Area of a Square

The area of a square is calculated by multiplying the length of one side by itself. If the side length is a cm, then the area A is given by:

$$A = a^2$$

This means the area is the square of the side length.

Formula Derivation

Since all sides of a square are equal, the area is:

$$\text{Area} = \text{side} \times \text{side} = a \times a = a^2$$

Worked Illustrations

- For side 1 cm: $1^2 = 1$ cm²
- For side 2 cm: $2^2 = 4$ cm²
- For side 3 cm: $3^2 = 9$ cm²
- For side 5 cm: $5^2 = 25$ cm²
- For side 8 cm: $8^2 = 64$ cm²

Solved Example

Example: Find the area of a square with side length 7 cm.

Solution:

Using the formula:

$$A = 7^2 = 49 \text{ cm}^2$$

Therefore, the area is 49 cm².

Practice Set

Level 1 – Easy

- Find the area of a square with side 4 cm.
- Calculate the area of a square with side 10 cm.

Level 2 – Moderate

- Find the area of a square with side length 12.5 cm.
- A square has an area of 81 cm². Find the length of its side.

Level 3 – Challenging

- The side of a square is increased by 20%. Find the percentage increase in its area.
- The area of a square garden is 196 m². A path of width 2 m is constructed outside the garden. Find the area of the path.

Answer Key

- Level 1: (1) 16 cm², (2) 100 cm²
- Level 2: (1) 156.25 cm², (2) 9 cm
- Level 3: (1) 44%, (2) 84 m²

Quick Reference

Quantity	Formula
Area of square	a^2

Glossary

- **Square:** A quadrilateral with four equal sides and four right angles.
- **Area:** The measure of the space inside a two-dimensional shape.
- **Side:** The length of one edge of the square.

Square Numbers

Square numbers are numbers that can be expressed as the product of a natural number multiplied by itself. For a natural number n, the squ number is n^2.

Examples of square numbers are:

$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \ldots$$

These are also called perfect squares.

Concept Explanation

If a natural number m can be written as $m = n^2$ where n is a natural number, then m is a square number.

For example, 25 is a square number because $25 = 5^2$.

Is 32 a square number? Since $5^2 = 25$ and $6^2 = 36$, and 32 lies between 25 and 36, but is not equal to any n^2, 32 is not a square number.

Worked Illustrations

Square numbers from 1 to 10:

Number n	Square n^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

Solved Example

Example: Is 49 a square number?

Solution: Since $7^2 = 49$, 49 is a square number.

Practice Set

Level 1 – Easy

- List all square numbers between 30 and 50.
- Is 64 a square number? Justify.

Level 2 – Moderate

- Find the square numbers between 50 and 100.
- Is 81 a square number? Explain.

Level 3 – Challenging

- Find all square numbers less than 200.
- Determine if 150 is a square number.

Answer Key

- Level 1: 36, 49; Yes, 64 = 8^2
- Level 2: 64, 81, 100; Yes, 81 = 9^2
- Level 3: 1,4,9,16,25,36,49,64,81,100,121,144,169,196; No, 150 is not a square number

Quick Reference

Term	Definition
Square Number	Number expressed as n^2 for some natural number n

Glossary

- **Perfect Square:** A number that is the square of a natural number.
- **Natural Number:** Positive integers starting from 1.

Properties of Square Numbers

Square numbers have specific properties related to their digits and factors.

Concept Explanation

1. The units digit (last digit) of a square number can only be 0, 1, 4, 5, 6, or 9.

2. Square numbers can only have an even number of zeros at the end.

3. The square of an even number is even, and the square of an odd number is odd.

Worked Illustrations

Examples of square numbers and their units digits:

Number	Square	Units Digit of Square
1	1	1
2	4	4
3	9	9
4	16	6
5	25	5
6	36	6
7	49	9
8	64	4
9	81	1
10	100	0

Solved Example

Example: Can a square number end with digit 3?

Solution: No, because square numbers only end with 0, 1, 4, 5, 6, or 9.

Practice Set

Level 1 – Easy

- List the possible units digits of square numbers.
- Is 45 a possible units digit of a square number?

Level 2 – Moderate

- Find the units digit of 37^2.
- Determine if 1234 can be a square number based on its units digit.

Level 3 – Challenging

- Explain why square numbers cannot end with digits 2, 3, 7, or 8.
- Find the units digit of the square of 999.

Answer Key

- Level 1: 0,1,4,5,6,9; No
- Level 2: 9; No
- Level 3: Explanation based on modular arithmetic; 1

Quick Reference

Property	Details
Units digit of square	0, 1, 4, 5, 6, or 9 only
Parity	Square of even number is even; square of odd number is odd

Glossary

- **Units Digit:** The last digit of a number.
- **Parity:** Whether a number is even or odd.

Finding the Square of a Number

To find the square of a number n, multiply n by itself:

$$n^2 = n \times n$$

For larger numbers, use algebraic expansion or special patterns.

Formula Derivation

For $n = a + b$,

$$(a + b)^2 = a^2 + 2ab + b^2$$

Worked Illustrations

Find 23^2:

$$23 = 20 + 3$$

$$23^2 = (20 + 3)^2 = 20^2 + 2 \times 20 \times 3 + 3^2 = 400 + 120 + 9 = 529$$

Solved Example

Example: Find 39^2 without direct multiplication.

Solution:

$$39 = 30 + 9$$

$$39^2 = 30^2 + 2 \times 30 \times 9 + 9^2 = 900 + 540 + 81 = 1521$$

Practice Set

Level 1 – Easy

- Find 15^2.
- Calculate 12^2.

Level 2 – Moderate

- Find 47^2 using algebraic expansion.
- Calculate 56^2 using the formula.

Level 3 – Challenging

- Find 105^2 using the pattern for numbers ending with 5.
- Calculate 95^2 using the special formula for numbers ending with 5.

Answer Key

- Level 1: (1) 225, (2) 144
- Level 2: (1) 2209, (2) 3136
- Level 3: (1) 11025, (2) 9025

Quick Reference

Formula	Explanation
$(a+b)^2 = a^2 + 2ab + b^2$	Square of sum of two numbers
$(10a+5)^2 = 100a(a+1) + 25$	Square of numbers ending with 5

Glossary

- **Algebraic Expansion:** Expressing powers of sums as sums of powers and products.
- **Binomial:** An expression with two terms.

Pythagorean Triplets

A Pythagorean triplet consists of three natural numbers a, b, and c such that:

$$a^2 + b^2 = c^2$$

These represent the sides of a right-angled triangle.

Concept Explanation

Examples of Pythagorean triplets:

- $3, 4, 5$ since $3^2 + 4^2 = 9 + 16 = 25 = 5^2$
- $6, 8, 10$ since $6^2 + 8^2 = 36 + 64 = 100 = 10^2$
- $5, 12, 13$ since $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

General formula for generating triplets:

For any natural number $m > 1$,

$$2m, m^2 - 1, m^2 + 1$$

forms a Pythagorean triplet.

Solved Example

Example: Find a Pythagorean triplet with smallest member 8.

Solution:

Try $2m = 8 \Rightarrow m = 4$.

Then:

$$m^2 - 1 = 16 - 1 = 15, \quad m^2 + 1 = 16 + 1 = 17$$

Triplet is $8, 15, 17$.

Practice Set

Level 1 – Easy

- Verify if $3, 4, 5$ is a Pythagorean triplet.
- Check if $5, 12, 13$ is a Pythagorean triplet.

Level 2 – Moderate

- Find a Pythagorean triplet with smallest member 6.
- Find a triplet with smallest member 10.

Level 3 – Challenging

- Find a Pythagorean triplet where one member is 14.
- Find a triplet where one member is 18.

Answer Key

- Level 1: Yes; Yes
- Level 2: (6,8,10); (10,24,26)
- Level 3: (14, 48, 50); (18, 80, 82)

Quick Reference

Formula	Triplet
$2m, m^2 - 1, m^2 + 1$	Pythagorean triplet for $m > 1$

Glossary

- **Right-angled triangle:** Triangle with one 90° angle.
- **Hypotenuse:** Side opposite the right angle, longest side.

Square Roots

The square root of a number x is a number y such that:

$$y^2 = x$$

For a perfect square x, there are two square roots: positive and negative. The principal (positive) square root is denoted by $\sqrt{x}$.

Concept Explanation

For example:

$$\sqrt{9} = 3 \quad \text{since} \quad 3^2 = 9$$

Both 3 and -3 satisfy $y^2 = 9$, but $\sqrt{9}$ denotes the positive root.

Worked Illustrations

Find the square root of 144:

$$\sqrt{144} = 12 \quad \text{since} \quad 12^2 = 144$$

Solved Example

Example: Find the length of the side of a square with area 144 cm².

Solution:

Given area $A = 144$, side length $a = \sqrt{A} = \sqrt{144} = 12$ cm.

Practice Set

Level 1 – Easy

- Find $\sqrt{81}$.
- Find $\sqrt{100}$.

Level 2 – Moderate

- Find the side length of a square with area 225 cm².
- Find the square root of 256.

Level 3 – Challenging

- Find the length of the diagonal of a square with side 8 cm.
- Find the missing side of a right triangle with hypotenuse 13 cm and one side 5 cm.

Answer Key

- Level 1: 9; 10
- Level 2: 15 cm; 16
- Level 3: $8\sqrt{2} \approx 11.31$ cm; 12 cm

Quick Reference

Concept	Formula
Square root	$y = \sqrt{x}$ where $y^2 = x$
Diagonal of square	$d = a\sqrt{2}$
Pythagoras theorem	$c^2 = a^2 + b^2$

Glossary

- **Square root:** Number which when squared gives the original number.
- **Principal square root:** The positive square root.
- **Hypotenuse:** Longest side of a right triangle.

Finding Square Roots

Square roots can be found by various methods:

- Repeated subtraction of odd numbers
- Prime factorisation
- Long division method

1. Repeated Subtraction of Odd Numbers

Every square number can be expressed as the sum of successive odd numbers starting from 1.

For example, to find $\sqrt{81}$, subtract successive odd numbers:

$$81 - 1 = 80, \quad 80 - 3 = 77, \quad 77 - 5 = 72, \quad 72 - 7 = 65, \quad 65 - 9 = 56, \quad 56 - 11 = 45, \quad 45 - 13 = 32, \quad 32 - 15 = 17, \quad 17 - 1$$

Since 0 is reached at the 9th step, $\sqrt{81} = 9$.

2. Prime Factorisation Method

Express the number as a product of prime factors. Pair the prime factors and multiply one from each pair to get the square root.

Example: Find $\sqrt{324}$.

Prime factorisation:

$$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^2 \times 3^4$$

Pairing:

$$\sqrt{324} = 2 \times 3^2 = 2 \times 9 = 18$$

3. Long Division Method

Used for large numbers. Steps:

1. Group digits in pairs from right to left.
2. Find the largest number whose square is less than or equal to the first group.
3. Subtract and bring down the next pair.
4. Double the quotient and find the next digit.
5. Repeat until all pairs are processed.

Solved Example

Find $\sqrt{529}$ by long division.

Step 1: Group digits: $\overline{5}\ \overline{2}\ \overline{9}$

Step 2: Largest square less than 5 is 2^2=4.

Step 3: Subtract 4 from 5, remainder 1, bring down 2 to make 12.

Step 4: Double quotient 2 to get 4_, find digit x such that $4x \times x \leq 12$. x=2.

Step 5: Multiply 42 × 2 = 84, subtract from 129, remainder 45, bring down 9.

Continue similarly to get quotient 23.

Practice Set

Level 1 – Easy

- Find $\sqrt{100}$ by prime factorisation.
- Find $\sqrt{81}$ by repeated subtraction.

Level 2 – Moderate

- Find $\sqrt{144}$ by long division.
- Find $\sqrt{256}$ by prime factorisation.

Level 3 – Challenging

- Find $\sqrt{2304}$ by long division.
- Find the smallest number to multiply 252 to get a perfect square.

Answer Key

- Level 1: 10; 9
- Level 2: 12; 16
- Level 3: 48; 2

Quick Reference

Method	Key Idea
Repeated subtraction	Subtract successive odd numbers until zero
Prime factorisation	Pair prime factors and multiply one from each pair
Long division	Divide in pairs, find digits stepwise

Glossary

- **Prime factorisation:** Expressing a number as product of primes.
- **Remainder:** Leftover after division.