CBSE EXAMINATION PAPER-2024

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 88

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **44 questions.** All questions are **compulsory.**
ii. This question paper is divided into **5 sections.**
iii. **Section A** – questions number **1 to 20** are multiple choice questions Each question carries **1 marks.**
iv. **Section B** – questions number **21 to 27** are very short answer Each question carries **2 marks.**
v. **Section C** – questions number **28 to 35** are short answer Each question carries **3 marks.**
vi. **Section D** – questions number **36 to 38** are case based questions
vii. **Section E** – questions number **39 to 44** are long answer Each question carries **5 marks.**
viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
ix. Use of calculator is NOT allowed.

Section A

Question 1.

The pair of linear equations $x + 2y + 5 = 0$ and $-3x = 6y - 1$ has

[1 Marks]

(A) exactly two solutions

(B) infinitely many solutions

(C) unique solution

(D) no solution

Explanation: To find the nature of solutions for the pair of equations, write both in standard form. The first equation is $x + 2y + 5 = 0$, and the second can be rewritten as $-3x - 6y + 1 = 0$. Comparing the ratios of coefficients, $(1 / -3) \neq (2 / -6)$ and $(5 / 1) \neq$ the same ratio, meaning the lines intersect at exactly one point. Therefore, the pair has a unique solution.

Question 2.

The common difference of the A.P.

$1/2x, 1 - 4x / 2x, 1 - 8x / 2x, \ldots$ is:

[1 Marks]

(A) $2x$

(B) -2

(C) $-2x$

(D) 2

Explanation: In an arithmetic progression (A.P.), the common difference (d) is the difference between any two consecutive terms. Here, the first term $a_1 = 1/2x$ and the second term $a_2 = (1 - 4x) / 2x$. Calculating the difference $d = a_2 - a_1 = [(1 - 4x) / 2x] - (1 / 2x) = (1 - 4x - 1) / 2x = (-4x) / 2x = -2$. Similarly, the difference between the second and third term is also -2, confirming that -2 is the common difference. Therefore, the correct option is -2.

Question 3. Two dice are thrown together. The probability that they show different numbers is:

[1 Marks]

(A) 1/6

(B) 5/6

(C) 1/3

(D) 2/3

Explanation: When two dice are thrown, there are total 36 possible outcomes. The number of outcomes where both dice show the same number is 6 (1-1, 2-2, 3-3, 4-4, 5-5, 6-6). Therefore, the number of outcomes where the dice show different numbers is 36 - 6 = 30. Hence, the probability that the two dice show different numbers is 30/36 = 5/6.

Question 4.

The probability of guessing the correct answer to a certain test question is x/6. If the probability of not guessing the correct answer is 2/3, then the value of x is:

[1 Marks]

(A) 4

(B) 6

(C) 3

(D) 2

Explanation:

The probability of guessing the correct answer is given as x/6, and the probability of not guessing correctly is 2/3. Since the probabilities of guessing correctly and not guessing correctly must add up to 1, we have (x/6) + (2/3) = 1. Converting 2/3 to 4/6 to have a common denominator, we get x/6 + 4/6 = 1, which means (x + 4)/6 = 1. Therefore, x + 4 = 6, and solving for x gives x = 2. Hence, the correct value of x is 2.

Question 5.

If $a = 2^2 \times 3^x$, $b = 2^2 \times 3 \times 5$, $c = 2^2 \times 3 \times 7$ and LCM(a, b, c) = 3780, then x is equal to

[1 Marks]

(A) 1

(B) 2

(C) 0

(D) 3

Explanation:

To find the value of x, we first write the prime factorization of each number: $a = 2^2 \times 3^x$, $b = 2^2 \times 3^1 \times 5^1$, $c = 2^2 \times 3^1 \times 7^1$. The LCM takes the highest powers of each prime from all numbers. Given LCM = 3780, which factorizes as $2^2 \times 3^3 \times 5 \times 7$. Since the highest power of 3 in the LCM is 3, and b and c have 3^1, the value of x must be 3 to achieve 3^3 in the LCM. Therefore, x = 3.

Question 6.

The zeroes of the quadratic polynomial $2x^2 - 3x - 9$ are:

[1 Marks]

(A) 3, −3/2

(B) 3, 3/2

(C) -3, -3/2

(D) −3, 3/2

Explanation:

To find the zeroes of the quadratic polynomial $2x^2 - 3x - 9$, we use the quadratic formula $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$, where a=2, b=-3, and c=-9. The discriminant $(b^2 - 4ac) = (-3)^2 - 4\times2\times(-9) = 9 + 72 = 81$. $\sqrt{81} = 9$. Therefore, the zeroes are $x = [3 \pm 9] / 4$, which gives x = 3 and x = -3/2. Hence, the correct option is 3 and -3/2.

Question 7.

From a point on the ground, which is 30 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is 60°. The height (in metres) of the tower is:

[1 Marks]

(A) 30

(B) 60

(C) $10\sqrt{3}$

(D) $30\sqrt{3}$

Explanation:

The correct answer is $30\sqrt{3}$. Using right triangle trigonometry, tan(angle) = height / distance. Here, tan 60° = height / 30. Since tan 60° = $\sqrt{3}$, height = $30 \times \sqrt{3}$.

Question 8.

If $\cos \theta = \sqrt{3}/2$ and $\sin \varnothing = 1/2$, then $\tan(\theta + \varnothing)$ is:

[1 Marks]

(A) √3

(B) 1

(C) not defined

(D) 1/√3

Explanation:

Given cos θ = √3/2 implies θ = 30° (or π/6 radians), and sin ∅ = 1/2 implies ∅ = 30° (or π/6 radians). The sum θ + ∅ = 30° + 30° = 60° (or π/3 radians). We know that tan 60° = √3. Hence, tan(θ + ∅) = √3. Therefore, the correct answer is √3.

Question 9. Maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is:

[1 Marks]

(A) 4

(B) 3

(C) 1

(D) 2

Explanation: When two circles intersect at two distinct points, they share exactly two points in common. In this case, the maximum number of common tangents that can be drawn is 2. This is because the circles intersect, so only the direct common tangents, which touch both circles externally without crossing the region between them, are possible. The other common tangents (internal tangents) do not exist as tangents because the circles overlap. Hence, the correct answer is 2.

Question 10.

In the given figure, if PT is a tangent to a circle with centre O and ∠TPO = 35°, then the measure of ∠x is:

[1 Marks]

(A) 115°

(B) 120°

(C) 125°

(D) 110°

Explanation:

In the given figure, PT is tangent at point P to the circle with centre O. The radius OP is perpendicular to the tangent PT at point P, so ∠OPT = 90°. Given ∠TPO = 35°, we can find ∠x = ∠POt by using the fact that triangle OPT is right angled at P. Therefore, ∠x = 90° + 35° = 125°. Hence, the correct option is 125°.

Question 11.

If the diagonals of a quadrilateral divide each other proportionally, then it is a:

[1 Marks]

(A) rectangle

(B) parallelogram

(C) trapezium

(D) square

Explanation: The correct answer is 'parallelogram'. According to the properties of quadrilaterals, if the diagonals bisect each other (divide each other proportionally), the quadrilateral must be a parallelogram. This means the diagonals cut each other into segments that are equal in length on both sides. Other options like rectangle, square, and trapezium have different diagonal properties: rectangles and squares have equal diagonals, and trapeziums do not necessarily have diagonals that bisect each other.

Question 12.

In ΔABC, DE ∥ BC (as shown in the figure). If AD = 2 cm, BD = 3 cm, BC = 7.5 cm, then the length of DE (in cm) is:

[1 Marks]

(A) 2.5

(B) 6

(C) 3

(D) 5

Explanation:

Since DE is parallel to BC, by the Basic Proportionality Theorem (Thales' theorem), DE divides sides AB and AC proportionally. Given AD = 2 cm and BD = 3 cm, the ratio of AD to

DB is 2:3. Using this ratio, the length of DE is (AD / (AD + BD)) × BC = (2 / (2 + 3)) × 7.5 = (2 / 5) × 7.5 = 3 cm. Therefore, the correct answer is 3 cm.

Question 13.

Given HCF(2520, 6600) = 40, LCM(2520, 6600) = 252 × k, then the value of k is:

[1 Marks]

(A) 1650

(B) 1600

(C) 165

(D) 1625

Explanation: We know that for any two numbers, Product of the numbers = HCF × LCM. Here, 2520 × 6600 = 40 × (252 × k). Calculating the left side, 2520 × 6600 = 16632000. The right side is 40 × 252 × k = 10080 × k. Equating, 16632000 = 10080 × k, so k = 16632000 ÷ 10080 = 1650. Therefore, the correct answer is 1650.

Question 14.

A pair of irrational numbers whose product is a rational number is:

[1 Marks]

(A) (√16,√4)

(B) (√3,√27)

(C) (√5,√2)

(D) (√36,√2

Explanation:

The correct pair is (√3, √27). Here, √3 is irrational and √27 = √(9 × 3) = 3√3, which is also irrational. Multiplying these two gives √3 × 3√3 = 3 × (√3 × √3) = 3 × 3 = 9, which is a rational number. Other options do not result in a rational product. This matches the concept that the product of two irrational numbers can sometimes be rational.

Question 15.

If a digit is chosen at random from the digits 1,2,3,4,5,6,7,8,9, then the probability that this digit is an odd prime number is:

[1 Marks]

(A) 2/3

(B) 5/9

(C) 4/9

(D) 1/3

Explanation: The digits from 1 to 9 are {1, 2, 3, 4, 5, 6, 7, 8, 9}. The odd prime numbers in this range are 3, 5, and 7. So there are 3 favorable outcomes. Since there are 9 digits in total, the probability is 3/9 = 1/3.

Question 16.

The mean of five observations is 15. If the mean of first three observations is 14 and that of last three observations is 17, then the third observation is

[1 Marks]

(A) 18

(B) 19

(C) 20

(D) 17

Explanation:

Let the five observations be a, b, c, d, e. Given, mean of five observations = 15, so sum = 15 × 5 = 75.\nMean of first three observations = 14, so a + b + c = 14 × 3 = 42.\nMean of last three observations = 17, so c + d + e = 17 × 3 = 51.\nFrom these, sum of all observations = (a + b + c) + (c + d + e) - c = 42 + 51 - c = 93 - c.\nBut sum of all observations = 75. So, 93 - c = 75 → c = 93 - 75 = 18.\nTherefore, the third observation is 18.

Question 17.

Perimeter of a sector of a circle whose central angle is 90° and radius 7 cm is:

[1 Marks]

(A) 35 cm

(B) 25 cm

(C) 11 cm

(D) 22 cm

Explanation:

The perimeter of a sector is the sum of the two radii and the length of the arc. The length of the arc for a sector with central angle 90° (which is 1/4th of the circle) is (1/4) × 2 × π × radius = (1/4) × 2 × (22/7) × 7 = 11 cm. Adding the two radii (7 cm + 7 cm = 14 cm), the total perimeter = 14 cm + 11 cm = 25 cm. Therefore, the correct option is 25 cm.

Question 18.

In the given figure, O is the centre of the circle. MN is the chord and the tangent ML at point M makes an angle of 70° with MN. The measure of ∠MON is:

[1 Marks]

(A) 120°

(B) 140°

(C) 70°

(D) 90°

Explanation:

Since ML is a tangent at point M and makes an angle of 70° with the chord MN, the angle between the tangent and the chord at point M is 70°. According to the tangent-chord angle theorem, this angle is equal to the angle in the alternate segment, which is the angle MON at the center of the circle. Therefore, the measure of ∠MON is 140°, as the central angle is twice the tangent-chord angle. Hence, the correct option is 140°.

Question 19.

Assertion (A) : The point which divides the line segment joining the points A (1, 2) and B(-1, 1) internally in the ratio 1 : 2 is (-1/3,5/3) Reason (R) : The coordinates of the point which divides the line segment joining the points A (x_1, y_1) and B(x_2, y_2) in the ratio m_1: m_2 are [m_1x_2 +m_2x_1 / m_1 + m_2 , m_1y_2+ m_2y_1 / m_1+m_2

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(B) Assertion (A) is false, but Reason (R) is true.

(C) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(D) Assertion (A) is true, but Reason (R) is false.

Explanation:

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). According to the section formula, the coordinates of the point dividing the line segment joining A(x1, y1) and B(x2, y2) internally in the ratio m1 : m2 are given by ((m1*x2 + m2*x1)/(m1 + m2), (m1*y2 + m2*y1)/(m1 + m2)). Using A(1, 2), B(-1, 1), and ratio 1:2, the point coordinates are ((1*(-1) + 2*1)/(1+2), (1*1 + 2*2)/(1+2)) = ((-1 + 2)/3, (1 + 4)/3) = (1/3, 5/3). However, the assertion states (-1/3, 5/3) which is incorrect for x-coordinate. Thus, the assertion is false and reason is true. Therefore, the correct option is: Assertion (A) is false, but Reason (R) is true.

Question 20.

Assertion (A) : In a cricket match, a batsman hits a boundary 9 times out of 45 balls he plays. The probability that in a given ball, he does not hit the boundary is 4/5. Reason (R) : P(E) + P(not E) =1

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion () is false, but Reason (R) is true.

Explanation:

The Assertion (A) is true because the probability of hitting a boundary in one ball is 9/45 = 1/5. Therefore, the probability of not hitting a boundary is 1 - 1/5 = 4/5, which matches the statement. The Reason (R) is also true as it states the basic probability rule that the sum of the probabilities of an event and its complement is always 1. Moreover, Reason (R) correctly explains Assertion (A) because the calculation of not hitting a boundary uses the formula P(not E) = 1 - P(E). Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Section B

Question 21.

One card is drawn at random from a well shuffled deck of 52 cards. Find the probability that the card drawn

(i) is queen of hearts:

(ii) is not a jack.

[2 Marks]

Answer:

(i) Total number of cards = 52.
Number of queen of hearts = 1.
Probability of drawing queen of hearts = 1/52.

(ii) Total number of jacks = 4.
Number of cards that are not jack = 52 - 4 = 48.
Probability of drawing a card that is not a jack = 48/52 = 12/13.

Question 22. If 2x + y = 13 and 4x – y = 17, find the value of (x – y).

[2 Marks]

Answer: To find the value of (x – y), we start by adding the two given equations:\n\n(2x + y) + (4x - y) = 13 + 17\n\nThis simplifies to:\n6x = 30\n\nDividing both sides by 6, we get:\nx = 5\n\nNext, substitute x = 5 into the first equation:\n2(5) + y = 13\n\nThis gives:\n10 + y = 13\n\nSo, y = 3\n\nFinally, calculate (x – y):\n5 – 3 = 2\n\nTherefore, the value of (x – y) is 2.

Question 23. Sum of two numbers is 105 and their difference is 45. Find the numbers.

[2 Marks]

Answer: Let the two numbers be x and y. According to the question, x + y = 105 and x - y = 45. Adding both equations: (x + y) + (x - y) = 105 + 45, which simplifies to 2x = 150. Therefore, x = 75. Substituting x = 75 in the first equation: 75 + y = 105, so y = 105 - 75 = 30. Hence, the two numbers are 75 and 30.

Question 24. Find a relation between x and y such that the point P(x, y) is equidistant from points A(7, 1) and B(3, 5).

[2 Marks]

Answer: Let the point P(x, y) be equidistant from points A(7, 1) and B(3, 5). Then, the distance from P to A equals the distance from P to B. Using the distance formula: distance PA = sqrt((x - 7)^2 + (y - 1)^2), and distance PB = sqrt((x - 3)^2 + (y - 5)^2). Since PA = PB, we take squares of both distances to avoid square roots: (x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2. Expanding and simplifying this equation gives the relation between x and y. After simplifying, the relation is x + y = 6.

Question 25. Points A(−1, y) and B(5, 7) lie on a circle with centre O(2, −3y) such that AB is a diameter of the circle. Find the value of y. Also, find the radius of the circle.

[2 Marks]

Answer: Since AB is the diameter of the circle, the centre O is the midpoint of A and B. The midpoint formula gives O as ((-1 + 5)/2 , (y + 7)/2) = (2, (y + 7)/2). Given O is (2, −3y), so (y + 7)/2 = −3y. Solving this gives y = -1. With y = -1, the points are A(-1, -1), B(5, 7), and O(2, 3). The radius is the distance from O to A = sqrt[(2 + 1)^2 + (3 + 1)^2] = sqrt[9 + 16] = 5. So, y = -1 and the radius is 5.

Question 26.

In the given figure, EA/EC = EB/ED, prove that ΔEAB ~ ΔECD.

[2 Marks]

Answer: Given that EA/EC = EB/ED, consider triangles EAB and ECD. Angle E is common to both triangles. Since EA/EC = EB/ED, by the Side-Side-Side (SSS) similarity criterion, the two triangles are similar. Therefore, ΔEAB ~ ΔECD.

Question 27.

Evaluate: cos 45° + sin 60° / sec 30° + cosec 30°.

[2 Marks]

Answer: Given the expression cos 45° + (sin 60°) / (sec 30° + cosec 30°), we first find the values of the trigonometric functions: cos 45° = √2 / 2, sin 60° = √3 / 2, sec 30° = 2 / √3, and cosec 30° = 2. Substituting these, the expression becomes (√2 / 2) + (√3 / 2) divided by (2 / √3) + 2. Simplifying the denominator, (2 / √3) + 2 = (2 + 2√3) / √3. Now, write the full expression as (√2 / 2) + (√3 / 2) multiplied by (√3) / (2 + 2√3). Calculating and simplifying, the final answer is approximately 1. The detailed arithmetic shows the expression simplifies to 1.

Section C

Question 28.

If the sum of first 7 terms of an A.P. is 49 and that of first 17 terms is 289, find the sum of its first 20 terms.

[3 Marks]

Answer:

Let the first term of the A.P be 'a' and the common difference be 'd'. The sum of first n terms of an A.P. is given by S_n = (n/2) [2a + (n-1)d].

Given, sum of first 7 terms S_7 = 49. Therefore, (7/2) [2a + 6d] = 49 which gives 7a + 21d = 49.

Also, sum of first 17 terms S_17 = 289. So, (17/2)[2a + 16d] = 289, which simplifies to 17a + 136d = 289.

Now, solve these two equations: 7a + 21d = 49 and 17a + 136d = 289.

Multiplying the first equation by 17 and the second by 7: 119a + 357d = 833 and 119a + 952d = 2023.

Subtracting the first from the second: 595d = 1190 gives d = 2.

Substitute d = 2 into 7a + 21d = 49: 7a + 42 = 49, so 7a = 7, hence a = 1.

Now, find sum of first 20 terms, S_20 = (20/2)[2a + 19d] = 10[2*1 + 19*2] = 10[2 + 38] = 10 * 40 = 400.

Therefore, the sum of the first 20 terms of the A.P. is 400.

Question 29. Find the zeroes of the quadratic polynomial $x^2 - 15$ and verify the relationship between the zeroes and the coefficients of the polynomial.

[3 Marks]

Answer: To find the zeroes of the polynomial $x^2 - 15$, we set the polynomial equal to zero: $x^2 - 15 = 0$. This gives $x^2 = 15$. Taking the square root of both sides, we get two zeroes: x = √15 and x = -√15. Now, the sum of zeroes is √15 + (-√15) = 0. The product of zeroes is √15 × (-√15) = -15. \n\nFor the quadratic polynomial in the form $x^2 + bx + c$, the sum of zeroes is -b and product of zeroes is c. Here, b = 0 and c = -15. So, sum of zeroes = -b = 0 and product of zeroes = c = -15, which matches our calculated values. Thus, the zeroes √15 and -√15 satisfy the relationships with the coefficients of the polynomial.

Question 30.

Solve the following system of linear equations graphically:

x – y + 1 = 0

x + y = 5

[3 Marks]

Answer: To solve the system graphically, first rewrite the equations in slope-intercept form. For the first equation x – y + 1 = 0, rearranged as y = x + 1. For the second equation x + y = 5, rearranged as y = 5 – x. Plot both lines on a graph by choosing values of x and finding corresponding y values. The point where the two lines intersect is the solution to the system. Upon plotting, the lines intersect at the point (2, 3), which means x = 2 and y = 3 satisfy both equations. Therefore, the solution of the system is x = 2 and y = 3.

Question 31.

Find the ratio in which the line segment joining the points (5, 3) and (–1, 6) is divided by Y-axis.

[3 Marks]

Answer: To find the ratio in which the y-axis divides the line segment joining points (5, 3) and (–1, 6), we first note that the y-axis is the line where x = 0. Let the point dividing the segment on the y-axis be P(0, y). Suppose the ratio in which P divides AB is k : 1. Using the section formula for the x-coordinate, we have:\n\n0 = (k * (–1) + 1 * 5) / (k + 1)\n=> 0 = (–k + 5) / (k + 1)\n=> –k + 5 = 0\n=> k = 5\n\nTherefore, the ratio is 5 : 1. To find y-coordinate of P, use the section formula for y:\n\ny = (k * 6 + 1 * 3) / (k + 1) = (5 * 6 + 3) / (5 + 1) = (30 + 3) / 6 = 33 / 6 = 5.5\n\nSo, the point P is (0, 5.5), and the y-axis divides the segment in the ratio 5 : 1.

Question 32.

Prove that $\sin \theta - 2 \sin^3 \theta / 2 \cos^3 \theta - \cos \theta = \tan \theta$.

[3 Marks]

Answer: To prove that $(\sin \theta - 2 \sin^3 \theta) / (2 \cos^3 \theta - \cos \theta) = \tan \theta$, start by factoring both numerator and denominator. The numerator $\sin \theta - 2 \sin^3 \theta$ can be written as $\sin \theta (1 - 2 \sin^2 \theta)$. Using the identity $\sin^2 \theta + \cos^2 \theta = 1$, we replace $\sin^2 \theta$ with $1 - \cos^2 \theta$. So, $1 - 2 \sin^2 \theta$ becomes $1 - 2(1 - \cos^2 \theta) = 1 - 2 + 2 \cos^2 \theta = 2 \cos^2 \theta - 1$. Thus, numerator equals $\sin \theta (2 \cos^2 \theta - 1)$. The denominator $2 \cos^3 \theta - \cos \theta$ can be factored as $\cos \theta (2 \cos^2 \theta - 1)$. Now the expression becomes $[\sin \theta (2 \cos^2 \theta - 1)] / [\cos \theta (2 \cos^2 \theta - 1)]$. The terms $(2 \cos^2 \theta - 1)$ cancel out, leaving $\sin \theta / \cos \theta$, which is $\tan \theta$. Hence, the given expression equals $\tan \theta$, as required.

Question 33. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

[3 Marks]

Answer: Consider a circle with center O and a chord AB. Tangents are drawn at points A and B, touching the circle at these points. We need to prove that the angles formed between the tangents and the chord AB at points A and B are equal. Since the tangent to a circle is perpendicular to the radius at the point of contact, the tangent at A is perpendicular to OA and the tangent at B is perpendicular to OB. In triangle OAB, OA and OB are equal as they are radii of the same circle. Therefore, triangle OAB is isosceles and angles OAB and OBA are equal. Since the tangent is perpendicular to the radius, each angle between the tangent and the chord is equal to the adjacent angle in triangle OAB. Hence, the two tangents at the end points of the chord AB make equal angles with the chord AB.

Question 34.

The ratio of the 10^{th} term to its 30th term of an A.P. is 1 : 3 and the sum of its first six terms is 42. Find the first term and the common difference of A.P.

[3 Marks]

Answer: Let the first term of the AP be a and the common difference be d. The 10th term (T10) is a + 9d and the 30th term (T30) is a + 29d. Given, the ratio T10 : T30 = 1 : 3, so (a + 9d) / (a + 29d) = 1/3. Cross-multiplying gives 3(a + 9d) = a + 29d, which simplifies to 3a + 27d = a + 29d or 2a = 2d, so a = d. The sum of the first six terms S6 = 6/2 [2a + (6 - 1)d] = 3[2a + 5d] = 42. Since a = d, substitute to get 3[2d + 5d] = 42, so 3 * 7d = 42, which means 21d = 42, giving d = 2. Therefore, a = 2. Hence, the first term is 2 and the common difference is 2.

Question 35.

P(−2, 5) and Q(3, 2) are two points. Find the coordinates of the point R on line segment PQ such that PR = 2QR.

[3 Marks]

Answer: Given points P(−2, 5) and Q(3, 2), we need to find point R on PQ such that PR = 2QR. Since R lies on the line segment PQ, its coordinates can be found using the section formula. Let point R divide PQ in the ratio m:n where PR = 2QR means m:n = 2:1. Using the section formula, the coordinates of R are ((2*3 + 1*(-2)) / (2+1), (2*2 + 1*5) / (2+1)) = ((6 - 2) / 3, (4 + 5) / 3) = (4/3, 3). Therefore, R has coordinates (4/3, 3).

Section D

Question 36.

A stable owner has four horses. He usually ties these horses with 7 m long rope to pegs at each corner of a square shaped grass field of 20 m length, to graze in his farm. But tying with rope sometimes results in injuries to his horses, so he decided to build a fence around the area so that each horse can graze.

Based on the above, answer the following questions :

> (1)
>
> **(a) Find the area of the total field in which these horses can graze.**
>
> [2 Marks]

Answer: The field is square-shaped with each side measuring 20 m. Therefore, the area of the field is given by side × side = 20 m × 20 m = 400 square meters. Since all four horses can graze freely inside this fenced field, the total grazing area for the horses is 400 square meters.

Key Points: The field is square shaped with side 20 m-The area of square = side × side-Area = 20 m × 20 m = 400 m^2-All four horses can graze in this entire field

(2) Find the area of the square shaped grass field.

[1 Marks]

Answer: The area of a square is calculated by squaring its side length. Here, the side of the square field is 20 meters. Therefore, the area = 20 meters × 20 meters = 400 square meters.

Key Points: Side of square field is 20 meters - Area of square = side × side - Calculate 20 × 20 to get 400 square meters

(3)

What is area of the field that is left ungrazed, if the length of the rope of each horse is 7 cm?

[1 Marks]

Answer: The length of the side of the square field is 20 m. Each horse is tied with a 7 m long rope at each corner. The total area of the field is 20 × 20 = 400 m^2. Each horse can graze a quarter circle area around the corner with radius equal to the length of the rope. Area accessible to one horse is $(1/4) \times \pi \times 7^2 = (1/4) \times 3.14 \times 49 \approx 38.465$ m^2. For all four horses, total grazing area = 4 × 38.465 ≈ 153.86 m^2. Area left ungrazed = Total field area - Total grazing area = 400 - 153.86 ≈ 246.14 m^2. So, approximately 246.14 m^2 of the field is left ungrazed.

Key Points: Calculate total area of the square field (side × side)-Calculate grazing area for one horse as quarter circle $(1/4 \times \pi \times r^2)$-Multiply by 4 for all horses-Subtract total grazing area from total field area to find ungrazed area-Mention approximate values and units clearly

(4)

If the length of the rope of each horse is increased from 7 m to 10 m, find the area grazed by one horse. (Use π = 3.14)

[2 Marks]

Answer: The area grazed by one horse tied with a 10 m long rope is the area of a circle with radius 10 m. Using the formula for the area of a circle, Area = π × radius × radius. Substituting the values, Area = 3.14 × 10 × 10 = 314 square meters. Therefore, the area grazed by one horse with a 10 m long rope is 314 m^2.

Key Points: Area grazed by horse is circular.-Use formula of area of circle: π × radius^2.-Radius is length of rope, which is 10 m.-Calculate area as 3.14 × 10 × 10=314 m^2.-Final answer: 314 square meters.

Question 37.

Vocational training complements traditional education by providing practical skills and hands-on experience. While education equips individuals with a broad knowledge base, vocational training focuses on job-specific skills, enhancing employability thus making the student selfreliant. Keeping this in view, a teacher made the following table giving the frequency distribution of students/adults undergoing vocational training from the training institute.

From the above answer the following questions :

(1) What is the lower limit of the modal class of the above data?

[1 Marks]

Answer: The modal class is the class interval with the highest frequency. From the given data, the highest frequency is 15 for the class interval 60-70. Therefore, the modal class is 60-70, and the lower limit of this modal class is 60.

Key Points: Modal class is the class interval with highest frequency - Identify the class interval with maximum frequency (60-70) - The lower limit is the first number in the modal class interval (60)

(2)

Find the median class of the above data.

[2 Marks]

Answer: To find the median class, first find the total number of students by adding all frequencies: 3 + 7 + 12 + 15 + 8 + 3 + 2 = 50. The median position is at (50 ÷ 2) = 25th student. Next, find the cumulative frequencies: 30-40: 3, 40-50: 3 + 7 = 10, 50-60: 10 + 12 = 22, 60-70: 22 + 15 = 37. Since the 25th student lies in the class 60-70 (as $22 < 25 \leq 37$), the median class is 60-70.

> Key Points: Calculate total frequency - Find median position (total frequency ÷ 2) - Calculate cumulative frequencies - Identify class interval containing the median position

(3) Give the empirical relationship between mean, median and mode.

[1 Marks]

Answer: The empirical relationship between mean, median and mode is given by the formula: Median = Mode + 2 × (Mean - Median). This helps to understand how these three measures of central tendency are related to each other in a data set.

> Key Points: Empirical relationship-Median equals Mode plus two times the difference of Mean and Median-Relation helps compare mean, median and mode

(4)

Find the number of participants of age less than 50 years who undergo vocational training.

[2 Marks]

Answer: To find the number of participants whose age is less than 50 years, we add the frequencies of all age groups below 50 years from the table. These age groups typically include 10-25, 25-40, and 40-50. By adding the frequencies in these intervals, we get the total number of participants under 50 years undergoing vocational training.

Key Points: Identify all age groups with upper limit less than 50 years - Sum the frequencies of these age groups - Write the total as the number of participants below 50 years

Question 38.

Teaching Mathematics through activities is a powerful approach that enhances students' understanding and engagement. Keeping this in mind, Ms. Mukta planned a prime number game for class 5 students. She announces the number 2 in her class and asked the first student to multiply it by a prime number and then pass it to second student. Second student also multiplied it by a prime number and passed it to third student. In this way by multiplying to a prime number, the last student got 173250.

Now, Mukta asked some questions as given below to the students :

(1) What is the least prime number used by students?

[1 Marks]

Answer: The least prime number used by the students is 2 because the teacher started the game by announcing the number 2, which is the smallest prime number.

Key Points: The game started with number 2 - 2 is the smallest prime number - Students multiplied by prime numbers starting from 2 - Therefore, the least prime number used is 2

(2) Which prime number has been used maximum times?

[1 Marks]

Answer: The prime number 5 has been used the maximum number of times because the number 173250 ends with a zero, indicating multiplication by 5 multiple times.

Key Points: The final number 173250 ends with zero which implies multiplication by 5s-The prime factor 5 is used multiple times to create factors ending with zero-Understanding prime factorization helps identify which prime number appears most frequently

(3)

How many students are in the class?

[2 Marks]

Answer: First, we express 173250 as a product of prime numbers. We start by dividing 173250 by 2 (the initial number). Then, we divide the result by prime numbers repeatedly until we reach 1. Counting how many prime numbers multiplied gives 173250 tells us how many students are in the class, as each student multiplied by one prime number. The prime factorization of 173250 is 2 × 3 × 3 × 5 × 5 × 7 × 11 × 13. There are 8 prime numbers multiplied after the initial 2, so there are 9 students in total (including the first student).

Key Points: Understand that each student multiplies by one prime number- Use prime factorization to express 173250 as a product of prime numbers- Count the number of prime factors to find how many students multiplied- Include the initial number 2 as the first student's contribution

(4)

What is the highest prime number used by students?

[2 Marks]

Answer: The highest prime number used by the students is 19. This is found by factorising the final number 173250 into its prime factors which includes 2, 3, 5, and 19. Among these, 19 is the largest prime number.

Key Points: The final number 173250 can be factorised into prime numbers - prime factorisation is used - identifying the largest prime factor gives the highest prime number used - the highest prime factor is 19

Section E

Question 39. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and by doing so, the time of flight is increased by 30 minutes. Find the original duration of the flight.

[5 Marks]

Answer: Let the original speed of the aircraft be S km/h, and the original time of the flight be T hours. We know the distance of the flight is 2800 km. Using the relation distance = speed × time, we have 2800 = S × T. When the average speed is reduced by 100 km/h, the speed becomes (S - 100) km/h, and the time increases by 30 minutes (which is 0.5 hours). So, the new time is (T + 0.5) hours. Using the distance formula again for the new conditions, 2800 = (S - 100) × (T + 0.5). Now, we have two equations: 1) 2800 = S × T, and 2) 2800 = (S - 100) × (T + 0.5). Substitute S from the first equation into the second: 2800 = (2800 / T - 100) × (T + 0.5). Expanding and simplifying this equation will give a quadratic equation in T. Solving it, we get two values, one of which is physically meaningful (positive time). The original duration of the flight, T, is 4 hours. Hence, the aircraft originally took 4 hours to complete the 2800 km flight.

Question 40.

The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is 2 X 16 / 21, find the fraction.

[5 Marks]

Answer:

Let the numerator of the fraction be x. Then the denominator is one more than twice the numerator, so the denominator is 2x + 1. Thus, the fraction is x / (2x + 1).

The reciprocal of this fraction is (2x + 1) / x.

The sum of the fraction and its reciprocal is given as 2 times 16/21, which is (2 * 16) / 21 = 32 / 21.

So, we write the equation:

(x / (2x + 1)) + ((2x + 1) / x) = 32 / 21

Multiply both sides by x(2x + 1) to clear the denominators:

x * x + (2x + 1) * (2x + 1) = (32 / 21) * x * (2x + 1)

Simplify left side: x squared + (2x + 1)^2 = (32 / 21) * x * (2x + 1)

Expand (2x + 1)^2: (2x)^2 + 2 * 2x * 1 + 1^2 = 4x^2 + 4x + 1

So, x^2 + 4x^2 + 4x + 1 = (32 / 21) * x * (2x + 1)

Combine like terms: 5x^2 + 4x + 1 = (32 / 21) * x * (2x + 1)

Expand right side: (32 / 21) * (2x^2 + x) = (64x^2 / 21) + (32x / 21)

Multiply both sides by 21 to remove denominator:

21 * (5x^2 + 4x + 1) = 64x^2 + 32x

105x^2 + 84x + 21 = 64x^2 + 32x

Bring all terms to one side:

105x^2 - 64x^2 + 84x - 32x + 21 = 0

41x^2 + 52x + 21 = 0

Use quadratic formula to solve for x:

x = [-52 ± sqrt(52^2 - 4 * 41 * 21)] / (2 * 41)

Calculate discriminant:

52^2 = 2704

4 * 41 * 21 = 3444

Discriminant = 2704 - 3444 = -740 (negative)

The negative discriminant suggests no real solution, so check calculations again.

Checking the step where equation was formed:

Sum of fraction and reciprocal = (x / (2x + 1)) + ((2x + 1) / x) = 32 / 21

Multiply both sides by x(2x + 1): x^2 + (2x + 1)^2 = (32 / 21) * x(2x + 1)

Left side: x^2 + (4x^2 + 4x + 1) = 5x^2 + 4x + 1

Right side: (32/21)(2x^2 + x) = (64x^2/21) + (32x/21)

Multiply both sides by 21:

21(5x^2 + 4x + 1) = 64x^2 + 32x

105x^2 + 84x + 21 = 64x^2 + 32x

Bring all terms to left:

105x^2 - 64x^2 + 84x - 32x + 21 = 0

41x^2 + 52x + 21 = 0

Discriminant is negative, so no real solution. Reassess the original problem or consider alternative approach.

Alternate method: Let fraction be x/(2x + 1). Sum with reciprocal is:

Fraction + Reciprocal = $(x / (2x + 1)) + ((2x + 1) / x) = (x^2 + (2x + 1)^2) / (x(2x + 1))$

Given sum = 32 / 21, so:

$(x^2 + (2x + 1)^2) / (x(2x +1)) = 32 / 21$

Cross multiply:

$21(x^2 + (2x + 1)^2) = 32 x (2x +1)$

Expand numerator:

$21[x^2 + 4x^2 + 4x + 1] = 32 x (2x +1)$

$21(5x^2 + 4x + 1) = 32(2x^2 + x)$

$105x^2 + 84x + 21 = 64x^2 + 32x$

Bring all to one side:

$41x^2 + 52x + 21 = 0$

Solve quadratic equation:

No real roots due to negative discriminant.

Thus, the fraction does not have real values for x.

But since the question expects a value, possibly the sum is 2 times (16/21) = (32/21). If the problem has a typo or expects approximate solution, this is the process.

Final answer: Fraction is $x / (2x + 1)$ where x satisfies $41x^2 + 52x + 21 = 0$. Since there's no real solution, the problem parameters may need revision.

Question 41. State and prove Basic Proportionality theorem.

[5 Marks]

Answer:

Basic Proportionality Theorem (Thales' Theorem): If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides those two sides proportionally.

Consider triangle ABC, with a line DE drawn parallel to side BC such that it intersects AB at D and AC at E.

According to the theorem, AD divided by DB equals AE divided by EC, or mathematically, AD / DB = AE / EC.

Proof:

Since DE is parallel to BC, by the corresponding angles postulate, angle ADE is equal to angle ABC, and angle DEA is equal to angle ACB.

Looking at triangles ADE and ABC, these two pairs of angles are equal; hence, triangle ADE is similar to triangle ABC by the AA (Angle-Angle) similarity criterion.

From the similarity of triangles ADE and ABC, corresponding sides are proportional:

AD / AB = AE / AC = DE / BC.

Rearranging the first two ratios, we get AD / DB = AE / EC.

This proves that the line DE divides sides AB and AC proportionally when drawn parallel to BC.

Question 42. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

[5 Marks]

Answer:

Let the transmission tower be fixed at the top of a 20 m high building. Denote the height of the tower as h meters. From a point on the ground, the angle of elevation to the bottom of the tower (which is the top of the building) is 45°, and to the top of the tower is 60°.

Let the horizontal distance from the observation point to the building be d meters.

Using the angle of elevation 45° to the bottom of the tower (top of the building):

$\tan 45° = \text{height of building} / \text{distance} = 20 / d$

Since $\tan 45° = 1$, we get d = 20 meters.

Using the angle of elevation 60° to the top of the tower (building + tower height):

$\tan 60° = (20 + h) / d$

We know $\tan 60° = \sqrt{3}$ and d = 20, therefore:

$\sqrt{3} = (20 + h) / 20$

$\Rightarrow 20 + h = 20\sqrt{3}$

$\Rightarrow h = 20(\sqrt{3} - 1)$

Using the value of $\sqrt{3} \approx 1.732$,

$h \approx 20 * (1.732 - 1) = 20 * 0.732 = 14.64$ meters.

Hence, the height of the transmission tower is approximately 14.64 meters.

Question 43. A solid iron pole consists of a solid cylinder of height 200 cm and base diameter 28 cm, which is surmounted by another cylinder of height 50 cm and radius 7 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass.

[5 Marks]

Answer: To find the mass of the iron pole, first calculate the volume of each cylindrical part and then add them to find the total volume. The first cylinder has a height of 200 cm and a diameter of 28 cm, so its radius is half of the diameter, which is 14 cm. The volume of a cylinder is given by $\pi \times \text{radius}^2 \times \text{height}$. Therefore, volume of the first cylinder = 3.14 × 14 × 14 × 200 = 123,176 cm^3. The second cylinder has a height of 50 cm and a radius of 7 cm. Its volume is 3.14 × 7 × 7 × 50 = 7,693 cm^3. Adding the two volumes gives the total volume = 123,176 + 7,693 = 130,869 cm^3. Given that 1 cm^3 of iron has a mass of 8 g, the total mass is volume × density = 130,869 × 8 = 1,046,952 grams, which is 1,046.952 kilograms. Hence, the mass of the iron pole is approximately 1,047 kilograms.

Question 44. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 4 mm. Find its surface area. Also, find its volume.

[5 Marks]

Answer:

The medicine capsule consists of a cylinder with two hemispheres attached at the ends. The total length of the capsule is 14 mm and the diameter is 4 mm, so the radius is half of the diameter, which is 2 mm.

First, we calculate the length of the cylindrical part. The capsule length includes two hemispheres, which together form a sphere of diameter 4 mm. The length of the cylinder is total length minus diameter, i.e., 14 mm - 4 mm = 10 mm.

Now, to find the surface area, we calculate the curved surface area of the cylinder and the surface area of the sphere formed by the two hemispheres.

Curved surface area of cylinder = $2 \times \pi \times \text{radius} \times \text{height} = 2 \times 3.14 \times 2 \times 10 = 125.6\ mm^2$.

Surface area of sphere = $4 \times \pi \times \text{radius}^2 = 4 \times 3.14 \times 2^2 = 50.24\ mm^2$.

Total surface area = curved surface area of cylinder + surface area of sphere = 125.6 + 50.24 = 175.84 mm^2.

Next, to find the volume:

Volume of cylinder = $\pi \times \text{radius}^2 \times \text{height} = 3.14 \times 2^2 \times 10 = 125.6\ mm^3$.

Volume of sphere = $(4/3) \times \pi \times \text{radius}^3 = (4/3) \times 3.14 \times 8 = 33.51\ mm^3 \times 4 = 33.51 \times 4 =$
Actually, $(4/3) \times \pi \times 2^3 = (4/3) \times 3.14 \times 8 = 33.51\ mm^3$.

Volume of both hemispheres = volume of the sphere = $33.51\ mm^3$.

Total volume = volume of cylinder + volume of sphere = $125.6 + 33.51 = 159.11\ mm^3$.

Therefore, the surface area of the capsule is approximately $175.84\ mm^2$ and the volume is approximately $159.11\ mm^3$.
