

CBSE EXAMINATION PAPER-2024

MATHEMATICS

(Solved)

Time allowed : 3 hours

Maximum Marks : 95

General Instructions :

Read the following instructions carefully and follow them :

- i. This question paper contains **45 questions**. All questions are **compulsory**.
- ii. This question paper is divided into **5 sections**.
- iii. **Section A** – questions number **1 to 3** are case based questions
- iv. **Section B** – questions number **4 to 22** are multiple choice questions
- v. **Section C** – questions number **23 to 29** are very short answer
- vi. **Section D** – questions number **30 to 38** are short answer
- vii. **Section E** – questions number **39 to 45** are long answer
- viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
- ix. Use of calculator is NOT allowed.

Section A

Question 1. A store has been selling calculators at ₹350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - (1/2)x$.

(1) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = x p(x)$. Also, verify the result.

[2 Marks]

(2) What rebate in price of calculator should the store give to maximise the revenue?

[2 Marks]

Question 2.

An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at $O (0, 0, 0)$ and the three stars have their locations at the points D , A and V having position vectors $2\hat{i}+3\hat{j}+4\hat{k}$, $7\hat{i}+5\hat{j}+8\hat{k}$ and $-3\hat{i}+7\hat{j}+11\hat{k}$ respectively.

(1)

Find a unit vector in the direction of vector D .

[1 Marks]

(2) How far is the star V from star A ?

[1 Marks]

(3)

Find the measure of angle $\angle VDA$.

[2 Marks]

(4)

What is the projection of vector D on vector D ?

[1 Marks]

Question 3. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $1/5$, Jaspreet's selection is $1/3$ and Alia's selection is $1/4$. The event of selection is independent of each other.

(1)

Find $P(G|H^c)$ where G is the event of Jaspreet's selection and H^c denotes the event that Rohit is not selected.

[1 Marks]

(2) What is the probability that at least one of them is selected?

[1 Marks]

(3)

Find the probability that exactly one of them is selected.

[2 Marks]

(4)

Find the probability that exactly two of them are selected.

[3 Marks]

Section B

Question 4.

[1 Marks]

(A) 0

(B) 10

(C) 5

(D) 25

Question 5.

[1 Marks]

(A)

(B)

(C)

(D)

Question 6.

[1 Marks]

(A) 100 l

(B) 10

(C) 10 l

(D) 1000

Question 7.

[1 Marks]

(A) 4

(B) 2

(C) -2

(D) -4

Question 8.

Derivative of e^{2x} with respect to e^x is:

[1 Marks]

(A) $2e^x$

(B) $2e^{3x}$

(C) e^x

(D) $2e^{2x}$

Question 9.

For what value of k , the function given below is continuous at $x = 0$?

[1 Marks]

(A) 0

(B) 1

(C) $1/4$

(D) 4

Question 10.

The value of

[1 Marks]

(A) $\pi/2$

(B) $\pi/4$

(C) $\pi/18$

(D) $\pi/6$

Question 11.

The general solution of the differential equation $x dy + y dx = 0$ is:

[1 Marks]

(A) $xy = c$

(B) $\log y = \log x + c$

(C) $x^2 + y^2 = c^2$

(D) $x + y = c$

Question 12.

The integrating factor of the differential equation $(x + 2y^2) dy/dx = y$ ($y > 0$) is:

[1 Marks]

(A) y

(B) $1/y$

(C) $1/x$

(D) x

Question 13.

If \vec{a} and \vec{b} are two vectors such that $|\vec{a}|=1$, $|\vec{b}|=2$ and $|\vec{a}+\vec{b}|=3$, then the angle between $2\vec{a}$ and $-\vec{b}$ is:

[1 Marks]

(A) $\pi/3$

(B) $11\pi/6$

(C) $5\pi/6$

(D) $\pi/6$

Question 14.

The vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $-3\hat{i}+4\hat{j}+4\hat{k}$ represents the sides of

[1 Marks]

(A) a right-angled triangle

(B) an isosceles triangle

(C) an obtuse-angled triangle

(D) an equilateral triangle

Question 15.

Let \vec{a} be any vector such that $|\vec{a}|=a$. The value of $|\vec{a}\hat{i}|^2+|\vec{a}\hat{j}|^2+|\vec{a}\hat{k}|^2$ is:

[1 Marks]

(A) 0

(B) $3a^2$

(C) a^2

(D) $2a^2$

Question 16.

The vector equation of a line passing through the point $(1, -1, 0)$ and parallel to Y-axis is:

[1 Marks]

(A)

(B)

(C)

(D)

Question 17.

The lines $1-x/2 = y-1/3 = z/1$ and $(2x-3)/2p = y/-1 = z-4/7$ are perpendicular to each other for p equal to:

[1 Marks]

(A) $-1/2$

(B) 2

(C) $1/2$

(D) 3

Question 18.

The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given is:

[1 Marks]

(A) 110

(B) 50

(C) 120

(D) 170

Question 19.

The probability distribution of a random variable X is:

where k is unknown.

The probability that the random variable X takes the value 2 is:

[1 Marks]

(A) $1/5$

(B) 1

(C) $2/5$

(D) $4/5$

Question 20. The function $f(x) = kx - \sin x$ is strictly increasing for:

[1 Marks]

(A) $k < 1$

(B) $k > 1$

(C) $k > -1$

(D) $k < -1$

Question 21. Assertion (A): The relation $R = \{(x, y) : (x+y) \text{ is a prime number and } x, y \in \mathbb{N}\}$ is not a reflexive relation. Reason (R): The number ' $2n$ ' is composite for all natural numbers n .

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Question 22.

Assertion (A): The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.

Reason (R): The optimal solution of a LPP having bounded feasible region must occur at corner points.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(B) Assertion (A) is true, but Reason (R) is false.

(C) Assertion (A) is false, but Reason (R) is true.

(D) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

Section C

Question 23.

Express $\tan^{-1}(\cos x / 1 - \sin x)$, where $-\pi/2 < x < \pi/2$ in the simplest form.

[2 Marks]

Question 24.

Find the principal value of $\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/\sqrt{2})$

[2 Marks]

Question 25.

If $y = \cos^3(\sec^2 2t)$, find dy/dt .

[2 Marks]

Question 26.

If $x^y = e^{x-y}$, prove that $dy/dx = \log x / (1 + \log x^2)$

[2 Marks]

Question 27.

Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

[2 Marks]

Question 28. The volume of a cube is increasing at the rate of $6 \text{ cm}^3/\text{s}$. How fast is the surface area of the cube increasing when the length of an edge is 8 cm ?

[2 Marks]

Question 29.

Find $\int \frac{1}{x(x^2-1)} dx$.

[2 Marks]

Section D

Question 30.

Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, find dy/dx .

[3 Marks]

Question 31.

Evaluate:

[3 Marks]

Question 32.

Find: $\int e^x [1/(1+x^2)^{3/2} + x/\sqrt{1+x^2}] dx$.

[3 Marks]

Question 33.

Find: $\int \frac{3x+5}{\sqrt{x^2+2x+4}} dx$.

[3 Marks]

Question 34.

Find the particular solution of the differential equation $(dy/dx) = y \cot 2x$, given $y(\pi/4) = 2$.

[3 Marks]

Question 35.

Find the particular solution of the differential equation

$(xe^{y/x} + y) dx = x dy$, given $y = 1$ when $x = 1$.

[3 Marks]

Question 36.

Solve the following linear programming problem graphically:

$$\text{Maximise } Z = 2x + 3y$$

subject to constraints:

$$x + y \leq 6,$$

$$x \geq 2,$$

$$y \leq 3,$$

$$x, y \geq 0.$$

[3 Marks]

Question 37. A card from a well-shuffled deck of 52 playing cards is lost. From the remaining cards, a card drawn at random is a King. Find the probability that the lost card is a King.

[3 Marks]

Question 38.

A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes and the distribution.

[3 Marks]

Section E

Question 39.

Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

[5 Marks]

Question 40.

Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis

[5 Marks]

Question 41.

Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f: A \rightarrow B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

[5 Marks]

Question 42.

Check whether the relation S in the set of real numbers \mathbb{R} defined by $S = \{(a,b) : \text{where } a - b + \sqrt{2} \text{ is an irrational number}\}$ is reflexive, symmetric or transitive.

[5 Marks]

Question 43.

$$2x + y - 3z = 13$$

$$3x + 2y + z = 4$$

$$x + 2y - z = 8$$

[5 Marks]

Question 44.

Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{z-1}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

[5 Marks]

Question 45.

If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3, -4, 7)$.

[5 Marks]
