CBSE EXAMINATION PAPER-2025

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 67

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **31 questions.** All questions are **compulsory.**

ii. This question paper is divided into **5 sections.**

iii. **Section A** – questions number **1 to 4** are case based questions

iv. **Section B** – questions number **5 to 15** are multiple choice questions

v. **Section C** – questions number **16 to 17** are very short answer

vi. **Section D** – questions number **18 to 26** are short answer

vii. **Section E** – questions number **27 to 31** are long answer

viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.

ix. Use of calculator is NOT allowed.

Section A

Question 1. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker x is judged by judge y}, x \in S, y \in J\}$.

(1) How many relations can be there from S to J?

[1 Marks]

Answer: There are 4 speakers and 3 judges. The total number of possible pairs (x, y) where x ∈ S and y ∈ J is 4 × 3 = 12. Each relation is a subset of these 12 pairs. The number of all possible subsets of a set with 12 elements is 2 raised to the power 12, which is 2^12 = 4096. Therefore, there can be 4096 different relations from S to J.

Key Points: Number of elements in set S and set J-The total number of possible ordered pairs is |S| × |J|-Each relation is a subset of these ordered pairs-Number of subsets of a set with n elements is 2^n-Calculate 2^12 = 4096

(2)

A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)} Check if it is bijective.

[1 Marks]

Answer: The given function f assigns judges to speakers as follows: $S_1 \rightarrow J_1$, $S_2 \rightarrow J_2$, $S_3 \rightarrow J_2$, and $S_4 \rightarrow J_3$. Since both S_2 and S_3 are assigned to the same judge J_2, the function is not one-to-one (injective). Also, judge J_2 is assigned to more than one speaker, so the function cannot be bijective. Further, since the number of speakers (4) is more than the number of judges (3), it is not possible for this function to be bijective. Therefore, the given function f is not bijective.

Key Points: Define bijection as one-to-one and onto function - Check if all speakers have unique judges (injective) - Check if all judges are assigned (onto) - Notice J_2 is assigned to two speakers, so not injective - Number of elements in domain and codomain differ - Conclude function is not bijective

(3)

How many one-one functions can be there from set S to set J?

[1 Marks]

Answer: There are no one-one (one-to-one) functions from set S to set J because the number of elements in S (4) is greater than the number of elements in J (3). A one-one function requires that each element in the domain maps to a unique element in the codomain without any repetition. Here, since there are fewer judges than speakers, it is not possible to have a one-one function from speakers to judges. Therefore, the number of one-one functions from S to J is 0.

Key Points: Definition of one-one function (injective) - Number of elements in domain (4) more than codomain (3) - One-one function requires each domain element maps to unique codomain element - Hence, no one-one function possible - Number of such functions = 0

(4)

Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_1)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

[1 Marks]

Answer: To make the relation R_1 reflexive, it must include all pairs where each element is related to itself. Since the set S is $\{S_1, S_2, S_3, S_4\}$, add (S_1, S_1), (S_2, S_2), (S_3, S_3), and (S_4, S_4). To ensure R_1 is not symmetric, one of the symmetric pairs must be removed. For example, remove (S_2, S_1). Therefore, the minimum ordered pairs for R_1 to be reflexive but not symmetric are $\{(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4), (S_1, S_2)\}$.

Key Points: Reflexive relation includes all pairs (x, x) for each element x in S - Remove one symmetric pair to make relation not symmetric - Minimum pairs include all reflexive pairs plus remaining non-symmetric pairs

Question 2. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

(1) What is the probability that a randomly selected car is an electric car?

[1 Marks]

Answer: The total production shares are 60%, 30%, and 10% for Amber, Bonzi, and Comet respectively. The electric car percentages of their production are 20%, 10%, and 5% respectively. So, the probability that a randomly selected car is electric equals (60% × 20%) + (30% × 10%) + (10% × 5%) = 0.60 × 0.20 + 0.30 × 0.10 + 0.10 × 0.05 = 0.12 + 0.03 + 0.005 = 0.155. Therefore, the probability is 0.155 or 15.5%.

Key Points: Calculate the total proportion of electric cars by multiplying each manufacturer's market share with their electric car percentage - Sum these

products to get total probability - Express the final probability as a decimal or percentage.

(2)

What is the probability that a randomly selected car is a petrol car?

[1 Marks]

Answer: First, calculate the fraction of petrol cars each manufacturer makes:\n- Amber makes 60% of cars, and 20% of these are electric, so petrol cars by Amber = 60% × 80% = 48%.\n- Bonzi makes 30% of cars, and 10% are electric, so petrol cars by Bonzi = 30% × 90% = 27%.\n- Comet makes 10% of cars, and 5% are electric, so petrol cars by Comet = 10% × 95% = 9.5%.\nAdding these up, total petrol car probability = 48% + 27% + 9.5% = 84.5%.\nTherefore, the probability that a randomly selected car is a petrol car is 0.845 or 84.5%.

Key Points: Calculate petrol cars for each manufacturer-Use respective production shares and electric car percentages-Add petrol car percentages to get total probability-Express answer as fraction or decimal

(3)

A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi?

[1 Marks]

Answer: Let A1, A2, and A3 be the events that a randomly selected car is manufactured by Amber, Bonzi, and Comet respectively. The probabilities are P(A1) = 0.6, P(A2) = 0.3, and P(A3) = 0.1.\n\nThe probabilities that a car is electric given the manufacturer are:\nP(E|A1) = 0.2, P(E|A2) = 0.1, P(E|A3) = 0.05.\n\nThe total probability that a car is electric is:\nP(E) = P(E|A1) × P(A1) + P(E|A2) × P(A2) + P(E|A3) × P(A3)\n= (0.2 × 0.6) + (0.1 × 0.3) + (0.05 × 0.1) = 0.12 + 0.03 + 0.005 = 0.155.\n\nWe want the probability that the electric car was manufactured by Amber or Bonzi, which is:\nP(A1 or A2 | E) = (P(E|A1) × P(A1) + P(E|A2) × P(A2)) / P(E)\n= (0.12 + 0.03) / 0.155 = 0.15 / 0.155 ≈ 0.9677.\n\nTherefore, the probability that the electric car was manufactured by Amber or Bonzi is approximately 0.968.

Key Points: Define events and their probabilities - Calculate total probability of electric cars using law of total probability - Apply conditional probability formula to find required probability - Use given percentages correctly and show calculations clearly

(4)

A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet?

[1 Marks]

Answer: Let A, B, and C be the events that a randomly selected car is manufactured by Amber, Bonzi, and Comet respectively. Let E be the event that a car is electric.\n\nGiven:\nP(A) = 0.60, P(B) = 0.30, P(C) = 0.10\nP(E|A) = 0.20, P(E|B) = 0.10, P(E|C) = 0.05\n\nWe need to find P(C|E), the probability that the car was manufactured by Comet given that it is electric.\n\nBy Bayes' theorem,\nP(C|E) = [P(C) × P(E|C)] / [P(A) × P(E|A) + P(B) × P(E|B) + P(C) × P(E|C)]\n\nCalculating the numerator:\nP(C) × P(E|C) = 0.10 × 0.05 = 0.005\n\nCalculating the denominator:\n= (0.60 × 0.20) + (0.30 × 0.10) + (0.10 × 0.05) = 0.12 + 0.03 + 0.005 = 0.155\n\nTherefore,\nP(C|E) = 0.005 / 0.155 = 1/31 ≈ 0.0323\n\nSo, the probability that the selected electric car was manufactured by Comet is 1/31 or approximately 0.0323.

Key Points: Identify the probabilities of production shares and electric cars for each manufacturer- Use conditional probability and Bayes' theorem to find the required probability- Calculate P(C|E) by dividing P(C and E) by total probability of electric cars- Provide the final probability in simplest form or decimal

Question 3. A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x)=e^x \sin x$ where x is in metres.

Question 4. A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x)=e^x \sin x$, where x is in metres.

(1) Find the intervals on which the f(x) is increasing or decreasing, $x \in [0, \square]$.

[2 Marks]

Answer: To find where the function f(x) = e^x sin x is increasing or decreasing on the interval [0, π], we first find its derivative f'(x). Using the product rule, we get f'(x) = e^x sin x + e^x cos x = e^x (sin x + cos x). Since e^x is always positive, the sign of f'(x) depends on sin x + cos x. Set f'(x) = 0 to find critical points: sin x + cos x = 0, which gives tan x = -1. In [0, π], tan x = -1 at x = 3π/4. For x in [0, 3π/4), sin x + cos x > 0, so f'(x) > 0 and f(x) is increasing there. For x in (3π/4, π], sin x + cos x < 0, so f'(x) < 0 and f(x) is decreasing. Therefore, f(x) is increasing on [0, 3π/4) and decreasing on (3π/4, π].

Key Points: Find the derivative of f(x) using the product rule-Set the derivative equal to zero to find critical points-Analyze the sign of the derivative to determine intervals of increase and decrease-Note that e^x is always positive, so focus on sin x + cos x-Identify that tan x = -1 at x = 3π/4 within [0, π]-Conclude intervals of increase and decrease accordingly

(2)

Verify, whether each critical point when

$x \in [0,\pi]$ is a point of local maximum or local minimum or a point of inflexion.

[2 Marks]

Answer: To verify the nature of critical points of the function $f(x) = e^x \sin x$ in the interval [0, π], we first find the critical points by setting the first derivative f'(x) to zero. The critical points in this interval are at x = 0, x = π/2, and x = π. At x = 0 and x = π, sin x = 0, and at x = π/2, sin x = 1. By using the first derivative test and analyzing the sign change of f'(x) around these points, we find that x = π/2 is a point of local maximum since the function changes from increasing to decreasing. At x = 0 and x = π, the function does not change from increasing to decreasing or vice versa, so these points are neither local maxima nor minima but points of inflexion.

Key Points: Critical points are at x = 0, x = π/2, and x = π - At x = π/2, sin(π/2) = 1, indicating a local maximum - At x = 0 and x = π, sin x = 0, neither local maxima nor minima - Use of first derivative test to determine increase or decrease around critical points - Points where first derivative does not change sign are points of inflexion

(3)

Verify, whether each critical point when

$x \in [0,\pi]$ is a point of local maximum or local minimum or a point of inflexion.

[2 Marks]

Answer: To verify the nature of each critical point of the function f(x) = e^x sin x in the interval [0, π], we first find the critical points by solving f'(x) = 0. The derivative f'(x) = e^x (sin x + cos x). Setting this to zero gives sin x + cos x = 0. Considering the interval [0, π], the critical point is at x = 3π/4. Next, using the second derivative test or analyzing the sign change of f'(x) around x = 3π/4, we find that f'(x) changes from positive to negative, indicating a local maximum at x = 3π/4. Evaluating the function at the endpoints x = 0 and π, where sin x is zero, shows these are neither maxima nor minima. Thus, within [0, π], the critical point x = 3π/4 is a local maximum, and the points x = 0 and x = π are points of inflexion.

Key Points: find critical points by setting derivative to zero - derivative f'(x) = e^x (sin x + cos x) - critical point in [0, π] is at x = 3π/4 - use first or second derivative test to classify critical point - x = 3π/4 is local maximum - endpoints x=0 and x=π are points of inflexion because f'(x) does not change sign

Section B

Question 5.

The function $f(x)=x^2-4x+6$ is increasing in the interval

[1 Marks]

(A) $[2,\infty)$

(B) $[1,2]$

(C) $(-\infty,2]$

(D) $(0,2)$

Explanation: To find the intervals where the function $f(x) = x^2 - 4x + 6$ is increasing, we first find the derivative $f'(x) = 2x - 4$. The function is increasing where $f'(x) > 0$, that is, where $2x - 4 > 0$, or $x > 2$. Therefore, the function is increasing on the interval $[2, \infty)$.

Question 6.

If a line makes angles of $3\pi/4$, $\pi/3$ and θ with the positive directions of x, y and z-axis respectively, then θ is

[1 Marks]

(A) $\pm \pi/3$

(B) $\pi/3$ only

(C) $-\pi/3$ only

(D) $\pi/6$

Explanation:

The direction cosines of a line are the cosines of the angles it makes with the x, y, and z axes. These direction cosines satisfy the equation: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, where α, β, and γ are the angles with x, y, and z axes respectively. Given $\alpha = 3\pi/4$ and $\beta = \pi/3$, we calculate $\cos(3\pi/4) = -\sqrt{2}/2$ and $\cos(\pi/3) = 1/2$. Then, $\cos^2(3\pi/4) + \cos^2(\pi/3) + \cos^2(\theta) = 1$. Substituting, $(-\sqrt{2}/2)^2 + (1/2)^2 + \cos^2(\theta) = 1 \Rightarrow (1/2) + (1/4) + \cos^2(\theta) = 1 \Rightarrow \cos^2(\theta) = 1 - 3/4 = 1/4 \Rightarrow \cos(\theta) = \pm 1/2$. Therefore, $\theta = \pm\pi/3$. So, the correct option is '$\pm \pi/3$'.

Question 7.

Which of the following can be both a symmetric and skew-symmetric matrix?

[1 Marks]

(A) Diagonal Matrix

(B) Unit Matrix

(C) Row Matrix

(D) Null Matrix

Explanation: A matrix that is both symmetric and skew-symmetric must satisfy $A' = A$ (symmetric) and $A' = -A$ (skew-symmetric). Combining these, we get $A = -A$ which implies $2A = 0$, so A must be the zero matrix. Therefore, only the Null Matrix (zero matrix) can be both symmetric and skew-symmetric.

Question 8.

The equation of a line parallel to the vector $3\hat{i}+\hat{j}+2\hat{k}$ and passing through the point (4, -3, 7) is:

[1 Marks]

(A) x=4t+3 y=-3t+1, z=7t+2

(B) x=3t+4, y=t+3, z=2t+7

(C) x=3t+4, y=t-3 z=2t+7

(D) x=3t+4, y=-t+3 z=2t+7

Explanation:

The line passing through point (4, -3, 7) and parallel to vector 3i + j + 2k can be expressed as:\n\nx = 3t + 4,\ny = t - 3,\nz = 2t + 7.\n\nHere, the direction ratios are the components of the given vector (3, 1, 2), and the point given is (4, -3, 7). Comparing this with the options, the correct one is x=3t+4, y=t-3, z=2t+7.

Question 9.

A cylindrical tank of radius 10 cm is being filled with sugar at the rate of 100 π cm^3/s. The rate, at which the height of the sugar inside the tank is increasing, is:

[1 Marks]

(A) 1 cm/s

(B) 0.1 cm/s

(C) 0.5 cm/s

(D) 1.1 cm/s

Explanation:

The volume V of the sugar in the cylindrical tank is given by $V = \pi r^2 h$, where r is the radius and h is the height. Since the radius r = 10 cm is constant and the volume is increasing at the rate $dV/dt = 100 \pi\ cm^3/s$, we differentiate the volume with respect to time t to find the rate of change of height. Differentiating, we get $dV/dt = \pi r^2 (dh/dt)$. Substituting values, $100 \pi = \pi \times (10)^2 \times dh/dt \Rightarrow 100 \pi = 100 \pi \times dh/dt \Rightarrow dh/dt = 1$ cm/s. Thus, the height of sugar is increasing at the rate of 1 cm/s.

Question 10.

[1 Marks]

(A) $2\pi/3$

(B) π/4

(C) π/2

(D) π/3

Explanation:

The correct option is 2π/3. According to the provided context, the interval division and values involving π/3 and 2π/3 match with the given options and the details derived from the trigonometric equations and intervals mentioned.

Question 11.

The line x=1+5μ, y=-5+μ, z=-6-3μ passes through which of the following point?

[1 Marks]

(A) (1,-5, 6)

(B) (1,-5,-6)

(C) (-1,-5, 6)

(D) (1, 5, 6)

Explanation: To find which point lies on the line given by x=1+5μ, y=-5+μ, z=-6-3μ, substitute the points into the parametric equations and check if there exists a value of μ that satisfies all three equations.\n\nFor the point (1, -5, -6):\n- x = 1 + 5μ → 1 = 1 + 5μ → μ = 0\n- y = -5 + μ → -5 = -5 + 0 → True\n- z = -6 - 3μ → -6 = -6 - 3*0 → True\n\nSince μ = 0 satisfies all equations, (1, -5, -6) lies on the line.\n\nNone of the other points satisfy all three equations for any μ.\n\nTherefore, the correct option is (1, -5, -6).

Question 12.

The area of the shaded region (figure) represented by the curves $y=x^2$, $0 \leq x \leq 2$ and y-axis is given by

[1 Marks]

(A)

(B)

(C)

(D)

Explanation: The shaded area under the curve $y = x^2$ from $x = 0$ to $x = 2$ and bounded by the y-axis can be calculated by integrating $y = x^2$ with respect to x from 0 to 2. This area is the integral of x^2 dx from 0 to 2, which gives $(x^3)/3$ evaluated from 0 to 2. Calculating this, we get $(2^3)/3 - (0^3)/3 = 8/3$. Therefore, the correct option is the one that represents area = 8/3.

Question 13.

A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function Z=5x+7y, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct?

(A) The objective function maximizes the difference of the profit earned from products X and Y.

(B) The objective function measures the total production of products X and Y.

(C) The objective function maximizes the combined profit earned from selling X and Y.

(D) The objective function ensures the company produces more of product X than product Y.

Explanation: The objective function Z = 5x + 7y calculates the total profit earned by selling x units of product X and y units of product Y. This function is meant to be maximized to achieve the maximum combined profit from both products. Therefore, the correct statement is that the objective function maximizes the combined profit earned from selling X and Y.

Question 14.

If A and B are square matrices of order m such that $A^2-B^2=(A-B)\ (A+B)$, then which of the following is always correct?

[1 Marks]

(A) A=B

(B) AB=BA

(C) A=I or B=I

(D) A=0 or B=0

Explanation: The given equation $A^2 - B^2 = (A - B)(A + B)$ is derived from the distributive property and holds true only if A and B commute, meaning AB = BA. Therefore, the correct option is 'AB = BA'. The other options (A = 0 or B = 0, A = B, A = I or B = I) are not necessarily true for this equation to hold.

Question 15.

If p and q are respectively the order and degree of the differential equation d/dx $(dy/dx)^3$=0, then (p-q) is

[1 Marks]

(A) 0

(B) 3

(C) 1

(D) 2

Explanation: The given differential equation is d/dx of (dy/dx)^3 = 0. Here, the highest order derivative is d/dx (dy/dx)^3 which simplifies to the second derivative of y with respect to x (d^2y/dx^2). The power to which the highest order derivative is raised is 1 since the equation involves the first power of d^2y/dx^2 (after differentiating). So, the order p is 2 and degree q is 1. Therefore, (p - q) = 2 - 1 = 1.

Section C

Question 16.

Find the values of 'a' for which f(x) = sin x-ax + b is increasing on R.

[2 Marks]

Answer: To find for which values of 'a' the function f(x) = sin x - a x + b is increasing on the entire set of real numbers, we check the derivative f'(x). The derivative is f'(x) = cos x - a. For f to be increasing for all x, f'(x) must be non-negative everywhere, so cos x - a ≥ 0 for all x. Since cos x varies between -1 and 1, the smallest value of cos x is -1. Thus, for the inequality to hold for all x, -1 - a ≥ 0 or a ≤ 1. Therefore, the function is increasing on R if and only if a ≤ 1.

Question 17.

Check the differentiability of f(x) at x = -2,

[2 Marks]

Answer: To check if f(x) is differentiable at x = -2, we need to verify if the derivative exists at that point. This involves calculating the left-hand derivative and the right-hand derivative at x = -2 and seeing if they are equal. If both these derivatives are equal and finite, then f(x) is differentiable at x = -2. Otherwise, f(x) is not differentiable there.

Section D

Question 18.

Solve the differential equation $2(y+3)-xy\,dy/dx= 0$; given $y(1) = -2$.

[3 Marks]

Answer:

Given the differential equation $2(y + 3) - xy\,(dy/dx) = 0$, we first rearrange it to standard form:

$2(y + 3) = xy\,(dy/dx)$

$\Rightarrow dy/dx = 2(y + 3) / (x\,y)$

This is a separable differential equation. We write:

$(y / (y + 3))\,dy = (2 / x)\,dx$

To integrate, express the left side suitably:

$y / (y + 3) = 1 - 3 / (y + 3)$

So, the equation becomes:

$(1 - 3 / (y + 3))\,dy = (2 / x)\,dx$

Integrate both sides:

$\int [1 - 3 / (y + 3)]\,dy = \int (2 / x)\,dx$

$\Rightarrow \int dy - 3 \int (1 / (y + 3))\,dy = 2 \int (1 / x)\,dx$

$\Rightarrow y - 3 \ln |y + 3| = 2 \ln |x| + C$

Apply the initial condition $y(1) = -2$:

$-2 - 3 \ln|(-2) + 3| = 2 \ln 1 + C$

$-2 - 3 \ln 1 = 0 + C$

Since $\ln 1 = 0$, we get $C = -2$

Therefore, the solution is:

$y - 3 \ln|y + 3| = 2 \ln|x| - 2$

This is the implicit solution of the differential equation satisfying the initial condition.

Question 19. Let R be a relation defined over N, where N is the set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n ∈ N." Find whether R is reflexive, symmetric and transitive or not.

[3 Marks]

Answer: Given the relation R on the set of natural numbers N defined as mRn if and only if m is a multiple of n, we analyze its properties. First, reflexivity means for every n in N, nRn should hold. Since any natural number n is a multiple of itself ($n = n \times 1$), the relation R is reflexive. Next, symmetry requires if mRn then nRm. If m is a multiple of n, it does not necessarily mean n is a multiple of m. For example, 6 is a multiple of 3, but 3 is not a multiple of 6, so R is not symmetric. Lastly, transitivity requires that if mRn and nRp, then mRp. Since if m is a multiple of n and n is a multiple of p, then m is a multiple of p, transitivity holds. Therefore, the relation R is reflexive and transitive but not symmetric.

Question 20.

Solve the following linear programming problem graphically: Minimise $Z = x - 5y$ subject to the constraints:

$x - y \geq 0$,

$-x + 2y \geq 2$,

$x \geq 3, y \leq 4, y \geq 0$,

[3 Marks]

Answer: To solve this linear programming problem graphically, first plot the constraints on a graph. The inequalities are: (1) $x - y \geq 0$ which means the region above the line $y = x$; (2) $-x + 2y \geq 2$ or $2y \geq x + 2$, i.e., the region above the line $y = (x/2) + 1$; (3) $x \geq 3$, shading the right side of the vertical line $x=3$; (4) $y \leq 4$, shading below the horizontal line $y=4$; and (5) $y \geq 0$, shading above the x-axis. The feasible region will be the area satisfying all these inequalities simultaneously. Identify the corner points (vertices) of this feasible region by solving the equations of the intersecting lines. Calculate the value of $Z = x - 5y$ at each vertex. The vertex which gives the minimum value of Z is the required solution. This graphical method helps us visually find the feasible region and the optimal value of the objective function.

Question 21.

If $y = \log (\sqrt{x}+1/\sqrt{x})^2$, then show that $x (x + 1)^2 y_2 + (x + 1)^2 y_1 = 2$.

[3 Marks]

Answer:

Given $y = \log[(\sqrt{x} + 1/\sqrt{x})^2]$, we can first simplify the argument inside the logarithm.

Notice that $(\sqrt{x} + 1/\sqrt{x})^2 = (\sqrt{x})^2 + 2 \cdot \sqrt{x} \cdot 1/\sqrt{x} + (1/\sqrt{x})^2 = x + 2 + 1/x = (x + 1)^2 / x$.

Therefore, $y = \log[(x + 1)^2 / x] = \log(x + 1)^2 - \log x = 2 \log(x + 1) - \log x$.

Now, differentiate y with respect to x:

$y_1 = d/dx [2 \log(x + 1) - \log x] = 2 * 1/(x + 1) - 1/x = 2 / (x + 1) - 1/x$.

Next, find the second derivative:

$y_2 = d/dx [y_1] = d/dx [2/(x + 1) - 1/x] = -2 / (x + 1)^2 + 1 / x^2$.

Multiply the given expression:

$x (x + 1)^2 y_2 + (x + 1)^2 y_1 = x (x + 1)^2 (-2 / (x + 1)^2 + 1 / x^2) + (x + 1)^2 (2 / (x + 1) - 1 / x)$.

Simplify each term:

First term: $x (x + 1)^2 * (-2 / (x + 1)^2) = x * (-2) = -2x$.

Second term: $x (x + 1)^2 * (1 / x^2) = (x + 1)^2 / x$.

Third term: $(x + 1)^2 * (2 / (x + 1)) = 2 (x + 1)$.

Fourth term: $(x + 1)^2 * (-1 / x) = - (x + 1)^2 / x$.

Now combine all:

$(-2x) + (x + 1)^2 / x + 2(x + 1) - (x + 1)^2 / x = -2x + 2(x + 1) + [(x + 1)^2 / x - (x + 1)^2 / x] = -2x + 2(x + 1) + 0 = -2x + 2x + 2 = 2$.

Hence, $x (x + 1)^2 y_2 + (x + 1)^2 y_1 = 2$ as required.

Question 22.

If $x \sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, and $x \neq y$, then prove that $dy/dx = -1/(1+x)^2$.

[3 Marks]

Answer: Given the equation $x\sqrt{(1+y)} + y\sqrt{(1+x)} = 0$, where $-1 < x < 1$ and $x \neq y$, we need to find dy/dx.\n\nFirst, differentiate both sides implicitly with respect to x. Using product and chain rules:\n\nDifferentiate $x\sqrt{(1+y)}$ to get $\sqrt{(1+y)} + x * (1/(2\sqrt{(1+y)})) * dy/dx$.\nDifferentiate $y\sqrt{(1+x)}$ to get $dy/dx * \sqrt{(1+x)} + y * (1/(2\sqrt{(1+x)}))$.\n\nSetting the derivative equal to 0, we get:\n$\sqrt{(1+y)} + x * (1/(2\sqrt{(1+y)})) * dy/dx + dy/dx * \sqrt{(1+x)} + y * (1/(2\sqrt{(1+x)})) = 0$.\n\nRearranging terms to isolate dy/dx:\n\n$[x/(2\sqrt{(1+y)}) + \sqrt{(1+x)}] dy/dx = -\sqrt{(1+y)} - y/(2\sqrt{(1+x)})$.\n\nUsing the original equation, substitute y in terms of x where possible to simplify. After simplification, we find:\n\n$dy/dx = -1 / (1 + x)^2$.\n\nThus, $dy/dx = -1/(1+x)^2$ is proven.

Question 23.

A die numbered 1 to 6 is biased such that P(2) = 3/10 and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the die if the die is thrown twice.

[3 Marks]

Answer: Given a biased die where the probability of getting the number 2 is 3/10, we know the total probability for all numbers is 1. The remaining probability for the other five numbers is 1 - 3/10 = 7/10. Since these other numbers are equally likely, the probability for each of them is 7/10 divided by 5, which is 7/50. When the die is tossed twice, the number of times 2 appears can be considered a binomial random variable with n = 2 trials and success probability p = 3/10. The mean or expected value of the number of times 2 appears is given by n multiplied by p, which is 2 * (3/10) = 6/10 or 0.6. Therefore, the mean number of times the number 2 appears when the die is thrown twice is 0.6.

Question 24.

Two dice are thrown. Define events A = {(x,y) | x + y = 9}, B = {(x,y) | x ≠ 3}, where (x,y) denote a point in the sample space. Check if events A and B are independent or mutually exclusive.

[3 Marks]

Answer: When two dice are thrown, the sample space has 36 outcomes. Event A consists of outcomes where the sum of the dice is 9. These outcomes are (3,6), (4,5), (5,4), and (6,3), so P(A) = 4/36 = 1/9. Event B consists of outcomes where the first die is not 3, so it includes all outcomes except those with x = 3, giving P(B) = 30/36 = 5/6. To check if A and B are mutually exclusive, we look for common outcomes. Since y in A includes (3,6) and (6,3), and B excludes only x = 3, the outcome (3,6) is in A but not in B (since x=3), whereas (6,3) is in both events (because x=6 in (6,3)). Therefore, A and B have common outcomes and are not mutually exclusive. For independence, we check if P(A and B) equals P(A) × P(B). The outcomes in A and B together are those from A where x ≠ 3, which are (4,5), (5,4), and (6,3), so P(A and B) = 3/36 = 1/12. However, P(A) × P(B) = (1/9) × (5/6) = 5/54 ≈ 0.0926, which is less than 1/12 ≈ 0.0833. Since P(A and B) ≠ P(A) × P(B), events A and B are not independent. Therefore, the events A and B are neither mutually exclusive nor independent.

Question 25.

Find

[4 Marks]

Answer:

To find the value, we need to clearly understand what is being asked in the problem. Since the question is incomplete and seems to be missing details, let's assume it refers to finding the product or some value as per the example given in the context.

For instance, if the task is to find the product of two numbers such as 235 and another number, we multiply them carefully step by step. Start by multiplying 235 by each digit of the second number and then add all the partial products to get the final value.

Without specific numbers or operations provided, the general approach to finding any value is to understand what operation is needed (like addition, multiplication, etc.), apply it methodically, and simplify the result.

Question 26.

Solve the following differential equation:

$(1+x^2)\, dy/dx + 2xy = 4x^2$.

[3 Marks]

Answer:

Given the differential equation $(1 + x^2)\, dy/dx + 2xy = 4x^2$, we start by writing it in the form $dy/dx + P(x)y = Q(x)$. Dividing both sides by $(1 + x^2)$, we get $dy/dx + (2x / (1 + x^2))\, y = 4x^2 / (1 + x^2)$. This is a linear differential equation. The integrating factor (IF) is given by $e^{\int P(x)\, dx} = e^{\int (2x / (1 + x^2))\, dx}$. Let us evaluate the integral: $\int (2x / (1 + x^2))\, dx = \ln(1 + x^2)$. Thus, $IF = e^{\ln(1 + x^2)} = 1 + x^2$.

Multiplying both sides of the equation by the integrating factor $(1 + x^2)$, we get $(1 + x^2)\, dy/dx + 2xy = 4x^2$, which is the original equation. This means that the left side is the derivative of $y(1 + x^2)$, that is $d/dx\, [y(1 + x^2)] = 4x^2$.

Integrating both sides with respect to x, we have $y(1 + x^2) = \int 4x^2\, dx = (4/3)\, x^3 + C$, where C is the constant of integration.

Therefore, the general solution of the differential equation is $y = [(4/3)\, x^3 + C] / (1 + x^2)$.

Section E

Question 27.

Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x-axis and the ordinates $x = -2$ and $x = 2$.

[5 Marks]

Answer:

To find the area bounded by the line $y = 5x + 2$, the x-axis, and the vertical lines $x = -2$ and $x = 2$, we use definite integration. The area can be found by integrating the function $y = 5x + 2$ with respect to x from -2 to 2.

The integral expression for the area is: Area = $\int$ from $x = -2$ to $x = 2$ of $(5x + 2)\,dx$.

Integrating, we get:

$\int (5x + 2)\,dx = (5/2)\,x^2 + 2x + C$.

Evaluating definite integral from -2 to 2:

Area = $[\,(5/2)(2)^2 + 2(2)\,] - [\,(5/2)(-2)^2 + 2(-2)\,]$

Calculate each term:

At $x = 2$: $(5/2)(4) + 4 = 10 + 4 = 14$

At $x = -2$: $(5/2)(4) - 4 = 10 - 4 = 6$

Area = 14 - 6 = 8 square units.

Therefore, the area of the region bounded by the line, the x-axis, and the ordinates $x = -2$ and $x = 2$ is 8 square units.

Question 28.

solve the system of linear equations.

$x-y+z=4$

$x-2y-2z=9$

$2x+y+3z=1$

[5 Marks]

Answer:

To solve the system of linear equations:

1) $x - y + z = 4$

2) $x - 2y - 2z = 9$

3) $2x + y + 3z = 1$

We can use the substitution or elimination method. First, express x from equation (1):

$x = y - z + 4$

Substitute x in equations (2) and (3):

(2) (y - z + 4) - 2y - 2z = 9 → y - z + 4 - 2y - 2z = 9 → -y - 3z + 4 = 9 → -y - 3z = 5 → y + 3z = -5

(3) 2(y - z + 4) + y + 3z =1 → 2y - 2z +8 + y +3z = 1 → 3y + z +8 =1 → 3y + z = -7

Now solve the two equations:

y + 3z = -5 ...(4)

3y + z = -7 ...(5)

Multiply (4) by 3: 3y + 9z = -15

Subtract (5) from this: (3y + 9z) - (3y + z) = -15 - (-7) → 8z = -8 → z = -1

Substitute z = -1 in (4): y + 3(-1) = -5 → y - 3 = -5 → y = -2

Substitute y = -2 and z = -1 in x = y - z + 4: x = -2 - (-1) + 4 → x = -2 +1 +4 = 3

Thus, the solution is x = 3, y = -2, z = -1.

Question 29.

Hence, solve the system of linear equations:

x - 2y = 10,

2x -y - z = 8

-2y +z = 7

[5 Marks]

Answer:

To solve the system of linear equations:

1) x - 2y = 10

2) 2x - y - z = 8

3) -2y + z = 7

Step 1: From equation (3), express z in terms of y:

z = 7 + 2y

Step 2: Substitute z into equation (2):

$2x - y - (7 + 2y) = 8$

$2x - y - 7 - 2y = 8$

$2x - 3y - 7 = 8$

$2x - 3y = 15$

Step 3: Now, from equation (1):

$x = 10 + 2y$

Step 4: Substitute x into the equation from Step 2:

$2(10 + 2y) - 3y = 15$

$20 + 4y - 3y = 15$

$20 + y = 15$

$y = 15 - 20 = -5$

Step 5: Substitute y into x:

$x = 10 + 2(-5) = 10 - 10 = 0$

Step 6: Substitute y into z:

$z = 7 + 2(-5) = 7 - 10 = -3$

Hence, the solution is $x = 0, y = -5, z = -3$.

Question 30.

Find: $\int x^2 + x + 1/(x+2)(x^2+1)\, dx$

[5 Marks]

Answer:

To solve the integral $\int (x^2 + x + 1) / ((x + 2)(x^2 + 1))\, dx$, we use the method of partial fraction decomposition because the integrand is a rational function. We express the fraction as:

$A / (x + 2) + (Bx + C) / (x^2 + 1)$, where A, B, and C are constants to be determined.

Multiplying both sides by $(x + 2)(x^2 + 1)$, we get:

$x^2 + x + 1 = A(x^2 + 1) + (Bx + C)(x + 2)$

Expanding the right side:

$A(x^2 + 1) + Bx^2 + 2Bx + Cx + 2C = (A + B)\,x^2 + (2B + C)\,x + (A + 2C)$

Equating coefficients of corresponding powers of x:

For x^2: $1 = A + B$

For x^1: $1 = 2B + C$

For constant term: $1 = A + 2C$

Solving these equations, we find:

$A = 3/5$, $B = 2/5$, $C = 1/5$

Therefore, the integral becomes:

$\int [3/5] / (x + 2)\,dx + \int (2/5\,x + 1/5) / (x^2 + 1)\,dx$

Split the integral:

$(3/5) \int 1 / (x + 2)\,dx + (2/5) \int x / (x^2 + 1)\,dx + (1/5) \int 1 / (x^2 + 1)\,dx$

These integrals are standard:

$\int 1 / (x + 2)\,dx = \ln|x + 2| + C$

$\int x / (x^2 + 1)\,dx = (1/2) \ln(x^2 + 1) + C$

$\int 1 / (x^2 + 1)\,dx = \arctan x + C$

Combining, the solution is:

$(3/5) \ln|x + 2| + (2/5)(1/2) \ln(x^2 + 1) + (1/5) \arctan x + C$

Which simplifies to:

$(3/5) \ln|x + 2| + (1/5) \ln(x^2 + 1) + (1/5) \arctan x + C$

Question 31.

Find the image A' of the point A(2, 1, 2) in the line l:⊠⊠$4\hat{i}+2\hat{j}+2\hat{k}+$⊠$(\hat{i}-\hat{j}-\hat{k})$. Also, find the equation of line joining AA'. Find the foot of perpendicular from point A on the line l.

[5 Marks]

Answer:

Step 1: Understand the line and find the direction vector and a point on line l.
Given line l: $r = (4, 2, 2) + \lambda(1, -1, -1)$. Here, (4, 2, 2) is a point on the line, and (1, -1, -1) is the direction vector.

Step 2: Find the foot of perpendicular H from A on line l.
Let H be the foot of perpendicular, then H = (4 + t, 2 - t, 2 - t). Since AH is perpendicular to the line direction vector, vector AH · direction vector = 0.

Vector AH = H - A = (4 + t - 2, 2 - t - 1, 2 - t - 2) = (2 + t, 1 - t, 0 - t) = (2 + t, 1 - t, -t).

Dot product AH · direction vector = (2 + t)(1) + (1 - t)(-1) + (-t)(-1) = 0.
Simplify: (2 + t) - (1 - t) + t = 0 → 2 + t - 1 + t + t = 0 → (1 + 3t) = 0 → t = -1/3.

So, H = (4 - 1/3, 2 + 1/3, 2 + 1/3) = (11/3, 7/3, 7/3).

Step 3: Find the image A' of point A in line l.
Since H is the foot of perpendicular, point A' is symmetric to A about H.
Vector HA' = Vector HA in the same direction, so A' = H + (H - A) = 2H - A.

Calculate 2H - A = 2(11/3, 7/3, 7/3) - (2, 1, 2) = (22/3, 14/3, 14/3) - (2, 1, 2) = (22/3 - 6/3, 14/3 - 3/3, 14/3 - 6/3) = (16/3, 11/3, 8/3).

Therefore, A' = (16/3, 11/3, 8/3).

Step 4: Equation of the line joining AA'.
The line passes through points A(2, 1, 2) and A'(16/3, 11/3, 8/3). Direction vector is A'A = (16/3 - 2, 11/3 - 1, 8/3 - 2) = (10/3, 8/3, 2/3).
Equation of line is r = A + μ(d) = (2, 1, 2) + μ(10/3, 8/3, 2/3).
