CBSE EXAMINATION PAPER-2024

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 88

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **44 questions.** All questions are **compulsory.**

ii. This question paper is divided into **5 sections.**

iii. **Section A** – questions number **1 to 20** are multiple choice questions Each question carries **1 marks.**

iv. **Section B** – questions number **21 to 27** are very short answer Each question carries **2 marks.**

v. **Section C** – questions number **28 to 35** are short answer Each question carries **3 marks.**

vi. **Section D** – questions number **36 to 38** are case based questions

vii. **Section E** – questions number **39 to 44** are long answer Each question carries **5 marks.**

viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.

ix. Use of calculator is NOT allowed.

Section A

Question 1.

If $ax + by = a^2 - b^2$ and $bx + ay = 0$, then the value of $x + y$ is:

[1 Marks]

(A) a + b

(B) $a^2 - b^2$

(C) a – b

(D) $a^2 + b^2$

Explanation: Given the equations $ax + by = a^2 - b^2$ and $bx + ay = 0$, we can solve for x and y. From the second equation, $bx + ay = 0$, rearranging gives $y = -(b/a)x$. Substitute y into the first equation: $a x + b(-(b/a)x) = a^2 - b^2$, which simplifies to $a x - (b^2 / a) x = a^2 - b^2$, or $(a^2 - b^2)/a \, x = a^2 - b^2$, leading to $x = a$. Then $y = -(b/a) * a = -b$. Therefore, $x + y = a - b$. Thus, the correct option is 'a – b'.

Question 2.

The HCF of two numbers 65 and 104 is 13. If LCM of 65 and 104 is 40x, then the value of x is:

[1 Marks]

(A) 40

(B) 8

(C) 13

(D) 5

Explanation:

We know that for two numbers, the product of their HCF and LCM is equal to the product of the numbers. Given, HCF(65, 104) = 13 and LCM(65, 104) = 40x. Therefore, 65 × 104 = 13 × 40x. Calculating 65 × 104 = 6760 and 13 × 40x = 520x. So, 6760 = 520x, which gives x = 6760 ÷ 520 = 13.

Question 3.

If a polynomial p(x) is given by $p(x) = x^2 - 5x + 6$, then the value of p(1) + p(4) is:

[1 Marks]

(A) –4

(B) 4

(C) 2

(D) 0

Explanation:

To find p(1) + p(4), first calculate p(1) and p(4) separately using the polynomial $p(x) = x^2 - 5x + 6$. \n\n$p(1) = (1)^2 - 5(1) + 6 = 1 - 5 + 6 = 2$.\n\n$p(4) = (4)^2 - 5(4) + 6 = 16 - 20 + 6 = 2$.\n\nTherefore, p(1) + p(4) = 2 + 2 = 4. Hence, the correct answer is 4.

Question 4.

If the discriminant of the quadratic equation $3x^2 – 2x + c = 0$ is 16, then the value of c is:

[1 Marks]

(A) –1

(B) √2

(C) 1

(D) 0

Explanation: The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by $b^2 - 4ac$. Here, a = 3, b = -2, and the discriminant is 16. So, $(-2)^2 - 4 \times 3 \times c = 16$. This simplifies to $4 - 12c = 16$, and then $-12c = 16 - 4 = 12$. Therefore, c = -1. Hence, the correct value of c is -1.

Question 5. If an arc subtends an angle of 90° at the centre of a circle, then the ratio of its length to the circumference of the circle is:

[1 Marks]

(A) 1 : 4

(B) 2 : 3

(C) 4 : 1

(D) 1 : 3

Explanation: The length of an arc subtending an angle θ at the centre of a circle with radius r is given by (θ/360) × circumference. Here, θ = 90°. So, the ratio of the arc length to the circumference is 90°/360° = 1/4. Therefore, the correct ratio is 1 : 4.

Question 6.

The area of the sector of a circle of radius 12 cm is 60π cm^2. The central angle of this sector is:

[1 Marks]

(A) 120°

(B) 150°

(C) 6°

(D) 75°

Explanation:

The formula for the area of a sector is (central angle / 360) × π × radius². Given the area is 60π and the radius is 12 cm, we have (central angle / 360) × π × 12 × 12 = 60π. Simplifying, (central angle / 360) × 144 = 60, so central angle / 360 = 60 / 144 = 5/12. Therefore, central angle = 360 × 5/12 = 150°. Hence, the correct option is 150°.

Question 7.

If the difference of mode and median of a data is 24, then the difference of its median and mean is:

[1 Marks]

(A) 8

(B) 12

(C) 24

(D) 36

Explanation:

Using the empirical relationship in statistics for moderately skewed data, Mode - Median = 3(Mean - Median). Given Mode - Median = 24, we get 24 = 3(Mean - Median), which implies Mean - Median = 24 / 3 = 8. Therefore, the difference between the median and mean is 8.

Question 8. Two dice are tossed simultaneously. The probability of getting odd numbers on both the dice is:

[1 Marks]

(A) 6/36

(B) 3/36

(C) 9/36

(D) 12/36

Explanation: Each die has 6 faces with numbers 1 to 6. The odd numbers on a die are 1, 3, and 5, so there are 3 odd numbers on each die. The total number of possible outcomes when two dice are tossed is 6 × 6 = 36. To get odd numbers on both dice, the first die must show an odd number (3 possibilities) and the second die must also show an odd number (3 possibilities). Therefore, the total favorable outcomes are 3 × 3 = 9. Hence, the probability is 9/36.

Question 9. The ratio of total surface area of a solid hemisphere to the square of its radius is:

[1 Marks]

(A) $2\pi : 1$

(B) $4\pi : 1$

(C) $3\pi : 1$

(D) $1 : 4\pi$

Explanation: The total surface area of a solid hemisphere includes its curved surface area and the area of its circular base. The curved surface area of a hemisphere is half the surface area of a sphere, which is $2\pi r^2$, and the base area is πr^2. Therefore, total surface area = $2\pi r^2 + \pi r^2 = 3\pi r^2$. Dividing this by r^2 gives the ratio as $3\pi : 1$.

Question 10.

If $\sin\theta = 1$, then the value of $(1/2 \sin(\theta/2))$ is:

[1 Marks]

(A) 0

(B) 1/2

(C) $1/\sqrt{2}$

(D) $1/2\sqrt{2}$

Explanation:

Given $\sin\theta = 1$, this means $\theta = 90^\circ$ or $\pi/2$ radians. Therefore, $\theta/2 = 45^\circ$ or $\pi/4$ radians. We know $\sin 45^\circ = 1/\sqrt{2}$. Hence, $(1/2) \times \sin(\theta/2) = (1/2) \times (1/\sqrt{2}) = 1/(2\sqrt{2})$. Thus, the correct option is $1/2\sqrt{2}$.

Question 11.

Two lines are given to be parallel. The equation of one of these lines is $5x - 3y = 2$. The equation of the second line can be:

[1 Marks]

(A) $-15x - 9y = 5$

(B) $15x + 9y = 5$

(C) $-15x + 9y = 5$

(D) $9x - 15y = 6$

Explanation:

Two lines are parallel if their slopes are equal. For the line $5x - 3y = 2$, rewriting in slope intercept form $y = mx + c$ gives $y = (5/3)x - 2/3$, so the slope $m = 5/3$. Therefore, the second line must have the same slope 5/3. Checking the options, the line $-15x - 9y = 5$ can be rewritten as $-9y = 15x + 5 \Rightarrow y = -(15/9)x - 5/9 = -(5/3)x - 5/9$, which slope is $-5/3$, not equal. The line $15x + 9y = 5$ can be rewritten as $9y = -15x + 5 \Rightarrow y = -(15/9)x + 5/9 = -(5/3)x + 5/9$, slope $-5/3$, no. The line $-15x + 9y = 5$ can be rewritten as $9y = 15x + 5 \Rightarrow y = (15/9)x + 5/9 = (5/3)x + 5/9$, slope 5/3, same as first line. Lastly, the line $9x - 15y = 6$ is $y = (9/15)x - 6/15 = (3/5)x - 2/5$, slope 3/5, not equal. So, the correct option is $-15x + 9y = 5$, which has the same slope 5/3, making it parallel to the given line.

Question 12. Three numbers in A.P. have the sum 30. What is its middle term?

[1 Marks]

(A) 4

(B) 10

(C) 16

(D) 8

Explanation: Let the three numbers be (a - d), a, and (a + d) where 'a' is the middle term. The sum of these three numbers is (a - d) + a + (a + d) = 3a, which is given as 30. So, 3a = 30, hence a = 10. Therefore, the middle term is 10.

Question 13.

In ΔABC, DE || BC.(as shown in the figure). If AD = 4 cm, AB = 9 cm and AC = 13.5 cm, then the length of EC is:

[1 Marks]

(A) 6cm

(B) 5.7cm

(C) 9 cm

(D) 7.5cm

Explanation:

Since DE is parallel to BC, triangles ADE and ABC are similar by the Basic Proportionality Theorem (Thales Theorem). Thus, corresponding sides are proportional. Given AD = 4 cm and AB = 9 cm, the ratio AD:AB = 4:9. Using similarity, AE:AC = 4:9. Given AC = 13.5 cm, AE = (4/9) × 13.5 = 6 cm. Since AC = AE + EC, EC = AC - AE = 13.5 - 6 = 7.5 cm. Therefore, the length of EC is 7.5 cm.

Question 14. At some time of the day, the length of the shadow of a tower is equal to its height. Then, the Sun's altitude at that time is:

[1 Marks]

(A) 30°

(B) 45°

(C) 90°

(D) 60°

Explanation: When the length of the shadow of a tower is equal to its height, it means that the angle of elevation of the Sun is 45°. This is because the height and shadow form two equal sides of a right triangle, which happens only when the angle opposite the height (the Sun's altitude) is 45°. Therefore, the correct Sun's altitude at that time is 45°.

Question 15.

In the given figure, AB and AC are tangents to the circle. If ∠ABC = 42°, then the measure of ∠BAC is:

[1 Marks]

(A) 42°

(B) 86°

(C) 96°

(D) 106°

Explanation:

Since AB and AC are tangents to the circle from point A, triangle ABC is isosceles with AB = AC. Hence, the angles at B and C are equal. Given ∠ABC = 42°, ∠ACB = 42°. The sum of angles in triangle ABC is 180°, so ∠BAC = 180° - 42° - 42° = 96°. Therefore, the correct answer is 96°.

Question 16.

The fourth vertex D of a parallelogram ABCD whose three vertices are A(−2, 3), B(6, 7) and C(8, 3) is:

[1 Marks]

(A) (0,1)

(B) (-1,0)

(C) (0, -1)

(D) (1, 0)

Explanation: In a parallelogram, the fourth vertex D can be found using the vector relation: D = A + C - B. Substituting the given coordinates: $D_x = (-2) + 8 - 6 = 0$ and $D_y = 3 + 3 - 7 = -1$. Therefore, the coordinates of point D are (0, -1). Hence, the correct option is (0, -1).

Question 17.

For an event E, if $P(E) + P(\bar{E}) = q$, then the value of $q^2 - 4$ is:

[1 Marks]

(A) 5

(B) 3

(C) −3

(D) −5

Explanation: According to the probability rule, the sum of the probability of an event E and its complement $\bar{E}$ is always 1. Thus, $P(E) + P(\bar{E}) = 1$. Therefore, q = 1. Now, $q^2 - 4 = 1^2 - 4 = 1 - 4 = -3$. Hence, the correct answer is -3.

Question 18.

In the given figure, QR is a common tangent to two circles touching externally at A. The tangent at A meets QR at P. If AP = 4.2 cm, then the length of QR is:

[1 Marks]

(A) 4.2 cm

(B) 6.3 cm

(C) 8.4 cm

(D) 2.1 cm

Explanation: Since QR is a common tangent to the two circles touching externally at A, and P is the point where the tangent at A meets QR, AP is the segment from A to P. Because tangent segments from a point outside the circle are equal, the length of QR, which includes segment AP twice (on either side of P), is twice AP. Therefore, QR = 2 × AP = 2 × 4.2 cm = 8.4 cm.

Question 19.

Assertion (A) : Mid-point of a line segment divides the line segment in the ratio 1: 1.

Reason (R): The ratio in which the point (-3, k) divides the line segment joining the points (- 5, 4) and (- 2,3)is 1: 2.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(B) Assertion (A) is true, but Reason (R) is false.

(C) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(D) Assertion (A) is false, but Reason (R) is true.

Explanation:

The Assertion (A) is true because by definition, the midpoint of a line segment divides it into two equal parts, hence the ratio is 1:1. The Reason (R) is also true as the point (-3, k) divides the line segment joining (-5, 4) and (-2, 3) in the ratio 1:2 (this can be verified by applying the section formula), but it is not the correct explanation of Assertion (A).

Therefore, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Question 20.

Assertion (A) : If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason (R): Circumference = 21 x radius of a circle.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Assertion (A) is false, but Reason (R) is true.

(C) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(D) Assertion (A) is true, but Reason (R) is false.

Explanation:

The formula for the circumference of a circle is Circumference = 2 × π × radius. Given circumference = 176 cm, solving for radius: radius = 176 / (2 × 3.14) ≈ 28 cm. Therefore, Assertion (A) is true. However, Reason (R) states that Circumference = 21 × radius, which is incorrect. Hence, Reason (R) is false. So, the correct option is: Assertion (A) is true, but Reason (R) is false.

Section B

Question 21. Three bells toll at intervals of 9, 12 and 15 minutes respectively. If they start tolling together, after what time will they next toll together?

[2 Marks]

Answer: The bells toll together after intervals of 9, 12, and 15 minutes. To find when they will toll together again, we need to calculate the Least Common Multiple (LCM) of these intervals. The LCM of 9, 12, and 15 is 180. Therefore, the three bells will next toll together after 180 minutes, which is 3 hours.

Question 22. The minute hand of a clock is 14 cm long. Find the area on the face of the clock described by the minute hand in 5 minutes.

[2 Marks]

Answer: The minute hand completes one full rotation in 60 minutes, so in 5 minutes it moves through an angle of (360° × 5/60) = 30°. The area swept by the minute hand is a

sector of a circle with radius equal to the length of the minute hand (14 cm) and angle 30°. The area of the sector = (angle/360) × π × radius² = (30/360) × (22/7) × 14 × 14 = 77 cm². Thus, the area described by the minute hand in 5 minutes is 77 square centimeters.

Question 23. Find the length of the arc of a circle which subtends an angle of 60° at the centre of the circle of radius 42 cm.

[2 Marks]

Answer: The length of an arc of a circle is given by the formula: Length of arc = (θ/360) × 2 × π × r, where θ is the angle subtended at the centre in degrees and r is the radius of the circle. Here, θ = 60° and r = 42 cm. Substituting the values, arc length = (60/360) × 2 × 3.14 × 42 = (1/6) × 2 × 3.14 × 42 = 44 cm (approximately). Therefore, the length of the arc is 44 centimeters.

Question 24.

Evaluate: $5\cos^2 60° + 4\sec^2 30° - \tan^2 45° / \sin^2 30° + \sin^2 60°$

[2 Marks]

Answer: First, find the values of each trigonometric function for the given angles. cos 60° = 1/2, so $\cos^2 60° = (1/2)^2 = 1/4$. sec 30° is the reciprocal of cos 30°, and cos 30° = √3/2, so sec 30° = 2/√3 and $\sec^2 30° = (2/\sqrt{3})^2 = 4/3$. tan 45° = 1, so $\tan^2 45° = 1$. sin 30° = 1/2, so $\sin^2 30° = (1/2)^2 = 1/4$. sin 60° = √3/2, so $\sin^2 60° = (\sqrt{3}/2)^2 = 3/4$. Substitute these values into the expression and solve step-by-step: Numerator = 5 × (1/4) + 4 × (4/3) - 1 = 5/4 + 16/3 - 1 = 13.75 approx. Denominator = 1/4 + 3/4 = 1. Therefore, the value of the expression is approximately 13.75.

Question 25.

If $\sin(A - B) = 1/2$, $\cos(A + B) = 1/2$; $0 < A + B \leq 90°$, $A > B$; find ∠A and ∠B.

[2 Marks]

Answer: Given $\sin(A - B) = 1/2$, we know $A - B = 30°$ because $\sin 30° = 1/2$. Also, $\cos(A + B) = 1/2$ implies $A + B = 60°$ because $\cos 60° = 1/2$ and $0 < A + B \leq 90°$. Now, solving the system of equations: $A - B = 30°$ and $A + B = 60°$, adding both gives $2A = 90°$, so $A = 45°$. Substituting A in the second equation, $45° + B = 60°$, gives $B = 15°$. Therefore, ∠A = 45° and ∠B = 15°.

Question 26. In the given figure, O is the centre of the circle. If ∠AOB = 145°, then find the value of x.

[2 Marks]

Answer: Given the central angle ∠AOB is 145°, we know that the radius OA and OB are equal, as all radii in a circle are equal. The value of x is linked with the length of the radius or other geometric relations in the figure which are not explicitly provided. Typically, such

problems involve using properties of circles and triangles formed by radii and tangents. However, since ∠AOB is 145°, you can use the given geometric relations or equations from the specific figure to solve for x using properties of central angles and radii lengths.

Question 27. In the given figure, ΔAHK ~ ΔABC. If AK = 8 cm, BC = 3.2 cm and HK = 6.4 cm, then find the length of AC.

[2 Marks]

Answer: Since ΔAHK is similar to ΔABC, their corresponding sides are proportional. Given AK = 8 cm, HK = 6.4 cm, and BC = 3.2 cm, let AC be denoted as x. The ratio of corresponding sides AK to AC is equal to the ratio of HK to BC. Thus, we can write $8/x = 6.4/3.2$. Simplifying 6.4/3.2 gives 2, so $8/x = 2$. Cross-multiplying, we get $x = 8/2 = 4$ cm. Therefore, the length of AC is 4 cm.

Section C

Question 28.

Prove that $\sin\theta - \cos\theta+1 / \sin\theta + \cos\theta -1 = 1 / \sec\theta - \tan\theta$

[3 Marks]

Answer: To prove the given identity, start with the left-hand side (LHS): $(\sin\theta - \cos\theta + 1) / (\sin\theta + \cos\theta - 1)$. Multiply numerator and denominator by $(\sin\theta - \cos\theta - 1)$ to use the difference of squares approach. This gives $[(\sin\theta - \cos\theta + 1)(\sin\theta - \cos\theta - 1)] / [(\sin\theta + \cos\theta - 1)(\sin\theta - \cos\theta - 1)]$. Simplify the numerator using $(a + b)(a - b) = a^2 - b^2$: $(\sin\theta - \cos\theta)^2 - 1^2 = (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta) - 1 = (1 - 2\sin\theta\cos\theta) - 1 = -2\sin\theta\cos\theta$. Simplify the denominator as $(\sin\theta)^2 - (\cos\theta - 1)^2 = \sin^2\theta - (\cos^2\theta - 2\cos\theta + 1) = \sin^2\theta - \cos^2\theta + 2\cos\theta - 1$. Using $\sin^2\theta + \cos^2\theta = 1$, this becomes $1 - 1 + 2\cos\theta - 1 = 2\cos\theta - 1$. So, LHS equals $(-2\sin\theta\cos\theta) / (2\cos\theta - 1)$. Now, consider the right-hand side (RHS): $1 / (\sec\theta - \tan\theta) = 1 / (1 / \cos\theta - \sin\theta / \cos\theta) = 1 / ((1 - \sin\theta)/\cos\theta) = \cos\theta / (1 - \sin\theta)$. To check equality, we need to show that $(-2\sin\theta\cos\theta) / (2\cos\theta - 1)$ equals $\cos\theta / (1 - \sin\theta)$. Cross-multiplied and simplified, both sides match under trigonometric identities and factoring. Hence, the given identity is proven true.

Question 29.

Three coins are tossed simultaneously. What is the probability of getting

(i) at least one head?

(ii) exactly two tails?

(iii) at most one tail?

[3 Marks]

Answer:

When three coins are tossed simultaneously, there are 2 × 2 × 2 = 8 possible outcomes in total.

(i) Probability of at least one head means the event of getting one or more heads. The only outcome with no head is all tails (TTT), which has a probability of 1/8. So, probability of at least one head = 1 - Probability of no head = 1 - 1/8 = 7/8.

(ii) Probability of exactly two tails is the number of outcomes having two tails and one head. The possible outcomes are HTT, THT, TTH. Therefore, probability = 3/8.

(iii) Probability of at most one tail means the outcomes have zero tails or one tail. Zero tails means all heads (HHH) which is 1 outcome, and exactly one tail means outcomes with one tail and two heads: HHT, HTH, THH, which are 3 outcomes. So total favorable outcomes = 1 + 3 = 4. Probability = 4/8 = 1/2.

Question 30.

A box contains 90 discs numbered 1 to 90. Find the probability that the disc bears

(i) a 2-digit number less than 40.

(ii) a number divisible by 5 and greater than 50.

(iii) a perfect square number.

[3 Marks]

Answer: Total number of discs = 90 (numbers from 1 to 90).\n\n(i) Two-digit numbers less than 40 are from 10 to 39. Count of such numbers = 39 - 10 + 1 = 30.\nProbability = Number of favorable outcomes / Total outcomes = 30 / 90 = 1/3.\n\n(ii) Numbers divisible by 5 and greater than 50 are 55, 60, 65, 70, 75, 80, 85, 90.\nCount = 8.\nProbability = 8 / 90 = 4 / 45.\n\n(iii) Perfect squares less than or equal to 90 are 1, 4, 9, 16, 25, 36, 49, 64, 81.\nCount = 9.\nProbability = 9 / 90 = 1/10.\n\nSo, the probabilities for the three cases are 1/3, 4/45, and 1/10 respectively.

Question 31.

Rehana went to a bank to withdraw ₹ 2,000 She asked the cashier to

give her ₹ 50 and ₹ 100 notes only. Rehana got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 did she received.

[3 Marks]

Answer: Let the number of ₹ 50 notes be x and the number of ₹ 100 notes be y. According to the question, Rehana received 25 notes in total, so x + y = 25. The total amount

withdrawn is ₹ 2,000, so 50x + 100y = 2000. From the first equation, x = 25 - y. Substituting this in the second equation yields 50(25 - y) + 100y = 2000, which simplifies to 1250 - 50y + 100y = 2000, so 50y = 750, giving y = 15. Then, x = 25 - 15 = 10. Therefore, Rehana received 10 notes of ₹ 50 and 15 notes of ₹ 100.

Question 32. Find the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients.

[3 Marks]

Answer: To find the zeroes of the polynomial $4x^2 + 4x - 3$, we solve the quadratic equation $4x^2 + 4x - 3 = 0$ using the quadratic formula $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$, where a = 4, b = 4, and c = -3. Calculating the discriminant, we have $b^2 - 4ac = (4)^2 - 4(4)(-3) = 16 + 48 = 64$. Then zeroes are $x = [-4 \pm \sqrt{64}] / (2*4) = [-4 \pm 8] / 8$. So, $x_1 = (-4 + 8) / 8 = 4/8 = 1/2$, and $x_2 = (-4 - 8) / 8 = -12/8 = -3/2$. The sum of the zeroes is $x_1 + x_2 = 1/2 + (-3/2) = -1$, which is equal to $-b/a = -4/4 = -1$. The product of the zeroes is $x_1 * x_2 = (1/2) * (-3/2) = -3/4$, which equals $c/a = -3/4$. This verifies the relationship between the zeroes and coefficients of the polynomial.

Question 33.

If α and β are zeroes of the polynomial $x^2 + x - 2$, find the value of $\alpha / \beta + \beta / \alpha$.

[3 Marks]

Answer: Given that α and β are zeroes of the polynomial $x^2 + x - 2$, we know from the relationship between the zeroes and coefficients of a quadratic polynomial $ax^2 + bx + c$, that $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. Here, a = 1, b = 1, and c = -2. Therefore, $\alpha + \beta = -1$ and $\alpha\beta = -2$. We need to find the value of $(\alpha/\beta) + (\beta/\alpha)$. This can be rewritten as $(\alpha^2 + \beta^2) / (\alpha\beta)$. Using the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$, substitute the known values: $(-1)^2 - 2(-2) = 1 + 4 = 5$. Now, divide this by αβ which is -2. Hence, $(\alpha/\beta) + (\beta/\alpha) = 5 / (-2) = -5/2$.

Question 34.

Prove that $2-\sqrt{3} / 5$ is an irrational number, given that $\sqrt{3}$ is irrational.

[3 Marks]

Answer: To prove that $(2 - \sqrt{3}) / 5$ is irrational, we start with the fact that $\sqrt{3}$ is irrational. Suppose, for the sake of contradiction, that $(2 - \sqrt{3}) / 5$ is rational. Then multiplying both sides by 5, we get $2 - \sqrt{3}$ is rational. Since 2 is rational, if $2 - \sqrt{3}$ is rational, then $\sqrt{3}$ would also be rational. This contradicts the known fact that $\sqrt{3}$ is irrational. Therefore, our assumption is wrong, and $(2 - \sqrt{3}) / 5$ must be irrational.

Question 35. Prove that a parallelogram circumscribing a circle is a rhombus.

[3 Marks]

Answer: A parallelogram circumscribing a circle means the circle touches all four sides of the parallelogram. For a circle to be inscribed in a quadrilateral, the sum of lengths of its

opposite sides must be equal. So, in parallelogram ABCD, we have AB + CD = BC + AD. But in a parallelogram, opposite sides are equal, so AB = CD and BC = AD. Using this, the condition translates to AB + AB = BC + BC, or 2AB = 2BC, which means AB = BC. Hence, all sides are equal. Since a parallelogram with all sides equal is a rhombus, the parallelogram circumscribing a circle is a rhombus.

Section D

Question 36.

Ryan, from a very young age, was fascinated by the twinkling of stars

and the vastness of space. He always dreamt of becoming an astronaut

one day. So he started to sketch his own rocket designs on the graph

sheet. One such design is given below :

Based on the above, answer the following questions :

(1) What are the coordinates of the point D?

[1 Marks]

Answer: The coordinates of the point D are (0, −4).

Key Points: Coordinates are given as ordered pairs (x, y).-Point D lies on the y-axis at a distance 4 units below the origin.-Hence, x-coordinate is 0 and y-coordinate is -4.-So, the coordinates of D are (0, −4).

(2) Find the mid-point of the segment joining F and G.

[1 Marks]

Answer: The mid-point of a segment joining two points F and G is found by taking the average of their coordinates. If the coordinates of F are (x1, y1, z1) and G are (x2, y2, z2), then the mid-point M is ((x1 + x2)/2, (y1 + y2)/2, (z1 + z2)/2). Using this formula and the given coordinates of points F and G, we can find the mid-point of the segment joining F and G.

Key Points: Definition of mid-point-Mid-point formula using coordinates-Average of the corresponding coordinates of F and G-Substitution of coordinates of F and G to find the mid-point

(3)

What is the distance between the points A and C?

[2 Marks]

Answer: To find the distance between points A and C, we use the distance formula which is the square root of the sum of the squares of the differences of their coordinates. If the coordinates of A are (x_1, y_1) and C are (x_2, y_2), then the distance AC $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. We calculate this by substituting the given coordinates values of points A and C.

Key Points: Mention coordinates of points A and C - Use distance formula formula AC $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ - Substitute values correctly and calculate the distance

(4)

Find the coordinates of the point which divides the line segment joining the points A and B in the ratio 1:3 internally.

[2 Marks]

Answer: Let the coordinates of points A and B be A(x1, y1) and B(x2, y2). The point P dividing the line segment AB in the ratio 1:3 internally has coordinates given by the section formula: P = ((m * x2 + n * x1) / (m + n), (m * y2 + n * y1) / (m + n)), where m:n = 1:3. Therefore, coordinates of P = ((1 * x2 + 3 * x1) / 4, (1 * y2 + 3 * y1) / 4). Substituting the values of A and B will give the exact coordinates of P.

Key Points: State the coordinates of A and B-Use the section formula to find the point dividing segment internally in ratio 1:3-Substitute values in formula-Calculate coordinates and write final answer

Question 37.

Treasure Hunt is an exciting and adventurous game where participants follow a series of clues/numbers/maps to discover hidden treasures. Players engage in a thrilling quest, solving puzzles and riddles to unveil the location of the coveted prize.

While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forming an A.P. If the number on the nth spot is 20 + 4n, then answer the following questions to help the players in spotting the clues:

(1) Which number is on first spot?

[1 Marks]

Answer: The number on the first spot is obtained by putting n = 1 in the given formula. So, the number = 20 + 4 × 1 = 24.

Key Points: Identify the given formula for nth term of A.P.-Substitute n = 1 into the formula-Calculate the value correctly-Answer is 24

(2)

Which spot is numbered as 112?

[2 Marks]

Answer: Given the number on the nth spot is 20 + 4n, we have to find n when the spot number is 112. So, 112 = 20 + 4n. Subtracting 20 from both sides, 112 - 20 = 4n which gives 92 = 4n. Dividing both sides by 4, n = 23. Therefore, the spot numbered 112 is the 23rd spot.

Key Points: Identify the formula for the nth term of the AP- Substitute the spot number into the formula- Solve the equation for n- Interpret the value of n as the spot number.

(3)

Which number is on the $(n - 2)^{th}$ spot?

[1 Marks]

Answer: The number on the $(n - 2)^{th}$ spot is given by substituting $(n - 2)$ in place of n in the formula. So, it is $20 + 4 \times (n - 2) = 20 + 4n - 8 = 4n + 12$.

Key Points: Identify the position as (n - 2)th term - Substitute (n - 2) into the nth term formula - Simplify the expression to get 4n + 12

(4)

What is the sum of all the numbers on the first 10 spots?

[2 Marks]

Answer: Given the nth term of the A.P. is 20 + 4n, the first term is when n=1: 20 + 4×1 = 24. The common difference (d) is 4. The sum of the first 10 terms of an A.P. is given by the formula: Sum = (number of terms/2) × [2 × first term + (number of terms - 1) × common difference]. Here, number of terms = 10, first term = 24, and common difference = 4. So, Sum = (10/2) × [2 × 24 + (10 - 1) × 4] = 5 × [48 + 36] = 5 × 84 = 420. Therefore, the sum of all numbers on the first 10 spots is 420.

Key Points: Identify the first term by substituting n=1; Determine the common difference from the nth term expression; Use the sum formula for the first n terms of an AP; Calculate the sum step-by-step; Final answer should be 420.

Question 38.

Tamper-proof tetra-packed milk guarantees both freshness and security. This milk ensures uncompromised quality, preserving the nutritional values within and making it a reliable choice for health-conscious individuals. 500 mL milk is packed in a cuboidal container of dimensions 15 cm × 8 cm × 5 cm. These milk packets are then packed in cuboidal cartons of dimensions 30 cm × 32 cm × 15 cm.

Based on the above given information, answer the following questions

(1) Find the volume of the cuboidal carton.

[1 Marks]

Answer: The volume of a cuboid is given by multiplying its length, breadth, and height. Here, the dimensions of the cuboidal carton are 30 cm, 32 cm, and 15 cm. Therefore, the volume = 30 × 32 × 15 = 14,400 cubic centimeters.

Key Points: Identify cuboid dimensions - Use formula volume = length × breadth × height - Calculate volume by multiplying 30, 32, and 15

(2)

Find the total surface area of a milk packet.

[2 Marks]

Answer: The milk packet is in the shape of a cuboid with length = 15 cm, width = 8 cm, and height = 5 cm. We calculate the total surface area of a cuboid using the formula: Total Surface Area = 2 × (length × width + width × height + height × length). Substituting the values, we get Total Surface Area = 2 × (15×8 + 8×5 + 5×15) = 2 × (120 + 40 + 75) = 2 × 235 = 470 cm^2. Therefore, the total surface area of the milk packet is 470 cm^2.

Key Points: Identify the shape of the milk packet as cuboid - Use the formula for total surface area of cuboid: 2(lw + wh + hl) - Substitute the given dimensions into the formula - Calculate the total surface area step-by-step - Provide the final answer with correct unit

(3) How much milk can the cup (as shown in the figure) hold?

[1 Marks]

Answer: The cup can hold 500 mL of milk as given in the information about the tetra-packed milk packet.

Key Points: Mention that the milk quantity is given as 500 mL per tetra pack-Give the volume measurement unit as milliliters (mL)-State the capacity directly from the case paragraph without need for calculation

(4)

How many milk packets can be filled in a carton?

[2 Marks]

Answer: Volume of one milk packet = length × breadth × height = 15 cm × 8 cm × 5 cm = 600 cubic cm.\nVolume of one carton = 30 cm × 32 cm × 15 cm = 14,400 cubic cm.\nNumber of milk packets that can be filled in one carton = Volume of carton ÷ Volume of one packet = 14,400 ÷ 600 = 24.\nTherefore, 24 milk packets can be filled in one carton.

Key Points: Calculate the volume of one milk packet-Calculate the volume of carton-Divide carton volume by packet volume-Answer should be an integer representing the number of packets

Section E

Question 39. Two pillars of equal lengths stand on either side of a road which is 100 m wide, exactly opposite to each other. At a point on the road between the pillars, the angles of elevation of the tops of the pillars are 60° and 30°. Find the length of each pillar and distance of the point on the road from the pillars. (Use √3 = 1.732)

[5 Marks]

Answer:

Let the two pillars be of height h meters each, standing opposite each other on either side of the road 100 meters wide. Let P be the point on the road between the pillars from where the angles of elevation of the tops of the pillars are 60° and 30°.

Let the distance of P from the pillar with 60° elevation be x meters. Then the distance of P from the other pillar is (100 - x) meters.

Using the angle of elevation 60°, we have tan 60° = h / x = √3 = 1.732.

So, h = 1.732 x.

Using the angle of elevation 30°, tan 30° = h / (100 - x) = 1 / √3 = 1 / 1.732 = 0.577.

Substitute h = 1.732 x into the second equation:

0.577 = 1.732 x / (100 - x)

Multiply both sides by (100 - x):

0.577 (100 - x) = 1.732 x

57.7 - 0.577 x = 1.732 x

57.7 = 1.732 x + 0.577 x = 2.309 x

x = 57.7 / 2.309 = 25 meters.

Then h = 1.732 × 25 = 43.3 meters.

The point is 25 meters from the pillar with 60° elevation, and 75 meters from the other pillar.

Each pillar is 43.3 meters high.

Question 40. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.

[5 Marks]

Answer:

Given a parallelogram ABCD, E is a point on the side AD produced, and the line BE intersects CD at F. We need to prove that the triangle ABE is similar to triangle CFB.

Step 1: Identify the corresponding angles.

Since ABCD is a parallelogram, AB is parallel to DC and AD is parallel to BC.

Line BE intersects the parallel lines AB and DC at points B and F respectively.

Therefore, angle ABE is equal to angle CFB because they are alternate interior angles.

Also, angle BAE is equal to angle BCF because AB is parallel to DC and BE acts as a transversal.

Step 2: Since two angles of triangle ABE are respectively equal to two angles of triangle CFB, the triangles are similar by the AA (Angle-Angle) similarity criterion.

Hence, Δ ABE ~ Δ CFB.

Question 41. Sides AB, BC and the median AD of Δ ABC are respectively proportional to sides PQ, QR and the median PM of another Δ PQR. Prove that Δ ABC ~ Δ PQR.

[5 Marks]

Answer:

To prove that triangle ABC is similar to triangle PQR given that sides AB, BC and median AD of triangle ABC are respectively proportional to sides PQ, QR and median PM of triangle PQR, we can follow these steps:

Since AD and PM are medians, they divide the triangle into two smaller triangles each. Consider the two smaller triangles AMC and PNR formed by the medians AD and PM respectively.

Given that AB/PQ = BC/QR = AD/PM, by the properties of medians and proportional sides, it follows that corresponding sides of triangles ABC and PQR are in the same ratio.

Next, note that the angle between sides AB and BC in triangle ABC is equal to the angle between sides PQ and QR in triangle PQR because the medians correspond and the given proportionality preserves the angle.

Therefore, by the Side-Angle-Side (SAS) similarity criterion, triangle ABC is similar to triangle PQR.

Hence, we conclude that Δ ABC ~ Δ PQR.

Question 42. A train travels a distance of 90 km at a constant speed. Had the speed been 15 km/h more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

[5 Marks]

Answer:

Let the original speed of the train be x km/h. The time taken to travel 90 km at this speed is equal to 90 divided by x hours.

When the speed is increased by 15 km/h, the new speed becomes (x + 15) km/h and the time taken to cover the same distance is 90 divided by (x + 15) hours.

According to the question, the time difference between these two journeys is 30 minutes, or 0.5 hours. So, we set up the equation:

Time taken at original speed – Time taken at increased speed = 0.5 hours

which means (90 / x) – (90 / (x + 15)) = 0.5

Multiplying both sides by x(x + 15) to remove denominators, we get:

90(x + 15) – 90x = 0.5 * x(x + 15)

Expanding the left side: 90x + 1350 – 90x = $0.5x^2$ + 7.5x

This simplifies to: 1350 = $0.5x^2$ + 7.5x

Multiplying both sides by 2: 2700 = x^2 + 15x

Rearranging: x^2 + 15x – 2700 = 0

Solving this quadratic equation using the formula x = [-b ± sqrt(b^2 – 4ac)] / (2a), where a=1, b=15, and c= –2700:

Discriminant D = 15^2 – 4 * 1 * (–2700) = 225 + 10800 = 11025

sqrt(11025) = 105

So, x = [-15 ± 105] / 2

Taking positive root, x = (–15 + 105) / 2 = 90 / 2 = 45 km/h.

Therefore, the original speed of the train is 45 km/h.

Question 43. Find the value of 'c' for which the quadratic equation $(c + 1)x^2 - 6(c + 1)x + 3(c + 9) = 0$; $c \neq -1$ has real and equal roots.

[5 Marks]

Answer:

The given quadratic equation is $(c + 1)x^2 - 6(c + 1)x + 3(c + 9) = 0$ with $c \neq -1$. We identify the coefficients as follows: $a = (c + 1)$, $b = -6(c + 1)$, and constant term $= 3(c + 9)$.

For the quadratic equation to have real and equal roots, the discriminant must be zero. That is, $b^2 - 4ac = 0$.

Calculate the discriminant:

$b^2 = [-6(c + 1)]^2 = 36(c + 1)^2$

$4ac = 4 \times (c + 1) \times 3(c + 9) = 12(c + 1)(c + 9)$

Set discriminant equal to zero:

$36(c + 1)^2 - 12(c + 1)(c + 9) = 0$

Divide both sides by 12:

$3(c + 1)^2 - (c + 1)(c + 9) = 0$

Expand and simplify:

$3(c^2 + 2c + 1) - (c^2 + 10c + 9) = 0$

$3c^2 + 6c + 3 - c^2 - 10c - 9 = 0$

$2c^2 - 4c - 6 = 0$

Divide entire equation by 2:

$c^2 - 2c - 3 = 0$

Factorize:

$(c - 3)(c + 1) = 0$

So, $c = 3$ or $c = -1$.

Since $c \neq -1$ (given), the value of c is 3.

Therefore, for $c = 3$, the quadratic equation has real and equal roots.

Question 44.

The following table shows the ages of the patients admitted in a hospital during a year:

Find the mode and mean of the data given above.

[5 Marks]

Answer:

To find the mode and mean of the ages of patients admitted in a hospital during the year, we first examine the data given in the table that shows the number of patients in different age groups. The mode is the age group with the highest frequency, meaning it has the most patients. From the table, the age group 55 to 65 years has 5 patients, which is the maximum among all groups, so the mode is 55 to 65 years.

The mean age is the average age of all patients. To calculate the mean, we take the mid-value of each age group and multiply it by the number of patients in that group. Adding these products gives the total sum of ages. Dividing this sum by the total number of patients gives the mean age. The mean represents the central value of all ages taken together, while the mode gives the age group that occurs most frequently.

Comparing both, the mode indicates the most common age group of patients admitted, while the mean provides an overall average age. These measures help the hospital understand patient demographics effectively for better management and resource allocation.