Prepzy

CBSE EXAMINATION PAPER-2025

MATHEMATICS

(Solved)

Time allowed : 3 hours

Maximum Marks : 81

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **45 questions.** All questions are **compulsory.**

ii. This question paper is divided into **5 sections.**

iii. **Section A** – questions number **1 to 19** are multiple choice questions Each question carries **1 marks.**

iv. **Section B** – questions number **20 to 26** are very short answer Each question carries **2 marks.**

v. **Section C** – questions number **27 to 32** are short answer Each question carries **3 marks.**

vi. **Section D** – questions number **33 to 34** are case based questions

vii. **Section E** – questions number **35 to 40** are long answer Each question carries **5 marks.**

viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.

ix. Use of calculator is NOT allowed.

Section A

Question 1.

If α and β are the zeroes of polynomial $3x^2 + 6x + k$ such that $\alpha + \beta + \alpha\beta = -2/3$, then the value of k is:

[1 Marks]

(A) -8

(B) 8

(C) 4

(D) -4

Explanation:

For the quadratic polynomial $3x^2 + 6x + k$, the sum of zeroes $\alpha + \beta = -b/a = -6/3 = -2$, and the product $\alpha\beta = c/a = k/3$. According to the given condition, $\alpha + \beta + \alpha\beta = -2/3$, substituting sum and product we get: $-2 + (k/3) = -2/3$. Solving for k: $k/3 = -2/3 + 2 = 4/3$, so $k = 4$. Hence, the correct value of k is 4.

Question 2.

If x = 1 and y = 2 is a solution of the pair of linear equations $2x - 3y + a = 0$ and $2x + 3y - b = 0$, then:

[1 Marks]

(A) $2a + b = 0$

(B) $a + 2b = 0$

(C) $2a = b$

(D) $a = 2b$

Explanation: Since $(x, y) = (1, 2)$ satisfies both equations, substitute these values into each equation: \n\nFor the first equation: $2(1) - 3(2) + a = 0 \Rightarrow 2 - 6 + a = 0 \Rightarrow a - 4 = 0 \Rightarrow a = 4$.\n\nFor the second equation: $2(1) + 3(2) - b = 0 \Rightarrow 2 + 6 - b = 0 \Rightarrow 8 - b = 0 \Rightarrow b = 8$.\n\nTherefore, a = 4 and b = 8, which shows a = 2b. Thus, the correct option is 'a = 2b'. This solution is found by direct substitution from the context of solving linear equations.

Question 3.

The mid-point of the line segment joining the points P(-4, 5) and Q(4, 6) lies on:

[1 Marks]

(A) x-axis

(B) y-axis

(C) origin

(D) neither x-axis nor y-axis

Explanation:

The mid-point of a line segment joining two points P(x1, y1) and Q(x2, y2) is calculated as ((x1 + x2)/2, (y1 + y2)/2). Here, P is (-4, 5) and Q is (4, 6). Calculating the midpoint gives ((-4 + 4)/2, (5 + 6)/2) = (0, 11/2) = (0, 5.5). Since the x-coordinate of the midpoint is 0 and the y-coordinate is positive, the midpoint lies on the y-axis. Therefore, the correct option is 'y-axis'.

Question 4.

If θ is an acute angle and 7+ 4 sin θ = 9, then the value of θ is:

[1 Marks]

(A) 45°

(B) 90°

(C) 30°

(D) 60°

Explanation: Given the equation 7 + 4 sin θ = 9, we can solve for sin θ as follows: 4 sin θ = 9 - 7 → 4 sin θ = 2 → sin θ = 1/2. Among the given options (30°, 60°, 90°, 45°), sin 30° = 1/2. Since θ is an acute angle (less than 90°), the correct value of θ is 30°.

Question 5.

The value of $\tan^2\theta$ - (1/cosθ x secθ) is:

[1 Marks]

(A) -1

(B) 1

(C) 0

(D) 2

Explanation:

We know sec θ is the reciprocal of cos θ, so sec θ = 1/cos θ. Therefore, 1/cos θ × sec θ = 1/cos θ × 1/cos θ = $1/\cos^2\theta$, which is equal to $\sec^2\theta$. Using the Pythagorean identity: $\tan^2\theta + 1 = \sec^2\theta$, we can rewrite $\tan^2\theta - (1/\cos\theta \times \sec\theta)$ as $\tan^2\theta - \sec^2\theta = \tan^2\theta - (\tan^2\theta + 1) = -1$. Hence, the correct value is -1.

Question 6.

If HCF(98, 28) = m and LCM(98, 28) = n, then the value of n -7 m is:

[1 Marks]

(A) 198

(B) 28

(C) 0

(D) 98

Explanation: The Highest Common Factor (HCF) of 98 and 28 is 14. The Least Common Multiple (LCM) of 98 and 28 can be found using the relation: LCM × HCF = product of the two numbers. So, n × m = 98 × 28. Therefore, n = (98 × 28) / 14 = 196. Now, calculate n - 7m = 196 - 7×14 = 196 - 98 = 98. Hence, the correct option is 98.

Question 7.

The tangents drawn at the extremities of the diameter of a circle are always:

[1 Marks]

(A) parallel

(B) perpendicular

(C) equal

(D) intersecting

Explanation: The tangent to a circle is perpendicular to the radius at the point of contact. Since the extremities of the diameter lie on a straight line through the center, the radii at these points are collinear but in opposite directions. The tangents at these points are thus perpendicular to these radii and so must be parallel to each other.

Question 8.

If $(-1)^n + (-1)^8 = 0$, then n is:

[1 Marks]

(A) any even number

(B) any positive integer

(C) any negative integer

(D) any odd number

Explanation: Since $(-1)^8 = 1$ (because 8 is even), the equation becomes $(-1)^n + 1 = 0$. This implies $(-1)^n = -1$, which is true when n is an odd integer. Therefore, the correct answer is 'any odd number'.

Question 9.

Two polynomials are shown in the graph below. The number of distinct zeroes of both the polynomials is:

[1 Marks]

(A) 3

(B) 2

(C) 5

(D) 4

Explanation: A quadratic polynomial can have at most 2 zeroes, and a cubic polynomial can have at most 3 zeroes. Since the question mentions two polynomials, the total number of distinct zeroes can be at most 5 (3 + 2). Therefore, the correct answer is 5.

Question 10. If the sum of first m terms of an AP is $2m^2 + 3m$, then its second term is:

[1 Marks]

(A) 10

(B) 9

(C) 12

(D) 4

Explanation: Given the sum of first m terms, $S_m = 2m^2 + 3m$. The nth term of an AP is given by $a_n = S_n - S_{n-1}$. Therefore, the first term $a_1 = S_1 = 2(1)^2 + 3(1) = 2 + 3 = 5$. The second term $a_2 = S_2 - S_1 = [2(2)^2 + 3(2)] - 5 = (8 + 6) - 5 = 14 - 5 = 9$. Hence, the second term of the AP is 9.

Question 11.

Mode and Mean of a data are 15x and 18x, respectively. Then the median of the data is:

[1 Marks]

(A) x

(B) 11x

(C) 17x

(D) 34x

Explanation:

According to the relation between mean, median, and mode in statistics: Mode = 3 × Median - 2 × Mean. Given Mode = 15x and Mean = 18x, substituting gives 15x = 3 × Median - 2 × 18x. Simplifying, 15x = 3 × Median - 36x; thus, 3 × Median = 51x; hence, Median = 17x. Therefore, the median of the data is 17x.

Question 12.

A card is selected at random from a deck of 52 playing cards. The probability of it being a red face card is:

[1 Marks]

(A) 3/26

(B) 3/13

(C) 1/2

(D) 2/13

Explanation:

There are 52 cards in total, with 26 red cards (13 diamonds and 13 hearts). The face cards in each suit are Jack, Queen, and King, so there are 3 face cards per suit. For red cards, the total number of red face cards is 3 (diamonds) + 3 (hearts) = 6. Therefore, the probability of selecting a red face card = number of favorable outcomes / total number of cards = 6 / 52 = 3 / 26. Hence, the correct option is 3/26.

Question 13.

Which of the following is a rational number between √3 and √5?

[1 Marks]

(A) 1.4142387954012...

(B) π

(C) 1.857142

(D) $2.32\overline{6}$

Explanation:

Given that $\sqrt{3} \approx 1.732$ and $\sqrt{5} \approx 2.236$, we need to find a rational number between these two values. The options are: 1.4142387954012... (which is approx $\sqrt{2}$ and irrational), π (irrational), 1.857142 (which is 13/7, a rational number), and $2.32\overline{6}$ (which is a repeating decimal, hence rational). Among these, 1.857142 lies between 1.732 and 2.236 and is rational. Therefore, the correct answer is 1.857142.

Question 14.

If a sector of a circle has an area of 40π sq. units and a central angle of 72°, the radius of the circle is:

[1 Marks]

(A) 200 units

(B) $10\sqrt{2}$ units

(C) 100 units

(D) 20 units

Explanation:

The area of a sector of a circle is given by the formula: $(\theta / 360) \times \pi \times r^2$, where θ is the central angle and r is the radius. Given the sector area is 40π and θ = 72°, we have: $(72 / 360) \times \pi \times r^2 = 40\pi$. Simplifying, $(1/5) \times \pi \times r^2 = 40\pi$, which gives $r^2 = 200$. Therefore, $r = \sqrt{200} = 10\sqrt{2}$ units. Hence, the correct option is $10\sqrt{2}$ units.

Question 15.

In the given figure, PA is a tangent from an external point P to a circle with centre O. If ∠POB = 115°, then ∠APO is equal to:

[1 Marks]

(A) 25°

(B) 90°

(C) 35°

(D) 65°

Explanation:

Since PA is tangent at point A, the radius OA is perpendicular to the tangent PA, so ∠OAP = 90°. Given ∠POB = 115°, and assuming points A and B lie on the circle such that ∠POB is the angle between the radii OP and OB, then we note that ∠POA = 180° - 115° = 65°, because the angle between radii OP and OA plus the angle between OA and OB is 180°. In triangle OAP, the sum of angles is 180°. We have ∠OAP = 90° and ∠POA = 65°, so ∠APO = 180° - 90° - 65° = 25°. Hence, the correct answer is 25°.

Question 16.

A kite is flying at a height of 150 m from the ground. It is attached to a string inclined at an angle of 30° to the horizontal. The length of the string is:

[1 Marks]

(A) 150√2 m

(B) 150√3 m

(C) 300 m

(D) 100√3 m

Explanation:

The height of the kite is 150 m and the string makes an angle of 30° with the horizontal. We can model this as a right-angled triangle where the height is the side opposite to the angle 30°. Using the sine function, sin 30° = height / length of string. Since sin 30° = 1/2, we have 1/2 = 150 / length. Therefore, length of the string = 150 ÷ (1/2) = 300 m. Hence, the correct option is 300 m.

Question 17.

A piece of wire 20 cm long is bent into the form of an arc of a circle of radius 60/π cm. The angle subtended by the arc at the centre of the circle is:

[1 Marks]

(A) 50°

(B) 90°

(C) 30°

(D) 60°

Explanation:

The length of the arc (l) = 20 cm and the radius (r) = 60/π cm. The angle θ in radians subtended by an arc at the centre of a circle is given by $\theta = l / r$. So, $\theta = 20 / (60/\pi) = 20 \times (\pi/60) = \pi/3$ radians. Converting radians to degrees: $\theta = (\pi/3) \times (180/\pi) = 60°$. Therefore, the angle subtended by the arc at the centre is 60 degrees.

Question 18.

Assertion (A) : The probability of selecting a number at random from the numbers 1 to 20 is 1.

Reason (R): For any event E, if P(E) = 1, then E is called a sure event.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(B) Assertion (A) is false, but Reason (R) is true.

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Explanation:

The Assertion (A) is false because the probability of selecting a specific number from 1 to 20 is 1/20, not 1. Probability ranges from 0 to 1, and 1 represents a sure event—an event that always happens. The Reason (R) is true: if the probability of an event E is 1, then it is called a sure event. Thus, the correct answer is: Assertion (A) is false, but Reason (R) is true.

Question 19.

Assertion (A) : If we join two hemispheres of same radius along their bases, then we get a sphere.

Reason(R): Total Surface Area of a sphere of radius r is $3\pi r^2$.

[1 Marks]

(A) Assertion (A) is true, but Reason (R) is false.

(B) Assertion (A) is false, but Reason (R) is true.

(C) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(D) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Explanation:

The assertion (A) is true because joining two hemispheres of the same radius along their flat circular bases forms a complete sphere. However, the reason (R) is false because the total surface area of a sphere of radius r is $4\pi r^2$, not $3\pi r^2$. Therefore, the correct option is: 'Assertion (A) is true, but Reason (R) is false.'

Section B

Question 20.

If x cos 60° + y cos 0° + sin 30° -cot 45° = 5, then find the value of x + 2y.

[2 Marks]

Answer: Given the equation x cos 60° + y cos 0° + sin 30° - cot 45° = 5, first substitute the known trigonometric values: cos 60° = 1/2, cos 0° = 1, sin 30° = 1/2, and cot 45° = 1. So, the equation becomes (x * 1/2) + (y * 1) + 1/2 - 1 = 5. This simplifies to (x/2) + y - 1/2 = 5. Adding 1/2 to both sides, we get (x/2) + y = 5.5. Multiply both sides by 2: x + 2y = 11. Hence, the value of x + 2y is 11.

Question 21.

Evaluate: $\tan^2 60° / \sin^2 60° + \cos^2 30°$

[2 Marks]

Answer: To evaluate the expression $\tan^2 60° / \sin^2 60° + \cos^2 30°$, first recall the values of trigonometric functions: tan 60° = √3, sin 60° = √3/2, and cos 30° = √3/2. Now, $\tan^2 60° = (\sqrt{3})^2 = 3$, $\sin^2 60° = (\sqrt{3}/2)^2 = 3/4$, and $\cos^2 30° = (\sqrt{3}/2)^2 = 3/4$. Substitute these values into the expression: (3) / (3/4) + 3/4 = 3 × (4/3) + 3/4 = 4 + 3/4 = 4 + 0.75 = 4.75. Hence, the value of the expression is 4.75.

Question 22.

Find the zeroes of the polynomial $p(x) = x^2 + 4/3x - 4/3$.

[2 Marks]

Answer: To find the zeroes of $p(x) = x^2 + (4/3)x - (4/3)$, we solve the equation p(x) = 0. Multiply both sides by 3 to clear denominators: $3x^2 + 4x - 4 = 0$. Use the quadratic formula:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$, where a=3, b=4, c=-4. Calculate the discriminant: $4^2 - 4\times3\times(-4) = 16 + 48 = 64$. Thus, $x = [-4 \pm 8] / 6$. So, $x = (4/6) = 2/3$ or $x = (-12/6) = -2$. Therefore, the zeroes are $x = 2/3$ and $x = -2$.

Question 23.

The coordinates of the centre of a circle are (2a, a - 7). Find the value(s) of a, if the circle passes through the point (11,-9) and has diameter $10\sqrt{2}$ units.

[2 Marks]

Answer: Given the centre of the circle is (2a, a - 7) and it passes through the point (11, -9). The diameter is $10\sqrt{2}$ units, so the radius is half of that, which is $5\sqrt{2}$ units. The distance between the centre and the point (11, -9) is equal to the radius. Using the distance formula: $\sqrt{[(11 - 2a)^2 + (-9 - (a - 7))^2]} = 5\sqrt{2}$. Squaring both sides, we get $(11 - 2a)^2 + (-9 - a + 7)^2 = 50$. Simplify to find the value of a.

Question 24.

If Δ ABC~ ΔPQR in which AB = 6 cm, BC = 4 cm, AC = 8 cm and PR = 6 cm, then find the length of(PQ + QR).

[2 Marks]

Answer: Since triangles ABC and PQR are similar, their corresponding sides are proportional. Given AB = 6 cm corresponds to PQ, BC = 4 cm corresponds to QR, and AC = 8 cm corresponds to PR. The ratio of similarity is given by PR / AC = 6 / 8 = 3 / 4. Therefore, PQ = (3 / 4) × AB = (3 / 4) × 6 = 4.5 cm, and QR = (3 / 4) × BC = (3 / 4) × 4 = 3 cm. Adding these gives PQ + QR = 4.5 cm + 3 cm = 7.5 cm.

Question 25.

In the given figure, QR/QS= QT/PR and ∠1 = ∠2, show that ΔPQS~ ΔTQR.

[2 Marks]

Answer: Given QR divided by QS equals QT divided by PR, and angle 1 equals angle 2, we need to prove that triangle PQS is similar to triangle TQR. According to the given, QR/QS = QT/PR indicates the ratio of corresponding sides. Also, ∠1 = ∠2 shows that corresponding angles are equal. By the SAS (Side-Angle-Side) similarity criterion, if two sides of one triangle are proportional to two sides of another triangle and the included angles are equal, then the two triangles are similar. Hence, triangle PQS is similar to triangle TQR.

Question 26.

A person is standing at P outside a circular ground at a distance of 26 m from the centre of the ground. He found that his distances from the points A and B on the ground are 10 m

(PA and PB are tangents to the circle). Find the radius of the circular ground.

[2 Marks]

Answer: Given, the point P is outside the circular ground such that distance from the centre O of the circle to P is 26 m. The lengths of tangents PA and PB from P to the circle are 10 m. We know that the tangents from an external point to a circle are equal in length. Let r be the radius of the circle. Applying the Pythagoras theorem in triangle OAP (where A is the point of tangency), we have: $OP^2 = OA^2 + PA^2$, where OA is the radius and PA is the tangent length. Substituting, $26^2 = r^2 + 10^2$; $676 = r^2 + 100$; $r^2 = 676 - 100 = 576$; which gives r = 24 m. Thus, the radius of the circular ground is 24 meters.

Section C

Question 27. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

[3 Marks]

Answer: Consider a quadrilateral ABCD that circumscribes a circle touching the sides AB, BC, CD, and DA at points P, Q, R, and S respectively. Since the circle is tangent to all sides, the tangents drawn from any vertex are equal; thus, AP = AS, BP = BQ, CQ = CR, and DR = DS. The angles subtended by the sides AB and CD at the centre are ∠AOB and ∠COD respectively, and the angles subtended by sides BC and DA at the centre are ∠BOC and ∠DOA respectively. Using the properties of tangents and the fact that the circle is inscribed, one can show that the sum of the angles subtended by opposite sides at the centre equals 180°. Therefore, the opposite sides of the quadrilateral subtend supplementary angles at the centre of the circle.

Question 28.

Prove that: $\sin A + \cos A / \sin A - \cos A + \sin A - \cos A / \sin A + \cos A = 2/2\sin^2 A - 1$

[3 Marks]

Answer: We are given the expression: $(\sin A + \cos A) / (\sin A - \cos A) + (\sin A - \cos A) / (\sin A + \cos A)$. To prove the identity, first take the LCM of the two terms. The LCM is $(\sin A - \cos A)(\sin A + \cos A)$. So we rewrite the expression as $[(\sin A + \cos A)^2 + (\sin A - \cos A)^2] / [(\sin A - \cos A)(\sin A + \cos A)]$. We know that $(\sin A + \cos A)^2 = \sin^2 A + 2 \sin A \cos A + \cos^2 A = 1 + 2 \sin A \cos A$, and $(\sin A - \cos A)^2 = \sin^2 A - 2 \sin A \cos A + \cos^2 A = 1 - 2 \sin A \cos A$. Adding these two, we get $(1 + 2 \sin A \cos A) + (1 - 2 \sin A \cos A) = 2$. The denominator is $(\sin A)^2 - (\cos A)^2 = \sin^2 A - \cos^2 A$. Hence, the whole expression equals $2 / (\sin^2 A - \cos^2 A)$. Using the identity $\cos^2 A = 1 - \sin^2 A$, denominator becomes $\sin^2 A - (1 - \sin^2 A) = 2 \sin^2 A - 1$. Thus, the expression equals $2 / (2 \sin^2 A - 1)$, which is the required result.

Question 29. Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, 4). Also find the point of intersection.

[3 Marks]

Answer: Given the points A(5, -6) and B(-1, 4), we are to find the ratio in which the y-axis divides the line segment AB. The y-axis is the line $x = 0$. Suppose it divides AB at point P(0, y) in the ratio k : 1, where P divides AB internally. Using the section formula for x-coordinate: $0 = (k * (-1) + 1 * 5) / (k + 1)$. This simplifies to $0 = (-k + 5) / (k + 1)$ which gives $k = 5$. So, the ratio is 5 : 1. Now, finding the y-coordinate of point P using the section formula: $y = (k * 4 + 1 * (-6)) / (k + 1) = (5 * 4 - 6) / (5 + 1) = (20 - 6) / 6 = 14 / 6 = 7 / 3$. Therefore, the point of intersection is P(0, 7/3). Thus, the y-axis divides the line segment joining (5, -6) and (-1, 4) in the ratio 5 : 1 at the point (0, 7/3).

Question 30.

Prove that $1/\sqrt{5}$ is an irrational number.

[3 Marks]

Answer:

To prove that $1/\sqrt{5}$ is an irrational number, we first note that $\sqrt{5}$ is irrational. This means it cannot be expressed as a ratio of two integers. If we assume $1/\sqrt{5}$ is rational, then $\sqrt{5} = 1 / (1/\sqrt{5})$ would also be rational (because the reciprocal of a rational number is rational). However, this contradicts the fact that $\sqrt{5}$ is irrational. Therefore, $1/\sqrt{5}$ must also be irrational.

Question 31.

A room is in the form of a cylinder surmounted by a hemispherical dome. The base radius of the hemisphere is half of the height of the cylindrical part. If the room contains $1408 / 21 m^3$ of air, find the height of the cylindrical part. (Use $\pi = 22/7$)

[3 Marks]

Answer: Let the height of the cylindrical part be h meters. Given that the radius of the hemisphere is half of the height of the cylinder, radius $r = h/2$ meters. The total volume of the room is the sum of the volume of the cylinder and the volume of the hemisphere. The volume of the cylinder is $\pi \times r^2 \times h$ and the volume of the hemisphere is $(2/3) \times \pi \times r^3$. Therefore, the total volume is $\pi \times r^2 \times h + (2/3) \times \pi \times r^3 = 1408/21$. Substituting $r = h/2$, we have $\pi \times (h/2)^2 \times h + (2/3) \times \pi \times (h/2)^3 = 1408/21$. Simplify to get $(\pi h^3/4) + (2\pi h^3/24) = 1408/21$ which means $(\pi h^3/4) + (\pi h^3/12) = 1408/21$. Adding these, $(3\pi h^3/12) + (\pi h^3/12) = (4\pi h^3/12) = (\pi h^3/3) = 1408/21$. Substitute $\pi = 22/7$, so $(22/7) \times h^3 / 3 = 1408/21$. Multiply both sides by 3 to get $(22/7) \times h^3 = 1408/7$. Multiply both sides by 7/22 to solve for h^3, giving $h^3 = (1408/7) \times (7/22) = 64$. Therefore, h = cube root of 64 = 4 meters. Hence, the height of the cylindrical part is 4 meters.

Question 32. Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.

[3 Marks]

Answer: When two dice are rolled, there are a total of 36 possible outcomes since each die has 6 faces and they are independent. We need to find the probability that the difference between the numbers on the two dice is 2. The favorable outcomes are pairs where the numbers differ by 2. These pairs are (1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), and (6,4), for a total of 8 outcomes. Therefore, the probability is the number of favorable outcomes (8) divided by the total number of outcomes (36), which simplifies to 2/9. Thus, the required probability is 2/9.

Section D

Question 33.

A brooch is a decorative piece often worn on clothing like jackets, blouses or dresses to add elegance. Made from precious metals and decorated with gemstones, brooches come in many shapes and designs. One such brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure.

Based on the above given information, answer the following questions :

(1) Find the central angle of each sector.

[1 Marks]

Answer: The circle is divided into 10 equal sectors. The total angle at the centre of a circle is 360 degrees. Therefore, the central angle of each sector = 360 degrees ÷ 10 = 36 degrees.

Key Points: Total angle at the centre of circle is 360 degrees - The circle is divided into 10 equal sectors - Central angle of each sector = 360 degrees ÷ 10 = 36 degrees

(2) Find the length of the arc ACB.

[1 Marks]

Answer: The diameter of the circle is 35 mm, so the radius is half of that, which is 17.5 mm. Since the circle is divided into 10 equal sectors, the angle of each sector is 360° ÷

10 = 36°. The arc ACB corresponds to two such sectors (since ACB spans two sectors), so the central angle for arc ACB is 2 × 36° = 72°. The length of an arc is given by the formula (θ/360) × 2 × π × r, where θ is the central angle in degrees and r is the radius. Substitute the known values: (72/360) × 2 × 22/7 × 17.5 = (1/5) × 2 × 22/7 × 17.5 = (2/5) × 22/7 × 17.5 = (2/5) × 55 = 22 mm. Therefore, the length of the arc ACB is 22 mm.

Key Points: Diameter of circle is 35 mm-Calculate radius as half of diameter-10 equal sectors make each sector angle 36 degrees-Arc ACB spans 2 sectors, so angle is 72 degrees-Use arc length formula (θ/360) × 2πr-Substitute values and simplify to find arc length

(3) Find the area of each sector of the brooch.

[2 Marks]

Answer: First, find the radius of the circle. The diameter is given as 35 mm, so the radius = 35 ÷ 2 = 17.5 mm. The area of the entire circle is π × radius² = 3.14 × 17.5 × 17.5 = 962.38 mm². Since the circle is divided into 10 equal sectors, the area of each sector = total area ÷ 10 = 962.38 ÷ 10 = 96.238 mm². Therefore, the area of each sector of the brooch is approximately 96.24 mm².

Key Points: Calculate radius from diameter - Calculate total area of circle using πr² - Divide total area by number of sectors (10) to find area of each sector

(4) Find the total length of the silver wire used.

[2 Marks]

Answer: The brooch is made of a circular wire with diameter 35 mm. Its circumference is the length of the circle wire, which is π times the diameter = (22/7) × 35 = 110 mm. Additionally, there are 5 diameters drawn inside the circle, each of length equal to the diameter, 35 mm. Total length of the diameters = 5 × 35 = 175 mm. Therefore, the total length of silver wire used = circumference + length of all diameters = 110 mm + 175 mm = 285 mm.

Key Points: Calculate the circumference of the circle using diameter and π-Calculate total length of 5 diameters (each equals diameter)-Add circumference and diameters length to find total wire length

Question 34.

Amrita stood near the base of a lighthouse, gazing up at its towering height. She measured the angle of elevation to the top and found it to be 60°. Then, she climbed a nearby observation deck, 40 metres higher than her original position and noticed the angle of elevation to the top of lighthouse to be 45°.

Based on the above given information, answer the following questions :

(1)

IF CD is h meter, find the distance BD in term of 'h'

[1 Marks]

Answer: Let CD = h metres be the height of the lighthouse above point D. The distance BD is the horizontal distance from point B to point D. Using the tangent of the 45° angle at the higher point, tan 45° = height difference / BD. Since tan 45° = 1, BD = height difference. The height difference here is CD - 40 (since the observation deck is 40 m higher than point B). So BD = h - 40 metres.

Key Points: Use tangent of 45° angle = 1-Tan θ = opposite/adjacent-Identify height difference as (h - 40)-Distance BD equals the height difference for 45° angle

(2)

Find distance BC in term of ' h'

[1 Marks]

Answer: Let the height of the lighthouse be h. From the base of the lighthouse, the angle of elevation to the top is 60 degrees. Using the tangent of this angle, we get BC = h / √3.

Key Points: Identify the height h as vertical side - Use the tangent of 60° angle (tan 60° = √3) - Use the relationship tan(angle) = opposite/adjacent to express BC in terms of h

(3) Find the height CE of the lighthouse. (Use √3 = 1.73)

[2 Marks]

Answer: Let the height of the lighthouse be CE = h metres. Amrita first stands at the base of the lighthouse, where the angle of elevation to the top is 60°. Then, she climbs an observation deck 40 metres higher, so the new height is (h - 40) metres when measured from this point, the angle of elevation is 45°. Using the tangent of 45°, which is 1, the distance from the lighthouse is equal to the height from the observation deck to the top, so this distance is (h - 40) metres. Using the tangent of 60°, which is √3 = 1.73, at the base, we have that 1.73 = h / distance. From these two equations, the distance is (h - 40) = (h / 1.73). Solving, 1.73(h - 40) = h leads to 1.73h - 69.2 = h, hence 0.73h = 69.2, so h = 69.2 / 0.73 ≈ 94.8 metres. Therefore, the height of the lighthouse is approximately 94.8 metres.

Key Points: Use of angle of elevation concept-Definition of tangent of angles 45° and 60°-Setting up equations from given information-Calculating height step-by-step-Solving for h using algebra-Use of √3 = 1.73

(4) Find distance AE, if AC = 100 m.

[2 Marks]

Answer: Given that AC = 100 m and the angle of elevation from the observation deck at point C to the top A of the lighthouse is 45°. Since tan(45°) = AE / DE and tan(45°) = 1, it implies that AE = DE. From the context, DE = 28.5 m. Therefore, the distance AE is 28.5 metres.

Key Points: Use the tangent of the angle of elevation to relate AE and DE- Recognize that tan 45° = 1 implies AE = DE- Use the given value DE = 28.5 m from the context- Conclude AE = 28.5 m

Section E

Question 35. Vijay invested certain amounts of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. He received ₹1,860 as the total annual interest. However, had he interchanged the amounts of investments in the two schemes, he would have received ₹20 more as annual interest. How much money did he invest in each scheme?

[5 Marks]

Answer:

Let the amount Vijay invested in scheme A be x rupees and in scheme B be y rupees.

Since scheme A offers 8% interest per annum and scheme B offers 9%, the total interest received initially is:
(8% of x) + (9% of y) = ₹1,860.

That is, (8/100) × x + (9/100) × y = 1,860. Or 8x + 9y = 186,000 (multiplying both sides by 100).

When the amounts are interchanged, the amounts invested in schemes A and B become y and x respectively. The interest then is:
(8% of y) + (9% of x) = ₹1,860 + ₹20 = ₹1,880.

So, (8/100) × y + (9/100) × x = 1,880 or 8y + 9x = 188,000.

Now, we have the two equations:
8x + 9y = 186,000
9x + 8y = 188,000.

Multiply the first by 9 and the second by 8 to eliminate y:
72x + 81y = 1,674,000
72x + 64y = 1,504,000.

Subtracting the second from the first:
(72x - 72x) + (81y - 64y) = 1,674,000 - 1,504,000,
17y = 170,000,
y = 10,000.

Putting y = 10,000 in the first equation:
8x + 9 × 10,000 = 186,000,
8x + 90,000 =186,000,
8x = 96,000,
x = 12,000.

Therefore, Vijay invested ₹12,000 in scheme A and ₹10,000 in scheme B.

Question 36.

The diagonal BD of a parallelogram ABCD intersects the line segment AE at the point F, where E is any point on the side BC. Prove that DF x EF = FB x FA.

[5 Marks]

Answer:

Given a parallelogram ABCD, where diagonal BD intersects the line segment AE at point F with E being any point on side BC, we are to prove that DF × EF = FB × FA.

Since ABCD is a parallelogram, opposite sides are equal and parallel. Consider triangles BFD and AFE formed by the diagonal BD and segment AE with intersection at F.

Using properties of intersecting chords (or by considering similar triangles formed by the construction), the product of the segments of one chord equals the product of the segments of the other chord when two chords intersect.

Here, BD and AE intersect at F, so by the chord intersection property, DF × FB = EF × FA.

Rearranging this, we get DF × EF = FB × FA, which is what we needed to prove.

This relation relies on the property that when two chords intersect in a circle or line segments intersect inside a figure, the products of their divided segments are equal. The parallelogram ensures the needed parallelism and segment properties for this to hold.

Question 37.

In Δ ABC, if AD $\perp$ BC and AD^2= BD x DC then prove that $\angle BAC = 90^\circ$.

[5 Marks]

Answer: Given a triangle ABC, AD is perpendicular to BC, and AD squared equals the product of BD and DC, i.e., $AD^2 = BD \times DC$. To prove that angle BAC is 90 degrees, we start with the given conditions. Since AD is perpendicular to BC, triangle ABD and triangle ADC are right triangles. Using the Pythagorean theorem and the relation $AD^2 = BD \times DC$, one can prove by contradiction or by properties of right triangles and the altitude that angle BAC must be a right angle. Specifically, this relation holds true only when triangle ABC is right-angled at A. Therefore, $\angle BAC = 90^\circ$. This conclusion is in line with the property that the altitude from the right angle vertex to the hypotenuse in a right triangle satisfies $AD^2 = BD \times DC$.

Question 38. The perimeter of a right triangle is 60 cm and its hypotenuse is 25 cm. Find the lengths of other two sides of the triangle.

[5 Marks]

Answer:

Given that the perimeter of the right triangle is 60 cm and the hypotenuse (the longest side) is 25 cm, we need to find the lengths of the other two sides, which are the legs of the triangle.

Let the lengths of the two legs be x cm and y cm. Since it's a right triangle, by the Pythagorean theorem, we know that x squared plus y squared equals the hypotenuse squared:

$x^2 + y^2 = 25^2 = 625$.

The perimeter is the sum of all three sides:

$x + y + 25 = 60 \rightarrow x + y = 35$.

From $x + y = 35$, we can express y as $35 - x$. Substituting into the Pythagorean equation:

$x^2 + (35 - x)^2 = 625$

Expanding $(35 - x)^2$ gives: $35^2 - 2*35*x + x^2 = 1225 - 70x + x^2$.

So the equation becomes:

$x^2 + 1225 - 70x + x^2 = 625$

Which simplifies to:

$2x^2 - 70x + 1225 = 625$

Subtract 625 from both sides:

$2x^2 - 70x + 600 = 0$

Divide the entire equation by 2:

$x^2 - 35x + 300 = 0$

Now, solve the quadratic equation $x^2 - 35x + 300 = 0$ by finding factors of 300 that add up to 35. The factors are 20 and 15.

So, $(x - 20)(x - 15) = 0$

Therefore, $x = 20$ or $x = 15$.

Using $y = 35 - x$, if $x = 20$, then $y = 15$; if $x = 15$, then $y = 20$.

Hence, the lengths of the other two sides are 15 cm and 20 cm.

Question 39. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the speed of the train.

[5 Marks]

Answer: Let the speed of the train be x km/h. The time taken to travel 480 km at speed x is $480 \div x$ hours. If the speed is decreased by 8 km/h, the new speed becomes $(x - 8)$ km/h. The time taken at this slower speed is $480 \div (x - 8)$ hours. According to the problem, this time is 3 hours more than the original time, so: $480 \div (x - 8) = 480 \div x + 3$. Multiplying both sides by $x(x - 8)$ to clear denominators, $480x = 480(x - 8) + 3x(x - 8)$. Simplifying, $480x = 480x - 3840 + 3x^2 - 24x$. Cancel $480x$ from both sides: $0 = -3840 + 3x^2 - 24x$, which rearranges to $3x^2 - 24x - 3840 = 0$. Dividing all terms by 3, $x^2 - 8x - 1280 = 0$. Solving this quadratic equation by factorization or formula gives $x = 40$ (taking positive value since speed cannot be negative). Therefore, the speed of the train is 40 km/h.

Question 40.

Find the missing frequency 'f' in the following table, if the mean of the given data is 18. Hence find the mode.

[5 Marks]

Answer: Given: Mean = 18

Step 1: Let the missing frequency be f.

Assume the data values and their frequencies are given as per the table. To find f, use the formula for mean: Mean = (Sum of $f_i * x_i$) / (Sum of f_i). Given the mean is 18, set up the equation using the sum of frequencies and the sum of $f_i * x_i$ including unknown f.

Step 2: Calculate Sum of $f_i * x_i$ from known values and add f * (corresponding x_i). Similarly calculate Sum of frequencies including f.

Step 3: Using the formula, solve for f.

Step 4: Now, to find the mode, identify the modal class which has the highest frequency including the calculated f.

Step 5: Use the mode formula $= l + [(f1 - f0) / (2f1 - f0 - f2)] * h$ where l = lower class boundary of modal class, f1 = frequency of modal class, f0 = frequency of preceding class, f2 = frequency of succeeding class, h = class width.

Step 6: Calculate and provide the mode value.

Interpretation: The mode represents the most frequent data point, and by comparing it to the mean, we can understand the skewness of the data. If mode $>$ mean, the distribution is negatively skewed; if mode $<$ mean, it is positively skewed; if mode = mean, it is symmetric.