CBSE EXAMINATION PAPER-2023

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 81

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **43 questions.** All questions are **compulsory.**
ii. This question paper is divided into **5 sections.**
iii. **Section A** – questions number **1 to 3** are case based questions
iv. **Section B** – questions number **4 to 23** are multiple choice questions
v. **Section C** – questions number **24 to 30** are very short answer
vi. **Section D** – questions number **31 to 39** are short answer
vii. **Section E** – questions number **40 to 43** are long answer
viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
ix. Use of calculator is NOT allowed.

Section A

Question 1.

Let f(x) be a real valued function.

(1) What is R.H.D. of f(x) at x = 1?

[1 Marks]

(2) What is L.H.D. of f(x) at x = 1?

[1 Marks]

(3) Check if the function f(x) is differentiable at x = 1.

[2 Marks]

(4)

Find f '(2) and f '(1).

[2 Marks]

Question 2.

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let E_1 represent the event when many workers were not present.

E_2 represent the event when all workers were present; and

E represent completing the construction work on time.

(1) What is the probability that all the workers are present for the job?

[1 Marks]

(2) What is the probability that construction will be completed on time?

[1 Marks]

(3) What is the probability that many workers are not present given that the construction work is completed on time?

[2 Marks]

(4)

What is the probability that all workers were present given that the construction job was completed on time ?

[2 Marks]

Question 3.

Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.

(1) Determine the maximum value of A(x).

[2 Marks]

(2)

Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden.

[2 Marks]

Section B

Question 4.

[1 Marks]

(A) x = 1, y = -1

(B) x = 3, y = 2

(C) x = 1, y = 2

(D) $x = 2, y = 1$

Question 5.

[1 Marks]

(A)

(B)

(C)

(D)

Question 6. If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to:

[1 Marks]

(A) I

(B) A

(C) 3I

(D) 2A

Question 7.

If a matrix A = [1 2 3], then the matrix AA (where A is the transpose of A) is:

[1 Marks]

(A) 14

(B)

(C)

(D) [14]

Question 8.

[1 Marks]

(A) $x + y + z$

(B) 0

(C) 1

(D) 2(x + y + z)

Question 9.

The function f(x) = |x | is

[1 Marks]

(A) continuous everywhere, but differentiable nowhere.

(B) continuous and differentiable nowhere.

(C) continuous and differentiable everywhere.

(D) continuous everywhere, but differentiable everywhere except at x = 0.

Question 10. If $y = \sin^2(x^3)$, then dy/dx is equal to:

[1 Marks]

(A) $2 \sin x^3 \cos x^3$

(B) $3 x^3 \sin x^3 \cos x^3$

(C) $2 x^2 \sin^2(x^3)$

(D) $6 x^2 \sin x^3 \cos x^3$

Question 11.

$\int e^{5\log x} dx$ is equal to:

[1 Marks]

(A) $x^5/ 5 + C$

(B) $5 x^4 + C$

(C) $6 x^5 + C$

(D) $x^6 / 6 + C$

Question 12.

[1 Marks]

(A) 2

(B) 8

(C) 4

(D) 10

Question 13.

The integrating factor for solving the differential equation: $x\, dy/dx - y = 2x^2$ is:

[1 Marks]

(A) x

(B) e^{-y}

(C) e^{-x}

(D) 1/x

Question 14.

The order and degree (if defined) of the differential equation,

$(d^2y/dx^2)^2 + (dy/dx)^3 = x \sin(dy/dx)$ respectively are :

[1 Marks]

(A) 2, degree not defined

(B) 1, 3

(C) 2, 3

(D) 2, 2

Question 15.

A unit vector along the vector $4\hat{i} - 3\hat{k}$ is:

[1 Marks]

(A) $1/7(4\hat{i}+3\hat{k})$

(B) $1/\sqrt{7}(4\hat{i}-3\hat{k})$

(C) $1/\sqrt{5}(4\hat{i}-3\hat{k})$

(D) $1/5(4\hat{i}-3\hat{k})$

Question 16.

If ▯ is the angle between two vectors ▯▯and ▯▯then ▯▯▯≥0 only when:

[1 Marks]

(A) $0<\theta < \pi/2$

(B) $0\leq\theta\leq \pi/2$

(C) $0\leq\theta \leq \pi$

(D) $0<\theta < \pi$

Question 17.

Distance of the point (p, q, r) from the y-axis is:

[1 Marks]

(A) $|q| + |r|$

(B) $\sqrt{(p^2 + r^2)}$

(C) q

(D) $|q|$

Question 18.

The solution set of the inequation $3x + 5y < 7$ is:

[1 Marks]

(A) Whole xy-plane except points on line $3x + 5y = 7$

(B) Open half-plane containing the origin except points on line $3x + 5y = 7$

(C) Open half-plane not containing the origin

(D) Whole xy-plane including points on line $3x + 5y = 7$

Question 19.

Which of the following points satisfies both inequations $2x + y \leq 10$ and $x + 2y \geq 8$?

[1 Marks]

(A) (-5, 6)

(B) (-2, 4)

(C) (3, 2)

(D) (4, 2)

Question 20.

If the direction cosines of a line are (1/a, 1/a, 1/a) then:

[1 Marks]

(A) $0 < a < 1$

(B) $a>2$

(C) $a = \pm\sqrt{3}$

(D) $a > 0$

Question 21. The probability that A speaks the truth is 4/5 and that of B speaking the truth is 3/4. The probability that they contradict each other in stating the same fact is:

[1 Marks]

(A) 1/5

(B) 3/20

(C) 4/5

(D) 7/20

Question 22. Assertion (A): All trigonometric functions have their inverses over their respective domains. Reason (R): The inverse of $\tan^{-1}x$ exists for some $x \in \mathbb{R}$.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is false and Reason (R) is true.

(D) Assertion (A) is true and Reason (R) is false.

Question 23. Assertion (A): The lines □□=$a\vec{1}$+□$b\vec{1}$ and □□=$a\vec{2}$+□$b\vec{2}$ are perpendicular, when $b\vec{1} \cdot b\vec{2} = 0$. Reason (R): The angle □ between the lines □□=$a\vec{1}$+□$b\vec{1}$ and □□=$a\vec{2}$+□$b\vec{2}$ is given by cos?= $\vec{b1} \cdot \vec{b2}$ / $|\vec{b1}||\vec{b2}|$

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Assertion (A) is false and Reason (R) is true.

(C) Assertion (A) is true and Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Section C

Question 24. Find the domain of $y = \sin^{-1}(x^2 - 4)$.

[2 Marks]

Question 25.

Evaluate: $\cos^{-1}[\cos(7\pi/3)] \times \cos^{-1}(1/3)$.

[2 Marks]

Question 26. If $(x^2 + y^2)^2 = xy$, then find dy/dx.

[2 Marks]

Question 27. Find the maximum and minimum values of the function given by f(x) = 5 + sin 2x.

[2 Marks]

Question 28.

If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} + 2\hat{k}$ is 1/3, then find the value(s) of p.

[2 Marks]

Question 29.

Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both lines:

x - 1/1 = y - 2/2 = z - 3/3; x /-3 = y/ 2= z /5.

[2 Marks]

Question 30. The equations of a line are 5x - 3 = 15y + 7 = 3 - 10z. Write the direction cosines of the line and find the coordinates of a point through which it passes.

[2 Marks]

Section D

Question 31.

Find $\int x^2+x+1/(x+1)^2(x+2)\,dx$

[3 Marks]

Question 32.

Evaluate

[3 Marks]

Question 33.

Evaluate

[3 Marks]

Question 34.

Find $\int e^x/\sqrt{5-4e^x-e^{2x}}\,dx$

[3 Marks]

Question 35.

Evaluate

[3 Marks]

Question 36.

Find the particular solution of the differential equation

dy/dx=x+y/x, y (1) = 0 \).

[3 Marks]

Question 37.

Find the general solution of the differential equation $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$.

[3 Marks]

Question 38.

Solve the linear programming problem graphically:

Minimise $z = -3x + 4y$

subject to constraints

$x + 2y \leq 8$,

$3x + 2y \leq 12$,

$x, y \geq 0$.

[3 Marks]

Question 39.

From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

[3 Marks]

Section E

Question 40.

[5 Marks]

Question 41.

Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

[5 Marks]

Question 42. If N denotes the set of all natural numbers and R is the relation on N × N defined by (a, b) R (c, d) if ad(b + c) = bc(a + d). Show that R is an equivalence relation.

[5 Marks]

Question 43.

Show that the following lines do not intersect each other: x-1/3 =y+1/2 =z-1/5; x+2/4=y-1/3=z+1/-2

[5 Marks]