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Introduction to Geometry

Geometry is the branch of mathematics concerned with the properties and relations of points, lines, surfaces, and solids. The term 'Geometry' originates from the Greek words 'Geo' meaning Earth and 'Metron' meaning Measurement. Historically, geometry developed from practical needs such as land measurement, art, and architecture.

Geometrical concepts are fundamental in various fields including engineering, design, and everyday observations of shapes and forms.

Points

A point represents an exact location in space and has no size, length, or breadth. It is denoted by a capital letter such as A, B, or C.

Examples of points include the tip of a compass, the sharpened end of a pencil, or the pointed end of a needle. These are precise locations used to mark positions.

Try These:

- Mark four points on a paper and name them A, C, P, H.
- Identify five examples of points in your daily life, such as stars in the sky.

Line Segments

A line segment is the shortest path connecting two points. It has two endpoints and a definite length.

For example, folding a paper creates a crease which represents a line segment with endpoints A and B.

Notation: The line segment joining points A and B is denoted as $\overline{AB}$ or $\overline{BA}$.

Examples of line segments include edges of a box, a tube light, or the border of a postcard.

Worked Example

Given points A and B on a plane, the length of the line segment $\overline{AB}$ can be calculated using the distance formula if coordinates are known:

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

Try These:

- Name the line segments in a given figure and identify their endpoints.

Lines

A line extends infinitely in both directions and is determined by any two distinct points.

Notation: A line passing through points A and B is denoted as $\overleftrightarrow{AB}$ or by a lowercase letter such as l.

Lines contain infinitely many points and have no endpoints.

Properties

- Two points determine a unique line.
- A line extends without end in both directions.

Intersecting Lines

Two lines that meet or cross at exactly one point are called intersecting lines. The point where they meet is called the point of intersection.

Notation: If lines l_1 and l_2 intersect at point P, we say l_1 and l_2 intersect at P.

Properties

- Intersecting lines form four angles at the point of intersection.
- Vertically opposite angles formed are equal.

Parallel Lines

Two lines in a plane that do not meet, no matter how far they are extended, are called parallel lines.

Notation: If lines l_1 and l_2 are parallel, we write $l_1 \parallel l_2$.

Examples include opposite edges of a ruler, cross-bars of a window, and railway tracks.

Rays

A ray is a part of a line that starts at a point and extends infinitely in one direction.

Notation: A ray starting at point A and passing through point P is denoted as $\overrightarrow{AP}$.

Examples include beams of light from a lighthouse, rays from a torch, and sun rays.

Properties

- Has a fixed starting point called the initial point.
- Extends infinitely in one direction.

Curves

A curve is a continuous and smooth flowing line without sharp angles. It can be open or closed.

Types of curves include:

- Simple closed curves (do not cross themselves)
- Closed curves with cusps or self-intersections
- Open curves (do not form a loop)

Examples include doodles, outlines of lakes, and figure-eight shapes.

Polygons

A polygon is a simple closed figure made up entirely of line segments called sides.

Properties:

- Vertices: Points where two sides meet.
- Adjacent sides: Two sides sharing a common vertex.
- Diagonals: Line segments joining non-adjacent vertices.

Examples include triangles, quadrilaterals, pentagons, and concave polygons.

Formula for Sum of Interior Angles

$$\text{Sum of interior angles} = (n-2) \times 180^\circ$$

where

n

is the number of sides.

Angles

An angle is formed by two rays with a common initial point called the vertex.

Notation: The angle formed by rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ with vertex O is denoted as $\angle AOB$.

Parts of an Angle

- Vertex: The common initial point of the two rays.
- Sides: The two rays forming the angle.
- Interior: The region between the two rays.
- Exterior: The region outside the angle.

Properties

- Angles can be named using three points, with the vertex as the middle letter.
- Angles divide the plane into interior, boundary, and exterior regions.

Worked Example

Given rays $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, the measure of angle $\angle POQ$ is the amount of rotation from one ray to the other, measured in degrees or radians.

Summary of Basic Geometrical Ideas

- A point determines a location and is denoted by a capital letter.
- A line segment is the shortest distance between two points, denoted by $\overline{AB}$.
- A line extends infinitely in both directions and is determined by two points.

- Intersecting lines meet at exactly one point.
- Parallel lines never meet.
- A ray starts at a point and extends infinitely in one direction.
- A curve is any continuous drawing without lifting the pencil; it can be open or closed.
- A simple curve does not cross itself.
- A polygon is a simple closed curve made of line segments.
- Angles are formed by two rays with a common vertex.
- Angles have interior, boundary, and exterior regions.