CBSE EXAMINATION PAPER-2022

MATHEMATICS

(Solved)

Time allowed : 3 hours Maximum Marks : 43

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **21 questions.** All questions are **compulsory.**
ii. This question paper is divided into **3 sections.**
iii. **Section A** – questions number **1 to 8** are very short answer Each question carries **2 marks.**
iv. **Section B** – questions number **9 to 13** are short answer Each question carries **3 marks.**
v. **Section C** – questions number **14 to 18** are case based questions
vi. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
vii. Use of calculator is NOT allowed.

Section A

Question 1.

Find the sum of first 30 terms of AP: -30, -24, -18, ...

[2 Marks]

Answer: The given AP is -30, -24, -18, ... Here, the first term a = -30 and the common difference d = -24 - (-30) = 6. The formula for the sum of first n terms of an AP is $S_n = n/2 \times [2a + (n-1)d]$. For n = 30, $S_{30} = 30/2 \times [2 \times (-30) + (30-1) \times 6] = 15 \times [-60 + 174] = 15 \times 114 = 1710$. Therefore, the sum of the first 30 terms is 1710.

Question 2.

In an AP if S_n= n (4n + 1), then find the AP.

[2 Marks]

Answer: Given the sum of the first n terms of an AP, S_n = n (4n + 1). To find the AP, first find the first term a by putting n = 1: a = S_1 = 1 × (4 × 1 + 1) = 5. Now, find the second term T_2 = S_2 - S_1 = 2 × (8 + 1) - 5 = 18 - 5 = 13. The common difference d = T_2 - a = 13 - 5 = 8. Therefore, the AP is 5, 13, 21, 29, ... with first term 5 and common difference 8.

Question 3. A solid metallic sphere of radius 10.5 cm is melted and recast into a number of smaller cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed.

[2 Marks]

Answer: First, calculate the volume of the original solid sphere. The volume of a sphere is given by (4/3) × π × r^3. Here, r = 10.5 cm, so volume of sphere = (4/3) × π × $(10.5)^3$. Next, calculate the volume of one small cone using the formula (1/3) × π × r^2 × h. Here, r = 3.5 cm and h = 3 cm. Then, divide the volume of the sphere by the volume of one cone to find the number of cones formed.

Question 4. Find the value of m for which the quadratic equation $(m - 1)x^2 + 2(m - 1)x + 1 = 0$ has two real and equal roots.

[2 Marks]

Answer: To find the value of m for which the given quadratic equation has two real and equal roots, we use the condition for equal roots: the discriminant (b^2 - 4ac) must be zero. Here, a = (m - 1), b = 2(m - 1), and c = 1. Calculating the discriminant, we get $[2(m - 1)]^2 - 4 * (m - 1) * 1 = 0$. This simplifies to $4(m - 1)^2 - 4(m - 1) = 0$, or 4(m - 1)[(m - 1) - 1] = 0. Therefore, either (m - 1) = 0, giving m = 1, or (m - 1) - 1 = 0, giving m = 2. Hence, the quadratic has two equal roots when m = 1 or m = 2.

Question 5.

Solve the following quadratic equation for x : $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

[2 Marks]

Answer: Given the quadratic equation $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$, we solve it by factorisation. First, rewrite the equation as $(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$. This implies $\sqrt{3}x - \sqrt{2} = 0$. Solving for x gives $x = \sqrt{2} / \sqrt{3}$, which can be simplified to $x = \sqrt{6} / 3$. Hence, the root of the equation is $x = \sqrt{6} / 3$.

Question 6.

Find the mode of the following frequency distribution:

[2 Marks]

Answer: The mode of a frequency distribution is the value or class interval that appears most frequently in the data set. To find the mode, first identify the class interval with the highest frequency, called the modal class. For example, if the class 40-55 has the highest frequency, then the mode lies in this class. Mode indicates the value that occurs most frequently and helps us understand the most common data point in the set.

Question 7.

The product of Rehan's age (in years) 5 years ago and his age 7 years from now, is one more than twice his present age. Find his present age.

[2 Marks]

Answer: Let Rehan's present age be x years. Five years ago, his age was (x - 5). Seven years from now, his age will be (x + 7). According to the problem, the product of these two ages is one more than twice his present age. So, (x - 5) multiplied by (x + 7) equals 2x + 1. Expanding the product: x squared plus 7x minus 5x minus 35 equals 2x + 1, which simplifies to x squared plus 2x minus 35 equals 2x plus 1. Subtract 2x from both sides: x squared minus 35 equals 1. Then subtract 1 from both sides: x squared minus 36 equals 0. This factors into (x - 6)(x + 6) equals 0. So, x equals 6 or x equals -6. Age cannot be negative, so Rehan's present age is 6 years.

Question 8. Two concentric circles are of radii 4 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

[2 Marks]

Answer: Given two concentric circles with radii 4 cm (larger circle) and 3 cm (smaller circle), a chord of the larger circle that touches the smaller circle is a tangent to the smaller circle. The distance from the center to the chord equals the radius of the smaller circle (3 cm). Using the Pythagoras theorem in the right triangle formed by the radius of the larger circle (4 cm), the distance to the chord (3 cm), and half the chord length, we find half the chord length = $\sqrt{(4^2 - 3^2)} = \sqrt{(16 - 9)} = \sqrt{7}$ cm. Therefore, the length of the chord = $2 \times \sqrt{7}$ cm.

Section B

Question 9.

For what value of x is the median of the following frequency distribution 34.5?

[3 Marks]

Answer: To find the value of x for which the median of the given frequency distribution is 34.5, first calculate the total frequency by adding all the frequencies including x. Next, find

N/2 where N is the total frequency. Then, find the cumulative frequencies until the class interval containing the median position is identified. The median class is the one where the cumulative frequency just exceeds N/2. Using the formula for median in grouped data: Median = L + [(N/2 - F) / f] × h, where L is the lower boundary of median class, F is cumulative frequency before the median class, f is frequency of the median class, and h is class width. Substitute the median value 34.5 into the formula and solve for x. This will give the required value of x that makes the median 34.5.

Question 10. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Construct tangents to the circle from these two points P and Q.

[3 Marks]

Answer: To draw the circle and construct tangents from points P and Q, follow these steps: First, draw a circle with center O and radius 3 cm using a compass. Next, draw a diameter and extend it on both sides. Mark points P and Q on this extended diameter such that both are 7 cm away from the center O. Now, to construct tangents from P to the circle, draw a circle centered at P with radius equal to the distance from P to the point of contact on the circle (this can be found using the perpendicular bisector method or by constructing right triangles). Repeat a similar procedure for point Q. The lines drawn from P and Q to the points where the smaller circles touch the original circle are the tangent lines. These tangents touch the circle exactly at one point each, fulfilling the definition of tangent lines from an external point.

Question 11. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building.

[3 Marks]

Answer: Let the height of the building be h meters and the distance between the building and the tower be d meters. Given the height of the tower is 50 meters. From the foot of the tower, the angle of elevation to the top of the building is 30°, so using the tangent function, tan 30° = h / d. From the foot of the building, the angle of elevation to the top of the tower is 60°, so tan 60° = 50 / d. We know tan 30° is 1/√3 and tan 60° is √3. From the second equation, d = 50 / √3. Substitute this d value into the first equation: (1/√3) = h / (50 / √3). Cross-multiplying, h = (1/√3) * (50 / √3) = 50 / 3 ≈ 16.67 meters. Therefore, the height of the building is approximately 16.67 meters.

Question 12. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, then find the width of the river.

[3 Marks]

Answer: Let the width of the river be AB. Let P be the point on the bridge which is 3 meters above the river banks. The angles of depression from P to the banks A and B are 30° and

45° respectively. Draw perpendiculars from P to the banks at points A and B, and let the foot of the perpendiculars be D on the line AB. Since the angles of depression are given, angle PAD is 30° and angle PBD is 45°. Using right triangle properties, For the bank with 45° angle, Height PD = 3 m, Let distance BD = x, Tan 45° = PD / BD = 3 / x, So, x = 3 m. For the bank with 30° angle, Height PD = 3 m, Let distance AD = y, Tan 30° = PD / AD = 3 / y, So, y = 3 / tan 30° = 3 / (1/√3) = 3√3 m. Therefore, the width of the river AB = AD + BD = 3√3 + 3 = 3 (√3 + 1) meters.

Question 13.

Following is the daily expenditure on lunch by 30 employees of a company:

Find the mean daily expenditure of the employees.

[3 Marks]

Answer:

To find the mean daily expenditure of the employees, we first organize the data of daily expenditures along with the number of employees corresponding to each expenditure. Using the method of the assumed mean or direct method, we multiply each expenditure by the number of employees to get the total expenditure for that group. Then, we add all these totals to get the sum of all daily expenditures of 30 employees. Finally, we divide the total expenditure by 30, which is the total number of employees. This gives the mean daily expenditure per employee. The mean is a measure of central tendency and shows the average amount spent daily on lunch by each employee.

Section C

Question 14.

In Mathematics, relations can be expressed in various ways. The matchstick patterns are based on linear relations. Different strategies can be used to calculate the number of matchsticks used in different figures.

One such pattern is shown below. Observe the pattern and answer the following questions using Arithmetic Progression.

> **(1)**
>
> **Write the AP for the number of triangles used in the figures. Also, write the n^{th} term of this AP.**

[2 Marks]

Answer: The number of triangles in the figures form an arithmetic progression. The first figure has 1 triangle, the second has 2 triangles, the third has 3 triangles, and so on. So, the AP is 1, 2, 3, 4, ... etc. The n^{th} term of this AP is given by: n.

Key Points: The numbers of triangles increase by 1 with each figure-The sequence is an Arithmetic Progression (AP)-First term a = 1-Common difference d = 1-n^{th} term = a + (n-1) × d = 1 + (n-1) × 1 = n

(2) Which figure has 61 matchsticks?

[2 Marks]

Answer: Let the number of 'F' shapes in the figure be n. From the pattern, we see that the number of matchsticks increases by 5 when we add one more figure: For 1 figure, matchsticks = 6; for 2 figures, matchsticks = 11; for 3 figures, matchsticks = 16, and so on. The rule to find the number of matchsticks is: Number of matchsticks = 6 + (n - 1) × 5. To find the figure with 61 matchsticks, set 6 + (n - 1) × 5 = 61. This gives (n - 1) × 5 = 55, so n - 1 = 11, and n = 12. Therefore, the figure with 61 matchsticks is the 12th figure.

Key Points: Identify number of matchsticks for initial patterns - Establish the arithmetic pattern and common difference (5) - Derive the formula for the number of matchsticks - Calculate 'n' by equating to 61 - Conclude the figure number is 12

Question 15.

Gadisar Lake is located in the Jaisalmer district of Rajasthan. It was built by the King of Jaisalmer and rebuilt by Gadsi Singh in the 14^{th} century. The lake has many Chhatris. One of them is shown below. observe the picture From a point A h m above water level, the angle of elevation of top of Chhatri (point B) is 45° and angle of depression of its reflection in water (point C) is 60°. If the height of Chhatri above water level is approximately 10 m, then

(1) Draw a well-labelled figure based on the above information.

[2 Marks]

Answer: Draw a vertical line representing the Chhatri, labelled as BC, with B at the top and C as its reflection in the water directly below the base. Mark the water surface as a horizontal line. Point A is drawn at a height h above the water level on the left side of the Chhatri. Draw the line of sight from A to B showing the angle of elevation 45°, and from A to C showing the angle of depression 60°. Label all points A (observer's eye), B (top of Chhatri), C (reflection of top in water), and water level. Include angle markings for 45° and 60° at point A. The figure should clearly show the observer's height h, Chhatri of height 10 m, and the angles mentioned.

Key Points: Draw vertical Chhatri with points B (top) and base on water level- Mark water surface as horizontal line- Point A above water level at height h- Draw line of sight from A to B with 45° angle of elevation- Draw line of sight from A to C (reflection point) with 60° angle of depression- Label all points A, B, C and angles clearly- Show height of Chhatri as 10 m

(2)

Find the height (h) of the point A above water level.

(Use √3 = 1.73)

[2 Marks]

Answer: Let the height of point A above water level be h meters. The height of the Chhatri above water level is 10 meters. From point A, angle of elevation to top of Chhatri (B) is 45°. Therefore, height of Chhatri minus h equals distance along horizontal (say x). Since tan 45° = 1 = (10 - h) / x, so x = 10 - h. Angle of depression to reflection of Chhatri in water (point C) is 60°. The reflection of Chhatri is 10 meters below the water surface, so the vertical distance from point A to point C is h + 10. Using tan 60° = √3 = 1.73 = (h + 10) / x. Substitute x = 10 - h, we get: 1.73 = (h + 10) / (10 - h) 1.73(10 - h) = h + 10 17.3 - 1.73h = h + 10 17.3 - 10 = h + 1.73h 7.3 = 2.73h h = 7.3 / 2.73 = 2.67 meters (approximately). Therefore, the height h of point A above the water level is approximately 2.67 meters.

Key Points: Define h as height of point A above water level-Understand angles of elevation and depression regarding points B and C-Use right triangle trigonometry with tan 45° = 1 and tan 60° = 1.73-Set up equations using tanθ = opposite/adjacent-Substitute and solve equation for h

Question 16.

Question 17.

Question 18.