Average and Central Tendency

Average is a number that represents the central tendency of a group of observations or data. It lies between the highest and lowest values of the data set. Average helps to understand the typical value in a data set, even if individual values vary.

For example, if Isha studies on average 5 hours daily, it means she usually studies around 5 hours, though some days she may study more or less.

Average is a measure of central tendency, which summarizes data by a single representative value.

Practice Set

- Think of three daily activities and estimate their average time spent.
- Collect temperature data for a week and find the average temperature.

Answer Key

- Answers will vary based on individual data.

Quick Reference

- Average = Central value representing data set.

Glossary

- **Average:** A number representing the central tendency of data.
- **Central Tendency:** A measure that identifies a single value as representative of the entire data set.

Arithmetic Mean

The arithmetic mean (or mean) is the most common measure of central tendency. It is calculated by adding all observations and dividing by the number of observations.

Formula Derivation

Given observations $x_1, x_2, \ldots, x_n$, the arithmetic mean $\bar{x}$ is:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Worked Illustration

Two vessels contain 20 litres and 60 litres of milk respectively. To find the average amount of milk per vessel:

$$\text{Average} = \frac{20 + 60}{2} = \frac{80}{2} = 40 \text{ litres}$$

Each vessel would have 40 litres if the milk is shared equally.

Solved Example 1

Ashish studies for 4, 5, and 3 hours on three consecutive days. Find his average daily study time.

Solution:

Total hours = $4 + 5 + 3 = 12$

Number of days = 3

Average study time = $\frac{12}{3} = 4$ hours

Ashish studies 4 hours daily on average.

Solved Example 2

A batsman scored runs in six innings: 36, 35, 50, 46, 60, 55. Find the mean runs per inning.

Solution:

Total runs = $36 + 35 + 50 + 46 + 60 + 55 = 282$

Number of innings = 6

Mean runs = $\frac{282}{6} = 47$

The batsman scores 47 runs on average per inning.

Practice Set

- Find the mean of the numbers: 12, 15, 18, 20, 25.
- Calculate the average marks of a student who scored 75, 80, 85, 90, and 95 in five tests.
- Find the mean of fractions $\frac{1}{2}$ and $\frac{1}{4}$.

Answer Key

- Mean = $\frac{12+15+18+20+25}{5} = \frac{90}{5} = 18$
- Mean marks = $\frac{75+80+85+90+95}{5} = \frac{425}{5} = 85$
- Mean of fractions = $\frac{\frac{1}{2}+\frac{1}{4}}{2} = \frac{\frac{3}{4}}{2} = \frac{3}{8}$

Quick Reference

- Arithmetic Mean: $\bar{x} = \frac{\sum x_i}{n}$

Glossary

- **Arithmetic Mean:** Sum of observations divided by number of observations.

Range

The range of a data set is the difference between the highest and lowest observations. It gives an idea of the spread of the data.

Formula

$$\text{Range} = \text{Highest value} - \text{Lowest value}$$

Solved Example

The ages of 10 teachers are: 32, 41, 28, 54, 35, 26, 23, 33, 38, 40.

(i) Find the oldest and youngest teacher's age.

(ii) Find the range of ages.

(iii) Find the mean age.

Solution:

Arrange in ascending order: 23, 26, 28, 32, 33, 35, 38, 40, 41, 54

(i) Oldest = 54 years, Youngest = 23 years

(ii) Range = 54 - 23 = 31 years

(iii) Mean age = $\frac{23+26+28+32+33+35+38+40+41+54}{10} = \frac{350}{10} = 35$ years

Practice Set

- Find the range of the data: 15, 22, 18, 30, 25.
- Calculate the range and mean of the numbers: 5, 10, 15, 20, 25, 30.

Answer Key

- Range = 30 - 15 = 15
- Range = 30 - 5 = 25; Mean = $\frac{5+10+15+20+25+30}{6} = \frac{105}{6} = 17.5$

Quick Reference

- Range = Highest value - Lowest value

Glossary

- **Range:** Difference between maximum and minimum values in data.

Mode

The mode of a data set is the observation that occurs most frequently.

Solved Example 1

Find the mode of the numbers: 1, 1, 2, 4, 3, 2, 1, 2, 2, 4.

Solution:

Arrange: 1, 1, 1, 2, 2, 2, 2, 3, 4, 4

Mode = 2 (occurs 4 times)

Solved Example 2

Find the mode of: 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 8.

Solution:

2 and 5 both occur 3 times.

Modes = 2 and 5 (bimodal data)

Practice Set

- Find the mode of: 4, 5, 6, 4, 7, 4, 8, 5, 6.
- Find the mode of: 10, 12, 12, 15, 15, 15, 18, 20.

Answer Key

- Mode = 4
- Mode = 15

Quick Reference

- Mode = Most frequent observation in data.

Glossary

- **Mode:** Value that appears most often in data.

Median

The median is the middle value of a data set when arranged in ascending or descending order. It divides the data into two equal halves.

Finding Median

- Arrange data in order.
- If number of observations n is odd, median is the $\frac{n+1}{2}$th value.
- If n is even, median is the average of $\frac{n}{2}$th and $\frac{n}{2}+1$th values.

Solved Example

Find the median of: 24, 36, 46, 17, 18, 25, 35.

Solution:

Arrange: 17, 18, 24, 25, 35, 36, 46

Number of observations = 7 (odd)

Median = 4th value = 25

Practice Set

- Find the median of: 12, 15, 11, 14, 13.
- Find the median of: 10, 20, 30, 40, 50, 60.

Answer Key

- Arrange: 11, 12, 13, 14, 15; Median = 13
- Arrange: 10, 20, 30, 40, 50, 60; Median = $\frac{30+40}{2} = 35$

Quick Reference

- Median = Middle value of ordered data.

Glossary

- **Median:** Middle observation dividing data into two equal parts.

Bar Graphs and Double Bar Graphs

A bar graph represents data using bars of uniform width. The length of each bar corresponds to the frequency or value of the data.

A double bar graph compares two sets of data side-by-side for each category.

Choosing a Scale

Choose a scale that fits the data range and makes the graph clear. For example, 1 unit can represent 10 students or 100 marks.

Solved Example 1

Represent the favorite colors of 200 students:

Color	Red	Green	Blue	Yellow	Orange
Number of Students	43	19	55	49	34

Scale: 1 unit = 10 students

Blue is the most preferred color; Green is the least preferred.

Solved Example 2

Marks obtained by six students:

Student	Ajay	Bali	Dipti	Faiyaz	Geetika	Hari
Marks	450	500	300	360	400	540

Scale: 1 unit = 100 marks

Draw bars accordingly to compare marks visually.

Solved Example 3

Double bar graph comparing average daily sunshine hours in Margate and Aberdeen for each month.

Bars for Margate and Aberdeen are placed side-by-side for each month to compare sunshine hours.

Solved Example 4

Double bar graph showing marks of five students in Quarterly and Half Yearly exams.

Bars for each exam are shown side-by-side for each student to compare performance.

Practice Set

- Draw a bar graph for the number of books sold in different genres.
- Draw a double bar graph comparing rainfall in two cities over six months.

Answer Key

- Answers will vary based on data collected.

Quick Reference

- Bar graph: Visual representation of data using bars.
- Double bar graph: Compares two data sets side-by-side.

Glossary

- **Bar Graph:** Graphical display of data using bars.

- **Double Bar Graph:** Bar graph comparing two data sets.