CBSE EXAMINATION PAPER-2024

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 90

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **44 questions.** All questions are **compulsory.**
ii. This question paper is divided into **5 sections.**
iii. **Section A** – questions number **1 to 3** are case based questions
iv. **Section B** – questions number **4 to 22** are multiple choice questions
v. **Section C** – questions number **23 to 29** are very short answer
vi. **Section D** – questions number **30 to 38** are short answer
vii. **Section E** – questions number **39 to 44** are long answer
viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
ix. Use of calculator is NOT allowed.

Section A

Question 1. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h. The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = 4/V - 500/V^2 + 14$.

> **(1) Find dF/dV.**

[1 Marks]

(2) (a) Find the speed V for which fuel consumption F is minimum.

[2 Marks]

(3) Find F, when V = 40 km/h.

[1 Marks]

(4)

Find the quantity of fuel required to travel 600 km at the speed V at which $dF/dV = -0 \cdot 01$

[2 Marks]

Question 2.

The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy. A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg), available at ₹16/kg and ₹20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

(1) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.

[2 Marks]

(2) Identify and write all the constraints which determine the given feasible region in Figure-2.

[2 Marks]

Question 3.

Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.

Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive. Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

(1) Find the probability that the airplane will not crash.

[1 Marks]

(2)

Find $P(A \mid E_1) + P(A \mid E_2)$

[1 Marks]

(3) (a) Find P(A).

[2 Marks]

(4)

Find $P(E_2|A)$.

[2 Marks]

Section B

Question 4.

If the sum of all the elements of a 3 ×3 scalar matrix is 9, then the product of all its elements is :

[1 Marks]

(A) 0

(B) 729

(C) 9

(D) 27

Question 5.

Let $f : R_+ \rightarrow [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :

[1 Marks]

(A) one-one

(B) neither one-one nor onto

(C) onto

(D) bijective

Question 6.

[1 Marks]

(A) 4

(B) 2

(C) 1

(D) 0

Question 7.

The number of points of discontinuity of f(x) =

[1 Marks]

(A) 1

(B) infinite

(C) 2

(D) 0

Question 8. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

[1 Marks]

(A) strictly decreasing on R

(B) strictly increasing on R

(C) strictly decreasing on $(-\infty, 0)$

(D) neither strictly increasing nor strictly decreasing on R

Question 9. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

[1 Marks]

(A) π

(B) Zero (0)

(C) $\pi^2/4$

(D)

Question 10. The differential equation $dy/dx = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is :

[1 Marks]

(A) $\cos x - \sin (y/x)$

(B) y/x

(C) $\cos^2 (x/y)$

(D) $x^2 + y^2/xy$

Question 11. For any two vectors ⊠⊠and ⊠⊠which of the following statements is always true?

[1 Marks]

(A) ⊠⊠ ⊠⊠≥ |⊠⊠| |⊠⊠|

(B) ⊠⊠ ⊠⊠= |⊠⊠| |⊠⊠|

(C) ⊠⊠ ⊠⊠≤ |⊠⊠| |⊠⊠|

(D) ⊠⊠ ⊠⊠< |⊠⊠| |⊠⊠|

Question 12.

The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :

[1 Marks]

(A) (2, 0, 0)

(B) (1, 0, 0)

(C) ($\sqrt{5}$, 0, 0)

(D) (0, 0, 0)

Question 13. The common region determined by all the constraints of a linear programming problem is called :

[1 Marks]

(A) an unbounded region

(B) an optimal region

(C) a bounded region

(D) a feasible region

Question 14.

Let E be an event of a sample space S of an experiment, then P(S|E) =

[1 Marks]

(A) 0

(B) 1

(C) $P(S \cap E)$

(D) P(E)

Question 15. If A = [aij] be a 3 ×3 matrix, where aij = i – 3j, then which of the following is false?

[1 Marks]

(A) $a_{11} < 0$

(B) $a_{13} > a_{31}$

(C) $a_{31} = 0$

(D) $a_{12} + a_{21} = -6$

Question 16. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :

[1 Marks]

(A) $x/1 + x^4$

(B) $2x/1 + x^4$

(C) $1/1 + x^4$

(D) $-2x/1 + x^4$

Question 17.

The degree of the differential equation $(y''^2)2 + (y')^3 = x \sin(y')$ is :

[1 Marks]

(A) not defined

(B) 2

(C) 3

(D) 1

Question 18. The unit vector perpendicular to both vectors $\hat{i}+\hat{k}$ and $\hat{i}-\hat{k}$ is

[1 Marks]

(A) $2\hat{j}$

(B) $\hat{j}$

(C) $\hat{i}+\hat{k}/\sqrt{2}$

(D) $\hat{i}-\hat{k}/\sqrt{2}$

Question 19. Direction ratios of a vector parallel to line $x - 1/2 = -y = 2z + 1/6$ are :

[1 Marks]

(A) 2, 1, 6

(B) 2, 1, 3

(C) 2, − 1, 6

(D) 2, – 1, 3

Question 20. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :

[1 Marks]

(A) 90°

(B) 60°

(C) 120°

(D) 0°

Question 21. Assertion (A) : For any symmetric matrix A, B¢AB is a skew-symmetric matrix.
Reason (R) : A square matrix P is skew-symmetric if P' = – P.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Question 22. Assertion (A) : For two non-zero vectors [illegible] and [illegible], [illegible] ≠ [illegible] Reason (R) : For two non-zero vectors [illegible] and [illegible], [illegible] ≠ [illegible]

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Section C

Question 23.

Find the value of $\tan^{-1}(1/\sqrt{3}) + \cot^{-1}(1/\sqrt{3}) + \tan^{-1}[\sin(-\pi/2)]$.

[2 Marks]

Question 24. Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$ and also find its range.

[2 Marks]

Question 25. If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \pi/3$.

[2 Marks]

Question 26.

If $y = \text{cosec}\,(\cot^{-1} x)$, then prove that $\sqrt{1+x^2}\, dy/dx - x = 0$.

[2 Marks]

Question 27. If M and m denote the local maximum and local minimum values of the function $f(x) = x + 1/x\ (x \neq 0)$ respectively, find the value of $(M - m)$.

[2 Marks]

Question 28.

Find : $\int e^{4x} - 1 / e^{4x} + 1\, dx$.

[2 Marks]

Question 29. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing on its domain.

[2 Marks]

Section D

Question 30.

If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$ prove that $dy/dx = - y \log x / x \log y$.

[3 Marks]

Question 31.

Show that $d/dx\, (|x|) = x/|x|,\ x \neq 0$

[3 Marks]

Question 32.

Evaluate

[3 Marks]

Question 33.

Find $\int 1/x\,[\log x^2 - 3\log x - 4]\,dx$

[3 Marks]

Question 34.

Find the particular solution of the differential equation $2xy + y^2 - 2x^2\,dy/dx = 0$ $y = 2$ when $x = 1$.

[3 Marks]

Question 35.

Find the general solution of the differential equation

$y\,dx = (x + 2y^2)\,dy$.

[3 Marks]

Question 36.

The position vectors of vertices of $\triangle$ ABC are $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$ and $C(3\hat{i}-4\hat{j}-4\hat{k})$. Find all the angles of $\triangle$ ABC.

[3 Marks]

Question 37. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on the top of the dice, then find the probability distribution of X.

[3 Marks]

Question 38.

Find $\int x^2 . \sin^{-1}(x^{3/2})dx$

[3 Marks]

Section E

Question 39.

Show that a function f : R → R defined by $f(x) = 2x/1+x^2\}$ is neither one-one nor onto. Further, find set A so that the given function f : R → A becomes an onto function.

[5 Marks]

Question 40.

A relation R is defined on N × N (where N is the set of natural numbers) as:

(a, b) R (c, d) ⇔ a − c = b − d.

Show that R is an equivalence relation.

[5 Marks]

Question 41.

Find the equation of the line which bisects the line segment joining points A(2, 3, 4) and B(4, 5, 8) and is perpendicular to the lines x-8/3 = y+19/-16 = z-10/ 7 and x-15/3= y -29/8= z-5/5

[5 Marks]

Question 42.

Solve the following system of equations, using matrices: 2/x+3/y+10/z=4 ,= 4/x-6/y+15/z=1, 6/x+9/y-20/z=2

where x, y, z ≠0

[5 Marks]

Question 43.

[5 Marks]

Question 44. If A_1 denotes the area of region bounded by $y^2 = 4x$, x = 1 and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, x = 4, find $A_1 : A_2$.

[5 Marks]