

- Introduction to Ratio and Proportion
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Introduction to Ratio and Proportion

In daily life, quantities of the same type are often compared by difference or division. For example, if Avnee collected 30 flowers and Shari collected 45 flowers, the difference is 15 flowers. However, for quantities like lengths of an ant and a grasshopper, difference is not meaningful; instead, comparison by division (ratio) is more appropriate.

Example: Cost of a car is ₹2,50,000 and a motorbike is ₹50,000. The difference is ₹2,00,000, but the ratio is $\frac{2,50,000}{50,000} = 5 : 1$, meaning the car costs five times the motorbike.

Concept Explanation

Ratio is a comparison of two quantities of the same kind by division, expressing how many times one quantity is of the other.

Formula Derivation

Given two quantities a and b (both in the same unit), the ratio of a to b is:

$$\text{Ratio} = \frac{a}{b} = a : b$$

Worked Illustrations

- Avnee's flowers to Shari's flowers: $30 : 45 = \frac{30}{45} = \frac{2}{3} = 2 : 3$
- Height of Rahim to Avnee: $150 \text{ cm} : 140 \text{ cm} = 150/140 = 15/14 = 15 : 14$

Solved Examples

Example: In a class, there are 20 boys and 40 girls. Find the ratio of girls to total students.

Solution: Total students = $20 + 40 = 60$

$$\text{Ratio of girls to total students} = \frac{40}{60} = \frac{2}{3} = 2 : 3$$

Practice Set

- Level 1 – Easy
 - Find the ratio of boys to girls in a class with 15 boys and 25 girls.
 - Find the ratio of cost of a pen (₹10) to a pencil (₹2).
- Level 2 – Moderate
 - Length of a lizard is 20 cm and a crocodile is 4 m. Find the ratio of their lengths.
 - Convert and find the ratio of diameter (8 mm) to length (18 cm) of a pencil.
- Level 3 – Challenging
 - In a school, 73 holidays occur in a year. Find the ratio of holidays to total days (365).
 - Ravi walks 6 km/hr and Roshan 4 km/hr. Find the ratio of distances covered in 3 hours.

Answer Key

- Level 1: $15:25 = 3:5$; $10:2 = 5:1$
- Level 2: Crocodile to lizard = $400 \text{ cm} : 20 \text{ cm} = 20:1$; Diameter to length = $8 \text{ mm} : 180 \text{ mm} = 2:45$
- Level 3: Holidays to days = $73:365 = 1:5$; Distances = $18 \text{ km} : 12 \text{ km} = 3:2$

Quick Reference

Quantity 1	Quantity 2	Ratio
Father's weight	Isha's weight	$75:25 = 3:1$
Cost of pen	Cost of pencil	$10:2 = 5:1$
Length of crocodile	Length of lizard	$400 \text{ cm} : 20 \text{ cm} = 20:1$

Glossary

- **Ratio:** A comparison of two quantities of the same kind by division.
- **Equivalent Ratios:** Ratios that represent the same relationship when simplified.

- **Unit:** Standard measure used to express quantities.

Ratio

Ratio compares two quantities of the same kind by division, expressing how many times one quantity is of the other. It is denoted by the symbol “:”.

Concept Explanation

To find the ratio of two quantities, ensure both are in the same unit, then divide the first quantity by the second.

Formula Derivation

For quantities a and b , ratio is:

$$\text{Ratio} = a : b = \frac{a}{b}$$

Worked Illustrations

- Weight of father to Isha: $75 : 25 = 3 : 1$
- Cost of pen to pencil: $10 : 2 = 5 : 1$

Solved Examples

Example: Find the ratio of number of girls to total students in a class with 20 boys and 40 girls.

Solution: Total students = $20 + 40 = 60$

$$\text{Ratio} = 40 : 60 = \frac{40}{60} = \frac{2}{3} = 2 : 3$$

Practice Set

- Level 1 – Easy
 - Find the ratio of boys to girls in a class with 20 boys and 40 girls.
 - Find the ratio of cost of a toffee (50 paise) to a chocolate (₹10).
- Level 2 – Moderate
 - Find the ratio of time taken by Saurabh (15 minutes) to Sachin (1 hour) to reach school.
 - Find the ratio of number of holidays (73) to total days (365) in a year.
- Level 3 – Challenging
 - Convert and find the ratio of length of pencil (18 cm) to diameter (8 mm).
 - Find the ratio of distance covered by Ravi (6 km/hr) to Roshan (4 km/hr) in 1 hour.

Answer Key

- Level 1: 20:40 = 1:2; 0.5:10 = 1:20
- Level 2: 15:60 = 1:4; 73:365 = 1:5
- Level 3: 180 mm : 8 mm = 45:2; 6:4 = 3:2

Quick Reference

Quantity 1	Quantity 2	Ratio
Number of girls	Total students	2:3
Cost of pen	Cost of pencil	5:1
Length of crocodile	Length of lizard	20:1

Glossary

- **Ratio:** Comparison of two quantities by division.
- **Equivalent Ratios:** Ratios that simplify to the same fraction.
- **Unit Conversion:** Changing quantities to the same unit before comparison.

Proportion

Proportion states that two ratios are equal. If $a : b = c : d$, then a, b, c, d are in proportion.

Concept Explanation

Two ratios $a : b$ and $c : d$ are in proportion if:

$$\frac{a}{b} = \frac{c}{d}$$

This is written as $a : b :: c : d$ or $a : b = c : d$.

Formula Derivation

Cross-multiplying gives:

$$a \times d = b \times c$$

Worked Illustrations

- Check if 3 : 10 and 15 : 50 are in proportion:

$$\frac{3}{10} = 0.3, \quad \frac{15}{50} = 0.3 \Rightarrow \text{Proportion holds}$$

Solved Examples

Example: Are 30, 40, 45, and 60 in proportion?

Solution:

$$\frac{30}{40} = \frac{3}{4}, \quad \frac{45}{60} = \frac{3}{4} \Rightarrow \text{Yes, in proportion}$$

Practice Set

- Level 1 – Easy
 - Check if 1 : 5 and 3 : 15 are in proportion.
 - Check if 2 : 9 and 18 : 81 are in proportion.
- Level 2 – Moderate
 - Check if 15 : 45 and 5 : 25 are in proportion.
 - Check if 4 : 12 and 9 : 27 are in proportion.
- Level 3 – Challenging
 - Are the ratios 25g : 30g and 40kg : 48kg in proportion?
 - Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?

Answer Key

- Level 1: Yes; Yes
- Level 2: Yes; Yes
- Level 3: Yes; No

Quick Reference

Ratio 1	Ratio 2	Proportion?
3:10	15:50	Yes
30:40	45:60	Yes
15 cm : 2 m	10 sec : 3 min	No

Glossary

- **Proportion:** Equality of two ratios.
- **Extreme Terms:** First and fourth terms in proportion.
- **Middle Terms:** Second and third terms in proportion.

Unitary Method

The unitary method finds the value of one unit first, then uses it to find the value of required units.

Concept Explanation

Given the total value for multiple units, divide by the number of units to find the value of one unit. Then multiply by the required number of units.

Formula Derivation

If n units cost C , then cost of 1 unit is $\frac{C}{n}$. Cost of m units is:

$$\frac{C}{n} \times m$$

Worked Illustrations

- Cost of 2 notebooks is ₹24. Cost of 1 notebook = $\frac{24}{2} = 12$. Cost of 5 notebooks = $12 \times 5 = 60$.
- A scooter uses 2 litres petrol for 80 km. Petrol for 1 km = $\frac{2}{80} = \frac{1}{40}$ litres. Petrol for 120 km = $\frac{1}{40} \times 120 = 3$ litres.

Solved Examples

Example: Cost of 6 cans of juice is ₹210. Find cost of 4 cans.

Solution:

$$\text{Cost of 1 can} = \frac{210}{6} = 35$$

$$\text{Cost of 4 cans} = 35 \times 4 = 140$$

Practice Set

- Level 1 – Easy
 - Find cost of 3 notebooks if 5 cost ₹75.
 - Find petrol needed for 50 km if 4 litres are needed for 160 km.
- Level 2 – Moderate
 - Find cost of 15 soaps if 12 cost ₹153.60.
 - Find number of envelopes purchasable for ₹100 if 105 cost ₹350.
- Level 3 – Challenging
 - A car travels 90 km in 2.5 hours. Find time to cover 30 km and distance in 2 hours.
 - Complete the distance table for Karan and Kriti given partial data.

Answer Key

- Level 1: ₹45; 1.25 litres
- Level 2: ₹192; 30 envelopes
- Level 3: (a) 50 minutes; (b) 72 km; Table completed as per speed calculations

Quick Reference

Given	Find	Method
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Cost of 6 cans = ₹210	Cost of 4 cans	Cost of 1 can = $210/6 = 35$; Cost of 4 cans = $35 \times 4 = 140$
Distance in 5 litres = 220 km	Distance in 1.5 litres	Distance per litre = $220/5 = 44$; Distance in 1.5 litres = $44 \times 1.5 = 66$ km

Glossary

- **Unitary Method:** Finding value of one unit first, then required units.
- **Unit:** Single quantity used as a basis for calculation.

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