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Introduction to Algebraic Expressions

Algebraic expressions are combinations of variables and constants using operations such as addition, subtraction, multiplication, and division. Variables like x, y, l, m represent quantities that can change, while constants like 4, 100, or -17 have fixed values. Expressions such as $4x + 5$ or $10y - 20$ are formed by multiplying variables by constants and then adding or subtracting constants.

Expressions can also combine variables with themselves or with other variables, for example, x^2, $2y^2$, $3x^2 - 5$, xy, and $4xy + 7$. Here, powers indicate repeated multiplication, e.g., $x^2 = x \times x$, and terms like xy represent the product of two variables.

Worked Illustration

- x^2 means $x \times x$. For $x = 3$, $x^2 = 9$.
- $2y^2 = 2 \times y \times y$. For $y = 2$, $2y^2 = 8$.
- $3x^2 - 5 = 3 \times x^2 - 5$. For $x = 2$, $3x^2 - 5 = 7$.
- $xy = x \times y$. For $x = 3, y = 4$, $xy = 12$.
- $4xy + 7 = 4 \times x \times y + 7$. For $x = 1, y = 5$, $4xy + 7 = 27$.

Practice Set

- Describe how the expressions $7xy + 5$, x^2y, and $4x^2 - 5x$ are formed.

Answer Key

Each expression is formed by multiplying variables and constants and combining terms by addition or subtraction as per the examples above.

Quick Reference

- Variables: letters representing changing values.
- Constants: fixed numerical values.
- Expressions: combinations of variables and constants using arithmetic operations.
- Power notation: $x^2 = x \times x$, $x^3 = x \times x \times x$.

Glossary

- **Variable:** A symbol representing a number that can vary.
- **Constant:** A fixed numerical value.

- **Expression:** A combination of variables and constants using arithmetic operations.
- **Term:** A part of an expression separated by + or - signs.
- **Power:** The number of times a variable is multiplied by itself.

Terms and Factors of an Expression

An algebraic expression is made up of terms, which are parts separated by addition or subtraction. Each term is a product of factors, which can be constants or variables. For example, in $4x + 5$, the terms are $4x$ and 5. In $3x^2 + 7y$, the terms are $3x^2$ and $7y$.

Factors of a term are the numbers and variables multiplied together to form the term. For example, the term $4x^2$ has factors 4, x, and x. The term $-3xy$ has factors -3, x, and y.

Worked Illustration

Consider the expression $4x^2 - 3xy$:

- Terms: $4x^2$ and $-3xy$.
- Factors of $4x^2$: 4, x, x.
- Factors of $-3xy$: -3, x, y.

Tree diagrams can be used to represent terms and their factors visually.

Practice Set

- Identify terms and factors in the expressions: $8y + 3x^2$, $7mn - 4$, $2x^2y$.
- Draw tree diagrams for the above expressions.

Answer Key

Terms and factors are identified by separating the expression at + or - signs and breaking down each term into multiplicative components.

Quick Reference

- Term: a part of an expression separated by + or -.
- Factor: a number or variable multiplied to form a term.
- Tree diagram: a visual representation of terms and factors.

Glossary

- **Term:** A part of an expression separated by + or -.
- **Factor:** A number or variable multiplied to form a term.
- **Coefficient:** The numerical factor of a term.

Coefficients in Algebraic Terms

The coefficient of a term is the numerical factor multiplying the variables. For example, in $5xy$, 5 is the coefficient. In $-7x^2y^2$, -7 is the coefficient. When the coefficient is 1, it is usually omitted, e.g., x means $1x$, and $-x$ means $-1x$.

Worked Illustration

Identify coefficients in the expressions:

- $4x - 3y$: coefficients of x and y are 4 and -3 respectively.
- $8 - x + y$: coefficient of x is -1, coefficient of y is 1.
- $y^2x - y$: coefficient of x in y^2x is y^2.
- $2z - 5xz$: coefficient of x in $-5xz$ is $-5z$.

Practice Set

- Find coefficients of x in: $4x - 3y$, $8 - x + y$, $y^2x - y$, $2z - 5xz$.
- Find coefficients of y in: $4x - 3y$, $8 + yz$, $yz^2 + 5$, $my + m$.

Answer Key

Coefficients are the numerical or algebraic factors multiplying the variable in each term.

Quick Reference

- Coefficient: numerical factor multiplying variables in a term.
- Coefficient 1 is usually omitted.
- Negative signs indicate negative coefficients.

Glossary

- **Coefficient:** The numerical factor of a term.
- **Term:** A product of factors including variables and coefficients.

Like and Unlike Terms

Terms are like terms if they have the same variables raised to the same powers, regardless of their coefficients. Terms with different variables or powers are unlike terms.

For example, in $2xy - 3x + 5xy - 4$, $2xy$ and $5xy$ are like terms, but $2xy$ and $-3x$ are unlike terms.

Worked Illustration

Pair	Like/Unlike	Reason
$7x, 12y$	Unlike	Different variables
$15x, -21x$	Like	Same variable and power
$-4ab, 7ba$	Like	Same variables (order does not matter)
$3xy, 3x$	Unlike	Different variables
$6xy^2, 9x^2y$	Unlike	Different powers of variables
$pq^2, -4pq^2$	Like	Same variables and powers
$mn^2, 10mn$	Unlike	Different powers of n

Practice Set

- Group like terms from: $12x, 12, -25x, -25, -25y, 1, x, 12y, y$.
- Classify pairs of terms as like or unlike.

Answer Key

Like terms have identical variables with the same powers; unlike terms differ in variables or powers.

Quick Reference

- Like terms: same variables and powers.
- Unlike terms: different variables or powers.
- Order of variables does not affect like terms.

Glossary

- **Like terms:** Terms with identical variables and powers.
- **Unlike terms:** Terms differing in variables or powers.

Finding the Value of an Expression

The value of an algebraic expression depends on the values assigned to its variables. To find the value, substitute the given values into the expression and simplify step-by-step.

Worked Illustrations

Find the value of the expressions for given values:

- For $x = 2$:
  - $x + 4 = 2 + 4 = 6$
  - $4x - 3 = 4 \times 2 - 3 = 8 - 3 = 5$
  - $19 - 5x^2 = 19 - 5 \times 2^2 = 19 - 20 = -1$
  - $100 - 10x^3 = 100 - 10 \times 2^3 = 100 - 80 = 20$
- For $n = -2$:
  - $5n - 2 = 5 \times (-2) - 2 = -10 - 2 = -12$
  - $5n^2 + 5n - 2 = 5 \times (-2)^2 + 5 \times (-2) - 2 = 20 - 10 - 2 = 8$
  - $n^3 + 5n^2 + 5n - 2 = (-2)^3 + 5 \times (-2)^2 + 5 \times (-2) - 2 = -8 + 20 - 10 - 2 = 0$
- For $a = 3, b = 2$:
  - $a + b = 3 + 2 = 5$
  - $7a - 4b = 7 \times 3 - 4 \times 2 = 21 - 8 = 13$
  - $a^2 + 2ab + b^2 = 3^2 + 2 \times 3 \times 2 + 2^2 = 9 + 12 + 4 = 25$
  - $a^3 - b^3 = 3^3 - 2^3 = 27 - 8 = 19$

Practice Set

- Evaluate $3x^2 + 2x - 5$ for $x = 1, 2, -1$.
- Find the value of $2m^3 - 3m^2 + m - 4$ for $m = 0, 1, -1$.
- Calculate $xy + x - y$ for $x = 2, y = 3$ and $x = -1, y = 4$.

Answer Key

Substitute the given values into the expressions and simplify step-by-step using powers and arithmetic operations.

Quick Reference

- Substitute variable values carefully.
- Calculate powers before multiplication.
- Perform arithmetic operations in order.

Glossary

- **Substitution:** Replacing variables with given values.
- **Evaluation:** Calculating the numerical value of an expression.
- **Power:** Repeated multiplication of a variable.