CBSE EXAMINATION PAPER-2024

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 91

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **45 questions.** All questions are **compulsory.**
ii. This question paper is divided into **5 sections.**
iii. **Section A** – questions number **1 to 3** are case based questions
iv. **Section B** – questions number **4 to 23** are multiple choice questions
v. **Section C** – questions number **24 to 30** are very short answer
vi. **Section D** – questions number **31 to 39** are short answer
vii. **Section E** – questions number **40 to 45** are long answer
viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
ix. Use of calculator is NOT allowed.

Section A

Question 1.

Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves. A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

where x denotes the number of hours.

> **(1) Express the probability distribution given above in the form of a probability distribution table.**
>
> [1 Marks]

> **(2) Find the value of k.**
>
> [1 Marks]

> **(3) Find $P(1 < X < 6)$.**
>
> [2 Marks]

> **(4) Find the mean number of hours spent by the student.**
>
> [2 Marks]

Question 2.

A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using an exponential growth model, the rate of growth of this sample of bacteria is calculated.

The differential equation representing the growth as: $dP/dt = kP$, where P is the population at any time t.

> **(1) Obtain the general solution of the given differential equation and express it as an exponential function of t.**

[2 Marks]

(2) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k.

[2 Marks]

Question 3.

A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on academic achievements, others based on financial needs. Every year a school offers scholarships to girl children and meritorious achievers based on criteria. In the session 2022–23, the school offered monthly scholarships of Rs. 3000 each to girl students and Rs. 4000 each to meritorious achievers in academics and sports.

In all, 50 students were given scholarships and the monthly expenditure was Rs. 1,80,000.

(1) Check whether the system of matrix equations so obtained is consistent or not.

[1 Marks]

(2) Express the given information algebraically using matrices.

[1 Marks]

(3) Had the amount of scholarship given to each girl child and meritorious achiever been interchanged, what would be the monthly expenditure incurred by the school?

[2 Marks]

(4) Find the number of scholarships of each kind given by the school, using matrices.

[2 Marks]

Section B

Question 4.

If A = [a_ij] is an identity matrix, then which of the following is true?

[1 Marks]

(A)

(B) a_ij = 0, ∀i, j

(C)

(D) a_ij = 1, ∀ i, j

Question 5.

Let R_+ denote the set of all non-negative real numbers. Then the function $f : R_+ \to R+$ defined as $f(x) = x^2 + 1$ is:

[1 Marks]

(A) neither one-one nor onto

(B) both one-one and onto

(C) onto but not one-one

(D) one-one but not onto

Question 6.

Let A = [[a, b], [c, d]] be a square matrix such that adj A = A. Then, (a + b + c + d) is equal to:

[1 Marks]

(A) 2a

(B) 2c

(C) 2b

(D) 0

Question 7. A function $f(x) = |1 - x + |x||$ is:

[1 Marks]

(A) discontinuous at x = 1 only

(B) discontinuous at x = 0 only

(C) continuous everywhere

(D) discontinuous at x = 0, 1

Question 8. If the sides of a square are decreasing at the rate of 1.5 cm/s, the rate of decrease of its perimeter is:

[1 Marks]

(A) 1.5 cm/s

(B) 6 cm/s

(C) 3 cm/s

(D) 2.25 cm/s

Question 9.

[1 Marks]

(A) f(−x) = f(x)

(B) f(a − x) = f(x)

(C) f(−x) = −f(x)

(D) f(a − x) = −f(x)

Question 10. x log x dy/dx + y = 2 log x is an example of a:

[1 Marks]

(A) variable separable differential equation

(B) homogeneous differential equation

(C) first order linear differential equation

(D) differential equation whose degree is not defined

Question 11.

if [illegible] $=2\hat{i}-\hat{j}+\hat{k}$ and [illegible] $=\hat{i}+\hat{j}-\hat{k}$, then [illegible] and [illegible] are:

[1 Marks]

(A) unit vectors

(B) perpendicular vectors

(C) collinear vectors which are not parallel

(D) parallel vectors

Question 12.

If α,β, and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is NOT true?

[1 Marks]

(A) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(C) $\cos \alpha + \cos \beta + \cos \gamma = 1$

(D) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Question 13. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called:

[1 Marks]

(A) feasible solutions

(B) constraints

(C) optimal solutions

(D) infeasible solutions

Question 14.

Let E and F be two events such that $P(E) = 0.1$, $P(F) = 0.3$, $P(E \cup F) = 0.4$, then $P(F|E)$ is:

[1 Marks]

(A) 0.6

(B) 0.5

(C) 0

(D) 0.4

Question 15. If A and B are two skew symmetric matrices, then (AB + BA) is:

[1 Marks]

(A) a skew symmetric matrix

(B) a null matrix

(C) an identity matrix

(D) a symmetric matrix

Question 16.

[1 Marks]

(A) ±2

(B) ∓2

(C) 2

(D) −2

Question 17.

The derivative of 2^x w.r.t. 3^x is:

[1 Marks]

(A) $(3/2)^x$ log 2/log3

(B) $(3/2)^x$ log 3/log2

(C) $(2/3)^x$ log 3/log2

(D) $(2/3)^x$ log 2/log3

Question 18.

If |⊠|=2 and-3≤k≤2, then|k⊠|∈:

[1 Marks]

(A) [−6, 4]

(B) [4, 6]

(C) [0, 4]

(D) [0, 6]

Question 19. If a line makes an angle of π/4 with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is:

[1 Marks]

(A) π/4

(B) 0

(C) π/2

(D) π

Question 20.

Of the following, which group of constraints represents the feasible region given below?

[1 Marks]

(A) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$

(B) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$

(C) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$

(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

Question 21.

[1 Marks]

(A)

(B)

(C)

(D)

Question 22. Assertion (A) : Every scalar matrix is a diagonal matrix. Reason (R) : In a diagonal matrix, all the diagonal elements are 0.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Assertion (A) is true, but Reason (R) is false.

(C) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(D) Assertion (A) is false, but Reason (R) is true

Question 23. Assertion (A): Projection of □□on □□is same as projection of □□on □□Reason (R): Angle between □□and □□is same as angle between □□and □□numerically.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Assertion (A) is true, but Reason (R) is false.

(C) Assertion (A) is false, but Reason (R) is true

(D) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Section C

Question 24.

Evaluate: $\sec^2(\tan^{-1}1/2) + \text{cosec}^2(\cot^{-1}1/3)$

[2 Marks]

Question 25.

If $x = e^{x/y}$, prove that (2) (dy/dx) =log x-1(log x) ^{2}

[2 Marks]

Question 26.

Check the differentiability of

[2 Marks]

Question 27.

Evaluate

[2 Marks]

Question 28.

Given d/dx F(x)= 1/√2x – x^2 and F (1) = 0, find F(x).

[2 Marks]

Question 29.

Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively in the ratio 4 : 1 externally. Further, find |A⊠⊠|:|B⊠⊠|

[2 Marks]

Question 30.

Let ⊠⊠ and ⊠⊠ be two non-zero vectors.

Prove that |⊠⊠×⊠⊠|≤|⊠⊠||⊠⊠|

State the condition under which equality holds, i.e., |⊠⊠×⊠⊠|=|⊠⊠||⊠⊠|

[2 Marks]

Section D

Question 31.

If x cos (p + y) + cos p sin (p + y) = 0, prove that

cos p (dy/dx) = – $\cos^2$ (p + y), where p is a constant.

[3 Marks]

Question 32.

Find the value of a and b so that the function f defined as

[3 Marks]

Question 33.

Find the intervals in which the function f(x) = x log/ x is strictly increasing or strictly decreasing.

[3 Marks]

Question 34.

Find the absolute maximum and absolute minimum values of the function f given by f(x) = x/2+ 2/x on the interval [1, 2].

[3 Marks]

Question 35.

Find: $\int x^2+1/(x^2+2)(x^2+4)\,dx$

[3 Marks]

Question 36.

Find $\int 2+\sin 2x /1 +\cos 2x\, e^x\, dx$.

[3 Marks]

Question 37.

Evaluate

[3 Marks]

Question 38.

Solve the following linear programming problem graphically:

Maximise z = 4x + 3y,

subject to the constraints

$x + y \leq 800$

$2x + y \leq 1000$

$x \leq 400$

$x, y \geq 0$.

[3 Marks]

Question 39.

The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

[3 Marks]

Section E

Question 40. A relation R on set A = {−4, −3, −2, −1, 0, 1, 2, 3, 4} is defined as R = {(x, y) : x + y is an integer divisible by 2}. Show that R is an equivalence relation. Also, write the equivalence class [2].

[5 Marks]

Question 41. It is given that function $f(x) = x^4 - 6x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

[5 Marks]

Question 42. The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

[5 Marks]

Question 43. Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines $x = -2$ and $x = 2$.

[5 Marks]

Question 44.

Find the equation of the line passing through the point of intersection of the lines x/1 = y-1/2=z-2/3 and x-1/0 = y/−3 = z-7/2 and perpendicular to these given lines.

[5 Marks]

Question 45.

Two vertices of parallelogram ABCD are A(−1, 2, 1) and B(1, −2, 5). If the equation of the line passing through C and D is x-4/1= y+7/-2 = z-8/2, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

[5 Marks]
