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Multiplying Fractions

Multiplying fractions involves multiplying the numerators together and the denominators together. For two fractions $\frac{a}{b}$ and $\frac{c}{d}$, the product is:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Formula Derivation

Given two fractions $\frac{a}{b}$ and $\frac{c}{d}$, multiplying them means finding a fraction representing the product of their values. Multiplying numerators and denominators separately preserves the value:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Worked Illustration

Multiply $\frac{2}{3}$ and $\frac{3}{4}$:

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

Solved Example

Example: A farmer distributes $\frac{2}{3}$ acre of land to each of her 5 grandchildren. How much land is given in total?

Solution:

Total land = $5 \times \frac{2}{3} = \frac{5 \times 2}{3} = \frac{10}{3} = 3\frac{1}{3}$ acres.

Practice Set

- **Level 1:** Multiply $\frac{1}{2} \times \frac{3}{5}$.
- **Level 2:** Multiply $\frac{4}{7} \times \frac{2}{3} \times \frac{5}{6}$.
- **Level 3:** A rectangle has length $\frac{5}{8}$ units and breadth $\frac{3}{10}$ units. Find its area.

Answer Key

- Level 1: $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$
- Level 2: $\frac{4}{7} \times \frac{2}{3} \times \frac{5}{6} = \frac{4 \times 2 \times 5}{7 \times 3 \times 6} = \frac{40}{126} = \frac{20}{63}$
- Level 3: Area = $\frac{5}{8} \times \frac{3}{10} = \frac{15}{80} = \frac{3}{16}$ square units.

Quick Reference

To multiply fractions, multiply numerators and denominators separately and simplify the result.

Glossary

- **Numerator:** The top number of a fraction.
- **Denominator:** The bottom number of a fraction.
- **Product:** The result of multiplication.

Area Interpretation of Fraction Multiplication

Multiplying two fractions can be visualized as finding the area of a rectangle with fractional side lengths. If length = $\frac{a}{b}$ units and breadth = $\frac{c}{d}$ units, then area is:

$$\text{Area} = \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \text{ square units}$$

Worked Illustration

Length = $\frac{2}{3}$ unit, Breadth = $\frac{3}{4}$ unit.

Area = $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$ square unit.

Solved Example

Example: Find the area of a rectangle with length $\frac{5}{6}$ units and breadth $\frac{2}{5}$ units.

Solution:

$$\text{Area} = \frac{5}{6} \times \frac{2}{5} = \frac{10}{30} = \frac{1}{3} \text{ square units}$$

Practice Set

- **Level 1:** Find the area of a rectangle with sides $\frac{1}{2}$ and $\frac{1}{3}$ units.
- **Level 2:** Calculate the area of a rectangle with sides $\frac{7}{8}$ and $\frac{4}{9}$ units.

- **Level 3:** A square has side length $\frac{3}{7}$ units. Find its area.

Answer Key

- Level 1: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ square units.
- Level 2: $\frac{7}{8} \times \frac{4}{9} = \frac{28}{72} = \frac{7}{18}$ square units.
- Level 3: Area = $\left(\frac{3}{7}\right)^2 = \frac{9}{49}$ square units.

Quick Reference

Area of rectangle with fractional sides = product of the fractions.

Glossary

- **Area:** The measure of the surface enclosed within a shape.
- **Rectangle:** A quadrilateral with four right angles.

Simplifying Fractions After Multiplication

After multiplying fractions, the result should be simplified to its lowest terms by dividing numerator and denominator by their greatest common factor (GCF).

Method: Cross-Cancellation

Before multiplying, simplify by canceling common factors between numerators and denominators across the fractions.

Solved Example

Multiply and simplify $\frac{2}{3} \times \frac{3}{4}$:

Cancel 3 in numerator and denominator:

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{4} = \frac{2}{1} \times \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Practice Set

- **Level 1:** Simplify $\frac{3}{5} \times \frac{10}{9}$.
- **Level 2:** Simplify $\frac{4}{7} \times \frac{14}{15} \times \frac{5}{8}$.
- **Level 3:** Simplify $\frac{6}{11} \times \frac{33}{18} \times \frac{9}{22}$.

Answer Key

- Level 1: $\frac{3}{5} \times \frac{10}{9} = \frac{3 \times 10}{5 \times 9} = \frac{30}{45} = \frac{2}{3}$
- Level 2: $\frac{4}{7} \times \frac{14}{15} \times \frac{5}{8} = \frac{4 \times 14 \times 5}{7 \times 15 \times 8} = \frac{280}{840} = \frac{1}{3}$
- Level 3: $\frac{6}{11} \times \frac{33}{18} \times \frac{9}{22} = \frac{6 \times 33 \times 9}{11 \times 18 \times 22} = \frac{1782}{4356} = \frac{3}{7}$

Quick Reference

Simplify fractions by dividing numerator and denominator by their GCF or by cross-cancellation before multiplying.

Glossary

- **Greatest Common Factor (GCF):** The largest number dividing two or more numbers exactly.
- **Cross-Cancellation:** Simplifying factors diagonally before multiplication.

Commutative Property of Fraction Multiplication

The order of multiplication of fractions does not affect the product. For fractions $\frac{a}{b}$ and $\frac{c}{d}$:

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

Solved Example

Verify $\frac{2}{3} \times \frac{4}{5} = \frac{4}{5} \times \frac{2}{3}$:

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}, \quad \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

Practice Set

- **Level 1:** Show that $\frac{1}{4} \times \frac{3}{7} = \frac{3}{7} \times \frac{1}{4}$.
- **Level 2:** Verify commutativity for $\frac{5}{6} \times \frac{2}{9}$.
- **Level 3:** Prove commutativity for three fractions $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.

Answer Key

- Level 1: Both products equal $\frac{3}{28}$.
- Level 2: Both products equal $\frac{10}{54} = \frac{5}{27}$.
- Level 3: Both orders yield $\frac{2\times3\times4}{3\times4\times5} = \frac{24}{60} = \frac{2}{5}$.

Quick Reference

Multiplication of fractions is commutative: order does not change the product.

Glossary

- **Commutative Property:** The property that changing the order of numbers does not change the result.

Division of Fractions

To divide one fraction by another, multiply the first fraction (dividend) by the reciprocal of the second fraction (divisor). For fractions $\frac{a}{b}$ and $\frac{c}{d}$:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$$

Formula Derivation

Division by a fraction is equivalent to multiplication by its reciprocal. This is because dividing by a number is the same as multiplying by its multiplicative inverse.

Solved Example

Divide $\frac{3}{4}$ by $\frac{2}{5}$:

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

Practice Set

- **Level 1:** Calculate $\frac{1}{2} \div \frac{3}{4}$.
- **Level 2:** Find $\frac{5}{6} \div \frac{2}{3}$.
- **Level 3:** Evaluate $\frac{7}{8} \div \frac{14}{15} \div \frac{3}{5}$.

Answer Key

- Level 1: $\frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$
- Level 2: $\frac{5}{6} \times \frac{3}{2} = \frac{15}{12} = \frac{5}{4}$
- Level 3: $\frac{7}{8} \times \frac{15}{14} \times \frac{5}{3} = \frac{7 \times 15 \times 5}{8 \times 14 \times 3} = \frac{525}{336} = \frac{25}{16}$

Quick Reference

Divide fractions by multiplying the dividend by the reciprocal of the divisor.

Glossary

- **Reciprocal:** The inverse of a fraction $\frac{a}{b}$ is $\frac{b}{a}$.
- **Dividend:** The number being divided.
- **Divisor:** The number by which division is performed.
- **Quotient:** The result of division.

Fractional Relations and Applications

Fractional relations compare parts of a whole or different quantities using fractions.

Worked Illustration

If a rectangle is divided into 8 equal parts and 3 parts are shaded, then:

- Shaded part = $\frac{3}{8}$
- Unshaded part = $\frac{5}{8}$
- Ratio of shaded to unshaded = $\frac{3}{5}$

Solved Example

Example: Leena used $\frac{1}{4}$ litre of milk to make 5 cups of tea. How much milk is in each cup?

Solution:

Milk per cup = $\frac{1}{4} \div 5 = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ litre.

Practice Set

- **Level 1:** Find the fraction of shaded area if 2 out of 5 parts are shaded.
- **Level 2:** If $\frac{3}{7}$ of a cake is eaten, what fraction remains?
- **Level 3:** A cistern is filled by four fountains with rates $1, 2, 4, 5$ cisterns/day respectively. Find the time to fill the cistern together.

Answer Key

- Level 1: $\frac{2}{5}$
- Level 2: Remaining fraction = $1 - \frac{3}{7} = \frac{4}{7}$
- Level 3: Total rate = $1 + 2 + 4 + 5 = 12$ cisterns/day; Time = $\frac{1}{12}$ day = 2 hours.

Quick Reference

Fractional relations express parts of a whole and their comparisons.

Glossary

- **Fractional Relation:** A comparison between two quantities expressed as a fraction.
- **Shaded Region:** The part of a figure highlighted or marked.