Introduction to Algebra

Algebra is a branch of mathematics that uses letters to represent numbers and express general rules and formulas. Unlike arithmetic, which deals with specific numbers, algebra allows us to work with unknown quantities and variables. This enables us to solve problems systematically by forming algebraic expressions and equations.

Concept Explanation

In algebra, letters such as x, y, n, m are used to denote variables, which can take on different numerical values. Operations like addition, subtraction, multiplication, and division can be performed on these variables just as on numbers.

Formula Derivation

Algebraic expressions are formed by combining variables and constants using arithmetic operations. For example, $2n$ represents two times the variable n, and $x + 10$ represents the variable x increased by 10.

Worked Illustrations

- Expression $2n$ means multiply n by 2.
- Expression $x + 10$ means add 10 to x.

Solved Examples

Example 1: If $n = 5$, find $2n$.

Solution:

Substitute $n = 5$ into $2n$:

$$2n = 2 \times 5 = 10$$

Example 2: If $x = 7$, find $x + 10$.

Solution:

Substitute $x = 7$ into $x + 10$:

$$x + 10 = 7 + 10 = 17$$

Practice Set

- Level 1 – Easy
  - Evaluate $3m$ when $m = 4$.
  - Find $y + 5$ if $y = 8$.
- Level 2 – Moderate

    - Express the total cost if one item costs ₹7 and n items are bought.
    - If x is a number, write an expression for 5 less than twice x.
- Level 3 – Challenging
    - Form an expression for the number of matchsticks needed to form n letter Ls, each requiring 2 matchsticks.
    - Given x is Raju's age, write an expression for Balu's age if he is 3 years younger.

Answer Key

- Level 1
    - $3m = 3 \times 4 = 12$
    - $y + 5 = 8 + 5 = 13$
- Level 2
    - Total cost = $7n$
    - Expression: $2x - 5$
- Level 3
    - Number of matchsticks = $2n$
    - Balu's age = $x - 3$

Quick Reference

Term	Meaning
Variable	A symbol representing a number that can change.
Expression	A combination of variables and numbers using operations.
Coefficient	The numerical factor of a variable in an expression.

Glossary

- **Algebra:** A branch of mathematics dealing with symbols and rules for manipulating those symbols.
- **Variable:** A letter representing a number that can vary.
- **Expression:** A mathematical phrase combining numbers, variables, and operations.
- **Coefficient:** The number multiplied by a variable.

Matchstick Patterns

Matchstick patterns are visual representations used to understand algebraic relationships by counting the number of matchsticks required to form repeated shapes.

Concept Explanation

By observing how matchsticks are arranged to form letters or shapes repeatedly, we can find a general formula relating the number of shapes to the total matchsticks used.

Formula Derivation

Consider the letter L formed by 2 matchsticks. For n Ls placed side by side, the total matchsticks required is:

$$M = 2n$$

where M is the number of matchsticks and n is the number of Ls.

Worked Illustrations

For 1 L, $M = 2 \times 1 = 2$ matchsticks.

For 3 Ls, $M = 2 \times 3 = 6$ matchsticks.

Solved Examples

Example: How many matchsticks are needed to make 7 Ls?

Solution:

Using the formula $M = 2n$, substitute $n = 7$:

$$M = 2 \times 7 = 14$$

So, 14 matchsticks are required.

Practice Set

- Level 1 – Easy
  - Find matchsticks needed for 5 Ls.
  - Find matchsticks needed for 10 Ls.
- Level 2 – Moderate
  - Find matchsticks needed for 15 Ls.
  - Write the formula for matchsticks needed for n Cs if one C requires 3 matchsticks.
- Level 3 – Challenging
  - Derive the formula for matchsticks needed to form n Fs where the first F requires 6 matchsticks and each additional F adds 5 matchsticks.
  - Calculate matchsticks needed for 20 Fs.

Answer Key

- Level 1
  - $2 \times 5 = 10$
  - $2 \times 10 = 20$
- Level 2
  - $2 \times 15 = 30$

    - $M = 3n$
- Level 3
    - Formula: $M = 5n + 1$
    - For $n = 20$, $M = 5 \times 20 + 1 = 101$

Quick Reference

Shape	Matchsticks per unit	Formula for n units
L	2	$2n$
C	3	$3n$
F	6 for first, 5 for each additional	$5n + 1$

Glossary

- **Pattern:** A repeated decorative design or sequence.
- **Variable:** A symbol representing a number that can change.
- **Formula:** A mathematical rule expressed using symbols.

Variables in Algebra

Variables are symbols used to represent numbers whose values can change. They are fundamental in expressing general mathematical relationships.

Concept Explanation

A variable can take any value from a set of numbers. It allows us to write formulas and expressions that apply to many cases instead of just one.

Formula Derivation

For example, if the cost of one notebook is ₹5, and m notebooks are bought, the total cost C is:

$$C = 5m$$

Here, m is a variable representing the number of notebooks.

Worked Illustrations

If $m = 4$, then $C = 5 \times 4 = 20$ rupees.

Solved Examples

Example: Sarita has 10 more marbles than Ameena. If Ameena has x marbles, express Sarita's marbles.

Solution:

Sarita's marbles = $x + 10$

If $x = 20$, then Sarita has $20 + 10 = 30$ marbles.

Practice Set

- Level 1 – Easy
  - Write an expression for the total cost if one pen costs ₹3 and p pens are bought.
  - Express the number of apples if a baskets have 5 apples each.
- Level 2 – Moderate

    - Write an expression for the age of Balu if Raju's age is x and Balu is 3 years younger.
    - Express the total number of students if there are r rows with 10 students each.
- Level 3 – Challenging
    - Form an expression for the total cost if n notebooks cost ₹5 each and there is a fixed charge of ₹20.
    - Express the number of matchsticks needed to form n squares joined side by side, each square requiring 4 matchsticks but sharing sides.

Answer Key

- Level 1
    - Total cost = $3p$
    - Number of apples = $5a$
- Level 2
    - Balu's age = $x - 3$
    - Total students = $10r$
- Level 3
    - Total cost = $5n + 20$
    - Matchsticks = $3n + 1$ (since each new square shares a side)

Quick Reference

Expression	Meaning
$x + 10$	Variable x plus 10
$5m$	5 times variable m
$x - 3$	Variable x minus 3

Glossary

- **Variable:** A symbol representing a number that can change.
- **Expression:** A combination of variables and numbers using operations.
- **Coefficient:** The number multiplied by a variable.
- **Constant:** A fixed number in an expression.