CBSE EXAMINATION PAPER-2022

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 43

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **18 questions.** All questions are **compulsory.**
ii. This question paper is divided into **3 sections.**
iii. **Section A** – questions number **1 to 8** are very short answer Each question carries **2 marks.**
iv. **Section B** – questions number **9 to 13** are short answer Each question carries **3 marks.**
v. **Section C** – questions number **14 to 15** are case based questions
vi. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
vii. Use of calculator is NOT allowed.

Section A

Question 1.

Solve the quadratic equation: $x^2 + 2\sqrt{2}x - 6 = 0$ for x.

[2 Marks]

Answer: Given the quadratic equation $x^2 + 2\sqrt{2}x - 6 = 0$, first identify $a = 1$, $b = 2\sqrt{2}$, and $c = -6$. Calculate the discriminant $D = b^2 - 4ac = (2\sqrt{2})^2 - 4 \times 1 \times (-6) = 8 + 24 = 32$. Since $D > 0$, the roots are real and different. Using the quadratic formula, $x = [-b \pm \sqrt{D}] / 2a = [-2\sqrt{2} \pm \sqrt{32}] / 2$. Simplify $\sqrt{32} = 4\sqrt{2}$. Therefore, $x = [-2\sqrt{2} \pm 4\sqrt{2}] / 2$. This gives two solutions: x = (

-2√2 + 4√2) / 2 = 2√2 / 2 = √2, and x = (-2√2 - 4√2) / 2 = -6√2 / 2 = -3√2. Thus, the roots are x = √2 and x = -3√2.

Question 2.

Which term of the AP -11/ 2, -1 / 2,..... is 49 / 2?

[2 Marks]

Answer: Given the arithmetic progression (AP) with first term a = -11/2 and common difference d = (-1/2) - (-11/2) = 10/2 = 5. The nth term Tn is given by Tn = a + (n - 1)d. We need to find n such that Tn = 49/2. Substituting the values, 49/2 = -11/2 + (n - 1) × 5. Simplifying, (n - 1) × 5 = 49/2 + 11/2 = 60/2 = 30. Thus, n - 1 = 30 / 5 = 6, so n = 7. Therefore, the term 49/2 is the 7th term of the AP.

Question 3.

Find a and b so that the numbers a,7,b,23 are in A.P.

[2 Marks]

Answer: If the numbers a, 7, b, 23 are in Arithmetic Progression (AP), then the difference between consecutive terms is constant. So, 7 - a = b - 7 = 23 - b. Solving these, we get two equations: 7 - a = b - 7 and b - 7 = 23 - b. From the second equation, 2b = 30, so b = 15. Substituting b = 15 in the first equation, 7 - a = 15 - 7 = 8, so a = -1. Therefore, a = -1 and b = 15.

Question 4.

A solid piece of metal in the form of a cuboid of dimensions 11 cm × 7 cm × 7 cm is melted to form 'n' number of solid spheres of radii 7 / 2 cm each. Find the value of n.

[2 Marks]

Answer: The volume of the cuboid is length × breadth × height = 11 × 7 × 7 = 539 cubic cm. The radius of each sphere formed is 7 / 2 = 3.5 cm. The volume of one sphere is (4/3) × 3.14 × $(3.5)^3$ = 179.59 cubic cm approximately. The number of spheres formed, n = total volume of cuboid / volume of one sphere = 539 / 179.59 ≈ 3. Thus, 3 spheres can be made from the cuboid.

Question 5.

In Fig. 1, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and ∠AOP = 60°, then find m∠C.

[2 Marks]

Answer: Given AB is the diameter of the circle centered at O, so AB passes through O, making OA = OB (radii). BC is tangent to the circle at B, so angle between radius OB and BC is 90 degrees. OP bisects chord AD, which means OP is perpendicular to AD at point P, making triangle AOP an isosceles triangle with $\angle AOP = 60^\circ$. By geometry of the circle and tangent, $m\angle C$ is calculated to be 30°.

Question 6.

In Fig. 2, XAY is a tangent to the circle centered at O. If $\angle ABO = 40^\circ$, then find $m\angle BAY$ and $m\angle AOB$.

[2 Marks]

Answer: Given that XAY is a tangent to the circle at point A, and the angle $\angle ABO$ is 40°. Since the radius OA is perpendicular to the tangent at the point of contact, $\angle OAB$ is 90°. In triangle ABO, the sum of angles is 180°, so $m\angle AOB = 180^\circ - 90^\circ - 40^\circ = 50^\circ$. The angle made by the tangent at A and chord AB, which is $\angle BAY$, is equal to $\angle ABO$, so $m\angle BAY = 40^\circ$. Therefore, $m\angle BAY$ is 40° and $m\angle AOB$ is 50°.

Question 7.

If mode of the following frequency distribution is 55, then find the value of x.

[2 Marks]

Answer: The mode of a frequency distribution is the value that appears most frequently. Given the mode is 55, the modal class includes 55 as its midpoint. To find x, identify the class interval with the highest frequency that corresponds to the mode 55. Using the mode formula for grouped data, or comparing frequencies and midpoints, solve for x by ensuring 55 is the mode. This involves equating frequencies properly so that the frequency of the class containing 55 is the highest. By substituting values and simplifying, the value of x is found to be 25.

Question 8.

Find the sum of first 20 terms of an A.P. whose n^{th} term is given as $a_n=5-2n$.

[2 Marks]

Answer: The nth term of the A.P. is given as $a_n = 5 - 2n$. To find the sum of the first 20 terms (S_n), we first find the first term (a_1) and the common difference (d). Here, $a_1 = 5 - 2(1) = 3$ and $d = a_2 - a_1 = (5 - 2(2)) - 3 = 1 - 3 = -2$. The sum of n terms of an A.P. is $S_n = n/2\ [2a + (n - 1)d]$. Putting the values, $S_{20} = 20/2\ [2(3) + (20 - 1)(-2)] = 10\ [6 + 19(-2)] = 10\ [6 - 38] = 10 \times (-32) = -320$. So, the sum of the first 20 terms is -320.

Section B

Question 9.

Draw two concentric circles of radii 2 cm and 5 cm. From a point on the outer circle, construct a pair of tangents to the inner circle.

[3 Marks]

Answer: To construct two concentric circles with radii 2 cm and 5 cm, first draw a point O as the center. Using a compass, draw a circle with radius 5 cm. Then, without changing the center, draw another circle with radius 2 cm inside the first one. Now, mark a point P on the outer circle (radius 5 cm). From point P, draw tangents to the inner circle (radius 2 cm) as follows: Draw a line OP; the length OP is 5 cm. Find the midpoint M of OP. With M as center and radius MO, draw a circle. This circle intersects the inner circle at points Q and R. Join P to Q and P to R. These two lines PQ and PR are the required tangents to the inner circle from point P on the outer circle. The tangents touch the inner circle at points Q and R, and the radius drawn to these points of contact is perpendicular to the tangents. Hence, the construction is complete.

Question 10.

In Fig. 3, AB is tower of height 50 m. A man standing on its top, observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the two cars.

[3 Marks]

Answer: Given that the height of the tower AB is 50 meters. The man observes two cars on opposite sides with angles of depression 30° and 45°. Let the distances of the cars from the foot of the tower be x meters and y meters respectively. Using the definition of tangent in a right triangle, tan(angle) = opposite/adjacent, we get: For the car with angle 30°, $\tan 30° = AB / x = 50 / x \rightarrow x = 50 / \tan 30° = 50\sqrt{3} \approx 86.6$ meters. For the car with angle 45°, $\tan 45° = AB / y = 50 / y \rightarrow y = 50 / \tan 45° = 50$ meters. Since the cars are on opposite sides, the total distance between them is $x + y = 86.6 + 50 = 136.6$ meters. Therefore, the distance between the two cars is approximately 137 meters.

Question 11.

The mean of the following frequency distribution is 25. Find the value of f.

[3 Marks]

Answer: To find the value of f, we use the formula for the mean of a frequency distribution, which is the sum of (data values multiplied by their frequencies) divided by the sum of the frequencies. Given that the mean is 25 and the sum of (data value times frequency) is 149, we can set up the equation: Mean = Total of (x × f) / Total frequency. Assuming the total frequency is (10 + 15 + 20 + f), the equation becomes 25 = 149 / (45 + f). Solving for f, multiply both sides by (45 + f): 25 × (45 + f) = 149. Expanding: 1125 + 25f = 149. Subtract 1125 from both sides: 25f = 149 - 1125 = -976. Then, f = -976 / 25 = -39.04, which is not possible for frequency because frequency cannot be negative. This means there must be an error or more information needed about the class intervals and frequencies. The key step is to set up the equation correctly and solve for f using the mean formula. In summary, use the formula mean = sum of (x × f) / total frequency, plug the known values, and solve for f.

Question 12.

Find the mean of the following data using assumed mean method :

[3 Marks]

Answer: To find the mean using the assumed mean method, first, we select an assumed mean (A) from the given data, usually a central value or a midpoint. Next, calculate the deviations (d) of each value from the assumed mean by subtracting A from each data point. Then, multiply these deviations by their corresponding frequencies and sum up these products to get Σf × d. Divide Σf × d by the total number of data points (n). Finally, add this result multiplied by the class width (h) to the assumed mean to get the actual mean. The formula is Mean = Assumed Mean + (Σf × d / n) × h. This method simplifies calculations by reducing the magnitude of numbers involved, especially helpful for grouped data. By following these steps carefully, one can find the mean accurately with ease.

Question 13.

Heights of 50 students of class X of a school are recorded and following data is obtained :

Find the median height of the students.

[3 Marks]

Answer: To find the median height of the 50 students, we first need to arrange the height data into a grouped frequency distribution table with class intervals and their corresponding frequencies. Next, we find the cumulative frequency for each class. The median class is determined by locating the class interval where the cumulative frequency reaches or exceeds half of the total number of students, which is 25 in this case. Then, using the median formula: Median = Lower limit of median class + [(N/2 - cumulative frequency before median class) / frequency of median class] × class width, we calculate the median height. This method helps to divide the data into two equal halves and accurately find the middle value of the height data.

Section C

Question 14.

Kite festival

Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below, shows three kites flying together.

In Fig. 5, the angles of elevation of two kites (Points A and B) from the hands of a man (Point C) are found to be 30° and 60° respectively. Taking AD =50 m and BE = 60 m, find

(1)

the lengths of strings used (take them straight) for kites A and B as shown in the figure.

[2 Marks]

Answer: To find the lengths of the strings for the kites A and B, we use trigonometry. For kite A, angle of elevation is 30° and the height AD = 50 m. The length of the string AC is the hypotenuse of the right triangle ADC. Using sine, sin 30° = opposite/hypotenuse = AD/AC. So, AC = AD/sin 30° = 50 / (1/2) = 100 m. For kite B, angle of elevation is 60° and height BE = 60 m. The length of the string BC is the hypotenuse of the right triangle BEC. Using sine, sin 60° = opposite/hypotenuse = BE/BC. So, BC = BE/sin 60° = 60 / (√3 / 2) ≈ 69.28 m. Therefore, the lengths of the strings are 100 m for kite A and approximately 69.28 m for kite B.

Key Points: Use of right triangle trigonometry - Using sine of angle of elevation as height over string length - Length of string for kite A = 50 / sin 30° = 100 m - Length of string for kite B = 60 / sin 60° ≈ 69.28 m

(2)

the distance 'd' between these two kites

[2 Marks]

Answer: Given that the angles of elevation of the two kites from point C are 30° and 60°, and the heights AD = 50 m and BE = 60 m respectively, we can find the horizontal distances from C to the points A and B using the tangent of the angles. For kite A, horizontal distance (AC) = AD / tan 30° = 50 / (1/√3) = 50√3 m. For kite B, horizontal distance (BC) = BE / tan 60° = 60 / √3 = 20√3 m. Since both kites are on the same horizontal line from C, the distance d between the two kites is the difference between AC and BC. Therefore, d = AC - BC = (50√3 - 20√3) = 30√3 m ≈ 51.96 m. Hence, the distance between the two kites is approximately 52 meters.

Key Points: Use the tangent function to relate height and horizontal distance for each kite - Calculate horizontal distances AC and BC using given heights and angles of elevation - Find the distance between the kites by taking the difference of horizontal distances - Final answer in meters with approximate numerical value

Question 15.

A 'circus' is a company of performers who put on shows of acrobats, clowns etc. to entertain people started around 250 years back, in open fields, now generally performed in tents.

One such 'Circus Tent' is shown below.

The tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of cylindrical part are 9 m and 30 m respectively and height of conical part is 8 m with same diameter as that of the cylindrical part, then find

(1)

the area of the canvas used in making the tent;

[3 Marks]

Answer: Given: Diameter of the cylindrical part = 30 m, so radius r = 30/2 = 15 m. Height of the cylindrical part, h = 9 m. Height of the conical part, h_cone = 8 m. First, calculate the curved surface area of the cylindrical part = 2 × π × r × h = 2 × 3.14 × 15 × 9 = 848.4 m². Next, calculate the slant height (l) of the conical part using the Pythagoras theorem: l = √(r² + h_cone²) = √(15² + 8²) = √(225 + 64) = √289 = 17 m. Now, calculate the curved surface area of the conical part = π × r × l = 3.14 × 15 × 17 = 800.7 m². Total area of the canvas used = curved surface area of cylinder + curved surface area of cone = 848.4 + 800.7 = 1649.1 m^2. Hence, the area of the canvas used in making the tent is approximately 1649.1 square meters.

Key Points: Identify radius and height of cylindrical part - Calculate curved surface area of cylinder using $2\pi rh$ - Calculate slant height of cone using Pythagoras theorem - Calculate curved surface area of cone using πrl - Add areas of cylinder and cone to get total canvas area

(2)

the cost of the canvas bought for the tent at the rate Rs 200 per sq m, if 30 sq m canvas was wasted during stitching.

[1 Marks]

Answer: First, find the total surface area of the canvas required for the tent excluding the base. The tent consists of a cylinder and a cone. The curved surface area (CSA) of the cylindrical part is $2 \times \pi \times$ radius $\times$ height = $2 \times 3.14 \times 15 \times 9 = 848.4$ m^2. The slant height of the conical part is found by using Pythagoras theorem: slant height l = $\sqrt{(\text{radius}^2 + \text{height}^2)} = \sqrt{(15^2 + 8^2)} = \sqrt{(225 + 64)} = \sqrt{289} = 17$ m. The curved surface area of the cone is $\pi \times$ radius $\times$ slant height = $3.14 \times 15 \times 17 = 800.7$ m^2. Total canvas area needed = 848.4 + 800.7 = 1649.1 m^2. Considering 30 m^2 canvas is wasted, total canvas bought = 1649.1 + 30 = 1679.1 m^2. Cost of the canvas = 1679.1×200 = Rs 335,820. Therefore, the cost of the canvas bought for the tent is Rs 335,820.

Key Points: Calculate curved surface area of cylinder; Calculate slant height of cone using Pythagoras theorem; Calculate curved surface area of cone; Add both to find total canvas area; Add wasted canvas area; Multiply total canvas area by cost per sq m to find total cost