a three-dimensional figure with a circular base and a vertex directly above the center of the base. The height h is the perpendicular distance from the base to the vertex. The radius r is the radius of the base. The slant height l is the distance from the vertex to any point on the edge of the base.

Formula Derivation:

Using the Pythagorean theorem in the right triangle formed by height, radius, and slant height:

$$l = \sqrt{h^2 + r^2}$$

Curved surface area (CSA):

$$\text{CSA} = \pi r l = \pi r \sqrt{h^2 + r^2}$$

Total surface area (TSA):

$$\text{TSA} = \text{CSA} + \text{Area of base} = \pi r l + \pi r^2 = \pi r(l + r)$$

Volume (V):

$$V = \frac{1}{3} \pi r^2 h$$

Worked Illustrations:

Given a cone with radius $r = 3$ cm and height $h = 4$ cm.

Calculate slant height:

$$l = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ cm}$$

Curved surface area:

$$\pi \times 3 \times 5 = 15\pi \text{ cm}^2$$

Total surface area:

$$15\pi + \pi \times 3^2 = 15\pi + 9\pi = 24\pi \text{ cm}^2$$

Volume:

$$\frac{1}{3}\pi \times 3^2 \times 4 = \frac{1}{3}\pi \times 9 \times 4 = 12\pi \text{ cm}^3$$

Solved Examples:

Example: Two cones have radii in the ratio 4:1 and the slant height of the second cone is twice that of the first. Find the ratio of their curved surface areas.

Solution:

Let r_1, l_1 be radius and slant height of first cone, r_2, l_2 for second cone.

Given $r_1 : r_2 = 4 : 1$ and $l_2 = 2l_1$.

Curved surface areas:

$$\text{CSA}_1 = \pi r_1 l_1, \quad \text{CSA}_2 = \pi r_2 l_2$$

Ratio:

$$\frac{CSA_1}{CSA_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{r_1}{r_2} \times \frac{l_1}{l_2} = 4 \times \frac{1}{2} = 2$$

Thus, the curved surface area of the first cone is twice that of the second.

Practice Set:

- **Level 1 – Easy:** Find the volume of a cone with radius 7 cm and height 10 cm.
- **Level 2 – Moderate:** Calculate the total surface area of a cone with radius 5 cm and slant height 13 cm.
- **Level 3 – Challenging:** Two cones have volumes in the ratio 27:64. If their heights are in the ratio 3:4, find the ratio of their radii.

Answer Key:

- Level 1: Volume = $\frac{1}{3}\pi(7)^2(10) = \frac{1}{3}\pi \times 49 \times 10 = \frac{490}{3}\pi \text{ cm}^3$
- Level 2: Total surface area = $\pi r(l + r) = \pi \times 5(13 + 5) = 5\pi \times 18 = 90\pi \text{ cm}^2$
- Level 3: Volume ratio = $\frac{27}{64} = \frac{r_1^2 h_1}{r_2^2 h_2}$, given $h_1 : h_2 = 3 : 4$
- So, $\frac{27}{64} = \frac{r_1^2 \times 3}{r_2^2 \times 4} \Rightarrow \frac{r_1^2}{r_2^2} = \frac{27}{64} \times \frac{4}{3} = \frac{27 \times 4}{64 \times 3} = \frac{108}{192} = \frac{9}{16}$
- Therefore, $r_1 : r_2 = 3 : 4$

Quick Reference:

Quantity	Formula
Slant height	$l = \sqrt{h^2 + r^2}$
Curved surface area	$\pi r l$

Total surface area	$\pi r(l + r)$
Volume	$\frac{1}{3}\pi r^2 h$

Glossary:

- **Radius (r):** Distance from the center to the edge of the base circle.
- **Height (h):** Perpendicular distance from the base to the vertex.
- **Slant height (l):** Distance from the vertex to any point on the base edge.
- **Curved Surface Area:** Area of the cone's lateral surface excluding the base.
- **Total Surface Area:** Sum of curved surface area and base area.
- **Volume:** Space occupied by the cone.

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