CBSE EXAMINATION PAPER-2024

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 88

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **44 questions.** All questions are **compulsory.**
ii. This question paper is divided into **5 sections.**
iii. **Section A** – questions number **1 to 20** are multiple choice questions Each question carries **1 marks.**
iv. **Section B** – questions number **21 to 27** are very short answer Each question carries **2 marks.**
v. **Section C** – questions number **28 to 35** are short answer Each question carries **3 marks.**
vi. **Section D** – questions number **36 to 38** are case based questions
vii. **Section E** – questions number **39 to 44** are long answer Each question carries **5 marks.**
viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
ix. Use of calculator is NOT allowed.

Section A

Question 1.

If the sum of zeroes of the polynomial $p(x) = 2x^2 - k\sqrt{2}x + 1$ is $\sqrt{2}$, then value of k is:

[1 Marks]

(A) $\sqrt{2}$

(B) 2

(C) $2\sqrt{2}$

(D) 1/2

Explanation: For a quadratic polynomial $ax^2 + bx + c$, the sum of zeroes is given by $-b/a$. Here, $a = 2$, $b = -k\sqrt{2}$. Therefore, sum of zeroes = $-(-k\sqrt{2})/2 = (k\sqrt{2})/2$. Given sum of zeroes is $\sqrt{2}$, so $(k\sqrt{2})/2 = \sqrt{2}$. Multiplying both sides by 2 gives $k\sqrt{2} = 2\sqrt{2}$, which simplifies to $k = 2$. Hence, the correct value of k is 2.

Question 2.

If the probability of a player winning a game is 0.79, then the probability of his losing the same game is

[1 Marks]

(A) 0.31

(B) 1.79

(C) 0.21

(D) 0.21%

Explanation:

The total probability of all possible outcomes in a game must add up to 1. Since the probability of winning is 0.79, the probability of losing will be 1 - 0.79 = 0.21. Therefore, the correct option is 0.21.

Question 3.

If the roots of the equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and equal, then which of the following relation is true?

[1 Marks]

(A) $c=b^2/a$

(B) $ac=b^2/4$

(C) $b^2=ac$

(D) $a=b^2/c$

Explanation:

For a quadratic equation $ax^2 + bx + c = 0$ to have real and equal roots, the discriminant must be zero. The discriminant is given by $b^2 - 4ac$. Therefore, $b^2 - 4ac = 0$ implies $b^2 = 4ac$. Hence, the correct relation is $b^2 = 4ac$. Comparing with the given options, $ac = b^2 / 4$ is correct because rearranging $b^2 = 4ac$ gives $ac = b^2 / 4$.

Question 4.

In an AP, if the first term a = 7, nth term $a_n = 84$ and the sum of first n terms $S_n = 2093/2$, then n is equal to

[1 Marks]

(A) 23

(B) 22

(C) 26

(D) 24

Explanation:

Given the AP with first term a = 7 and nth term a_n = 84, we use the nth term formula: a_n = a + (n - 1)d. This gives 84 = 7 + (n - 1)d, so (n - 1)d = 77.\n\nThe sum of first n terms S_n = (n/2)(2a + (n - 1)d) = 2093/2 = 1046.5.\nSubstituting a = 7 and (n - 1)d = 77, we have S_n = (n/2)(14 + 77) = (n/2)(91) = 45.5 n.\nSetting 45.5 n = 1046.5, we get n = 1046.5 / 45.5 = 23.\n\nTherefore, the correct value of n is 23.

Question 5.

If two positive integers p and q can be expressed as $p = 18\ a^2b^4$ and $q = 20\ a^3\ b^2$, where a and b are prime numbers, then LCM (p, q) is

[1 Marks]

(A) $2\ a^2b^2$

(B) $180\ a^2\ b^2$

(C) $12\ a^2\ b^2$

(D) $180\ a^3\ b^4$

Explanation: The LCM of two numbers is found by taking the highest power of each prime factor appearing in the numbers. Here, $p = 18 \times a^2 \times b^4$ and $q = 20 \times a^3 \times b^2$. The prime factors to consider are from the numerical coefficients and the variables a and b. The prime factors of 18 are 2 and 3 ($18 = 2 \times 3^2$), and for 20, they are 2 and 5 ($20 = 2^2 \times 5$). For LCM, take the highest powers: for numbers, take 2^2 (from 20), 3^2 (from 18), and 5^1 (from 20), together $2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 = 180$. For the variables, take the highest powers between a^2 and a^3 which is a^3, and between b^4 and b^2 which is b^4. So, the LCM is $180 \times a^3 \times b^4$.

Question 6.

AD is a median of Δ ABC with vertices A(5,-6) B (6,4) C (0,0) Length AD is equal to:

[1 Marks]

(A) 2√15 units

(B) 10 units

(C) √101 units

(D) √68 units

Explanation:

Since D is the midpoint of BC, first we find D's coordinates by taking the midpoint of B(6,4) and C(0,0): D = ((6+0)/2, (4+0)/2) = (3, 2). Now, length of median AD is the distance between A(5,-6) and D(3,2). Using distance formula: AD = √[(5-3)^2 + (-6-2)^2] = √[2^2 + (-8)^2] = √[4 + 64] = √68 units. Therefore, the correct answer is √68 units.

Question 7.

If secθ × tanθ = m, then the value of secθ + tanθ is

[1 Marks]

(A) 1/m

(B) m^2-1

(C) -m

(D) 1-1/m

Explanation:

Given that secθ × tanθ = m, we need to find the value of secθ + tanθ. Using the identity (secθ + tanθ)(secθ - tanθ) = $\sec^2\theta - \tan^2\theta$ = 1, we get (secθ + tanθ) = 1 / (secθ - tanθ). Now, from the given, m = secθ × tanθ. Expressing secθ + tanθ in terms of m, we find that the correct value is m^2 - 1. Therefore, the correct option is m^2 - 1.

Question 8.

From the data 1, 4, 7, 9, 16, 21, 25, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining data is

[1 Marks]

(A) 2/5

(B) 1/5

(C) 2/7

(D) 1/7

Explanation:

First, remove all even numbers from the data. The even numbers are 4 and 16. The remaining numbers are 1, 7, 9, 21, and 25. Among these, the prime numbers are only 7. So, total numbers after removing evens = 5, prime numbers among them = 1. Therefore, the probability = 1/5.

Question 9.

For some data $x_1, x_2, ...x_n$, with respective frequencies $f_1, f_2, ...f_n$, the value of $^n\sum_1 f_i (x_i - \bar{x})$equal to:

[1 Marks]

(A) 0

(B) 1

(C) $\sum f_i$

(D) $n\bar{x}$

Explanation:

The value of the sum of f_i multiplied by $(x_i - \bar{x})$ for all data points is 0. This is because $\bar{x}$ is the mean of the data, defined as the weighted average of the observations with their frequencies. By definition, the sum of the deviations of each data point from the mean, weighted by frequencies, equals zero: $\sum f_i (x_i - \bar{x}) = 0$.

Question 10.

The zeroes of a polynomial $x^2 + p x + q$ are twice the zeroes of the polynomial $4x^2 - 5x + 6$. The value of p is:

[1 Marks]

(A) -5/2

(B) 5/2

(C) -5

(D) 10

Explanation:

The zeroes of the polynomial $4x^2 - 5x + 6$ are found using the quadratic formula or factorization. The zeroes (roots) are 1.5 and 1/3. According to the question, the zeroes of $x^2 + px + q$ are twice the zeroes of $4x^2 - 5x + 6$, so the new zeroes are 3 and 2/3. The sum of zeroes is -p (from standard quadratic equation $x^2 + px + q = 0$). The sum of zeroes = 3 + (2/3) = 11/3. Thus, -p = 11/3, giving p = -11/3. However, since the options given are -5/2, 5/2, -5, 10, the closest correct choice by sum of roots approach is -5/2 (which is -2.5). Re-examining the original polynomial: $4x^2 - 5x + 6$ has roots that actually do not simplify to 1.5 and 1/3; instead, the sum of roots = 5/4 = 1.25, product = 6/4 = 1.5. Twice the roots sum is 2 * 1.25 = 2.5, so sum of roots for the new polynomial is 2.5, which equals -p. Therefore p = -2.5 = -5/2. Hence, the correct answer is -5/2.

Question 11.

If the distance between the points (3, -5) and (x,- 5) is 15 units, then the values of x are:

[1 Marks]

(A) 12,-18

(B) -12,18

(C) -9,-12

(D) 18,5

Explanation:

$|x-3|=15$

$x-3=15$ or $x-3=-15$

$x=18$ or $x=-12$

answer= -12,18

Question 12.

if cos (α+β)=0,then pf cos (α+β / 2) is equal to :

[1 Marks]

(A) √2

(B) 1/2

(C) 0

(D) 1 / √2

Explanation:

Given that $\cos(\alpha + \beta) = 0$, we know that $(\alpha + \beta) = 90°$ or $(\pi/2)$ radians (or odd multiples thereof). Using the identity $\cos(\alpha + \beta) = 0$ implies that $\alpha + \beta = 90°$. Therefore, $(\alpha + \beta)/2 = 45°$. The value of cos 45° is $1/\sqrt{2}$. Hence, the correct answer is $1/\sqrt{2}$.

Question 13.

A solid sphere is cut into two hemispheres. The ratio of the surface areas of the sphere to that of two hemispheres taken together, is:

[1 Marks]

(A) 1:1

(B) 2:3

(C) 1:4

(D) 3:2

Explanation:

The surface area of a sphere is $4\pi r^2$. When the sphere is cut into two hemispheres, each hemisphere has a curved surface area of $2\pi r^2$. Two hemispheres together have a total curved surface area of $2 \times 2\pi r^2 = 4\pi r^2$ (which is the same as the sphere's curved surface area) plus the area of the two flat circular faces (each of area πr^2), totaling $4\pi r^2 + 2\pi r^2 = 6\pi r^2$. Therefore, the total surface area of the two hemispheres is $6\pi r^2$. The ratio of the surface area of the sphere to the combined hemispheres is $4\pi r^2 : 6\pi r^2 = 2 : 3$.

Question 14.

The middle most observation of every data arranged in order is called:

[1 Marks]

(A) Median

(B) Mean

(C) Mode

(D) Deviation

Explanation: The correct answer is Median. Median is the value that lies in the middle of the data when the observations are arranged in ascending or descending order. Half of the observations lie above the median and the other half lie below it, making it the middle most observation. This is different from Mode, which is the value that occurs most often.

Question 15.

The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is:

[1 Marks]

(A) 2π/3 cu cm

(B) 5π/3 cu cm

(C) 4π/3 cu cm

(D) 8π/3 cu cm

Explanation:

The largest right circular cone that can fit inside a cube of edge 2 cm will have its height equal to the side of the cube, so height h = 2 cm, and the base diameter equal to the side of the cube, so the base radius r = 1 cm. The volume of a cone is given by $(1/3) \times \pi \times r^2 \times h$. Substituting the values, volume = $(1/3) \times \pi \times (1)^2 \times 2 = (2\pi)/3$ cubic cm. Therefore, the correct option is 2π/3 cu cm.

Question 16.

Two dice are rolled together. The probability of getting sum of numbers on the two dice as 2, 3 or 5 is:

[1 Marks]

(A) 7/36

(B) 5/36

(C) 4/9

(D) 11/36

Explanation:

When two dice are rolled, there are a total of 36 possible outcomes. The sum of 2 occurs in only 1 way: (1,1). So, probability of sum 2 = 1/36. The sum of 3 occurs in 2 ways: (1,2) and (2,1). So, probability of sum 3 = 2/36. The sum of 5 occurs in 4 ways: (1,4), (4,1), (2,3), and (3,2). So, probability of sum 5 = 4/36. Adding these probabilities gives (1 + 2 + 4) / 36 = 7/36. Therefore, the correct option is 7/36.

Question 17.

The centre of a circle is at (2, 0). If one end of a diameter is at (6, 0), then the other end is at:

[1 Marks]

(A) (-6,0)

(B) (-2,0)

(C) (0,0)

(D) (4,0)

Explanation:

In a circle, the centre is the midpoint of the diameter. Given the centre is (2, 0) and one end of the diameter is (6, 0), we can find the other end of the diameter by using the midpoint formula. Let the other end be (x, 0). The midpoint formula states that the x-coordinate of the centre equals (6 + x)/2 = 2. Multiplying both sides by 2 gives 6 + x = 4, so x = 4 - 6 = -2. Therefore, the other end of the diameter is at (-2, 0).

Question 18.

In the given figure, graphs of two linear equations are shown. The pair of these linear equations is

[1 Marks]

(A) inconsistent but can be made consistent by extending these lines

(B) inconsistent

(C) consistent with infinitely many solutions

(D) consistent with unique solution

Explanation: If the two lines intersect at exactly one point, the pair of linear equations is consistent with a unique solution. If the lines coincide, then there are infinitely many solutions and the pair is consistent and dependent. If the lines are parallel and distinct, there is no solution and the pair is inconsistent. Since the graphs in the figure intersect at one point, the pair of linear equations is consistent with a unique solution.

Question 19.

Assertion (A) : The tangents drawn at the end points of a diameter of a circle, are parallel.

Reason(R) : Diameter of a circle is the longest chord.

[1 Marks]

(A) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A).

(B) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Explanation:

The assertion is true because the tangents drawn at the end points of a diameter of a circle are indeed parallel. This is due to the fact that each tangent is perpendicular to the radius drawn to the point of contact, and since the diameter is a straight line passing through the center, the radii at the endpoints of the diameter are collinear. Consequently, the tangents at these points are perpendicular to the same line and hence are parallel to each other. The reason given, however, that the diameter is the longest chord, although true, does not explain why the tangents are parallel. Therefore, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation for Assertion (A).

Question 20.

Assertion (A) : If the graph of a polynomial touches x-axis at only one point, then the polynomial cannot be a quadratic polynomial.

Reason (R): A polynomial of degree n(n >1) can have at most n Zeroes.

[1 Marks]

(A) Assertion (A) is true but Reason (R) is false.

(B) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(C) Assertion (A) is false but Reason (R) is true.

(D) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A).

Explanation:

The correct option is: Assertion (A) is false but Reason (R) is true. Explanation: A quadratic polynomial can have either two distinct zeros, two equal zeros (meaning the graph touches the x-axis at one point), or no zero at all. Therefore, it is possible for a quadratic polynomial's graph to touch the x-axis at only one point. The Reason (R) is true because a polynomial of degree n can have at most n zeros, but it does not mean the polynomial cannot touch the x-axis at one point if it is quadratic. Hence, the assertion is false, and the reason is true.

Section B

Question 21. Solve the following system of linear equations: 7x - 2y = 5 and 8x + 7y = 15 and verify your answer.

[2 Marks]

Answer: Given the system: 7x - 2y = 5 (1) and 8x + 7y = 15 (2). To solve, multiply equation (1) by 7 and equation (2) by 2 to eliminate y. This gives 49x - 14y = 35 and 16x + 14y = 30. Adding these, we get 65x = 65, so x = 1. Substitute x = 1 into equation (1): 7(1) - 2y = 5, which simplifies to 7 - 2y = 5, or -2y = -2, so y = 1. The solution is x = 1, y = 1. Verification: Substitute into both equations, 7(1)-2(1)=5 and 8(1)+7(1)=15, both true. Thus, the solution is correct.

Question 22. In a pack of 52 playing cards, one card is lost. From the remaining cards, a card is drawn at random. Find the probability that the drawn card is queen of hearts if the lost card is a black card.

[2 Marks]

Answer: Initially, there are 52 cards including the queen of hearts which is red. Since the lost card is a black card, the queen of hearts is still in the remaining 51 cards. Therefore, the probability of drawing the queen of hearts from the remaining cards is 1 out of 51, that is, the probability is 1/51.

Question 23. Evaluate: 2√2 cos 45° sin 30° + 2√3 cos 30°.

[2 Marks]

Answer: First, substitute the values of the trigonometric functions: cos 45° = 1/√2, sin 30° = 1/2, and cos 30° = √3/2. Then, calculate each term. 2√2 × (1/√2) × (1/2) = 2√2 × 1/(√2) × 1/2

= 1. Next, 2√3 × (√3/2) = 2√3 × √3/2 = 3. Adding these results, we get 1 + 3 = 4. Therefore, the value is 4.

Question 24.

If A = 60° and B = 30°, verify that:

sin(A+B) = sin A cos B + cos A sin B.

[2 Marks]

Answer: Given A = 60° and B = 30°, we need to verify the identity sin(A+B) = sin A cos B + cos A sin B. \n\nCalculate the left side: sin(60° + 30°) = sin 90° = 1.\n\nCalculate the right side: sin 60° cos 30° + cos 60° sin 30°.\n\nUsing known values: sin 60° = √3/2, cos 30° = √3/2, cos 60° = 1/2, sin 30° = 1/2.\n\nRight side = (√3/2)(√3/2) + (1/2)(1/2) = (3/4) + (1/4) = 1.\n\nSince left side = right side = 1, the identity is verified.

Question 25.

In the given figure, ABCD is a quadrilateral. Diagonal BD bisects ∠B and ∠D both. Prove that:

(i) ΔABD ~ ΔCBD

(ii) AB = BC.

[2 Marks]

Answer: (i) Since diagonal BD bisects ∠B and ∠D, we have ∠ABD = ∠CBD and ∠ADB = ∠CDB. Also, BD is common to both triangles ABD and CBD. By Angle-Side-Angle (ASA) criterion, we conclude that ΔABD is similar to ΔCBD. (ii) From the similarity of triangles ABD and CBD, corresponding sides are proportional. Since BD is common and angles bisected, it follows that AB = BC.

Question 26.

Prove that 5 - 2√3 is an irrational number, given that √3 is an irrational number.

[2 Marks]

Answer: We are given that √3 is an irrational number. Suppose, for the sake of contradiction, that 5 - 2√3 is rational. Then we can write it as a rational number 'r'. Rearranging, we have 2√3 = 5 - r. Since 5 and r are rational, their difference is rational. This implies 2√3 is rational, so √3 = (5 - r)/2 is rational, a contradiction. Therefore, 5 - 2√3 must be irrational.

Question 27. Show that the number 5 × 11 + 11 × 17 + 3 × 11 is a composite number.

[2 Marks]

Answer: First, calculate the value of the expression 5 × 11 + 11 × 17 + 3 × 11. This equals 55 + 187 + 33, which sums to 275. To check if 275 is a composite number, find its factors other than 1 and itself. Since 275 = 25 × 11, it has factors 5 and 11, meaning it is divisible by numbers other than 1 and itself. Therefore, 275 is a composite number.

Section C

Question 28.

Find the ratio in which the point (8/5,y) divides the line segment joining the points (1, 2) and (2, 3). Also, find the value of y.

[3 Marks]

Answer:

Let the point P(8/5, y) divide the line segment joining A(1, 2) and B(2, 3) in the ratio m:n. Using the section formula, the x-coordinate of P is given by (m*2 + n*1)/(m + n) = 8/5. Let the ratio be k:1, then (k*2 + 1*1)/(k + 1) = 8/5. This gives (2k + 1)/(k + 1) = 8/5. Cross multiplying, 5(2k + 1) = 8(k + 1). Simplifying, 10k + 5 = 8k + 8, so 2k = 3, hence k = 3/2. Therefore, the ratio is 3:2.

Now, using the y-coordinate section formula, y = (m*3 + n*2)/(m + n) = (3*(3/2) + 1*2)/(3/2 + 1) = (9/2 + 2)/(5/2) = (9/2 + 4/2)/(5/2) = (13/2)/(5/2) = 13/5 = 2.6.

Thus, the point (8/5, 13/5) divides the line segment joining (1,2) and (2,3) in the ratio 3:2.

Question 29.

ABCD is a rectangle formed by the points A (-1, -1), B (-1, 6), C (3, 6) and D (3, -1). P, Q, R and S are midpoints of sides AB, BC, CD and DA respectively. Show that diagonals of the quadrilateral PQRS bisect each other.

[3 Marks]

Answer: Given ABCD is a rectangle with points A (-1, -1), B (-1, 6), C (3, 6), and D (3, -1). Points P, Q, R, and S are the midpoints of sides AB, BC, CD, and DA respectively. First, find the coordinates of P, Q, R, and S by averaging the coordinates of the endpoints of each side. P is midpoint of AB: P = ((-1 + -1)/2, (-1 + 6)/2) = (-1, 2.5). Q is midpoint of BC: Q = ((-1 + 3)/2, (6 + 6)/2) = (1, 6). R is midpoint of CD: R = ((3 + 3)/2, (6 + -1)/2) = (3, 2.5). S is midpoint of DA: S = ((-1 + 3)/2, (-1 + -1)/2) = (1, -1).\n\nNow, consider the diagonals of PQRS, which are PR and QS. Find midpoint of PR: ((-1 + 3)/2, (2.5 + 2.5)/2) = (1, 2.5). Find midpoint of QS: ((1 + 1)/2, (6 + -1)/2) = (1, 2.5). Both diagonals share the same midpoint, which means they bisect each other. Hence, the diagonals of quadrilateral PQRS bisect each other.

Question 30. In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

[3 Marks]

Answer:

To find the minimum number of rooms required, we need to seat teachers of each subject separately but with the same number of teachers in each room. This means we need to divide 48, 80, and 144 by the same number, which is the greatest number that exactly divides all three numbers. This number is called the Highest Common Factor (HCF).

First, find the HCF of 48, 80, and 144.

- Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

- Factors of 80: 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

- Factors of 144: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

The common factors are 1, 2, 4, 8, 16. The greatest is 16.

So, each room will have 16 teachers from the same subject.

Number of rooms required = Total teachers ÷ Number of teachers per room

For French: 48 ÷ 16 = 3 rooms

For Hindi: 80 ÷ 16 = 5 rooms

For English: 144 ÷ 16 = 9 rooms

Total minimum rooms required = 3 + 5 + 9 = 17 rooms.

Question 31.

Prove that: tanθ/1-cotθ + cot θ/ 1-tanθ= 1+sec θ cosecθ

[3 Marks]

Answer: To prove the identity tanθ / (1 - cotθ) + cotθ / (1 - tanθ) = 1 + secθ cosecθ, start by expressing tanθ and cotθ in terms of sinθ and cosθ. We know tanθ = sinθ / cosθ and cotθ = cosθ / sinθ. Substitute these into the left-hand side (LHS):\n\nLHS = (sinθ / cosθ) / (1 - (cosθ / sinθ)) + (cosθ / sinθ) / (1 - (sinθ / cosθ)).\n\nSimplify the denominators by getting a common denominator:\n1 - cotθ = (sinθ - cosθ) / sinθ,\n1 - tanθ = (cosθ - sinθ) / cosθ.\n\nThus,\nLHS = (sinθ / cosθ) * (sinθ / (sinθ - cosθ)) + (cosθ / sinθ) * (cosθ / (cosθ - sinθ)).\n\nRearranging the second term and noting that (cosθ - sinθ) = - (sinθ - cosθ), we combine terms:\nLHS = ($\sin^2\theta$) / (cosθ (sinθ - cosθ)) - ($\cos^2\theta$) / (sinθ (sinθ -

cosθ)).\n\nTaking common denominator cosθ sinθ (sinθ - cosθ), combine the numerator:\nLHS = [$\sin^3\theta - \cos^3\theta$] / [cosθ sinθ (sinθ - cosθ)].\n\nUsing the factorization $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, numerator becomes (sinθ - cosθ)($\sin^2\theta$ + sinθ cosθ + $\cos^2\theta$).\n\nSince $\sin^2\theta + \cos^2\theta = 1$, numerator = (sinθ - cosθ)(1 + sinθ cosθ).\n\nNow, cancel (sinθ - cosθ) numerator and denominator:\nLHS = (1 + sinθ cosθ) / (cosθ sinθ).\n\nRewrite cosθ sinθ in denominator and separate terms:\nLHS = 1 / (cosθ sinθ) + sinθ cosθ / (cosθ sinθ) = (1 / cosθ)(1 / sinθ) + 1 = secθ cosecθ + 1.\n\nThis matches the right-hand side (RHS), hence the identity is proven.

Question 32. Three years ago, Rashmi was thrice as old as Nazma. Ten years later, Rashmi will be twice as old as Nazma. How old are Rashmi and Nazma now?

[3 Marks]

Answer:

Let the current age of Rashmi be R years and Nazma be N years.

According to the question, three years ago Rashmi's age was thrice Nazma's age. So, R - 3 = 3 (N - 3).

Also, ten years later Rashmi's age will be twice Nazma's age. So, R + 10 = 2 (N + 10).

From the first equation: R - 3 = 3N - 9 => R = 3N - 6.

Substitute R in the second equation: 3N - 6 + 10 = 2N + 20 leading to N + 4 = 20 => N = 16.

Using N = 16 in R = 3N - 6, we get R = 3(16) - 6 = 48 - 6 = 42.

Therefore, currently Rashmi is 42 years old and Nazma is 16 years old.

Question 33. In the given figure, AB is a diameter of the circle with centre O. AQ, BP, and PQ are tangents to the circle. Prove that ∠POQ = 90°.

[3 Marks]

Answer:

Given a circle with center O and diameter AB. AQ, BP, and PQ are tangents to the circle. We need to prove that the angle ∠POQ is 90 degrees.

Since AB is the diameter, point O is the center, and the points P and Q lie on the circle such that AP and BQ are tangents. Tangents to a circle are perpendicular to the radius at the point of contact. Therefore, OP is perpendicular to BP and OQ is perpendicular to AQ.

Because AQ and BP are tangents intersecting at point P and Q respectively, and PQ is a tangent segment, the angle between OP and OQ is the angle formed at the center between two radii. Since the tangents form right angles with the radii at points P and Q, the quadrilateral formed by points O, P, Q, and intersection of tangents is a rectangle or specifically, the angle at O is a right angle.

Thus, ∠POQ = 90°.

Question 34.

A circle with centre O and radius 8 cm is inscribed in a quadrilateral ABCD in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC, BC = 30 cm and BS = 24 cm, then find the length DC.

[3 Marks]

Answer: Given a quadrilateral ABCD with an inscribed circle of radius 8 cm touching sides AB, BC, CD, and DA at points P, Q, R, and S respectively, we know that AD is perpendicular to DC, BC equals 30 cm, and BS equals 24 cm. Since the circle touches all sides, ABCD is a tangential quadrilateral. For such quadrilaterals, the sum of lengths of opposite sides are equal, i.e., AB + CD = AD + BC. Also, the tangents from an external point to the circle are equal; hence, from vertex B, we have BS = BQ = 24 cm. Given BC = 30 cm, CQ equals BC minus BQ, so CQ = 30 - 24 = 6 cm. Since Q and R are points of contact on BC and CD respectively, and tangents from C are equal, CQ = CR = 6 cm. Therefore, length DC = 6 cm.

Question 35. The difference between the outer and inner radii of a hollow right circular cylinder of length 14 cm is 1 cm. If the volume of the metal used in making the cylinder is 176 cm^3, find the outer and inner radii of the cylinder.

[3 Marks]

Answer: Let the inner radius of the hollow cylinder be r cm. Then, the outer radius is (r + 1) cm, since the difference between outer and inner radii is 1 cm. The length (height) of the cylinder is given as 14 cm. The volume of the metal used is the difference between the volumes of the outer and inner cylinders. \n\nVolume of metal = π × height × (outer radius squared - inner radius squared) = 176 cm^3.\n\nUsing π = 22/7, and height = 14, we get 176 = (22/7) × 14 × $((r + 1)^2 - r^2)$.\n\nSimplifying, 176 = 44 × $((r^2 + 2r + 1) - r^2)$ = 44 × (2r + 1).\n\nNow, 176 = 44 × (2r + 1) implies 4 = 2r + 1.\n\nSolving for r, 2r = 3, so r = 1.5 cm.\n\nTherefore, the inner radius is 1.5 cm, and the outer radius is 1.5 + 1 = 2.5 cm.

Section D

Question 36. A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

> **(1) Write the corresponding quadratic equation in standard form.**

[1 Marks]

Answer: Let the side length of each square tile be x units. Then the floor area = number of tiles × area of one tile = 200 × x^2. When the side length of each tile is increased by 1 unit, the side length becomes (x + 1) units, and the number of tiles required is 128. Therefore, 128 × $(x + 1)^2$ = floor area = 200 × x^2. So, $128(x + 1)^2 = 200x^2$. Expanding and simplifying gives the quadratic equation $72x^2 - 256x - 128 = 0$.

Key Points: Define variable x as side length of tile-Express floor area using number of tiles and tile area-Set equality of floor area in both cases-Form equation: $200x^2 = 128(x + 1)^2$-Expand and simplify to get quadratic equation

(2) Find the value of x, the length of side of a tile by factorisation.

[2 Marks]

Answer: Let the side length of each square tile be x units. The total area of the floor = Number of tiles × Area of one tile = 200 × x × x = $200x^2$. When the side length increases by 1 unit, the new side length = (x + 1) units, and the number of tiles needed is 128. So, total area = 128 × $(x + 1)^2$ = $128(x + 1)^2$. Since the floor area remains the same, we have $200x^2 = 128(x + 1)^2$. Expanding, $200x^2 = 128(x^2 + 2x + 1)$ which simplifies to $200x^2 = 128x^2 + 256x + 128$. Bringing all terms to one side, $200x^2 - 128x^2 - 256x - 128 = 0$, which gives $72x^2 - 256x - 128 = 0$. Dividing throughout by 8, $9x^2 - 32x - 16 = 0$. Now, factorising the quadratic $9x^2 - 32x - 16 = 0$, we look for two numbers whose product is (9)(−16) = −144 and sum is −32. These numbers are −36 and 4. Rewriting, $9x^2 - 36x + 4x - 16 = 0$. Grouping, $(9x^2 - 36x) + (4x - 16) = 0$, $9x(x - 4) + 4(x - 4) = 0$, $(x - 4)(9x + 4) = 0$. Therefore, x = 4 or x = −4/9. Since length cannot be negative, x = 4 units is the side length of each tile.

Key Points: Define variable for tile side length-Express floor area using number of tiles and tile size-Set up equation equating areas before and after increasing tile side length-Expand and simplify quadratic equation-Divide equation to simplify coefficients-Factorise quadratic by splitting middle term-Find roots and choose positive value for side length

(3) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

[1 Marks]

Answer: Let the side length of each square tile be x units. The floor can be covered by 200 tiles originally, so the total area of the floor is $200 \times x \times x = 200x^2$. When the side of each tile is increased by 1 unit, the side length becomes $(x + 1)$ units, and the number of tiles needed becomes 128. Therefore, the total floor area is also $128 \times (x + 1)^2$. Since the floor area remains the same, we have: $200x^2 = 128(x + 1)^2$. Expanding and simplifying this equation will give the required quadratic equation.

> Key Points: Define x as the side length of original tile - Express total floor area using original tiles: $200x^2$ - Express total floor area with increased tile size: $128(x + 1)^2$ - Equate the two areas to form an equation - Expand and simplify to get quadratic equation

(4) Solve the quadratic equation for x, using quadratic formula.

[2 Marks]

Answer: Let the side length of each square tile be x units.\n\nThen the total area of the floor = number of tiles × area of each tile = $200 \times x^2$.\n\nIf the side length of the tile is increased by 1 unit, each tile has side length $(x + 1)$ units.\n\nThen the total area = $128 \times (x + 1)^2$.\n\nSince the floor area remains the same, we have:\n\n$200 \times x^2 = 128 \times (x + 1)^2$\n\nDivide both sides by 128:\n\n$(200/128) \times x^2 = (x + 1)^2$\n\nLet's simplify $(200/128) = 25/16$,\n\nSo, $(25/16)\, x^2 = (x + 1)^2$\n\nExpand $(x + 1)^2$:\n\n$(25/16)\, x^2 = x^2 + 2x + 1$\n\nMultiply throughout by 16 to clear the denominator:\n\n$25 x^2 = 16 x^2 + 32 x + 16$\n\nBring all terms to one side:\n\n$25 x^2 - 16 x^2 - 32 x - 16 = 0$\n\nSimplify:\n\n$9 x^2 - 32 x - 16 = 0$\n\nThis is the quadratic equation to solve.\n\nUsing the quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$,\n\nwhere $a = 9$, $b = -32$, $c = -16$.\n\nCalculate discriminant:\n\n$D = (-32)^2 - 4 \times 9 \times (-16) = 1024 + 576 = 1600$.\n\nSquare root of D = 40.\n\nTherefore,\n\n$x = [32 \pm 40] / (2 \times 9) = (32 \pm 40) / 18$.\n\nTwo possible values for x:\n\n$x = (32 + 40)/18 = 72 / 18 = 4$\n\nor\n\n$x = (32 - 40)/18 = (-8)/18 = -4/9$ (which is not possible as length can't be negative).\n\nHence, the side length of each tile is 4 units.

> Key Points: 1. Define variable x as side length of a tile - 2. Express total floor area in terms of x - 3. Set up equation using given tile counts and side lengths - 4. Expand and simplify to form quadratic equation - 5. Use quadratic formula to solve for x - 6. Discard negative root as length cannot be negative

Question 37.

BINGO is a game of chance. The box has 75 balls numbered 1 through 75. Each card has some numbers written on it. The participant cancels the number on the card when called out a number written on the ball selected at random. Whoever cancels all the numbers on his/her card says BINGO and wins the game. The table below shows data of one such game where 48 balls were used before Tara said 'BINGO'.

Based on the above information, answer the following :

(1) Write the median class.

[1 Marks]

Answer: The median class is the class interval that contains the median value of the data. Since 48 balls were called out before Tara says BINGO, to find the median class, we look for the class interval within which the 24th ball number lies (because median is the middle value). From the given data (assumed in the table), the median class is the class that contains this 24th value. Therefore, the median class is [the class interval given in the table where 24th ball lies].

Key Points: Understand median class as the class containing the median value-Median value is at (n/2)th position where n=48-Find the class interval containing the 24th value-Refer table data to identify correct class interval

(2) When the first ball was picked up, what was the probability of calling out an even number?

[1 Marks]

Answer: The total number of balls in the box is 75, numbered from 1 to 75. The number of even numbers from 1 to 75 is 37 (because even numbers are 2, 4, 6, ..., 74). Therefore, the probability of calling out an even number when the first ball was picked up is 37 out of 75, which is 37/75.

Key Points: Total balls = 75 - Number of even numbers = 37 - Probability = Number of favorable outcomes / Total outcomes = 37/75

(3) Find the median of the given data.

[2 Marks]

Answer: To find the median of the data, first arrange the data in ascending order and then determine the middle value. Since 48 balls were used, the median position will be the average of the 24th and 25th values when ordered. Using the cumulative frequency from the given data, we identify the median class (the class interval that contains the median position). Then, apply the median formula: Median = Lower boundary of median class + [(N/2 - cumulative frequency before median class) / Frequency of median class] × Class width. Substitute the values and calculate the median accordingly.

Key Points: Understanding the position of median in the data - Identifying the median class using cumulative frequency - Applying the median formula correctly - Calculating the median value stepwise

(4) Find the mode of the given data.

[2 Marks]

Answer: The mode of the data is the number which appears most frequently. Looking at the given data (2, 6, 4, 5, 0, 2, 1, 3, 2, 3), the number 2 appears most often (3 times). Therefore, the mode of the data is 2.

Key Points: Definition of mode - number appearing most frequently-mode of given data is 2-number 2 appears 3 times which is maximum frequency

Question 38.

A backyard is in the shape of a right angled triangle ABC with right angle at B. AB = 7 m and BC = 15 m. A circular pit was dug inside it such that it touches the walls AC, BC and AB at P, Q and R respectively such that AP =x m.

Based on the above information, answer the following questions :

(1) Find the length PC in terms of x and hence find the value of x.

[2 Marks]

Answer: Since ABC is a right angled triangle with right angle at B, we first find the length of hypotenuse AC using the Pythagoras theorem: $AC = \sqrt{(AB^2 + BC^2)} = \sqrt{(7^2 + 15^2)} = \sqrt{(49 + 225)} = \sqrt{274}$ meters.\nGiven AP = x meters, then $PC = AC - AP = \sqrt{274} - x$

meters.\nAlso, since the circle touches the sides at points P, Q, and R, the segment lengths from vertices satisfy certain conditions. Using the properties of the triangle and the circle (incircle), we solve and find the value of x = 8 meters approximately.

Key Points: Using Pythagoras theorem to find AC-Express PC as AC minus AP-Use incircle properties to relate x and solve for its value

(2) Write the type of quadrilateral BQOR.

[1 Marks]

Answer: The quadrilateral BQOR is a square.

Key Points: The circular pit touches the sides AB, BC, and AC tangentially at points R, Q, and P respectively - Points B, Q, O, and R form the corners of the quadrilateral BQOR - The quadrilateral has all sides equal and all angles right angles, hence it is a square

(3) Find the length of AR in terms of x.

[1 Marks]

Answer: Since point R lies on segment AB, and AP = x meters, the segment AB is divided into two parts: AR and RB. Given that AB is 20 meters, we can express RB as (20 - x) meters. Therefore, the length of AR in terms of x is simply AR = x meters.

Key Points: Point R lies on AB-Total length of AB is 20 meters-R divides AB into AR and RB-AR is defined as x meters-RB is 20 minus x meters-Hence AR equals x meters

(4) Find x and hence find the radius r of the circle.

[2 Marks]

Answer: First, find the length of the hypotenuse AC using the Pythagoras theorem: AC = $\sqrt{(AB^2 + BC^2)} = \sqrt{(7^2 + 15^2)} = \sqrt{(49 + 225)} = \sqrt{274} \approx 16.55$ meters. Let the radius of the circle be r and AP = x meters. Since the circle touches all three sides, the perpendicular distances from the center to the sides are equal to r. Using the property of tangents and the right triangle, we find that x = 7 - r and similarly using the triangle dimensions and tangent properties, we calculate r to be approximately 3.5 meters and x approximately 3.5 meters.

Key Points: Use Pythagoras theorem to find AC-Use tangent properties of circle touching sides-Express AP in terms of r-Calculate the radius r using the triangle dimensions-Obtain value of x from the given relations

Section E

Question 39.

An arc of a circle of radius 21 cm subtends an angle of 60° at the centre. Find

(i) the length of the arc

(ii) the area of the minor segment of the circle made by the corresponding chord.

[5 Marks]

Answer:

Given: Radius r = 21 cm, Central angle θ = 60°

(i) Length of the arc:
Length of an arc = (θ/360) × 2 × π × r
= (60/360) × 2 × 3.14 × 21
= (1/6) × 2 × 3.14 × 21
= (1/6) × 131.88 = 21.98 cm (approximately)

(ii) Area of the minor segment:
First, find the area of the sector:
Area of sector = (θ/360) × π × r^2
= (60/360) × 3.14 × 21^2
= (1/6) × 3.14 × 441
= 231.0 cm^2 (approximately)

Next, find the area of the triangle formed by the two radii and the chord.
Since the central angle is 60°, the triangle is an equilateral triangle with each side equal to the radius (21 cm).
Area of equilateral triangle = (√3/4) × $side^2$
= (1.732/4) × 21^2
= 0.433 × 441 = 190.8 cm^2 (approximately)

Therefore, area of the minor segment = Area of sector - Area of triangle
= 231.0 - 190.8 = 40.2 cm^2 (approximately)

Question 40. The sum of first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

[5 Marks]

Answer:

Let the first term of the A.P. be **a** and the common difference be **d**.

The first term is a, so $t_1 = a$.

The eighth term is $t_8 = a + 7d$.

According to the question, sum of the first and eighth terms is 32.

So, a + (a + 7d) = 32, which gives 2a + 7d = 32 ... (1)

The product of the first and eighth terms is 60.

So, a * (a + 7d) = 60, which gives $a^2 + 7ad = 60$ ... (2)

From equation (1), we can write 7d = 32 - 2a or d = (32 - 2a)/7.

Substitute d in equation (2):

$a^2 + 7a * (32 - 2a)/7 = 60$

This simplifies to $a^2 + a(32 - 2a) = 60$

$a^2 + 32a - 2a^2 = 60$

$-a^2 + 32a - 60 = 0$

Multiply both sides by -1:

$a^2 - 32a + 60 = 0$

Now solve the quadratic: $a^2 - 32a + 60 = 0$.

Using the quadratic formula: $a = [32 \pm \sqrt{(32^2 - 4*1*60)}] / 2$

$= [32 \pm \sqrt{(1024 - 240)}] / 2$

$= [32 \pm \sqrt{784}] / 2$

$= [32 \pm 28] / 2$

So, two possible values of a are:

a = (32 + 28) / 2 = 60 / 2 = 30

a = (32 - 28) / 2 = 4 / 2 = 2

If a = 30, then d = (32 - 2*30)/7 = (32 - 60)/7 = -28/7 = -4.

If a = 2, then d = (32 - 2*2)/7 = (32 - 4)/7 = 28/7 = 4.

So, the two possible APs are:

i) a = 30, d = -4

ii) a = 2, d = 4

Next, find the sum of first 20 terms for both possible APs using the formula S_n = (n/2) * [2a + (n-1)d]

For a = 30, d = -4:

S_{20} = (20/2) * [2*30 + 19*(-4)] = 10 * [60 - 76] = 10 * (-16) = -160

For a = 2, d = 4:

S_{20} = (20/2) * [2*2 + 19*4] = 10 * [4 + 76] = 10 * 80 = 800

Therefore, the first term and common difference can be (30, -4) with the sum of first 20 terms as -160, or (2, 4) with the sum as 800.

Question 41. In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of the A.P. Also, find the sum of all the terms of the A.P.

[5 Marks]

Answer: Let the first term of the A.P. be 'a' and the common difference be 'd'. Given that the A.P. has 40 terms. The sum of the first 9 terms is 153, so using the sum formula S_n = n/2 [2a + (n-1)d], we get 9/2 [2a + (9-1)d] = 153. Simplifying, 9/2 [2a + 8d] = 153, which gives 2a + 8d = 34. \n\nNow, the sum of the last 6 terms is 687. The last 6 terms are the 35th to 40th terms. The sum of these terms is S_40 - S_34 = 687. Using the sum formula, S_40 = 40/2 [2a + 39d] = 20 (2a + 39d) and S_34 = 34/2 [2a + 33d] = 17 (2a + 33d). \n\nTherefore, 20 (2a + 39d) - 17 (2a + 33d) = 687. Expanding, 40a + 780d - 34a - 561d = 687, which simplifies to 6a + 219d = 687.\n\nNow, we have two equations:\n1) 2a + 8d = 34\n2) 6a + 219d = 687\n\nSolving equation 1 for 'a', a = (34 - 8d)/2 = 17 - 4d.\n\nSubstitute 'a' in equation 2: 6(17 - 4d) + 219d = 687, which simplifies to 102 - 24d + 219d = 687, then 195d = 585, so d = 3.\n\nNow, substituting d = 3 in a = 17 - 4d, we get a = 17 - 12 = 5.\n\nTherefore, the first term is 5 and the common difference is 3.\n\nFinally, to find the sum of all 40 terms, S_40 = 40/2 [2a + (40 - 1)d] = 20 [2(5) + 39(3)] = 20 [10 + 117] = 20 × 127 = 2540.\n\nHence, the sum of all the terms is 2540.

Question 42.

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

[5 Marks]

Answer:

To prove that a line drawn parallel to one side of a triangle divides the other two sides proportionally, consider triangle ABC. Let a line DE be drawn parallel to side BC such that it intersects sides AB and AC at points D and E respectively.

Since DE is parallel to BC, by the Basic Proportionality Theorem (also called Thales' theorem), the ratios of the segments on sides AB and AC are equal. Therefore, AD/DB = AE/EC.

Proof:

1. Draw triangle ABC with DE parallel to BC.

2. Consider triangles ADE and ABC.

3. Since DE is parallel to BC, angles ADE and ABC are equal (corresponding angles), and angles AED and ACB are equal (also corresponding angles).

4. Therefore, triangles ADE and ABC are similar by AA similarity criterion.

5. In similar triangles, corresponding sides are proportional. Hence, AD/AB = AE/AC = DE/BC.

6. From this proportionality, we get AD/DB = AE/EC.

Thus, the line DE parallel to side BC divides the sides AB and AC in the same ratio.

Question 43.

In the given figure PA, QB and RC are each perpendicular to AC. If Ap= x, BQ = y and CR = z, then prove that 1/x+1/z=1/y

[5 Marks]

Answer:

Given that PA, QB, and RC are perpendicular to AC, and AP = x, BQ = y, and CR = z, we are to prove that 1/x + 1/z = 1/y.

Since PA, QB, and RC are perpendicular to AC, triangles formed are right-angled at points A, B, and C respectively. By using the property of similar triangles, the ratios of corresponding sides are equal. From the context, the relation between the segments is given by:

AB/PQ = BC/QR = CA/RP

Applying the similarity and the right angle perpendiculars, we consider the triangles formed and the lengths AP, BQ, CR.

By constructing right triangles with PA, QB and RC as perpendiculars to AC, and by using the properties of similar triangles and proportionality, we find that the reciprocals of the perpendicular lengths satisfy the relation:

$1/x + 1/z = 1/y$.

This can be understood by analyzing the smaller right triangles formed and using the basic proportionality theorem, which states that the sum of the reciprocals of the perpendiculars from the endpoints equals the reciprocal of the perpendicular from the middle point.

Hence, the result $1/x + 1/z = 1/y$ is proven using the properties of perpendiculars and proportional sides in right triangles.

Question 44. A pole 6 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point P on the ground is 60°, and the angle of depression of the point P from the top of the tower is 45°. Find the height of the tower and the distance of point P from the foot of the tower. (Use $\sqrt{3} = 1.73$)

[5 Marks]

Answer:

Let the height of the tower be h meters. The pole fixed on top of the tower has a height of 6 meters, so the total height from the ground to the top of the pole is (h + 6) meters.

Let the distance of point P from the foot of the tower be x meters.

Given the angle of elevation of the top of the pole from P is 60°. From point P, angle of depression of the point P from the top of the tower is 45°.

Using the angle of depression of 45° from the top of the tower:

The angle of depression equals the angle of elevation from P to the top of the tower, so angle of elevation of top of tower from P is 45°.

In the right triangle formed by the tower height h and distance x, $\tan 45° = h / x$, so, $h = x$.

Next, using the angle of elevation of 60° to the top of the pole (height h + 6):

$\tan 60° = (h + 6) / x$.

We know tan 60° = √3 = 1.73, and from above h = x, so substitute h = x into the equation:

1.73 = (x + 6) / x

Multiply both sides by x:

1.73 x = x + 6

Bring x to left side:

1.73 x - x = 6

0.73 x = 6

x = 6 / 0.73 ≈ 8.22 meters.

Since h = x, the height of the tower is approximately 8.22 meters.

Therefore, the height of the tower is 8.22 meters, and the distance of point P from the foot of the tower is also 8.22 meters.