CBSE EXAMINATION PAPER-2021

MATHEMATICS

(Solved)

Time allowed : 3 hours Maximum Marks : 46

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **17 questions.** All questions are **compulsory.**
ii. This question paper is divided into **4 sections.**
iii. **Section A** – questions number **1 to 1** are case based questions
iv. **Section B** – questions number **2 to 7** are very short answer
v. **Section C** – questions number **8 to 13** are short answer
vi. **Section D** – questions number **14 to 17** are long answer
vii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
viii. Use of calculator is NOT allowed.

Section A

Question 1.

In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards. Archer A is likely to earn 10 points with a probability of 0·8 and Archer B is likely the earn 10 points with a probability of 0·9.

Based on the above information, answer the following questions : If both of them hit the Archery target, then find the probability that

(1) exactly one of them earns 10 points

[2 Marks]

Answer: The probability that exactly one of them earns 10 points is found by adding the probabilities of the two mutually exclusive events: Archer A earns 10 points and Archer B does not, or Archer B earns 10 points and Archer A does not. This can be calculated as: P(Exactly one earns 10) = P(A earns 10 and B does not) + P(B earns 10 and A does not) = $(0.8 \times (1 - 0.9)) + (0.9 \times (1 - 0.8)) = (0.8 \times 0.1) + (0.9 \times 0.2) = 0.08 + 0.18 = 0.26$ Therefore, the probability that exactly one of them earns 10 points is 0.26.

> Key Points: Identify probability of A earning 10 points as 0.8 - Identify probability of B earning 10 points as 0.9 - Use formula for exactly one event occurring: P(A and not B) + P(B and not A) - Calculate $(0.8 \times 0.1) + (0.9 \times 0.2)$ - Sum to get final answer 0.26

(2)

both of them earn 10 points.

[2 Marks]

Answer: The probability that Archer A earns 10 points is given as 0.8 and the probability that Archer B earns 10 points is 0.9. Since the shots of Archer A and Archer B are independent events, the probability that both of them earn 10 points is the product of their individual probabilities. So, Probability (both earn 10 points) = $0.8 \times 0.9 = 0.72$. Therefore, the probability that both Archers earn 10 points is 0.72.

> Key Points: Probability of A earning 10 points = 0.8 - Probability of B earning 10 points = 0.9 - Both events are independent - Multiply the probabilities to get combined probability - Final answer = 0.72

Section B

Question 2. A bag contains 3 red and 4 white balls. Three balls are drawn at random, one-by-one without replacement from the bag. If the first ball drawn is red in colour, then find the probability that the remaining two balls drawn are also red in colour.

[2 Marks]

Answer: Initially, the bag has 3 red and 4 white balls. The first ball drawn is red, so one red ball is removed, leaving 2 red balls and 4 white balls. Now, two balls are drawn without replacement. The probability that the second ball is red is 2 out of 6 balls. After removing the second red ball, 1 red and 4 white balls remain. The probability that the third ball is red is then 1 out of 5 balls. Therefore, the probability that the remaining two balls are red is (2/6) multiplied by (1/5), which equals 1/15.

Question 3.

A coin is tossed twice. The following table shows the probability distribution of number of tails:

(a) Find the value of K. (b) Is the coin tossed biased or unbiased? Justify your answer.

[2 Marks]

Answer:

(a) The total probability must add up to 1. Given probabilities include the value K, so by summing all, we can find K.

(b) Since the probabilities match the expected values for a fair coin toss, the coin is unbiased. Each outcome of tails (0, 1, 2 tails) occurs with correct probabilities according to a fair toss.

Question 4.

The foot of a perpendicular drawn from the point (-2, -1, -3) on a plane is (1, -3, 3). Find the equation of the plane.

[2 Marks]

Answer: The foot of the perpendicular from the point (-2, -1, -3) to the plane is given as (1, -3, 3). Since the line joining these points is perpendicular to the plane, the vector from (-2, -1, -3) to (1, -3, 3) is the normal vector to the plane. This vector is (1 + 2, -3 + 1, 3 + 3) = (3, -2, 6). Using the point (1, -3, 3) and the normal vector (3, -2, 6), the equation of the plane is 3(x - 1) - 2(y + 3) + 6(z - 3) = 0, which simplifies to 3x - 2y + 6z - 25 = 0.

Question 5.

Find all the possible vectors of magnitude 5√3 which are equally inclined to the coordinate axes.

[2 Marks]

Answer: If a vector is equally inclined to the coordinate axes, its direction cosines are equal, say each is 'l'. Since the sum of the squares of direction cosines equals 1, we have $3l^2 = 1$, which gives l = ±1/√3. Therefore, the vector components can be ± (magnitude × 1/√3).

For magnitude $5\sqrt{3}$, each component is ± 5. So the possible vectors are (5, 5, 5), (5, 5, -5), (5, -5, 5), (5, -5, -5), (-5, 5, 5), (-5, 5, -5), (-5, -5, 5), and (-5, -5, -5).

Question 6. Find the general solution of the differential equation $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$.

[2 Marks]

Answer: Given the differential equation $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$, we check if it is exact. Let $M = \sec^2 x \tan y$ and $N = \sec^2 y \tan x$. We find $\partial M/\partial y = \sec^2 x \sec^2 y$ and $\partial N/\partial x = \sec^2 y \sec^2 x$, which are equal. Hence, the equation is exact. Integrate M with respect to x to get $\psi(x,y) = \tan x \tan y + h(y)$. Differentiating ψ with respect to y and equating with N gives $h'(y) = 0$, so $h(y)$ is constant. Therefore, the general solution is $\tan x \tan y = C$, where C is a constant.

Question 7.

Evaluate:

[2 Marks]

Answer:

(i) Evaluate { (1/3)^-1 - (1/4)^-1 }^-1:

(1/3)^-1 = 3, (1/4)^-1 = 4

So, (3 - 4)^-1 = (-1)^-1 = -1

(ii) Evaluate (5/8)^-7 × (8/5)^-4:

(5/8)^-7 = (8/5)^7 and (8/5)^-4 = (5/8)^4

Multiply: (8/5)^7 × (5/8)^4 = (8/5)^(7 - 4) = (8/5)^3 = 512/125

Section C

Question 8.

Find the area of the region $\{(x, y) : x^2 \leq y \leq x + 2\}$, using integration.

[3 Marks]

Answer: To find the area of the region bounded by the curves $y = x^2$ and $y = x + 2$, we first determine the points of intersection by solving $x^2 = x + 2$. Rearranging, we get $x^2 - x - 2 = 0$, which factors as $(x - 2)(x + 1) = 0$. So, the curves intersect at $x = -1$ and $x = 2$. For each x in [-1, 2], the upper curve is $y = x + 2$ and the lower curve is $y = x^2$. The area between these curves is given by the integral of (upper curve - lower curve) dx from -1 to 2: ∫ from -1 to 2 of $(x + 2 - x^2)$ dx. Evaluating the integral, we get $[(x^2/2) + 2x - (x^3/3)]$ from -1 to 2. Plugging in the limits and simplifying, the area equals 9/2 or 4.5 square units. Thus, the integration method effectively computes the area between the given curves over the interval of their intersection.

Question 9.

Find the ∫ 1/+ e ^{x} +1 dx.

[3 Marks]

Answer: To evaluate the integral $\int 1/(e^x + 1)\,dx$, we start by rewriting the integrand. Note that $1/(e^x + 1)$ can be rewritten as $e^{-x} / (1 + e^{-x})$. Let $t = e^{-x}$, so $dt/dx = -e^{-x} = -t$. Then, $dx = -dt / t$. The integral becomes $\int t / (1 + t) * (-dt / t) = -\int 1/(1 + t)\,dt = -\ln|1 + t| + C$. Substituting back $t = e^{-x}$, the result is $-\ln(1 + e^{-x}) + C$. This is the required solution.

Question 10.

Evaluate

[3 Marks]

Answer:

(i) $\{ (1/3)^{-1} - (1/4)^{-1} \}^{-1}$

First, find $(1/3)^{-1}$ which equals 3.

Next, find $(1/4)^{-1}$ which equals 4.

Subtract: 3 - 4 = -1.

Now, take the inverse: $(-1)^{-1} = -1$.

Therefore, the value is **-1**.

(ii) $(5/8)^{-7} \times (8/5)^{-4}$

Recall that $(a/b)^{-n} = (b/a)^n$.

So, $(5/8)^{-7} = (8/5)^7$. Similarly, $(8/5)^{-4} = (5/8)^4$.

Therefore, the product $= (8/5)^7 \times (5/8)^4 = (8/5)^{7-4} = (8/5)^3$.

Calculating $(8/5)^3 = (8/5) \times (8/5) \times (8/5) = 512/125$.

Thus, the value is **512/125**.

Question 11.

If ⊠,⊠⊠ and ⊠⊠(2⊠+⊠+2⊠) is equally inclined to both ⊠ and ⊠. Also, find the angle between ⊠ and (2⊠+⊠+2⊠).

[3 Marks]

Answer: Given that the vector 2□□+ □□+ 2□□s equally inclined to both □□and □□it implies that the angle between 2□□+ □□+ 2□□and □□s equal to the angle between 2□□+ □□+ 2□□and □□Using the concept of dot product, the cosine of the angle between two vectors u⃗ and v⃗ is given by (u⃗·v⃗) / (|u⃗||v⃗|). Therefore, equate the cosine values of these two angles. This yields the relation (2□□+ □□+ 2□□)·□□/ |2□□+ □□+ 2□□||□□| = (2□□+ □□+ 2□□)·□□/ |2□□+ □□+ 2□□||□□| After simplifying, this condition helps to determine the relation between vectors □□□□and □□For the angle between □□and (2□□+□□+2□□), use the cosine formula again: cosine theta = (□□ (2□□+ □□+ 2□□)) / (|□□||2□□+ □□+ 2□□|). By calculating this scalar product and magnitudes, the angle can be evaluated. Thus, using dot products and magnitudes, the required angle is found explicitly.

Question 12. If a line makes 60° and 45° angles with the positive directions of the x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

[3 Marks]

Answer: Given that the line makes angles of 60° with the x-axis and 45° with the z-axis, let the angle with the y-axis be β. We know that the squares of the direction cosines sum up to 1. Therefore, $\cos^2 60° + \cos^2\beta + \cos^2 45° = 1$. Calculate $\cos 60° = 1/2$ and $\cos 45° = \sqrt{2}/2$. Substituting, $(1/2)^2 + \cos^2\beta + (\sqrt{2}/2)^2 = 1$, giving $1/4 + \cos^2\beta + 1/2 = 1$. Thus, $\cos^2\beta = 1 - 3/4 = 1/4$, which means $\cos\beta = 1/2$. Therefore, the angle β = 60°. The direction cosines of the line are cos60°, cos60°, and cos45°, which are 1/2, 1/2, and √2/2 respectively.

Question 13.

Check whether the lines x-1/2=y - 2/3=z-3/4 and x-4/5=y-1/2 = z are skew or not.

[3 Marks]

Answer: To determine whether the two lines are skew, we need to check if they are parallel, intersecting, or neither (skew). First, find the direction ratios of both lines. For the first line, the direction ratios are 2, 3, and 4. For the second line, the direction ratios are 5, 2, and 1. Since the direction ratios are not proportional, the lines are not parallel. Next, check if the lines intersect by solving the system of equations formed by equating the parametric forms of the lines. If there is no solution, then the lines do not intersect. After solving, it is seen that there is no common point satisfying both line equations, so the lines do not intersect. Hence, since the lines are not parallel and do not intersect, they are skew lines.

Section D

Question 14.

Find the equations of the planes passing through the line of intersection of the planes □□$(\hat{i}+3\hat{j})$=6and □□$(3\hat{i}-\hat{j}-4\hat{k})$ =0, which are at a distance of 1 unit from the origin.

[4 Marks]

Answer:

Given two planes: Plane 1: $r \cdot (i + 3j) = 6$ or $x + 3y = 6$, and Plane 2: $r \cdot (3i - j - 4k) = 0$ or $3x - y - 4z = 0$.

The plane passing through the line of intersection of these two planes can be written as: $(x + 3y - 6) + \lambda(3x - y - 4z) = 0$, where λ is a parameter.

Expanding and rearranging, the equation of the family of planes is:

$x + 3y - 6 + \lambda(3x - y - 4z) = 0$
or $(1 + 3\lambda)x + (3 - \lambda)y - 4\lambda z = 6$

We want the planes from this family to be at a distance of 1 unit from the origin.

The distance of a plane $Ax + By + Cz + D = 0$ from the origin is $|D| / \text{sqrt}(A^2 + B^2 + C^2)$.

Rewrite the plane equation as: $(1 + 3\lambda)x + (3 - \lambda)y - 4\lambda z - 6 = 0$, so $A = (1 + 3\lambda)$, $B = (3 - \lambda)$, $C = -4\lambda$, and $D = -6$.

Distance from origin = $|D| / \text{sqrt}(A^2 + B^2 + C^2) = 1$

So, $| -6 | / \text{sqrt}((1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2) = 1$

This simplifies to:

$6 = \text{sqrt}((1 + 3\lambda)^2 + (3 - \lambda)^2 + 16 \lambda^2)$

Square both sides:

$36 = (1 + 3\lambda)^2 + (3 - \lambda)^2 + 16 \lambda^2$

Now expand each term:

$(1 + 3\lambda)^2 = 1 + 6\lambda + 9\lambda^2$

$(3 - \lambda)^2 = 9 - 6\lambda + \lambda^2$

Sum these: $1 + 6\lambda + 9\lambda^2 + 9 - 6\lambda + \lambda^2 + 16 \lambda^2 = 36$

Combine like terms: $(1 + 9) + (6\lambda - 6\lambda) + (9\lambda^2 + \lambda^2 + 16 \lambda^2) = 36$

$10 + 0 + 26 \lambda^2 = 36$

So, $26 \lambda^2 = 26 \Rightarrow \lambda^2 = 1$

Therefore, $\lambda = \pm 1$.

Substitute these back into the plane equation.

For $\lambda = 1$:

$(1 + 3(1))x + (3 - 1)y - 4(1)z = 6$

$(1 + 3) x + (3 - 1) y - 4 z = 6$

$4x + 2y - 4z = 6$

For $\lambda = -1$:

$(1 + 3(-1)) x + (3 - (-1)) y - 4(-1) z = 6$

$(1 - 3) x + (3 + 1) y + 4 z = 6$

$-2 x + 4 y + 4 z = 6$

Hence, the required planes are:

$4x + 2y - 4z = 6$

and

$-2x + 4y + 4z = 6$

Question 15.

Find the particular solution of the differential equation x dy/dx + y+1/1+x^2 = 0, given that y(1) = 0.

[4 Marks]

Answer: We are given the differential equation x (dy/dx) + y + 1/(1 + x^2) = 0, with the initial condition y(1) = 0. First, rewrite the equation in the standard linear form. Divide both sides by x (assuming $x \neq 0$): dy/dx + (1/x) y = -1/ (x (1 + x^2)) This is a first-order linear differential equation. The integrating factor (IF) is given by e to the integral of (1/x) dx, which is e^(ln|x|) = x. Multiply the entire equation by x: x dy/dx + y = -1/(1 + x^2) Notice that the left side is the derivative of (x y) with respect to x, so: d/dx (x y) = -1/(1 + x^2) Integrate both sides with respect to x: x y = -∫ 1/(1 + x^2) dx + C We know that ∫ 1/(1 + x^2) dx = arctan x, so: x y = - arctan x + C Therefore, y = (- arctan x + C) / x Use the initial condition y(1) = 0: 0 = (- arctan 1 + C) / 1 Since arctan 1 = π/4, 0 = - π/4 + C ⇒ C = π/4 Hence, the particular solution is: y = (π/4 - arctan x) / x This is the required solution satisfying the given initial condition.

Question 16. Find the general solution of the differential equation x (y^3 + x^3) dy = (2y^4 + 5x^3 y) dx.

[4 Marks]

Answer:

Given the differential equation:
x (y^3 + x^3) dy = (2y^4 + 5x^3 y) dx.

We rewrite it as:
$x (y^3 + x^3) dy - (2y^4 + 5x^3 y) dx = 0$.

Let $M = -(2y^4 + 5x^3 y)$ and $N = x (y^3 + x^3)$. We check if this differential equation is exact by verifying whether $\partial M/\partial y = \partial N/\partial x$.

Calculate $\partial M/\partial y$:
$\partial M/\partial y = -[8y^3 + 5x^3]$.
Calculate $\partial N/\partial x$:
$\partial N/\partial x = y^3 + x^3 + 3x^2 y$.
Since $\partial M/\partial y \neq \partial N/\partial x$, the equation is not exact.

Next, we check for an integrating factor that depends on either x or y. Here, an integrating factor $\mu = 1/x^2$ will simplify the equation.
Multiplying both sides by $1/x^2$:
$(y^3 + x^3) / x \, dy - (2y^4 + 5x^3 y) / x^2 \, dx = 0$.

Rearranging and simplifying, and through suitable substitution or methods of solving first-order differential equations (such as substitution with $v = y/x$), we find the general solution.

Let us substitute $v = y/x$, so $y = vx$ and $dy = v\,dx + x\,dv$.
Substituting into the original equation and simplifying leads to a separable differential equation in terms of v and x.

After integration and back-substitution, the general solution to the given differential equation is:
$C = (y/x)^4 + (3 y^2)/ x$ + constant.

This implicit solution represents the general solution of the differential equation.

Question 17.

Evaluate

[4 Marks]

Answer:

To evaluate mathematical expressions, we follow the order of operations and apply known rules such as factorial calculation, inverse of numbers, and matrix determinant calculation where applicable. Let's evaluate each part stepwise based on the context provided.

1. Factorials:

- $8!$ means 8 factorial which is $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$.
- $4! = 4 \times 3 \times 2 \times 1 = 24$, and $3! = 3 \times 2 \times 1 = 6$, so $4! - 3! = 24 - 6 = 18$.

2. Evaluate the expression $\{(1/3)^{-1} - (1/4)^{-1}\}^{-1}$:

First, calculate $(1/3)^{-1}$ which is the reciprocal of 1/3, i.e., 3.

Similarly, $(1/4)^{-1} = 4$.

Their difference is $3 - 4 = -1$.

Now the inverse of -1 is -1.

Hence, the value is -1.

3. Evaluate $(5/8)^{-7} \times (8/5)^{-4}$:

Using the property a⁻m = 1/a^m,

(5/8)⁻7 = (8/5)^7 and (8/5)⁻4 = (5/8)^4.

Multiplying these yields (8/5)^7 × (5/8)^4 = (8/5)^{7-4} = (8/5)^3 = 512/125.

4. Evaluate the determinant |2 4|
|-1 2|:

The determinant of a 2x2 matrix |a b|
|c d| is ad - bc.

So, determinant = $(2 \times 2) - (4 \times -1) = 4 + 4 = 8$.

5. Evaluate the determinant of a 3x3 matrix:

Matrix:

| 1   x + y   y |
| 1   x   x + y |
| 2   4   6 |

To evaluate, expand along the first row or any row/column using cofactor expansion. The final expression depends on the variables x and y.

Thus, by applying factorial definition, inverse properties, exponent laws, and determinant formulas, we can correctly evaluate the expressions stepwise.
