Introduction to Plane Figures

In geometry, plane figures are flat shapes that lie on a plane. These figures have boundaries and enclosed regions. To analyze and compare these figures, we use concepts such as interior and exterior points, angles, and diagonals.

Concave Polygon

A concave polygon has at least one interior angle greater than 180 degrees. This means there exists a "dent" or inward curve in the polygon. For example, a pentagon with a concave indentation is a concave polygon. A key property is that a line segment joining two points inside the polygon may pass outside the polygon.

Interior and Exterior Points

Given a closed shape, points can be classified as:

- **Interior point:** A point lying inside the boundary of the shape.
- **Exterior point:** A point lying outside the boundary of the shape.

For example, in a closed irregular shape, point A inside is an interior point, while points B and C outside are exterior points.

Angles and Rays

An angle is formed by two rays sharing a common endpoint called the vertex. For example, rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ with common vertex A form an angle.

Diagonals of a Polygon

A diagonal of a polygon is a line segment connecting two non-adjacent vertices. For a polygon with n sides, the number of diagonals is given by the formula:

$$\text{Number of diagonals} = \frac{n(n-3)}{2}$$

For example, a pentagon $(n = 5)$ has:

$$\frac{5 \times (5-3)}{2} = \frac{5 \times 2}{2} = 5$$

diagonals.

Summary

- Concave polygons have at least one interior angle greater than 180 degrees.
- Interior points lie inside a closed figure; exterior points lie outside.
- Angles are formed by two rays with a common vertex.
- Number of diagonals in a polygon with n sides is $\frac{n(n-3)}{2}$.

Perimeter

The perimeter of a closed figure is the total distance around its boundary. It is the length of wire or string needed to form the figure.

Concept Explanation

To find the perimeter, sum the lengths of all sides of the figure. For example, if a figure has sides AB, BC, CD, DA, then:

$$\text{Perimeter} = AB + BC + CD + DA$$

Perimeter is widely used in real life, such as fencing fields, building compound walls, or preparing sports tracks.

Perimeter of a Rectangle

Consider a rectangle with length l and breadth b. The perimeter P is:

$$P = 2(l + b)$$

This is because opposite sides are equal, so:

$$P = l + b + l + b = 2l + 2b = 2(l + b)$$

Worked Examples

Example 1: Find the perimeter of a rectangle with length 15 cm and breadth 9 cm.

Solution:

$$P = 2(15 + 9) = 2(24) = 48 \text{ cm}$$

Example 2: Find the perimeter of a square with side 5 cm.

Solution:

$$P = 4 \times 5 = 20 \text{ cm}$$

Practice Set

- *Level 1 – Easy*
- Find the perimeter of a rectangle with length 10 cm and breadth 6 cm.
- Find the perimeter of a square with side 7 cm.
- *Level 2 – Moderate*
- Calculate the perimeter of a regular pentagon with side length 8 cm.
- A farmer wants to fence a rectangular field 240 m long and 180 m wide. Find the length of the fence needed.
- *Level 3 – Challenging*
- An athlete runs 10 rounds around a rectangular park 50 m long and 25 m wide. Find the total distance covered.
- Find the perimeter of an irregular hexagon with sides 100 m, 120 m, 90 m, 45 m, 60 m, and 80 m.

Answer Key

- 1. $2(10 + 6) = 32 \text{ cm}$
- 2. $4 \times 7 = 28 \text{ cm}$

- 3. $5 \times 8 = 40$ cm
- 4. $2(240 + 180) = 840$ m
- 5. One round perimeter: $2(50 + 25) = 150$ m, total distance: $10 \times 150 = 1500$ m
- 6. Sum of sides: $100 + 120 + 90 + 45 + 60 + 80 = 495$ m

Quick Reference

Shape	Perimeter Formula
Rectangle	$2(l + b)$
Square	$4 \times \text{side}$
Equilateral Triangle	$3 \times \text{side}$
Regular Polygon (n sides)	$n \times \text{side}$

Glossary

- **Perimeter:** Total length around a closed figure.
- **Regular Polygon:** Polygon with all sides and angles equal.
- **Concave Polygon:** Polygon with at least one interior angle greater than 180°.
- **Diagonal:** Line segment connecting two non-adjacent vertices of a polygon.

Area

Area is the amount of surface enclosed by a closed figure. It is measured in square units such as square centimeters (cm^2) or square meters (m^2).

Concept Explanation

To estimate the area of irregular shapes, we can place them on graph paper with 1 cm × 1 cm squares and count the number of squares covered. The following conventions are used:

- Ignore portions less than half a square.
- Count portions more than half a square as one full square.
- Count exactly half a square as $\frac{1}{2}$ square unit.

Area of a Rectangle

For a rectangle with length l and breadth b, the area A is:

$$A = l \times b$$

This formula can be verified by counting squares on graph paper.

Area of a Square

For a square with side length s, the area A is:

$$A = s \times s = s^2$$

Worked Examples

Example 1: Find the area of a rectangle with length 5 cm and breadth 3 cm.

Solution:

$$A = 5 \times 3 = 15 \text{ cm}^2$$

Example 2: Find the area of a square with side 4 cm.

Solution:

$$A = 4^2 = 16 \text{ cm}^2$$

Example 3: A rectangular piece of cardboard has area 36 cm^2 and length 9 cm. Find its width.

Solution:

$$\text{Width} = \frac{\text{Area}}{\text{Length}} = \frac{36}{9} = 4 \text{ cm}$$

Practice Set

- *Level 1 – Easy*
- Calculate the area of a rectangle with length 6 cm and breadth 4 cm.
- Calculate the area of a square with side 7 cm.
- *Level 2 – Moderate*
- Estimate the area of an irregular shape by counting squares on graph paper.
- Find the area of a rectangular floor 4 m long and 3 m wide.
- *Level 3 – Challenging*
- Calculate the number of square tiles of side 0.5 m needed to cover a room 4 m by 3 m.
- Find the area of a piece of cloth 2 m long and 1.25 m wide.

Answer Key

- 1. $6 \times 4 = 24 \text{ cm}^2$
- 2. $7^2 = 49 \text{ cm}^2$
- 3. Area estimation depends on counting squares.
- 4. $4 \times 3 = 12 \text{ m}^2$
- 5. Number of tiles = $\frac{12}{0.25} = 48$

- 6. $2 \times 1.25 = 2.5\ \mathrm{m}^2$

Quick Reference

Shape	Area Formula
Rectangle	$l \times b$
Square	s^2

Glossary

- **Area:** The measure of the surface enclosed by a closed figure.
- **Square Unit:** Unit of area measurement, e.g., cm^2, m^2.
- **Graph Paper:** Paper printed with a grid of squares used for plotting and measuring.