Closure Under Addition of Integers

Integers are said to be closed under addition if the sum of any two integers is always an integer. This means that when you add any two integers, the result is also an integer.

Formula Derivation

Let a and b be any two integers. Then their sum $a + b$ is also an integer.

Worked Illustrations

- $17 + 23 = 40$ (integer)
- $(-10) + 3 = -7$ (integer)
- $(-75) + 18 = -57$ (integer)
- $19 + (-25) = -6$ (integer)
- $27 + (-27) = 0$ (integer)
- $(-20) + 0 = -20$ (integer)
- $(-35) + (-10) = -45$ (integer)

Solved Examples

Example: Find the sum of (-15) and 20.

Solution:

$$(-15) + 20 = 5$$

Since 5 is an integer, the sum is an integer.

Practice Set

Level 1 – Easy

- Calculate $5 + (-3)$.
- Calculate $(-7) + 0$.
- Calculate $12 + (-12)$.

Level 2 – Moderate

- Calculate $(-25) + 30$.
- Calculate $(-40) + (-15)$.
- Calculate $100 + (-50)$.

Level 3 – Challenging

- Calculate $(-123) + 456$.
- Calculate $(-200) + (-300)$.

- Calculate $0 + (-999)$.

Answer Key

- 2
- -7
- 0
- 5
- -55
- 50
- 333
- -500
- -999

Quick Reference

For any integers a and b, $a + b$ is an integer.

Glossary

- **Closure Property:** A set is closed under an operation if performing that operation on members of the set always produces a member of the same set.
- **Integer:** A whole number that can be positive, negative, or zero.

Closure Under Subtraction of Integers

Integers are closed under subtraction if the difference of any two integers is always an integer.

Formula Derivation

For any integers a and b, $a - b$ is also an integer.

Worked Illustrations

- $7 - 9 = -2$ (integer)
- $17 - (-21) = 38$ (integer)
- $(-8) - (-14) = 6$ (integer)
- $(-21) - (-10) = -11$ (integer)
- $32 - (-17) = 49$ (integer)
- $(-18) - (-18) = 0$ (integer)
- $(-29) - 0 = -29$ (integer)

Solved Examples

Example: Find $-5 - 7$.

Solution:

$$-5 - 7 = -12$$

The difference is an integer.

Practice Set

Level 1 – Easy

- Calculate $10 - 5$.
- Calculate $(-3) - 0$.
- Calculate $0 - (-4)$.

Level 2 – Moderate

- Calculate $15 - (-10)$.
- Calculate $(-20) - 15$.
- Calculate $(-7) - (-3)$.

Level 3 – Challenging

- Calculate $(-100) - (-50)$.
- Calculate $200 - 350$.
- Calculate $(-500) - 0$.

Answer Key

- 5
- -3
- 4
- 25
- -35
- -4
- -50
- -150
- -500

Quick Reference

For any integers a and b, $a - b$ is an integer.

Glossary

- **Difference:** The result of subtracting one number from another.

Commutative Property of Integers

The commutative property states that the order of addition or multiplication does not affect the result. For integers, addition is commutative but subtraction is not.

Formula Derivation

For any integers a and b,

$$a + b = b + a$$

But,

$$a - b \neq b - a$$

Worked Illustrations

- $5 + (-6) = -1$ and $(-6) + 5 = -1$
- $(-8) + (-9) = -17$ and $(-9) + (-8) = -17$
- $5 - (-3) = 8$ but $(-3) - 5 = -8$

Solved Examples

Example: Verify if $7 + (-4) = (-4) + 7$.

Solution:

$$7 + (-4) = 3$$

$$(-4) + 7 = 3$$

Both are equal, so addition is commutative.

Check subtraction:

$$7 - (-4) = 11$$

$$(-4) - 7 = -11$$

Not equal, so subtraction is not commutative.

Practice Set

Level 1 – Easy

- Check if $3 + 5 = 5 + 3$.
- Check if $(-2) + 4 = 4 + (-2)$.

Level 2 – Moderate

- Check if $10 - 3 = 3 - 10$.

- Check if $(-7) - 2 = 2 - (-7)$.

Level 3 – Challenging

- Check if $(-15) + 20 = 20 + (-15)$.
- Check if $(-10) - (-5) = (-5) - (-10)$.

Answer Key

- True
- True
- False
- False
- True
- False

Quick Reference

Addition of integers is commutative: $a + b = b + a$. Subtraction is not commutative.

Glossary

- **Commutative Property:** Changing the order of numbers does not change the result.

Associative Property of Integers

The associative property states that when adding or multiplying three or more integers, the way in which the numbers are grouped does not change the result.

Formula Derivation

For any integers a, b, and c,

$$a + (b + c) = (a + b) + c$$

and

$$(a \times b) \times c = a \times (b \times c)$$

Worked Illustrations

Consider -5, -3, and -2:

$$(-5) + [(-3) + (-2)] = (-5) + (-5) = -10$$

$$[(-5) + (-3)] + (-2) = (-8) + (-2) = -10$$

Both sums are equal, demonstrating associativity of addition.

Consider -3, -2, and 5:

$$[(-3) \times (-2)] \times 5 = 6 \times 5 = 30$$

$$(-3) \times [(-2) \times 5] = (-3) \times (-10) = 30$$

Both products are equal, demonstrating associativity of multiplication.

Solved Examples

Example: Verify associativity for 2, 3, and 4 in addition.

Solution:

$$2 + (3 + 4) = 2 + 7 = 9$$

$$(2 + 3) + 4 = 5 + 4 = 9$$

Both are equal.

Practice Set

Level 1 – Easy

- Verify $1 + (2 + 3) = (1 + 2) + 3$.
- Verify $4 + (5 + 6) = (4 + 5) + 6$.

Level 2 – Moderate

- Verify $(-2) + [3 + (-4)] = [(-2) + 3] + (-4)$.
- Verify $2 \times (3 \times 4) = (2 \times 3) \times 4$.

Level 3 – Challenging

- Verify $(-3) \times [(-2) \times 5] = [(-3) \times (-2)] \times 5$.
- Verify $5 + [(-7) + 2] = [5 + (-7)] + 2$.

Answer Key

- True
- True
- True
- True
- True
- True

Quick Reference

Grouping of integers in addition or multiplication does not affect the result.

Glossary

- **Associative Property:** Changing the grouping of numbers does not change the result.

Additive Identity of Integers

The additive identity is the number which, when added to any integer, leaves it unchanged. For integers, zero is the additive identity.

Formula Derivation

For any integer a,

$$a + 0 = a = 0 + a$$

Worked Illustrations

- $(-8) + 0 = -8$
- $0 + (-8) = -8$
- $(-23) + 0 = -23$
- $0 + (-37) = -37$
- $0 + (-59) = -59$
- $0 + (-43) = -43$
- $(-61) + 0 = -61$

Solved Examples

Example: Find $15 + 0$.

Solution:

$$15 + 0 = 15$$

The integer remains unchanged.

Practice Set

Level 1 – Easy

- Calculate $0 + 7$.
- Calculate $(-5) + 0$.

Level 2 – Moderate

- Calculate $0 + (-12)$.
- Calculate $(-20) + 0$.

Level 3 – Challenging

- Calculate $0 + (-100)$.
- Calculate $(-999) + 0$.

Answer Key

- 7
- -5
- -12
- -20
- -100
- -999

Quick Reference

Zero is the additive identity for integers.

Glossary

- **Additive Identity:** A number which when added to any integer leaves it unchanged (zero for integers).

Multiplication of Integers

Multiplication of integers follows specific rules depending on the signs of the integers involved.

Multiplication of a Positive and a Negative Integer

Multiplying a positive integer by a negative integer results in a negative integer. The product is the product of their absolute values with a negative sign.

For example,

$$3 \times (-5) = -(3 \times 5) = -15$$

Similarly,

$$5 \times (-4) = -20$$

Multiplication of Two Negative Integers

Multiplying two negative integers results in a positive integer. The product is the product of their absolute values.

For example,

$$(-3) \times (-2) = 6$$

$$(-10) \times (-12) = 120$$

General Rule

For any positive integers a and b,

$$a \times (-b) = (-a) \times b = -(a \times b)$$

and

$$(-a) \times (-b) = a \times b$$

Worked Illustrations

- $4 \times (-8) = -32$
- $(-3) \times 7 = -21$
- $(-5) \times (-6) = 30$

Solved Examples

Example: Find $(-4) \times 9$.

Solution:

$$(-4) \times 9 = -(4 \times 9) = -36$$

Practice Set

Level 1 – Easy

- Calculate $3 \times (-5)$.
- Calculate $(-2) \times 4$.

Level 2 – Moderate

- Calculate $(-7) \times (-3)$.
- Calculate $6 \times (-8)$.

Level 3 – Challenging

- Calculate $(-15) \times (-12)$.
- Calculate $(-25) \times 20$.

Answer Key

- -15
- -8
- 21
- -48

- 180
- -500

Quick Reference

Multiplying integers: same signs give positive product, different signs give negative product.

Glossary

- **Product:** The result of multiplication.

Properties of Multiplication of Integers

Closure Under Multiplication

The product of any two integers is always an integer.

For any integers a and b,

$$a \times b \text{ is an integer}$$

Commutativity of Multiplication

Multiplication of integers is commutative, meaning:

$$a \times b = b \times a$$

Multiplication by Zero

Any integer multiplied by zero is zero:

$$a \times 0 = 0$$

Multiplicative Identity

Multiplying any integer by 1 leaves it unchanged:

$$a \times 1 = a$$

Associativity of Multiplication

Grouping of integers in multiplication does not affect the product:

$$(a \times b) \times c = a \times (b \times c)$$

Distributive Property

Multiplication distributes over addition:

$$a \times (b + c) = a \times b + a \times c$$

and over subtraction:

$$a \times (b - c) = a \times b - a \times c$$

Worked Illustrations

- $(-3) \times 4 = -12$
- $3 \times (-4) = -12$
- $(-3) \times (-4) = 12$
- $5 \times 0 = 0$
- $7 \times 1 = 7$
- $(2 \times 3) \times 4 = 2 \times (3 \times 4) = 24$
- $3 \times (4 + 5) = 3 \times 4 + 3 \times 5 = 27$

Solved Examples

Example: Verify distributive property for 2, 3, and 4.

Solution:

$$2 \times (3 + 4) = 2 \times 7 = 14$$

$$2 \times 3 + 2 \times 4 = 6 + 8 = 14$$

Both sides are equal.

Practice Set

Level 1 – Easy

- Calculate 5×0.
- Calculate $1 \times (-7)$.

Level 2 – Moderate

- Verify $3 \times (4 + (-2)) = 3 \times 4 + 3 \times (-2)$.
- Verify associativity for 2, 3, and 5.

Level 3 – Challenging

- Verify distributive property for (-4), 5, and (-3).
- Verify commutativity for (-7) and 8.

Answer Key

- 0
- -7
- True
- True
- True
- True

Quick Reference

Multiplication of integers is closed, commutative, associative, distributive over addition and subtraction, has identity 1, and zero annihilates.

Glossary

- **Multiplicative Identity:** Number 1, which leaves any integer unchanged when multiplied.
- **Distributive Property:** Multiplication distributes over addition and subtraction.

Division of Integers

Division is the inverse operation of multiplication. Division of integers follows sign rules similar to multiplication.

Rules for Division

- Dividing a negative integer by a positive integer results in a negative integer.
- Dividing a positive integer by a negative integer results in a negative integer.
- Dividing a negative integer by a negative integer results in a positive integer.
- Division by zero is undefined.

Formula Derivation

For positive integers a and b, with $b \neq 0$,

$$a \div (-b) = -(a \div b)$$

$$(-a) \div b = -(a \div b)$$

$$(-a) \div (-b) = a \div b$$

Worked Illustrations

- $(-12) \div 2 = -6$
- $(-20) \div 5 = -4$
- $72 \div (-8) = -9$
- $(-36) \div (-4) = 9$
- $0 \div 5 = 0$

Solved Examples

Example: Find $(-48) \div 8$ and $48 \div (-8)$.

Solution:

$$(-48) \div 8 = -6$$

$$48 \div (-8) = -6$$

Both are equal.

Practice Set

Level 1 – Easy

- Calculate $(-100) \div 5$.

- Calculate $81 \div (-9)$.

Level 2 – Moderate

- Calculate $(-75) \div 5$.
- Calculate $(-32) \div (-8)$.

Level 3 – Challenging

- Calculate $(-201) \div (-3)$.
- Calculate $125 \div (-25)$.

Answer Key

- -20
- -9
- -15
- 4
- 67
- -5

Quick Reference

Division of integers follows sign rules: same signs give positive quotient, different signs give negative quotient.

Glossary

- **Quotient:** The result of division.
- **Undefined:** Division by zero is not defined.