A Quick Recap of Decimals

Decimals extend the Indian place value system to represent decimal fractions such as $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, and their sums. For example, the decimal number 27.53 represents:

- 2 Tens
- 7 Units (Ones)
- 5 Tenths
- 3 Hundredths

To convert a fraction with denominator 10, 100, 1000, etc., into a decimal, divide the numerator by the denominator. For example, $\frac{3}{10} = 0.3$, $\frac{4}{100} = 0.04$, $\frac{67}{1000} = 0.067$.

Rule for dividing by powers of 10: To divide a number by $10, 100, 1000, \ldots$, move the decimal point to the left by as many places as there are zeros in the divisor. For example, $123 \div 10 = 12.3$, $24 \div 100 = 0.24$, $678 \div 1000 = 0.678$.

Worked Illustration

Express the following quantities in kilograms as fractions and decimals:

- 50 g = $\frac{50}{1000} = 0.05$ kg
- 100 g = $\frac{100}{1000} = 0.1$ kg
- 25 g = $\frac{25}{1000} = 0.025$ kg
- 250 g = $\frac{250}{1000} = 0.25$ kg

Practice Set

1. Convert the following fractions to decimals:

- $\frac{7}{10}$
- $\frac{45}{100}$
- $\frac{123}{1000}$

2. Divide the following numbers by powers of 10:

- 345 ÷ 10
- 6789 ÷ 100
- 12345 ÷ 1000

Answer Key

- $\frac{7}{10} = 0.7$
- $\frac{45}{100} = 0.45$
- $\frac{123}{1000} = 0.123$
- 345 ÷ 10 = 34.5
- 6789 ÷ 100 = 67.89
- 12345 ÷ 1000 = 12.345

Quick Reference

- Decimal fractions are fractions with denominators 10, 100, 1000, etc.

- To convert $\frac{a}{10^n}$ to decimal, divide a by 10^n.
- Dividing by 10^n moves the decimal point n places to the left.

Glossary

- **Decimal fraction:** A fraction with denominator 10, 100, 1000, etc.
- **Dividend:** The number to be divided.
- **Divisor:** The number by which division is performed.
- **Decimal point:** The dot separating the integer and fractional parts of a decimal number.

Decimal Multiplication

To multiply decimals, convert each decimal to a fraction, multiply the numerators and denominators, then convert back to decimal.

For example, to multiply 9.5×5:

$$9.5 = \frac{95}{10}, \quad 5 = \frac{5}{1}$$

Multiply:

$$\frac{95}{10} \times \frac{5}{1} = \frac{95 \times 5}{10 \times 1} = \frac{475}{10} = 47.5$$

Similarly, for 12.5×7.5:

$$12.5 = \frac{125}{10}, \quad 7.5 = \frac{75}{10}$$

Multiply:

$$\frac{125}{10} \times \frac{75}{10} = \frac{125 \times 75}{100} = \frac{9375}{100} = 93.75$$

Rule for decimal multiplication: Multiply the numbers ignoring the decimal points, then place the decimal point in the product so that the number of decimal places equals the sum of decimal places in the factors.

Worked Example

Find the area of a rectangle with length 13.3 cm and width 5.7 cm.

Area = length × width = 13.3×5.7

Convert to fractions:

$$13.3 = \frac{133}{10}, \quad 5.7 = \frac{57}{10}$$

Multiply:

$$\frac{133}{10} \times \frac{57}{10} = \frac{133 \times 57}{100} = \frac{7581}{100} = 75.81 \text{ cm}^2$$

Practice Set

1. Multiply:

- 5.8 × 1.24
- 0.75 × 0.4
- 2.25 × 8

2. Find the product of 5.96 and 24.8 using the decimal multiplication rule.

Answer Key

- 5.8 × 1.24 = 7.192
- 0.75 × 0.4 = 0.3
- 2.25 × 8 = 18
- 5.96 × 24.8 = 147.808

Quick Reference

- Multiply decimals as whole numbers.
- Count total decimal places in both numbers.
- Place decimal point in product accordingly.

Glossary

- **Multiplicand:** The number to be multiplied.
- **Multiplier:** The number by which another number is multiplied.

- **Product:** The result of multiplication.

Decimal Division

Decimal division is performed using place value and long division. When dividing decimals, regroup the dividend and divisor to eliminate decimals by multiplying numerator and denominator by powers of 10.

For example, to divide $3.9 \div 10$:

Convert $3.9 = \frac{39}{10}$, so

$$3.9 \div 10 = \frac{39}{10} \times \frac{1}{10} = \frac{39}{100} = 0.39$$

Rule: Dividing a decimal by $10, 100, 1000, \ldots$ moves the decimal point to the left by the number of zeros in the divisor.

Long division can be used to divide any two numbers, including decimals, by regrouping place values and placing the decimal point in the quotient when regrouping ones into tenths.

Worked Example

Divide 1325 by 4 using long division:

Step 1: Divide 13 hundreds by 4 → 3 hundreds, remainder 1 hundred.

Step 2: Regroup 1 hundred as 10 tens, add 2 tens → 12 tens.

Step 3: Divide 12 tens by 4 → 3 tens, remainder 0.

Step 4: Divide 5 ones by 4 → 1 one, remainder 1 one.

Step 5: Regroup 1 one as 10 tenths.

Step 6: Divide 10 tenths by 4 → 2 tenths, remainder 2 tenths.

Step 7: Regroup 2 tenths as 20 hundredths.

Step 8: Divide 20 hundredths by 4 → 5 hundredths, remainder 0.

Quotient = 331.25

Practice Set

1. Find the quotient:

- 237 ÷ 8
- 9.5 ÷ 4
- 0.06 ÷ 5

2. Divide 4.68 by 1.3 and 0.13.

Answer Key

- 237 ÷ 8 = 29.625
- 9.5 ÷ 4 = 2.375
- 0.06 ÷ 5 = 0.012
- 4.68 ÷ 1.3 = 3.6
- 4.68 ÷ 0.13 = 36

Quick Reference

- Multiply dividend and divisor by the same power of 10 to make divisor a whole number.
- Use long division to find quotient.
- Place decimal point in quotient when regrouping ones into tenths.

Glossary

- **Quotient:** The result of division.
- **Remainder:** The amount left over after division.
- **Long division:** A method of division using place value and repeated subtraction.

Look Before You Leap

The Earth takes approximately 365.2422 days to orbit the Sun, but the calendar year is 365 days. This discrepancy accumulates over years, causing the calendar to drift relative to the seasons.

To correct this, a leap year system is used:

- Every 4th year is a leap year with 366 days.
- Years divisible by 100 are not leap years.
- Years divisible by 400 are leap years.

This system keeps the calendar aligned with the Earth's orbit.

Worked Illustration

Calculate the number of days in 100 years with leap year adjustments:

Number of leap years = $\frac{100}{4} - \frac{100}{100} = 25 - 1 = 24$

Number of normal years = 100 - 24 = 76

Total days = $24 \times 366 + 76 \times 365 = 36524$ days

Actual days in 100 Earth revolutions = $100 \times 365.2422 = 36524.22$ days

The calendar is very close to the actual time.

Practice Set

1. Determine if the following years are leap years: 1900, 2000, 2024, 2100.

2. Calculate the number of days in 400 years using the leap year rules.

Answer Key

- 1900: Not a leap year (divisible by 100 but not 400)
- 2000: Leap year (divisible by 400)
- 2024: Leap year (divisible by 4, not 100)
- 2100: Not a leap year (divisible by 100 but not 400)

- Days in 400 years = $303 \times 365 + 97 \times 366 = 146097$ days

Quick Reference

- Leap year if divisible by 4 but not by 100, unless divisible by 400.
- Leap years have 366 days; normal years have 365 days.

Glossary

- **Leap year:** A year with 366 days to keep the calendar aligned with Earth's orbit.
- **Gregorian calendar:** The calendar system currently used internationally.
- **Century year:** A year ending with 00, e.g., 1900, 2000.