Factors and Multiples

To understand factors and multiples, consider arranging marbles in rows such that each row has the same number of marbles. For example, 6 marbles can be arranged as:

- 1 marble per row: 6 rows ($1 \times 6 = 6$)
- 2 marbles per row: 3 rows ($2 \times 3 = 6$)
- 3 marbles per row: 2 rows ($3 \times 2 = 6$)
- 6 marbles per row: 1 row ($6 \times 1 = 6$)

Numbers like 1, 2, 3, and 6 that exactly divide 6 are called factors or exact divisors of 6.

Concept Explanation

A factor of a number is a number that divides it exactly without leaving a remainder. A multiple of a number is the product of that number and an integer.

Formula Derivation

If a and b are integers, then a is a factor of b if and only if there exists an integer k such that:

$$b = a \times k$$

Here, b is a multiple of a.

Worked Illustration

For 4, factors are 1, 2, and 4 because:

- $4 = 1 \times 4$
- $4 = 2 \times 2$

Multiples of 3 are 3, 6, 9, 12, 15, ...

Solved Example

Example: Find factors of 18.

Solution: 18 can be expressed as:

- 1×18
- 2×9
- 3×6

Thus, factors of 18 are 1, 2, 3, 6, 9, and 18.

Practice Set

- Level 1: Find factors of 12, 20, and 30.
- Level 2: List first five multiples of 7 and 9.
- Level 3: Determine if 45 is a multiple of 3 and 5.

Answer Key

- Factors of 12: 1, 2, 3, 4, 6, 12
- Factors of 20: 1, 2, 4, 5, 10, 20
- Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
- Multiples of 7: 7, 14, 21, 28, 35
- Multiples of 9: 9, 18, 27, 36, 45
- 45 is a multiple of both 3 and 5.

Quick Reference

Term	Definition
Factor	Number that divides another number exactly
Multiple	Product of a number and an integer

Glossary

- **Exact Divisor:** A number that divides another number without remainder.
- **Factor:** Same as exact divisor.
- **Multiple:** Product of a number and any integer.

Prime and Composite Numbers

Numbers can be classified based on their factors:

- **Prime Numbers:** Numbers greater than 1 with exactly two factors: 1 and itself.
- **Composite Numbers:** Numbers with more than two factors.
- Number 1 is neither prime nor composite.

Concept Explanation

Prime numbers have only two factors, 1 and the number itself. Composite numbers have additional factors.

Worked Illustration

For example, 7 is prime because its factors are 1 and 7 only. 8 is composite because its factors are 1, 2, 4, and 8.

Solved Example

Example: Identify if 15 is prime or composite.

Solution: Factors of 15 are 1, 3, 5, and 15. Since it has more than two factors, 15 is composite.

Practice Set

- Level 1: List prime numbers between 1 and 20.
- Level 2: Find factors of 21 and classify as prime or composite.
- Level 3: Determine if 49 is prime or composite.

Answer Key

- Prime numbers between 1 and 20: 2, 3, 5, 7, 11, 13, 17, 19

- Factors of 21: 1, 3, 7, 21; composite
- 49 factors: 1, 7, 49; composite

Quick Reference

Number	Factors	Type
2	1, 2	Prime
4	1, 2, 4	Composite
5	1, 5	Prime

Glossary

- **Prime Number:** Number with exactly two factors.
- **Composite Number:** Number with more than two factors.

Tests for Divisibility of Numbers

Divisibility tests help determine if a number is divisible by another without performing full division.

Concept Explanation

Divisibility rules are shortcuts to check if a number is divisible by 2, 3, 4, 5, 6, 8, 9, 10, or 11.

Rules and Formulae

- **Divisible by 2:** If the last digit is 0, 2, 4, 6, or 8.
- **Divisible by 3:** If the sum of digits is divisible by 3.
- **Divisible by 4:** If the last two digits form a number divisible by 4.

- **Divisible by 5:** If the last digit is 0 or 5.
- **Divisible by 6:** If divisible by both 2 and 3.
- **Divisible by 8:** If the last three digits form a number divisible by 8.
- **Divisible by 9:** If the sum of digits is divisible by 9.
- **Divisible by 10:** If the last digit is 0.
- **Divisible by 11:** If the difference between the sum of digits at odd and even places is 0 or divisible by 11.

Worked Illustration

Check if 219 is divisible by 3:

Sum of digits = 2 + 1 + 9 = 12, which is divisible by 3, so 219 is divisible by 3.

Solved Example

Example: Is 1331 divisible by 11?

Solution: Sum of digits at odd places (from right): 1 + 3 = 4

Sum of digits at even places: 3 + 1 = 4

Difference = 4 - 4 = 0, which is divisible by 11, so 1331 is divisible by 11.

Practice Set

- Level 1: Check divisibility of 246 by 2, 3, and 5.
- Level 2: Determine if 4620 is divisible by 4 and 8.
- Level 3: Verify if 758 is divisible by 9 and 11.

Answer Key

- 246 is divisible by 2 and 3 but not by 5.
- 4620 is divisible by 4 and 8.
- 758 is not divisible by 9 or 11.

Quick Reference

Divisor	Divisibility Rule
2	Last digit 0,2,4,6,8
3	Sum of digits divisible by 3
4	Last two digits divisible by 4
5	Last digit 0 or 5
6	Divisible by 2 and 3
8	Last three digits divisible by 8
9	Sum of digits divisible by 9
10	Last digit 0
11	Difference of sums of digits at odd and even places divisible by 11

Glossary

- **Divisibility:** A number a is divisible by b if $a \div b$ leaves no remainder.
- **Remainder:** The amount left over after division.

Prime Factorisation

Prime factorisation is expressing a number as a product of its prime factors.

Concept Explanation

Every composite number can be uniquely expressed as a product of prime numbers raised to powers.

Formula Derivation

For a number n, prime factorisation is:

$$n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_k^{a_k}$$

where p_i are prime factors and a_i are their exponents.

Worked Illustration

Prime factorisation of 24:

- 24 = 2 × 12
- 12 = 2 × 6
- 6 = 2 × 3
- Thus, 24 = 2 × 2 × 2 × 3 = $2^3 \times 3$

Solved Example

Example: Find prime factorisation of 90.

Solution:

- 90 = 10 $\times$ 9
- 10 = 2 $\times$ 5
- 9 = 3 $\times$ 3
- Prime factors: 2, 3, 3, 5
- Prime factorisation: $2 \times 3^2 \times 5$

Practice Set

- Level 1: Prime factorise 16, 28, and 38.
- Level 2: Prime factorise 56 and 84.
- Level 3: Prime factorise 980.

Answer Key

- 16 = 2^4
- 28 = $2^2 \times 7$
- 38 = 2×19
- 56 = $2^3 \times 7$
- 84 = $2^2 \times 3 \times 7$
- 980 = $2^2 \times 5 \times 7^2$

Quick Reference

Number	Prime Factorisation
24	$2^3 \times 3$
90	$2 \times 3^2 \times 5$
980	$2^2 \times 5 \times 7^2$

Glossary

- **Prime Factorisation:** Expressing a number as a product of prime numbers.
- **Factor Tree:** A diagram used to break down a number into its prime factors.

Highest Common Factor

The Highest Common Factor (HCF) of two or more numbers is the greatest number that divides all of them exactly.

Concept Explanation

HCF is the largest common factor shared by the numbers.

Formula Derivation

Given numbers a and b with prime factorisations:

$$a = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}, \quad b = p_1^{b_1} p_2^{b_2} \cdots p_k^{b_k}$$

The HCF is:

$$\text{HCF}(a, b) = p_1^{\min(a_1, b_1)} p_2^{\min(a_2, b_2)} \cdots p_k^{\min(a_k, b_k)}$$

Worked Illustration

Find HCF of 20 and 28:

- 20 = $2^2 \times 5$
- 28 = $2^2 \times 7$
- Common prime factors: 2^2
- HCF = 4

Solved Example

Example: Find HCF of 850 and 680.

Solution:

- 850 = $2 \times 5^2 \times 17$
- 680 = $2^3 \times 5 \times 17$
- Common prime factors: 2, 5, 17
- HCF = $2 \times 5 \times 17 = 170$

Practice Set

- Level 1: Find HCF of 24 and 36.
- Level 2: Find HCF of 15, 25, and 30.
- Level 3: Find HCF of 20, 28, and 36.

Answer Key

- HCF(24, 36) = 12
- HCF(15, 25, 30) = 5
- HCF(20, 28, 36) = 4

Quick Reference

Numbers	HCF
20, 28	4
12, 18	6
24, 36	12

Glossary

- **HCF (Highest Common Factor):** Largest factor common to all numbers.
- **GCD (Greatest Common Divisor):** Another name for HCF.

Lowest Common Multiple

The Lowest Common Multiple (LCM) of two or more numbers is the smallest number that is a multiple of all of them.

Concept Explanation

LCM is the smallest number divisible by all given numbers.

Formula Derivation

Given numbers a and b with prime factorisations:

$$a = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}, \quad b = p_1^{b_1} p_2^{b_2} \cdots p_k^{b_k}$$

The LCM is:

$$\text{LCM}(a, b) = p_1^{\max(a_1, b_1)} p_2^{\max(a_2, b_2)} \cdots p_k^{\max(a_k, b_k)}$$

Worked Illustration

Find LCM of 12 and 18:

- 12 = $2^2 \times 3$
- 18 = 2×3^2
- LCM = $2^2 \times 3^2 = 36$

Solved Example

Example: Find LCM of 24 and 90.

Solution:

- 24 = $2^3 \times 3$
- 90 = $2 \times 3^2 \times 5$
- LCM = $2^3 \times 3^2 \times 5 = 360$

Practice Set

- Level 1: Find LCM of 8 and 12.
- Level 2: Find LCM of 40, 48, and 45.
- Level 3: Find LCM of 20, 25, and 30.

Answer Key

- LCM(8, 12) = 24
- LCM(40, 48, 45) = 720
- LCM(20, 25, 30) = 300

Quick Reference

Numbers	LCM

12, 18	36
24, 90	360
20, 25, 30	300

Glossary

- **LCM (Lowest Common Multiple):** Smallest multiple common to all numbers.

Problems on HCF and LCM

Concept Explanation

HCF and LCM concepts are applied in real-life problems involving measurements, scheduling, and grouping.

Solved Examples

Example 1: Two tankers contain 850 litres and 680 litres of kerosene. Find the maximum capacity of a container that can measure both exactly.

Solution: The container capacity must be the HCF of 850 and 680.

- 850 = $2 \times 5^2 \times 17$
- 680 = $2^3 \times 5 \times 17$
- HCF = $2 \times 5 \times 17 = 170$ litres

The container capacity is 170 litres.

Example 2: Three persons step off together with steps measuring 80 cm, 85 cm, and 90 cm. Find the minimum distance each should walk to cover the same distance in complete steps.

Solution: Find LCM of 80, 85, and 90.

LCM = 12240 cm

Example 3: Find the least number which when divided by 12, 16, 24, and 36 leaves a remainder 7.

Solution: Find LCM of 12, 16, 24, and 36.

LCM = 144

Required number = 144 + 7 = 151

Practice Set

- Level 1: Find HCF of 36 and 48.
- Level 2: Find LCM of 15, 20, and 25.
- Level 3: A problem involving scheduling events with different intervals.

Answer Key

- HCF(36, 48) = 12
- LCM(15, 20, 25) = 300

Quick Reference

- Use prime factorisation to find HCF and LCM.
- HCF is product of common prime factors with minimum exponents.
- LCM is product of all prime factors with maximum exponents.

Glossary

- **Scheduling Problem:** Application of LCM to find common intervals.
- **Measurement Problem:** Application of HCF to find maximum size of measuring unit.