CBSE EXAMINATION PAPER-2022

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 43

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **19 questions.** All questions are **compulsory.**
ii. This question paper is divided into **3 sections.**
iii. **Section A** – questions number **1 to 8** are very short answer Each question carries **2 marks.**
iv. **Section B** – questions number **9 to 13** are short answer Each question carries **3 marks.**
v. **Section C** – questions number **14 to 16** are case based questions
vi. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
vii. Use of calculator is NOT allowed.

Section A

Question 1.

How many odd numbers are there between 1 and 1000 which are divisible by 5 but not by 2 ?

[2 Marks]

Answer: Odd numbers are numbers which are not divisible by 2. Any number divisible by 5 ends with either 0 or 5. If it ends with 0, it is even, and if it ends with 5, it is odd. To find how many odd numbers between 1 and 1000 are divisible by 5 but not by 2, we consider those ending with 5. The series would be 5, 15, 25, ..., 995. There are 100 such numbers because the numbers increase by 10 for each step from 5 to 995.

Question 2.

If the sum of the roots of the quadratic equation $ky^2 - 11y + (k - 23) = 0$ is 13/21 more than the product of the roots, then find the value of k.

[2 Marks]

Answer: Given the quadratic equation $ky^2 - 11y + (k - 23) = 0$, let the roots be α and β. The sum of roots (α + β) is given by 11/k, and the product of roots (αβ) is (k - 23)/k. According to the problem, the sum of roots is 13/21 more than the product of the roots; so, 11/k = (k - 23)/k + 13/21. Multiplying both sides by k gives 11 = k - 23 + (13k)/21. Simplifying further, multiply all terms by 21 to eliminate the denominator: 231 = 21k - 483 + 13k. Combining like terms: 231 = 34k - 483. Adding 483 on both sides: 714 = 34k. Dividing both sides by 34: k = 714 / 34 = 21. Therefore, the value of k is 21.

Question 3.

If x= -2 s the common solution of quadratic equations $ax^2+x-3a= 0$,and $x^2+ bx + b = 0$, then find the value of a^2b.

[2 Marks]

Answer: The first quadratic equation $x^2 + 5x + 6 = 0$ factors as (x + 2)(x + 3) = 0, giving roots -2 and -3. For the second quadratic $x^2 + bx + b = 0$ to have real roots, its discriminant must be non-negative, i.e., $b^2 - 4b \geq 0$, which implies $b(b - 4) \geq 0$. The roots of this equation are $(-b + \sqrt{(b^2 - 4b)})/2$ and $(-b - \sqrt{(b^2 - 4b)})/2$. For the sequences to form geometric progressions with the same common ratio, the ratios of consecutive roots must be equal. Solving this condition leads to the common ratio being -3/2.

Question 4.

If in Fig. 1 there are two concentric circles with centre O. If ARC and AQB are tangents to the smaller circle from the point A lying on the larger circle, find the length of AC, if AQ =5 cm.

[2 Marks]

Answer: Given two concentric circles with center O, point A lies on the larger circle. From A, tangents ARC and AQB are drawn to the smaller circle. Since AQ is a tangent segment from A to the smaller circle and its length is 5 cm, by the property of tangents from an external point to a circle, the tangents from A are equal in length. Therefore, AC is also equal to 5 cm.

Question 5. The curved surface area of a right circular cylinder is 176 cm^2 and its volume is 1232 cm^3. What is the height of the cylinder?

[2 Marks]

Answer: Given the curved surface area (C.S.A) of the cylinder is 176 cm^2 and volume is 1232 cm^3. The formulas to use are: C.S.A = 2 × π × r × h and Volume = π × r^2 × h. From the first formula, 176 = 2 × 3.1416 × r × h which gives 176 = 6.2832 × r × h. From the second formula, 1232 = 3.1416 × r^2 × h. Dividing volume by curved surface area formula: (1232 / 176) = (3.1416 × r^2 × h) / (6.2832 × r × h) which simplifies to 7 = r / 2, so r = 14 cm. Substituting r back in the curved surface area formula: 176 = 2 × 3.1416 × 14 × h gives h = 176 / (87.9648) ≈ 2 cm. Therefore, the height of the cylinder is approximately 2 cm.

Question 6.

The largest sphere is carved out of a solid cube of side 21 cm. Find the volume of the sphere.

[2 Marks]

Answer: The largest sphere carved out of a cube will have a diameter equal to the side of the cube, which is 21 cm. Thus, the radius of the sphere is 21 divided by 2, which is 10.5 cm. The volume of a sphere is given by (4/3) × π × r^3. Using π as 22/7, the volume is (4/3) × (22/7) × 10.5 × 10.5 × 10.5 = 4841.25 cubic centimeters. Therefore, the volume of the largest sphere is 4841.25 cm^3.

Question 7.

Find the mean of the following frequency distribution :

[2 Marks]

Answer: To find the mean of a frequency distribution, first determine the midpoint of each class interval. Then multiply each midpoint by its corresponding frequency. Add all these products together to get the sum of values times frequencies. Next, add all the frequencies to get the total frequency. Finally, divide the total sum of values times frequencies by the total frequency. This quotient is the mean of the frequency distribution.

Question 8.

The mode of a grouped frequency distribution is 75 and the modal class is 65-80. The frequency of the class preceding the modal class is 6 and the frequency of the class succeeding the modal class is 8. Find the frequency of the modal class.

[2 Marks]

Answer: Given the mode (Mo) of the grouped data is 75 and the modal class is 65-80. The frequency of the class preceding the modal class (f_0) is 6, and the frequency of the class succeeding the modal class (f_2) is 8. The modal class frequency is denoted as f_1. Using the mode formula for grouped data: Mode = L + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] × h, where L = lower limit of modal class = 65, h = class size = 15, and Mo = 75. Plugging in values: 75 = 65 + [(f_1 - 6) / ($2f_1$ - 6 - 8)] × 15. Solving this equation gives the frequency of the modal class (f_1) as 14.

Section B

Question 9. Construct a pair of tangents to a circle of radius 4 cm which are inclined to each other at an angle of 60°.

[3 Marks]

Answer: To construct a pair of tangents to a circle of radius 4 cm inclined at 60°, first draw a circle with radius 4 cm. Choose the point from where tangents are to be drawn outside the circle. Let this point be T. The tangents from T will touch the circle at points P and Q. Since the angle between tangents is 60°, the angle subtended by the line segment joining P and Q at the centre O will be 120°, because the angle between two tangents is supplementary to the angle at the centre. Draw radii OP and OQ to the points of tangency. Then, draw the pair of tangents TP and TQ touching the circle at P and Q respectively. Remember that the radius is perpendicular to the tangent at the point of contact, so OP and OQ are perpendicular to tangents TP and TQ. This construction uses the properties of circles and tangents including equal tangents from an external point and perpendicularity of radius to tangent.

Question 10.

Find the value of 'p' for which the quadratic equation $p(x-4)(x-2) + (x-1)^2 = 0$ is shas real and equal roots.

[3 Marks]

Answer: The given quadratic equation is $p(x-4)(x-2) + (x-1)^2 = 0$. First, expand both terms: $p(x^2 - 6x + 8) + (x^2 - 2x + 1) = 0$. This simplifies to $p x^2 - 6p x + 8p + x^2 - 2x + 1 = 0$. Combining like terms gives $(p + 1) x^2 - (6p + 2) x + (8p + 1) = 0$. For the quadratic to have real and equal roots, the discriminant D = 0. The discriminant $D = [-(6p + 2)]^2 - 4(p + 1)(8p + 1)$. Simplify D to find the value of p. Setting D = 0 leads to an equation in p which can be solved to find the required value. Solving yields p = -1 or p = 1/3. However, if p = -1, the coefficient of x^2 is zero, so the equation is not quadratic. Thus, the required value is p = 1/3. Hence, when p = 1/3, the given equation has real and equal roots.

Question 11. Had Aarush scored 8 more marks in a Mathematics test, out of 35 marks, 7 times these marks would have been 4 less than the square of his actual marks. How many marks did he get in the test?

[3 Marks]

Answer: Let the actual marks scored by Aarush be x. According to the question, if Aarush had scored 8 more marks, his marks would be x + 8. Seven times these marks would then be 7 times (x + 8), which is equal to 7(x + 8). This quantity is said to be 4 less than the square of his actual marks, which means it is equal to $x^2 - 4$. Therefore, we have the equation: $7(x + 8) = x^2 - 4$. Expanding the left side, we get $7x + 56 = x^2 - 4$. Rearranging the equation to bring all terms to one side gives $x^2 - 7x - 60 = 0$. This is a quadratic equation. Solving it by factorization, we look for two numbers that multiply to -60 and add to -7, which are -12 and 5. So, the equation factors into $(x - 12)(x + 5) = 0$. Setting each factor to zero, we get x = 12 or x = −5. Since marks cannot be negative, the actual marks scored by Aarush is 12. Hence, Aarush scored 12 marks in the test.

Question 12.

If the last term of an AP. of 30 terms is 119 and the 8^{th} term from the end (towards the first term) is 91, then find the common difference of the AP. Hence find the sum of all the terms of the AP.

[3 Marks]

Answer:

Given that the AP has 30 terms and the last term (30th term) is 119. The 8th term from the end means the (30 - 8 + 1) = 23rd term from the beginning is 91.

Let the first term be 'a' and the common difference be 'd'. Using the nth term formula of AP: $T_n = a + (n - 1)d$,

For the 30th term: $a + 29d = 119$ ...(1)

For the 23rd term: $a + 22d = 91$ ...(2)

Subtracting equation (2) from (1), we get:

$(a + 29d) - (a + 22d) = 119 - 91$

$7d = 28$

Therefore, d = 4

Substitute d = 4 into equation (2):

$a + 22 \times 4 = 91 \Rightarrow a + 88 = 91 \Rightarrow a = 3$

Now the sum of the AP of 30 terms is given by $S = (n/2) \times (2a + (n - 1)d)$

$S = (30/2) \times [2 \times 3 + (30 - 1) \times 4] = 15 \times (6 + 116) = 15 \times 122 = 1830$

Hence, the common difference is 4 and the sum of all terms is 1830.

Question 13.

An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the two planes at that instant.

[3 Marks]

Answer: Let the horizontal distance from the observation point on the ground to the planes be d. Given are two planes flying vertically aligned but at different heights. The lower plane is at 3125 m. The two angles of elevation from the same point are 30° for the higher plane and 60° for the lower plane. Because the planes are vertically above one another, the horizontal distance to both planes is the same. Using the angle of elevation and height of the lower plane, we find that d = height / tan(angle). So, d = 3125 / tan 60°, which equals 3125 / √3 ≈ 1804.25 m. Next, calculate the height of the higher plane using the 30° angle, h = d × tan 30°. So, h = 1804.25 × 1/√3 ≈ 1041.38 m. Since the higher plane is at a greater height than the lower plane, but the height found is less, this implies the given heights correspond differently. Actually, the 60° corresponds to the higher angle and thus the lower height, so the higher plane is at 3125 m, and the one at 60° is the lower one. Then, using 30° angle and horizontal distance d, height of higher plane H = d × tan 30°. Using d = 3125 / tan 60°, we get H = d × (1/√3). Finally, distance between planes is difference in heights, which is H - 3125 m. Calculating gives the required distance between the planes at that instant. This approach uses the relationship between height, horizontal distance, and angle of elevation with the tangent function in right-angled triangles.

Section C

Question 14.

Khurja is a city in the Indian state of Uttar Pradesh famous for the pottery. Khurja pottery is traditional Indian pottery work which has attracted Indians as well as foreigners with a variety of tea-sets, crockery and ceramic tile works. A huge portion of the ceramics used in the country is supplied by Khurja and is also referred as 'The Ceramic Town'. One of the private schools of Bulandshahr organised an Educational Tour for class 10 students to Khurja. Students were very excited about the trip. Following are the few pottery objects of Khurja.

Students found the shapes of the objects very interesting and they could easily relate them with mathematical shapes viz sphere, hemisphere, cylinder etc. Maths teacher who was accompanying the students asked following questions :

(1)

The internal radius of hemispherical bowl (filled completely with water) in I is 9 cm and radius and height of cylindrical jar in II is 1.5 cm and 4 cm respectively. If the hemispherical bowl is to be emptied in cylindrical jars, then how many cylindrical jars are required ?

[2 Marks]

Answer: First, find the volume of the hemispherical bowl. Volume of a sphere = $(4/3) \times \pi \times r^3$, so volume of hemisphere = $(1/2) \times (4/3) \times \pi \times r^3 = (2/3) \times \pi \times r^3$. Here, r = 9 cm. Volume of hemisphere = $(2/3) \times \pi \times 9 \times 9 \times 9 = (2/3) \times \pi \times 729 = 486 \pi \text{ cm}^3$. Next, find the volume of one cylindrical jar. Volume of cylinder = $\pi \times r^2 \times h$. Given r = 1.5 cm, h = 4 cm. Volume of jar = $\pi \times (1.5)^2 \times 4 = \pi \times 2.25 \times 4 = 9 \pi \text{ cm}^3$. Number of jars required = Volume of hemisphere ÷ Volume of one jar = $486 \pi \div 9 \pi = 54$. Therefore, 54 cylindrical jars are required to empty the water from the hemispherical bowl.

> Key Points: Calculate the volume of the hemispherical bowl using the formula $(2/3) \times \pi \times r^3$-Calculate the volume of the cylindrical jar using the formula $\pi \times r^2 \times h$-Calculate the number of jars by dividing the volume of the hemisphere by the volume of one jar-Use given dimensions for radius and height clearly

(2)

If in the cylindrical jar full of water, a conical funnel of same height and same diameter is immersed, then how much water will flow out of the jar ?

[2 Marks]

Answer: Since the conical funnel has the same height and diameter as the cylindrical jar, its volume will be one-third of the volume of the cylinder. When the cone is immersed in the jar full of water, it will displace water equal to its own volume. Therefore, one-third of the water from the jar will flow out.

> Key Points: Volume of cone = (1/3) × Volume of cylinder - Cone and cylinder have same height and diameter - Water displaced equals volume of cone - Water flowing out equals displaced water volume which is one-third of the cylinder's volume

Question 15. Yoga is an ancient practice which is a form of meditation and exercise. By practising yoga, we not even make our body healthy but also achieve inner peace and calmness. The International Yoga Day is celebrated on 21st of June every year since 2015. To promote Yoga, Green park society in Pune organised a 7-day Yoga camp in their society. The number of people of different age groups who enrolled for this camp is given as follows : Based on the above, find the following :

Question 16. Yoga is an ancient practice which is a form of meditation and exercise. By practising yoga, we not even make our body healthy but also achieve inner peace and calmness. The International Yoga Day is celebrated on 21^{st} of June every year since 2015. To promote Yoga, Green park society in Pune organised a 7-day Yoga camp in their society. The number of people of different age groups who enrolled for this camp is given as follows :

(1) Find the median age of people enrolled for the camp.

[2 Marks]

Answer: To find the median age of people enrolled for the camp, first arrange the age groups in order and note the number of people in each group. Then find the total number of people enrolled. The median is the middle value when all the ages are arranged in increasing order. If the total number of people is odd, the median is the (n + 1)/2 th value. If the total number is even, the median is the average of the n/2 th and (n/2 + 1) th values. Using this method, we can determine the median age of the enrolled people.

Key Points: Define median - Arrange data in order - Calculate total number - Use median position formula - Identify median value from data

(2)

If x more people of age group 65 — 75 had enrolled for the camp, the mean age would have been 58. Find the value of x.

[2 Marks]

Answer: Let the total number of people originally enrolled be n and their total age sum be S. If x more people of age group 65-75 enrolled, their total age sum increases by 70x (assuming the average age in that group is 70). Then, the new total number of people will be n + x and the new total age sum will be S + 70x. Given that the new mean age is 58, we have the equation: (S + 70x) ÷ (n + x) = 58. From this equation, the value of x can be found by substituting the known values of n and S from the data. Solving the equation will give the value of x (the number of additional people aged 65-75 who need to join to make the mean age 58).

Key Points: Define variables for the number of people and total age sum-Assume average age for 65-75 group as 70-Set up the equation for new mean age after adding x people-Use the equation (S + 70x) / (n + x) = 58-Solve the equation to find x