$\therefore$ L.H.S = R.H.S

proved.

(b) let, $u = f(x,y) = ax^2 + 2hxy + b$

For testing homogeneous f^n

Replace x by tx and y by tx

$u = a t^2 x^2 + 2h t^2 xy + b t^2 y^2$

$u = t^2 (ax^2 + 2hxy + by^2)$

$f(tx, ty) = t^2 f(x,y)$

This shows that the f^n is homogeneous function of degree 2.

$n = 2$.

by^2

To verify Euler's theorem,

wrt,

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = nu.$$

$$x \cdot \frac{\partial}{\partial x}(ax^2+2hxy+by^2) + y \cdot \frac{\partial}{\partial y}(ax^2+2hxy+by^2) = 2u.$$

$$x.(2ax+2hy) + y.(2hx+2by) = 2u$$

$$\Rightarrow 2ax^2+2hxy+2hxy+2by^2 = 2u.$$

$$\Rightarrow \cancel{2}(ax^2+2hxy+by^2) = \cancel{2}u$$

$$\Rightarrow ax^2+2hxy+by^2 = u$$

$$\therefore u = u$$

L.H.S = R.H.S

proved.

Bachelor

ıme: BCA

Mathematics I (New)

POKHARA UNIVERSITY

Semester: Spring

Year : 2024

Full Marks: 100

Pass Marks: 45

Time : 3hrs.

ndidates are required to give their answers in their own words as far practicable.

e figures in the margin indicate full marks.

Attempt all the questions.

1.a) In an examination 52% failed in English, 42% in Maths, 28% in Psychology, 35% in English and Maths, 18% in English and Psychology, 20% in Maths and Psychology and 14% in all three subjects then find — 5

   i. What percent passed in all three subjects?

   ii. What percent failed in exactly two subjects?

   iii. Show above information in Venn-diagram.

b) If $A = [-3, 5)$ and $B = [4, 6]$. Find $A \cup B$ and $A - B$. Also, show then in the number line. — 5

c) Find the domain and range of the function $f(x) = \sqrt{25 - x^2}$ — 5

2.a) Shade on the graph the area which jointly satisfies the inequality: $2x - 3y \le 5,\ 2x + y \le 4,\ x \ge 0,\ y \ge 0$ — 5

b) Evaluate: $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{\sqrt{3x-2} - \sqrt{x+2}}$. — 5

c) Define continuity of a function at $x = a$. Discuss the continuity of the function defined by $f(x) = \begin{cases} x - x^2 & \text{for } x < 2 \\ x & \text{for } x = 2 \\ x - 4 & \text{for } x > 2 \end{cases}$. — 5

3.a) Find $\frac{dy}{dx}$ of: **(Any two)** — 10

   i. $y = x^2 \log x$

   ii. $x^3 + y^3 = 3axy$

   iii. $x = \frac{2t}{1+t^2}$ and $y = \frac{2t^2}{1+t^2}$

b) A monopolist has a demand curve $q = 400 - 20p$ and the cost curve $C = 5q + \frac{q^2}{50}$. What is the profit maximizing level of output when monopolist fixes its output? Also, find the maximum profit. — 5

4. a) Determine the interval where the function $f(x) = x^3 - 6x^2 + 9x - 2$ is concave upward or concave downward. — 5

b) If $u = \ln\sqrt{x^2 + y^2 + z^2}$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{x^2+y^2+z^2}$ — 5

c) State Euler's theorem. Verify Euler's theorem for the function $f(x, y) = x^3 + 3x^2y - 2xy^2 + 4y^3$. — 5

5. a) Use Lagrange's multiplier method to optimize the function $f(x, y) = 36 - x^2 - y^2$ subject to the constraints $x + 7y = 25$. — 5

OR

Determine the location and nature of all critical points of the function $f(x, y) = -x^3 + 2y^3 + 147x - 54y + 12$.

b) Define implication in logic. Find the negation, converse, inverse and contrapositive of the following statement, 'If $2 + 3 = 5$, then Pokhara is the capital of Nepal.' — 5

c) Use principles of mathematical induction to prove that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ — 5

6. a) Solve system of linear equation by Cramer's rule or matrix inverse method: $x - 2y + z = -4$, $2x + y - z = 5$ and $3x + 2y + 4z = 3$. — 7

b) i) Prove that: — 4

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

ii) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$ then prove that $(AB)^T = B^T A^T$. — 4

7. Attempt all the questions: — 2×5 2.5 × 4

   a) Rewrite the inequality $|5x - 6| \le 4$ without using absolute value sign.

   b) If $u = x^2y^2 + xy$, then verify that $u_{xy} = u_{yx}$

   c) Constructing truth table, show that $p \wedge (\sim p)$ is a contradiction.

   d) If $M = \begin{bmatrix} 8 & 2x \\ x & 9 \end{bmatrix}$ is a singular matrix, then find all possible values of x.

POKHARA UNIVERSITY

:l: Bachelor
ramme: BCA
rse: Mathematics I

Semester: Spring

Year : 2024
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1.a) Of a group of 200 students, 100 are interested in music, 70 are Interested in photography, and 40 likes swimming: furthermore 40 are interested in both music and photography, 30 are interested in both music and swimming 20 are interested in both photography and swimming and 10 are interested in all three activities. Find the number of students that are interested in: 7

i. Photography but not music and swimming
ii. Exactly one activity
iii. At least one activity
iv. None of the activity

b) i) Shade on a graph of the region which jointly satisfies the following inequalities: 5

$$2y + 3x \le 18, x - y \le 3, and y \le 6 ; x, y \ge 0$$

ii) Solve the inequality $x^2 - 3x + 2 \le 0$. 3

2.a) State the principle of mathematical induction using the principle, prove that $1 + 2 + 3 + \cdots \ldots + n = n(\frac{n+1}{2})$ 5

b) What are indeterminate forms? Evaluate $\lim_{x \to \infty} \sqrt{x}\,(\sqrt{x} - \sqrt{x-a})$. 5

c) A function $f: R \to R$ is defined by $f(x) = \begin{cases} 2x+3 & for\ x>3 \\ 3x & for\ x = 3 \\ 4x-3 & for\ x<3 \end{cases}$ 5

Test the continuity of the function $f(x)$ at $x = 3$

3.a) For the demand function $p = \frac{a}{x+b}$(where p is price, x is the quantity demanded and a and b are positive constants). Show by using the concept of limit, the demand increases to infinitely large amount as the price falls. Also, show that total revenue reaches a limiting value 5

as the quantity demanded increases indefinitely.

b) Find $\frac{dy}{dx}$ of: **(Any two)** 5+5

i. $4yx^3 = x^2 + y^2$
ii. $x^y = y^x$
iii. $y = \frac{e^x}{e^x + \log x}$

4. a) If $u = \sqrt{x^2 + y^2 + z^2}$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$. 5

b) Find the second order differentials of the function 5

$$u = x^2y^2 + y^2z^2 + x^2z^2$$

c) Very Euler's theorem for the function $u = \log(x^2 + y^2)$ 5

5. a) If p and q are two statements then show 7

i) $\sim (p \Rightarrow q) \cong p \wedge \sim q$ ii) $\sim p \vee q \Leftrightarrow p \vee q$

b) Define Tautology and contradiction with examples and give the converse, inverse and contra-positive of the implication; If ABCD is a square, then ABCD is a rectangle 8

6. a) Find the asymptotes of the curve 7

$x^2(x-y)^2 - a^2(x^2 + y^2) = 0.$

b) Define asymptote to a curve. Write down the types of asymptote of algebraic curve with one example of each type. Find the asymptotes of the curve $y = \frac{x^2+1}{x^2-4x+3}$ 8

No need

7. Attempt all the questions 4×2.5

a) Define power set. If A={a, b} then construct a power set of A.

b) Compute $A \cup B$ and $A \cap B$, where $A = [-2,4)$ and $B = [3,5]$.

c) If the cost function is $C = 5 - 3q + 2q^2$, find the expression for marginal cost and average cost.

d) Rewrite by using absolute value sign: $-3 \le x \le 5$.

b) If the average cost function of a firm is $AC = \frac{1}{3}q^2 - 5q + 30 + \frac{10}{q}$ where AC is the Average Cost and q is the output and price under perfect competition is $ 6. Verify that the profit will be maximum for the output of 6 units. 5

4. a) Use Lagrange's Multiplier method to find the extreme values for the function $f(x, y) = x^2 + 3xy - y^2$ subject to constraint $x + y = 42$ 5

OR

Determine the location and nature of all critical points of the function $f(x, y) = x^3 + y^3 - 3x - 27y + 24$.

b) i) *If* $u = \sqrt{x^2 + y^2 + z^2}$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$. 5

ii) Verify Euler's theorem for the function $f(x, y) = \frac{x^3 - y^3}{x - y}$. 5

5. a) If p and q are two statements, then show that 5

$\sim[(\sim p)\wedge q] \equiv p\vee(\sim q)$

b) Use principles of mathematical induction to prove that 5

$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

c) Sketch the graph of $f(x) = x^2 + 4x + 3$ with different characteristics. 5

6. a) Solve system of linear equation by Cramer's rule OR inverse matrix method. 7

$2x - y + 3z = 9$, $x + y + z = 6$ and $x - y + z = 2$

b) i) If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ then prove that $(AB)^T = B^T A^T$ 4

ii) Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ 4

7. Attempt **all** questions: 2.5×4

a) If A= $[-1,3]$ and B=(2, 7) Find A∩ B and A −B

b) Determine the function $f(x)=x^2 - 10x + 3$ is increasing or decreasing at x=5

c) If u= $x^2y + xy^2$, verify that $u_{xy} = u_{yx}$

d) Make truth table of $(\sim p\wedge q)\vee p$