- Fundamental Facts
- Introduction to Prime and Composite Numbers
- Factors and Multiples
- Prime Factorisation

Fundamental Facts

Understanding the basic properties of factors and multiples is essential in number theory.

Concept Explanation

- A factor of a number is an exact divisor of that number.
- 1 is a factor of every number, and every number is a factor of itself.
- Every factor is less than or equal to the given number.
- The number of factors of a given number is finite.
- Every multiple of a number is greater than or equal to that number.
- The number of multiples of a given number is infinite.
- Every number is a multiple of itself.

Formula Derivation

Not applicable for fundamental facts.

Worked Illustrations

Consider the number 12:

- Factors: 1, 2, 3, 4, 6, 12
- Multiples: 12, 24, 36, 48, 60, ...

Solved Examples

Example: Find factors and multiples of 10.

Solution:

- Factors of 10: 1, 2, 5, 10
- Multiples of 10: 10, 20, 30, 40, 50, ...

Practice Set

- **Level 1 – Easy:** List factors of 15.
- **Level 2 – Moderate:** Find first five multiples of 7.
- **Level 3 – Challenging:** Prove that the number of factors of a perfect square is odd.

Answer Key

- Factors of 15: 1, 3, 5, 15
- Multiples of 7: 7, 14, 21, 28, 35
- Proof: For perfect squares, factors come in pairs except the square root, making total factors odd.

Quick Reference

- Factor: Divides number exactly.
- Multiple: Product of number and integer.

Glossary

- **Factor:** A number dividing another without remainder.
- **Multiple:** Product of a number and an integer.

Introduction to Prime and Composite Numbers

Understanding prime and composite numbers is fundamental in number theory.

Concept Explanation

- **Prime Number:** A natural number greater than 1 with exactly two factors: 1 and itself.
- **Composite Number:** A natural number greater than 1 with more than two factors.
- **Even Number:** Multiples of 2.
- **Odd Number:** Numbers not divisible by 2.
- **Twin Primes:** Two consecutive odd primes differing by 2.
- **Prime Triplet:** Three consecutive primes differing by 2; only (3,5,7) exists.
- **Co-primes:** Two numbers with only 1 as common factor.
- **Perfect Numbers:** Numbers whose factors sum to twice the number.

Formula Derivation

Not applicable; definitions and properties.

Worked Illustrations

Check if 7 is prime:

- Factors: 1 and 7 only.
- Therefore, 7 is prime.

Check if 15 is prime:

- Factors: 1, 3, 5, 15.
- More than two factors, so composite.

Solved Examples

Example: Are 7 and 18 co-prime?

Solution:

- Factors of 7: 1, 7
- Factors of 18: 1, 2, 3, 6, 9, 18
- Common factor: 1 only
- Hence, co-prime.

Practice Set

- **Level 1 – Easy:** List prime numbers less than 20.
- **Level 2 – Moderate:** Find if 21 is prime or composite.
- **Level 3 – Challenging:** Prove that any two consecutive integers are co-prime.

Answer Key

- Primes less than 20: 2, 3, 5, 7, 11, 13, 17, 19
- 21 is composite (factors: 1, 3, 7, 21)
- Proof: Consecutive integers differ by 1, so no common factors other than 1.

Quick Reference

- Prime: Exactly two factors.
- Composite: More than two factors.
- Co-prime: Only 1 as common factor.

Glossary

- **Prime Number:** Number with exactly two factors.

- **Composite Number:** Number with more than two factors.
- **Co-prime:** Numbers with no common factors except 1.

Factors and Multiples

Understanding factors and multiples is key to solving many mathematical problems.

Concept Explanation

- A factor divides a number exactly without remainder.
- A multiple is the product of a number and an integer.

Formula Derivation

Not applicable; procedural understanding.

Worked Illustrations

Factors of 12:

- Check divisibility: 12 ÷ 1 = 12 remainder 0
- 12 ÷ 2 = 6 remainder 0
- 12 ÷ 3 = 4 remainder 0
- 12 ÷ 4 = 3 remainder 0
- 12 ÷ 6 = 2 remainder 0
- 12 ÷ 12 = 1 remainder 0
- Factors: 1, 2, 3, 4, 6, 12

Multiples of 12:

- 12 × 1 = 12
- 12 × 2 = 24
- 12 × 3 = 36
- 12 × 4 = 48
- 12 × 5 = 60

Solved Examples

Example: Find factors and multiples of 15.

Solution:

- Factors: 1, 3, 5, 15
- Multiples: 15, 30, 45, 60, 75, ...

Practice Set

- **Level 1 – Easy:** List factors of 20.
- **Level 2 – Moderate:** Find first five multiples of 9.
- **Level 3 – Challenging:** Prove that if a number is divisible by 4, it is divisible by 2.

Answer Key

- Factors of 20: 1, 2, 4, 5, 10, 20
- Multiples of 9: 9, 18, 27, 36, 45
- Proof: 4 = 2 × 2, so divisibility by 4 implies divisibility by 2.

Quick Reference

- Factor: Divides number exactly.

- Multiple: Product of number and integer.

Glossary

- **Factor:** Number dividing another without remainder.
- **Multiple:** Product of a number and an integer.

Prime Factorisation

Prime factorisation expresses a number as a product of prime numbers.

Concept Explanation

Every composite number can be expressed uniquely as a product of prime factors.

Formula Derivation

Example: Prime factorisation of 12

⊠egin{align*} 12 &= 2 \times 6 \ &= 2 \times 2 \times 3 \end{align*}

Hence, prime factorisation of 12 is $2 \times 2 \times 3$.

Worked Illustrations

Factor tree for 45:

- 45 = 3 × 15

- 15 = 3 × 5
- Prime factors: 3, 3, 5

Prime factorisation of 80:

- 80 ÷ 2 = 40
- 40 ÷ 2 = 20
- 20 ÷ 2 = 10
- 10 ÷ 2 = 5
- 5 ÷ 5 = 1
- Prime factors: 2, 2, 2, 2, 5

Solved Examples

Example: Find prime factorisation of 840.

Solution:

- 840 ÷ 2 = 420
- 420 ÷ 2 = 210
- 210 ÷ 2 = 105
- 105 ÷ 3 = 35
- 35 ÷ 5 = 7
- 7 ÷ 7 = 1
- Prime factors: 2, 2, 2, 3, 5, 7

Therefore, prime factorisation of 840 is $2 \times 2 \times 2 \times 3 \times 5 \times 7$.

Practice Set

- **Level 1 – Easy:** Prime factorise 36.
- **Level 2 – Moderate:** Prime factorise 150.

- **Level 3 – Challenging:** Prove the uniqueness of prime factorisation.

Answer Key

- 36 = 2 × 2 × 3 × 3
- 150 = 2 × 3 × 5 × 5
- Proof: Fundamental Theorem of Arithmetic states uniqueness of prime factorisation.

Quick Reference

- Prime factorisation: Expressing number as product of primes.
- Use factor trees or successive division by primes.

Glossary

- **Prime Factorisation:** Writing a number as product of prime numbers.
- **Factor Tree:** Diagram to find prime factors.