Percentage - Another Way Of Comparing Quantities

Percentage is a method to compare quantities by expressing them as parts of 100. It is especially useful when the total quantities differ, allowing for a fair comparison.

Concept Explanation

Percentage means "per hundred" and is denoted by the symbol %. For example, 80% means 80 out of 100.

Formula Derivation

Percentage is the numerator of a fraction with denominator 100. If a quantity is $\frac{a}{b}$, its percentage is given by:

$$\text{Percentage} = \frac{a}{b} \times 100\%$$

Worked Illustration

Anita scored 320 out of 400 marks. Her percentage is:

$$\frac{320}{400} \times 100 = 80\%$$

Rita scored 300 out of 360 marks. Her percentage is:

$$\frac{300}{360} \times 100 = 83.3\%$$

Though Anita scored more marks, Rita has a higher percentage, indicating better performance.

Solved Example

Rina has 100 tiles of different colors. The number of yellow tiles is 14. Find the percentage of yellow tiles.

Solution:

$$\text{Percentage of yellow tiles} = \frac{14}{100} \times 100 = 14\%$$

Practice Set

- Level 1: Find the percentage of green tiles if there are 26 green tiles out of 100.
- Level 2: A class has 25 students, 15 are girls. Find the percentage of girls.

- Level 3: A shop has 72 shoes of different sizes: 20 size 2, 30 size 3, 14 size 5, 28 size 4, 8 size 6. Find the percentage of each size.

Answer Key

- Level 1: 26%
- Level 2: $\frac{15}{25} \times 100 = 60\%$
- Level 3: Total shoes = 20+30+14+28+8=100; percentages are size 2: 20%, size 3: 30%, size 5: 14%, size 4: 28%, size 6: 8%

Quick Reference

Percentage = (Part / Whole) × 100%

Glossary

- **Percentage:** A number or ratio expressed as a fraction of 100.
- **Fraction:** A part of a whole expressed as $\frac{a}{b}$.
- **Numerator:** The top number in a fraction.
- **Denominator:** The bottom number in a fraction.

Converting Fractions and Decimals to Percentages

Converting fractions and decimals to percentages allows easier comparison and understanding of quantities.

Concept Explanation

To convert a fraction to a percentage, multiply it by 100. To convert a decimal to a percentage, multiply by 100 and add the % symbol.

Formula Derivation

For fraction $\frac{a}{b}$:

$$\text{Percentage} = \frac{a}{b} \times 100\%$$

For decimal d:

$$\text{Percentage} = d \times 100\%$$

Worked Illustrations

1. Convert $\frac{1}{3}$ to percentage:

$$\frac{1}{3} \times 100 = 33\frac{1}{3}\%$$

2. Convert 0.75 to percentage:

$$0.75 \times 100 = 75\%$$

Solved Examples

Out of 25 children, 15 are girls. Find the percentage of girls.

Solution:

$$\frac{15}{25} \times 100 = 60\%$$

Practice Set

- Level 1: Convert $\frac{5}{4}$ to percentage.
- Level 2: Convert 0.09 to percentage.
- Level 3: Out of 32 students, 8 are absent. Find the percentage absent.

Answer Key

- Level 1: 125%
- Level 2: 9%
- Level 3: $\frac{8}{32} \times 100 = 25\%$

Quick Reference

Fraction to Percentage: $\frac{a}{b} \times 100\%$

Decimal to Percentage: $d \times 100\%$

Glossary

- **Decimal:** A number expressed in the scale of tens, e.g., 0.75.
- **Improper Fraction:** A fraction where numerator > denominator.
- **Proper Fraction:** A fraction where numerator < denominator.

Converting Percentages to Fractions and Decimals

Percentages can be converted back to fractions and decimals for calculations and comparisons.

Concept Explanation

To convert a percentage to a fraction, write it over 100 and simplify. To convert to decimal, divide by 100.

Formula Derivation

For percentage $p\%$:

$$\text{Fraction} = \frac{p}{100} \quad \text{and} \quad \text{Decimal} = \frac{p}{100}$$

Worked Illustration

Convert 25% to fraction and decimal:

$$\frac{25}{100} = \frac{1}{4}, \quad 0.25$$

Solved Example

Convert 125% to fraction and decimal.

Solution:

$$\frac{125}{100} = \frac{5}{4}, \quad 1.25$$

Practice Set

- Level 1: Convert 50% to fraction and decimal.
- Level 2: Convert 90% to fraction and decimal.
- Level 3: Convert 250% to fraction and decimal.

Answer Key

- Level 1: $\frac{1}{2}, 0.5$
- Level 2: $\frac{9}{10}, 0.9$
- Level 3: $\frac{5}{2}, 2.5$

Quick Reference

Percentage to Fraction: $\frac{p}{100}$

Percentage to Decimal: $\frac{p}{100}$

Glossary

- **Simplify:** To reduce a fraction to its lowest terms.
- **Decimal:** A number expressed in base 10.

Use of Percentages

Percentages are used to interpret data, convert ratios, calculate increases or decreases, and understand real-life quantities.

Concept Explanation

Percentages express parts of a whole and are used in various contexts like savings, discounts, and ratios.

Formula Derivation

To find how many parts correspond to a percentage:

$$\text{Number} = \frac{\text{Percentage}}{100} \times \text{Total}$$

Worked Illustration

25% of 40 children like football:

$$\frac{25}{100} \times 40 = 10$$

Solved Example

Rahul saved ₹200 when a 25% discount was given. Find the original price.

Solution:

$$25\% \times P = 200 \Rightarrow \frac{25}{100} \times P = 200 \Rightarrow P = 200 \times \frac{100}{25} = 800$$

Practice Set

- Level 1: Find 50% of 164.
- Level 2: 9 is 25% of what number?
- Level 3: A shopkeeper bought an item for ₹375 and sold it for ₹400. Find the gain percentage.

Answer Key

- Level 1: 82
- Level 2: 36
- Level 3: Gain = 400 - 375 = 25; Gain% = $\frac{25}{375} \times 100 = 6.67\%$

Quick Reference

Number from Percentage: $\frac{p}{100} \times \text{Total}$

Glossary

- **Discount:** Reduction in price.
- **Gain:** Profit made on selling.
- **Loss:** Amount lost on selling.

Increase or Decrease as Per Cent

Percentage increase or decrease expresses how much a quantity has changed relative to its original value.

Concept Explanation

Percentage change is calculated by dividing the amount of change by the original amount and multiplying by 100.

Formula Derivation

$$\text{Percentage change} = \frac{\text{Amount of change}}{\text{Original amount}} \times 100$$

Worked Illustration

Wins increased from 4 to 6:

$$\text{Increase} = 6 - 4 = 2$$

$$\text{Percentage increase} = \frac{2}{4} \times 100 = 50\%$$

Solved Example

Illiterate persons decreased from 150 lakhs to 100 lakhs:

$$\text{Decrease} = 150 - 100 = 50$$

$$\text{Percentage decrease} = \frac{50}{150} \times 100 = 33\frac{1}{3}\%$$

Practice Set

- Level 1: Price of shirt decreased from ₹280 to ₹210. Find percentage decrease.
- Level 2: Petrol price increased from ₹1 to ₹52 per litre. Find percentage increase.
- Level 3: Population increased from 5,50,000 to 6,05,000. Find percentage increase.

Answer Key

- Level 1: $\frac{280-210}{280} \times 100 = 25\%$
- Level 2: $\frac{52-1}{1} \times 100 = 5100\%$
- Level 3: $\frac{605000-550000}{550000} \times 100 = 10\%$

Quick Reference

Percentage change = $\frac{\text{Change}}{\text{Original}} \times 100$

Glossary

- **Percentage Increase:** When new value > original value.
- **Percentage Decrease:** When new value < original value.

Prices Related to an Item or Buying and Selling

Understanding cost price, selling price, profit, and loss is essential in commerce.

Concept Explanation

Cost Price (CP) is the price at which an item is bought. Selling Price (SP) is the price at which it is sold.

If SP > CP, profit = SP − CP.

If SP < CP, loss = CP − SP.

Formula Derivation

Profit or Loss percentage is calculated on CP:

$$\text{Profit } \% = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$\text{Loss } \% = \frac{\text{Loss}}{\text{CP}} \times 100$$

Worked Illustration

Toy bought for ₹72, sold for ₹80:

$$\text{Profit} = 80 - 72 = 8$$

$$\text{Profit } \% = \frac{8}{72} \times 100 = 11\frac{1}{9}\%$$

Solved Example

Flower vase cost ₹120, sold at 10% loss. Find selling price.

Solution:

$$\text{Loss} = 10\% \times 120 = 12$$

$$\text{SP} = 120 - 12 = 108$$

Practice Set

- Level 1: Find gain percentage if CP = ₹375 and SP = ₹400.
- Level 2: Find selling price if CP = ₹50 and profit = 12%.
- Level 3: Find CP if SP = ₹540 and profit = 20%.

Answer Key

- Level 1: Gain = 25; Gain% = $\frac{25}{375} \times 100 = 6.67\%$
- Level 2: SP = 50 + 12% of 50 = 50 + 6 = 56
- Level 3: CP = $\frac{540 \times 100}{120} = 450$

Quick Reference

Profit or Loss % = (Profit or Loss / CP) × 100

Glossary

- **Cost Price (CP):** Price at which item is bought.
- **Selling Price (SP):** Price at which item is sold.
- **Profit:** When SP > CP.
- **Loss:** When SP < CP.

Simple Interest

Simple interest is the extra amount paid on borrowed money, calculated only on the principal.

Concept Explanation

Interest is charged as a percentage of the principal per year.

Formula Derivation

Simple Interest (SI) for principal P, rate $R\%$, and time T years is:

$$SI = \frac{P \times R \times T}{100}$$

Amount to be paid after T years is:

$$A = P + SI$$

Worked Illustration

Anita borrows ₹5000 at 15% per annum for 1 year:

$$SI = \frac{5000 \times 15 \times 1}{100} = 750$$

$$A = 5000 + 750 = 5750$$

Solved Example

Manohar pays ₹750 interest for 2 years on ₹4500. Find rate of interest.

Solution:

$$750 = \frac{4500 \times 2 \times R}{100} \Rightarrow 750 = 90R \Rightarrow R = \frac{750}{90} = 8\frac{1}{3}\%$$

Practice Set

- Level 1: Find interest on ₹10,000 at 5% p.a. for 1 year.
- Level 2: Find interest on ₹3500 at 7% p.a. for 2 years.
- Level 3: Find amount to be paid on ₹6050 borrowed at 6.5% p.a. for 3 years.

Answer Key

- Level 1: ₹500
- Level 2: $\frac{3500 \times 7 \times 2}{100} = 490$
- Level 3: SI = $\frac{6050 \times 6.5 \times 3}{100} = 1179.75$, Amount = 6050 + 1179.75 = 7229.75

Quick Reference

Simple Interest = $\frac{P \times R \times T}{100}$

Glossary

- **Principal (P):** Initial amount borrowed or invested.
- **Rate (R):** Interest rate per annum.
- **Time (T):** Duration in years.
- **Simple Interest (SI):** Interest calculated on principal only.
- **Amount (A):** Total to be paid = Principal + Interest.