

- Introduction to Exponents
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Introduction to Exponents

Large numbers such as the mass of Earth (5,970,000,000,000,000,000,000 kg) and Uranus (86,800,000,000,000,000,000,000 kg) are difficult to read and compare. To simplify, we use exponents to express such numbers in a compact form.

For example, 10,000 can be written as 10^4 , which means $10 \times 10 \times 10 \times 10$. Here, 10 is the base and 4 is the exponent.

Similarly, $1,000 = 10^3$, $100,000 = 10^5$, and so on. The base can be any number, not just 10. For instance, $81 = 3^4$ because $3 \times 3 \times 3 \times 3 = 81$.

Worked Examples

- $5^3 = 5 \times 5 \times 5 = 125$
- $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
- $243 = 3^5$
- $64 = 2^6$
- $625 = 5^4$

Practice Set

- Express 729 as a power of 3.
- Express 128 as a power of 2.
- Express 343 as a power of 7.
- Express 256 as a power of 2.
- Calculate $(-2)^3$ and $(-2)^4$.

Answer Key

- $729 = 3^6$
- $128 = 2^7$
- $343 = 7^3$
- $256 = 2^8$
- $(-2)^3 = -8, (-2)^4 = 16$

Quick Reference

a^n means multiplying base a by itself n times.

Special names: 10^2 is "10 squared", 10^3 is "10 cubed".

Glossary

- **Base:** The number being multiplied.
- **Exponent:** The number of times the base is multiplied.
- **Power:** The result of the base raised to the exponent.

Laws of Exponents

Multiplying Powers with the Same Base

When multiplying powers with the same base, add the exponents:

$$a^m \times a^n = a^{m+n}$$

Example: $2^2 \times 2^3 = 2^{2+3} = 2^5 = 32$

Dividing Powers with the Same Base

When dividing powers with the same base, subtract the exponents:

$$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$$

Example: $3^7 \div 3^4 = 3^{7-4} = 3^3 = 27$

Power of a Power

When raising a power to another power, multiply the exponents:

$$(a^m)^n = a^{m \times n}$$

Example: $(2^3)^2 = 2^{3 \times 2} = 2^6 = 64$

Multiplying Powers with the Same Exponent

When multiplying powers with different bases but the same exponent, multiply the bases and keep the exponent:

$$a^m \times b^m = (a \times b)^m$$

Example: $2^3 \times 3^3 = (2 \times 3)^3 = 6^3 = 216$

Dividing Powers with the Same Exponent

When dividing powers with different bases but the same exponent, divide the bases and keep the exponent:

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Example: $\frac{2^4}{3^4} = \left(\frac{2}{3}\right)^4$

Zero Exponent

Any non-zero number raised to the power zero is 1:

$$a^0 = 1 \quad (a \neq 0)$$

Example: $2^0 = 1, \quad 7^0 = 1$

Worked Illustrations

- $2^3 \times 2^5 = 2^{3+5} = 2^8 = 256$
- $5^6 \div 5^2 = 5^{6-2} = 5^4 = 625$
- $(3^2)^4 = 3^{2 \times 4} = 3^8 = 6561$
- $4^3 \times 2^3 = (4 \times 2)^3 = 8^3 = 512$
- $\frac{9^5}{3^5} = \left(\frac{9}{3}\right)^5 = 3^5 = 243$

Practice Set

- Simplify $2^4 \times 2^3$.
- Simplify $10^7 \div 10^4$.
- Simplify $(5^3)^2$.
- Simplify $3^5 \times 4^5$.
- Simplify $\frac{7^6}{2^6}$.
- Find the value of a^0 for $a = 5, -3, 10$.

Answer Key

- $2^7 = 128$
- $10^3 = 1000$
- $5^6 = 15625$
- $(3 \times 4)^5 = 12^5 = 248832$
- $\left(\frac{7}{2}\right)^6 = \frac{117649}{64}$
- All equal 1.

Quick Reference

Operation	Rule
Multiply powers with same base	$a^m \times a^n = a^{m+n}$
Divide powers with same base	$\frac{a^m}{a^n} = a^{m-n}$
Power of a power	$(a^m)^n = a^{m \times n}$

Multiply powers with same exponent	$a^m \times b^m = (ab)^m$
Divide powers with same exponent	$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
Zero exponent	$a^0 = 1$

Glossary

- **Exponent:** The power to which a base is raised.
- **Base:** The number being multiplied.
- **Power:** The result of raising a base to an exponent.
- **Zero exponent:** Any non-zero base raised to zero equals 1.

Prime Factorization and Exponents

Prime factorization is expressing a number as a product of prime numbers raised to powers.

Example: Express 72 as a product of powers of prime factors.

Step 1: Divide 72 by smallest prime 2 repeatedly:

$$72 = 2 \times 36 = 2 \times 2 \times 18 = 2 \times 2 \times 2 \times 9$$

Step 2: Factor 9 as 3×3 :

$$72 = 2^3 \times 3^2$$

Similarly,

- $432 = 2^4 \times 3^3$
- $1000 = 2^3 \times 5^3$
- $16,000 = 2^7 \times 5^3$

Worked Examples

- Express 1000 as product of prime powers: $2^3 \times 5^3$
- Express 16,000 as product of prime powers: $2^7 \times 5^3$

Practice Set

- Express 360 as product of prime powers.
- Express 540 as product of prime powers.
- Express 1,000,000 as product of prime powers.

Answer Key

- $360 = 2^3 \times 3^2 \times 5$
- $540 = 2^2 \times 3^3 \times 5$
- $1,000,000 = 2^6 \times 5^6$

Quick Reference

Prime factorization breaks a number into primes raised to powers, useful for simplifying expressions and finding greatest common divisors.

Glossary

- **Prime factorization:** Expressing a number as a product of prime numbers.
- **Prime number:** A number greater than 1 with no divisors other than 1 and itself.

Expressing Large Numbers in Standard Form

Large numbers can be expressed in standard form as a decimal number between 1 and 10 multiplied by a power of 10.

For example:

- $59 = 5.9 \times 10^1$
- $590 = 5.9 \times 10^2$
- $5900 = 5.9 \times 10^3$
- $59000 = 5.9 \times 10^4$

Similarly,

- $300,000,000,000,000,000 = 3.0 \times 10^{20}$
- $40,000,000,000 = 4.0 \times 10^{10}$
- Mass of Earth = $5,976,000,000,000,000,000,000 = 5.976 \times 10^{24}$ kg
- Mass of Uranus = 8.68×10^{25} kg

Comparing powers of 10 helps determine which number is larger.

Worked Examples

- Express 5985.3 as standard form: 5.9853×10^3
- Express 65,950 as standard form: 6.595×10^4
- Express 3,430,000 as standard form: 3.43×10^6
- Express 70,040,000,000 as standard form: 7.004×10^{10}

Practice Set

- Express 172 in exponential form.
- Express 5,643 in exponential form.
- Express 56,439 in exponential form.
- Express 176,428 in exponential form.
- Express 1,433,500,000,000 in standard form.
- Express 1,439,000,000,000 in standard form.
- Express 149,600,000,000 in standard form.

Answer Key

- $172 = 1.72 \times 10^2$
- $5,643 = 5.643 \times 10^3$
- $56,439 = 5.6439 \times 10^4$
- $176,428 = 1.76428 \times 10^5$
- $1,433,500,000,000 = 1.4335 \times 10^{12}$
- $1,439,000,000,000 = 1.439 \times 10^{12}$
- $149,600,000,000 = 1.496 \times 10^{11}$

Quick Reference

Standard form = decimal number between 1 and 10 \times power of 10.

Exponent of 10 = (number of digits to left of decimal point) - 1.

Glossary

- **Standard form:** A way to express large or small numbers as a decimal between 1 and 10 multiplied by a power of 10.
- **Decimal number system:** Base-10 number system using digits 0-9.