

*not enough time*

# ~~Spatial and Temporal Modeling~~ with Bayesian Networks

Reasoning About Distances in Populations

April 30, 2020

# Agenda

## Introduction & Resources

## Motivation

- What is our distance?
- What should be our distance?

## Tutorial

- Distribution of distances in a unit square
- Distribution of distances in the continental U.S.
- BayesiaLab & GIS Mapping
- Spatial Learning & Optimization



# Introduction



STEFAN CONRADY  
Managing Partner

BAYESIA USA, LLC

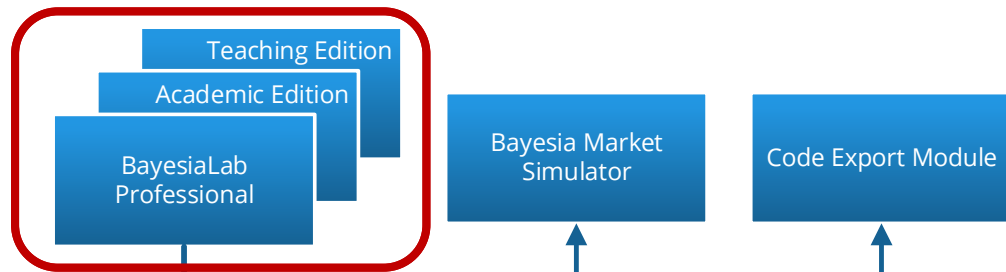
stefan.conrad@bayesia.us  
+1 888.386.8383 main  
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305 Lockhart Court  
Franklin, TN 37069  
BayesiaLab.com

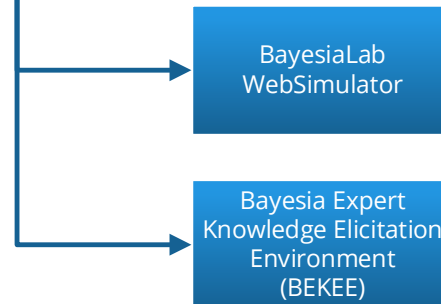




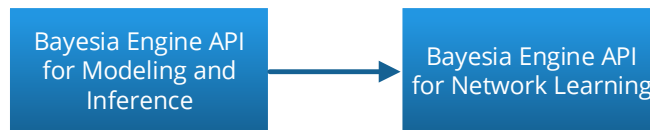
## Desktop Software



## Web Application



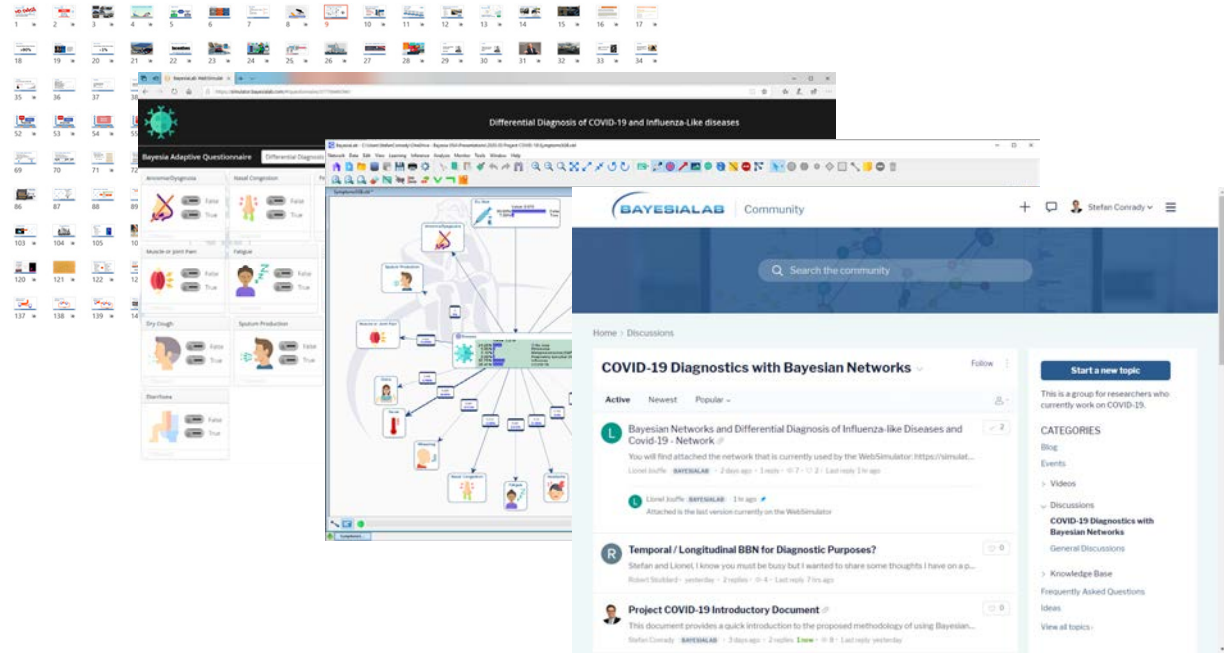
## API



# Resources

## Webinar Materials Available in the BayesiaLab Community

- Slides
- Network Model
- Webinar Recording
- Q&A



Search the community

Question?  
“Start a new topic”

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## COVID-19 Diagnostics with Bayesian Networks

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### Call for Expertise — Help Improve the COVID-19 Knowledge Base

2

Please join our volunteer group of infectious disease specialists, clinicians, epidemiologists, biost...

Stefan Conrady · BAYESIALAB · 4 days ago · 2 replies · 101 · 2 · Last reply 3 days ago



### Oral polio vaccine as infection control?

2

Chinese study theorizes that oral polio vaccine can reduce probability of Covid19 in under-30 dem...

Kurt Schulzke · 4 days ago · 2 replies · 5 · 2 · Last reply 4 days ago



Start a new topic

This is a group for researchers who currently work on COVID-19.

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**COVID-19 Diagnostics with Bayesian Networks**

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## Updated 7 days ago



COVID-19 WebSimulator

## From Local Insight to Worldwide Diagnostic Practice

## Call for Expertise

## Overcoming Human Challenges in Reasoning

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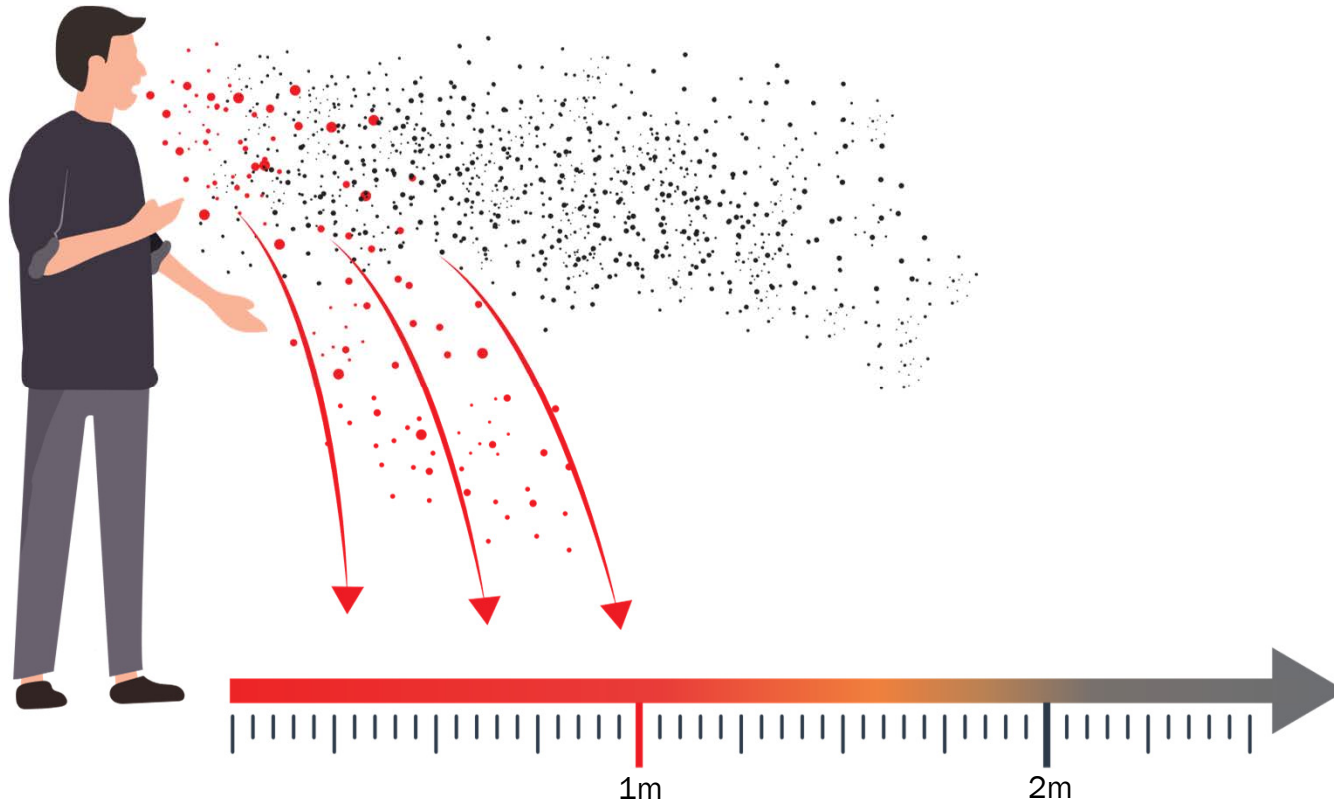
2



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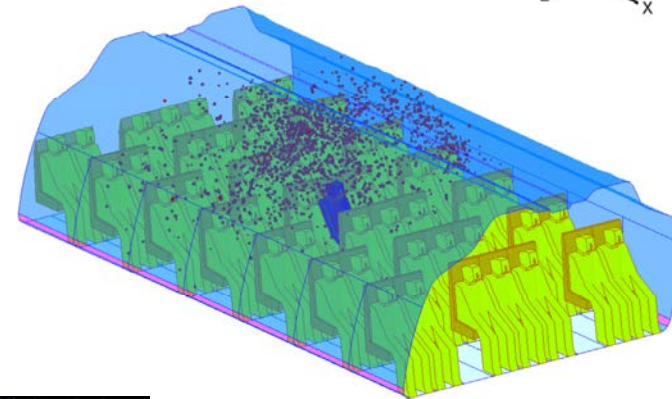
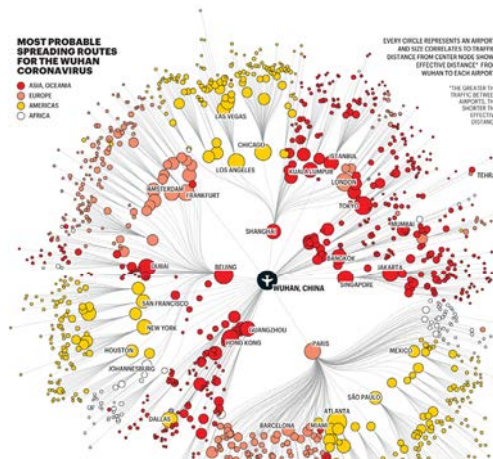
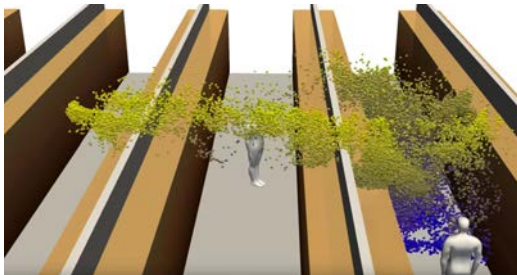


# Motivation



# Motivation

## Spreading Virus

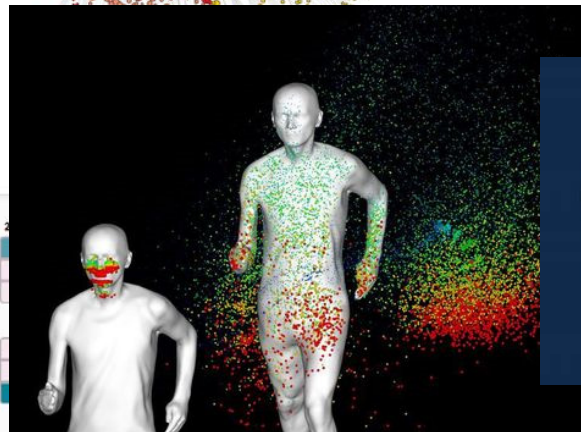
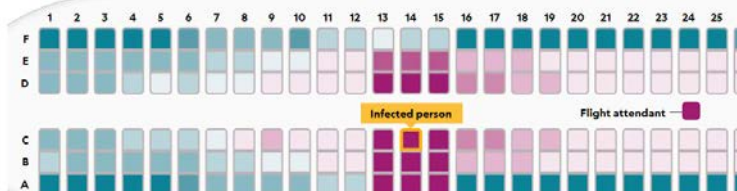


Probability of direct contact with the infected person

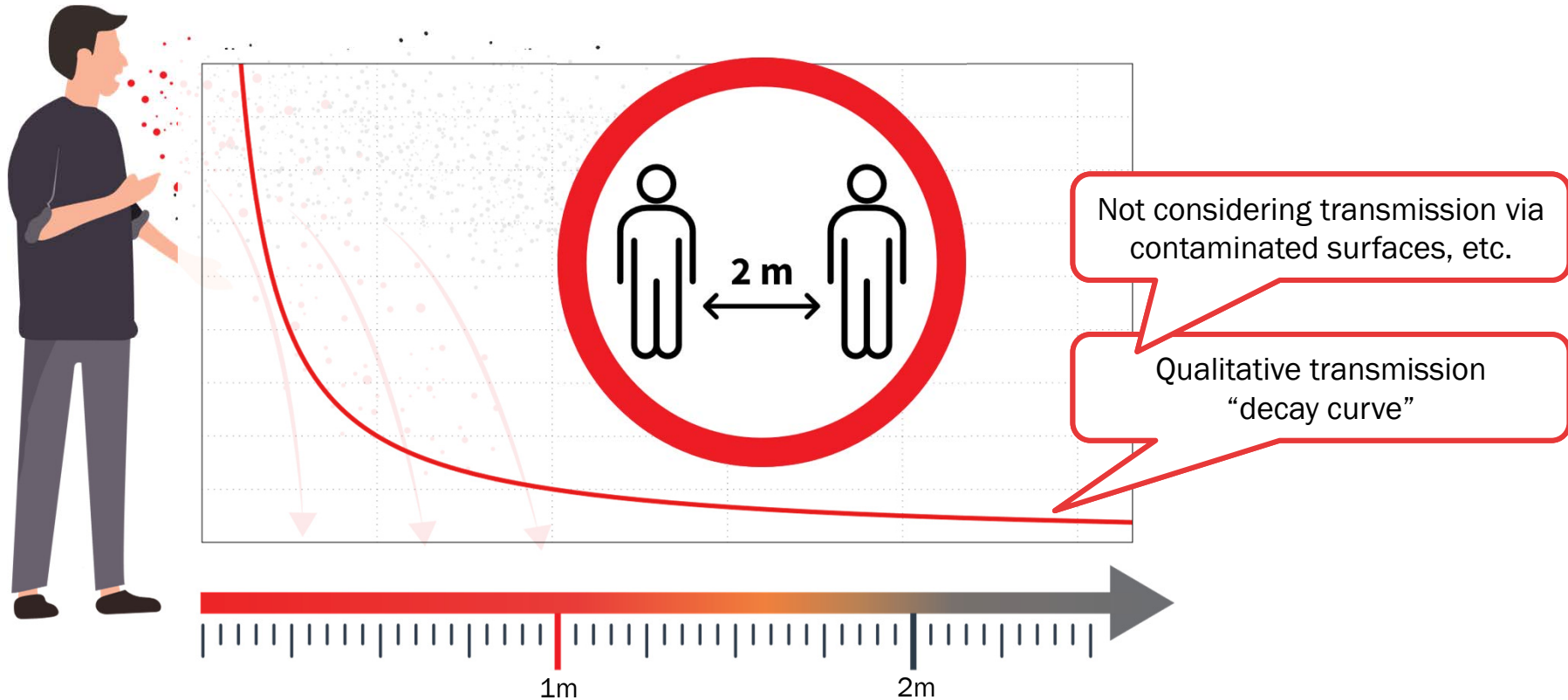
0 10 20 30 40 50 60 70 80 90 100%



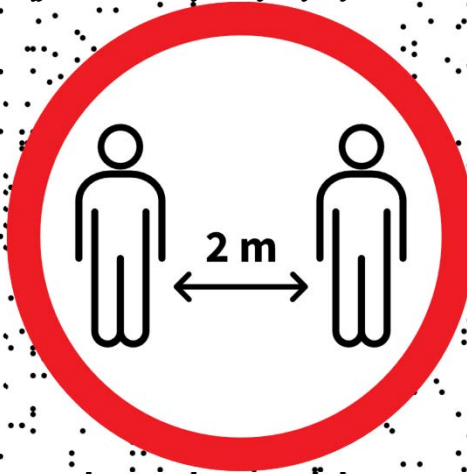
More likely than not to have contact



# Motivation



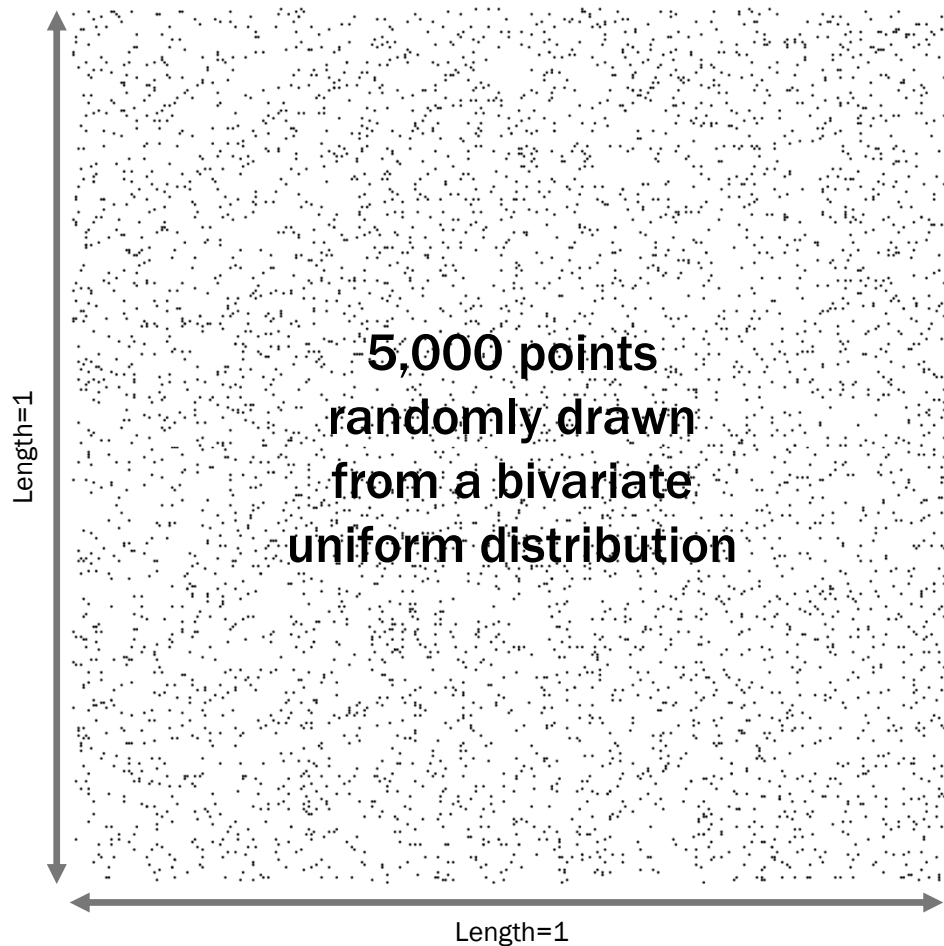
# Motivation





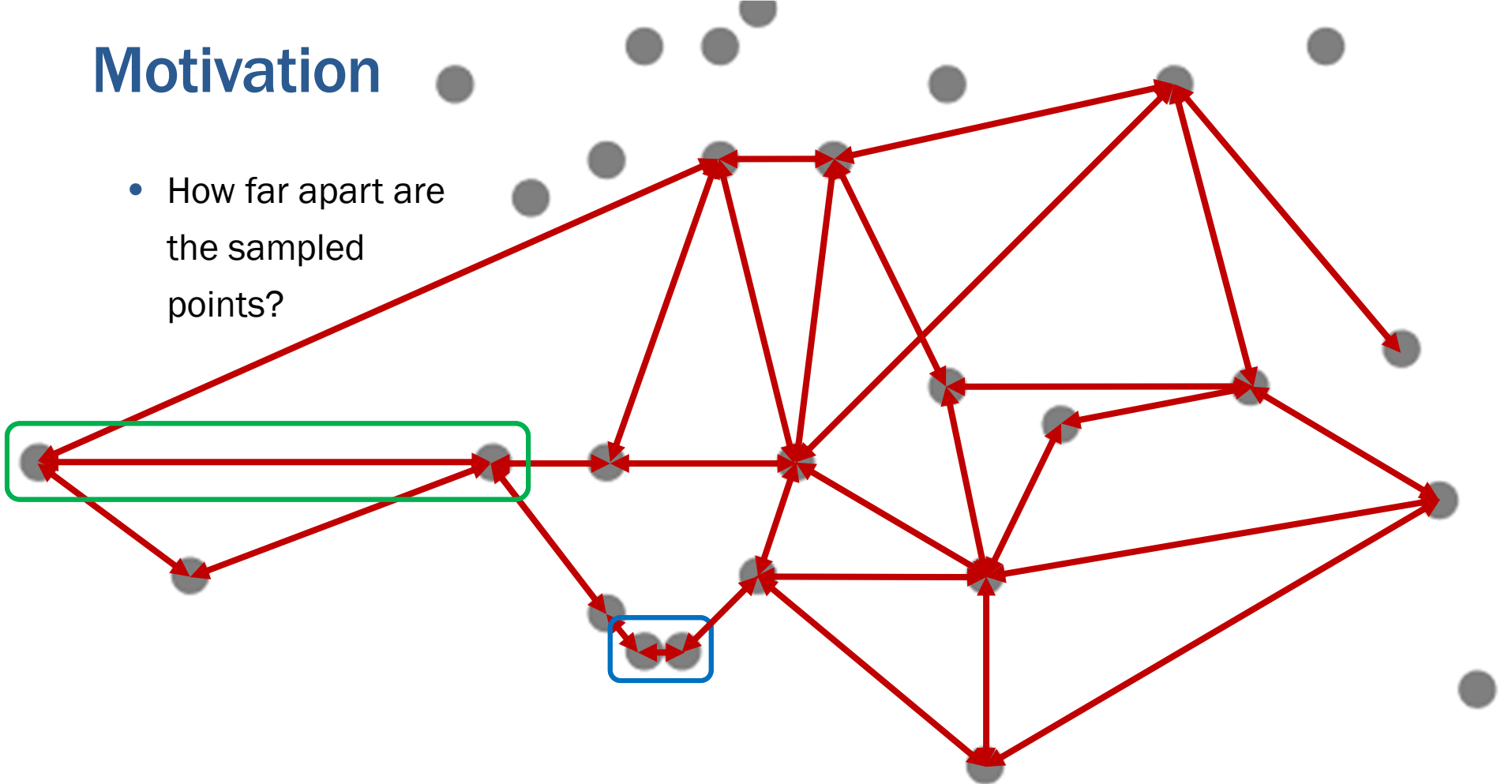
# Motivation

- How far apart are uniformly distributed points in a unit square?
- How many are in “spitting distance”?



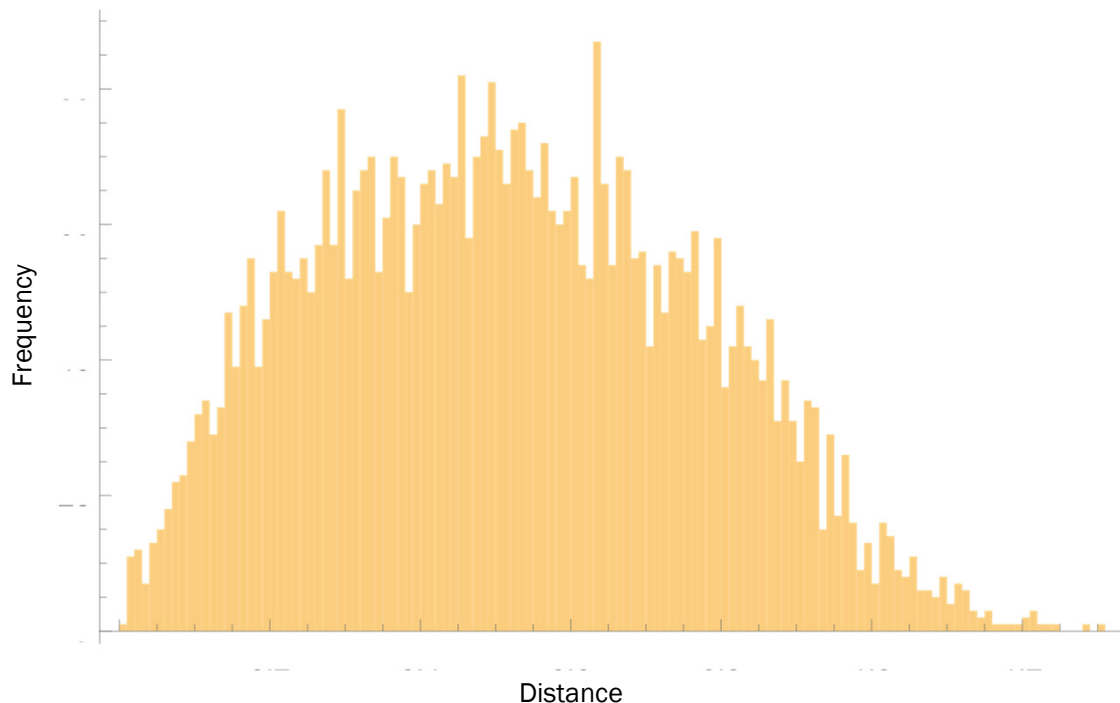
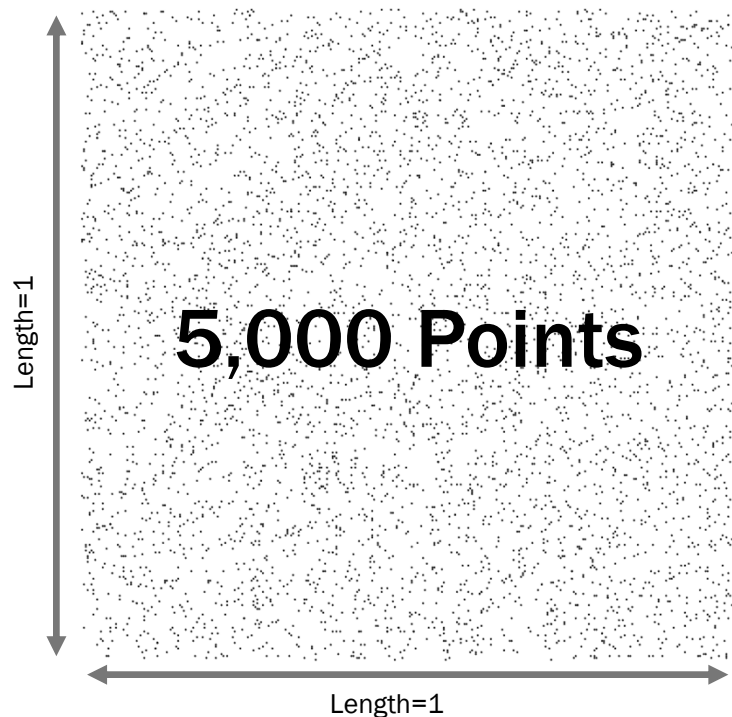
# Motivation

- How far apart are the sampled points?



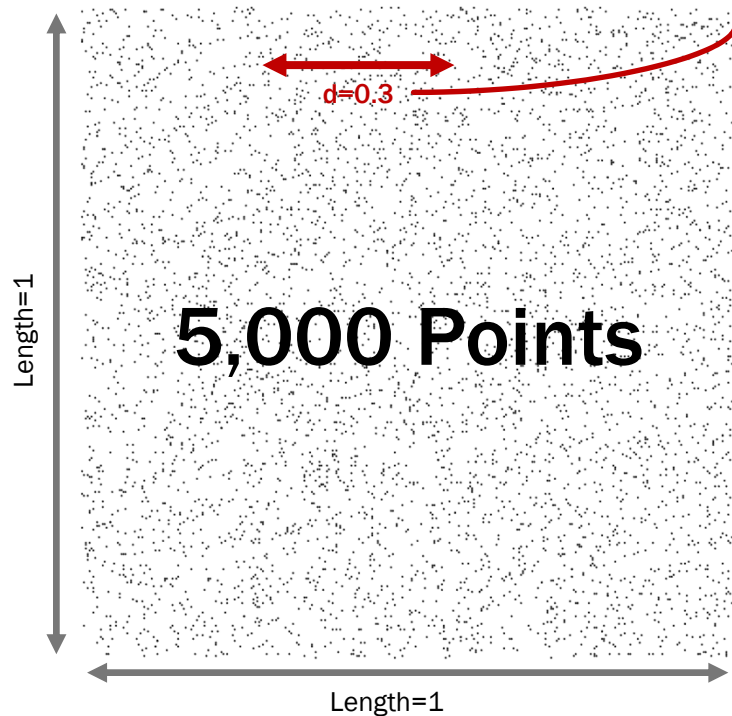
# Motivation

How far apart are these points?

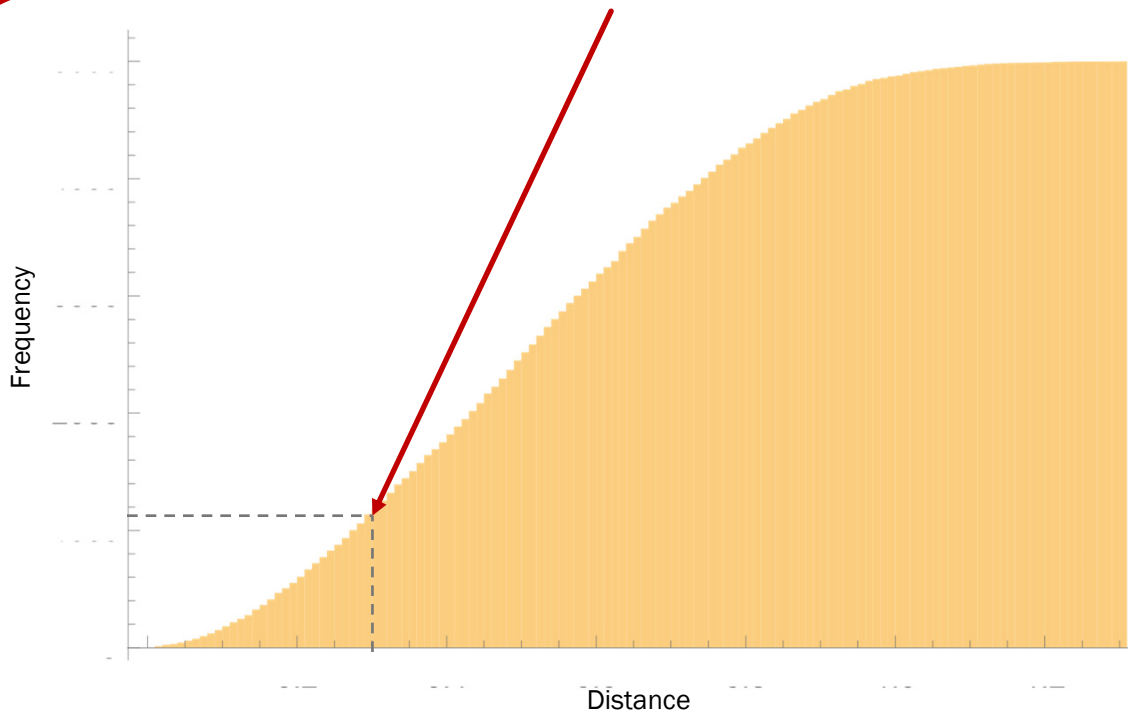


# Motivation

How far apart are these points?



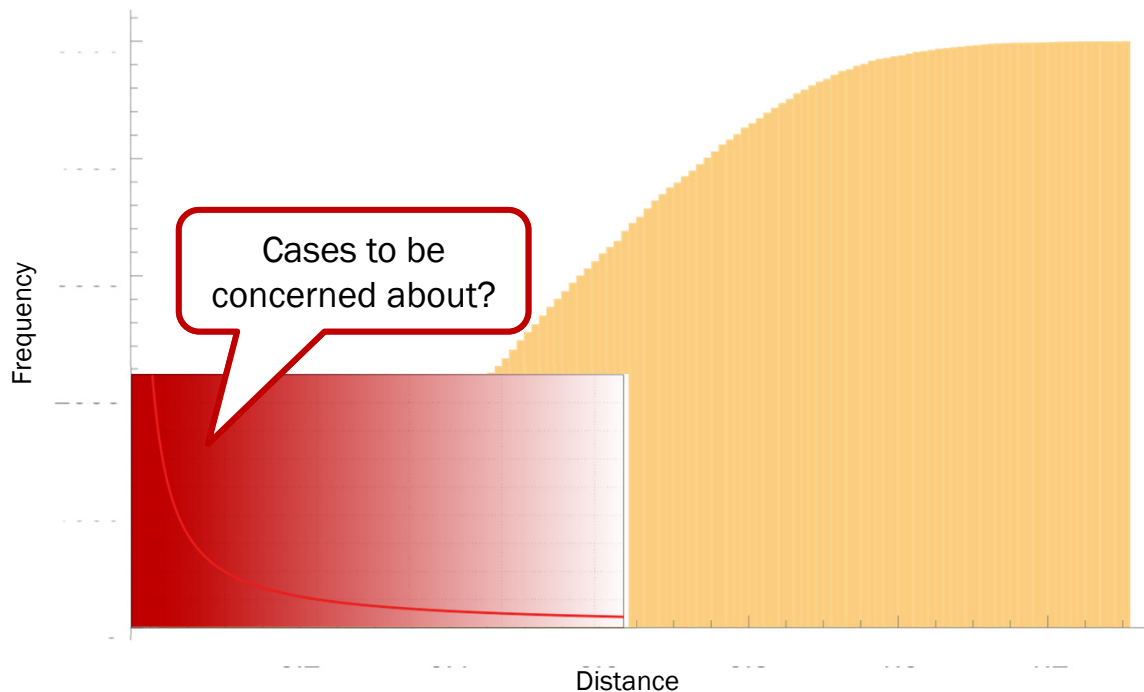
Between these 5,000 points, approx. 1,200 are less than 0.3 units apart.





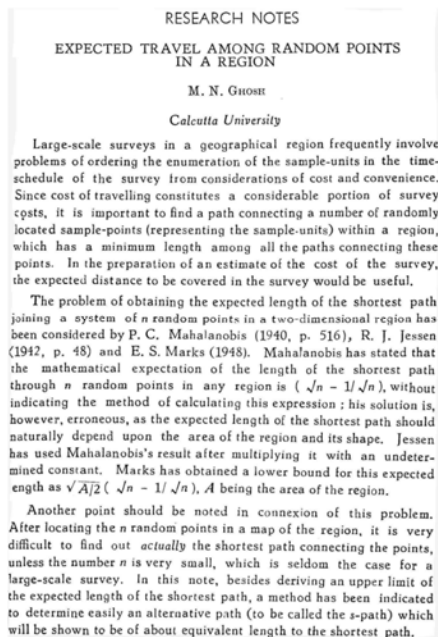
# Motivation

How far apart are these points?



# Motivation

## CALCUTTA STATISTICAL ASSOCIATION BULLETIN

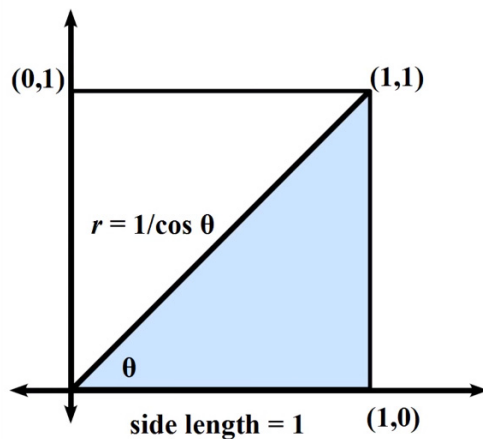


- B. Ghosh, "On the distribution of random distances in a rectangle," Science and Culture, vol. 8 (9), p. 388, 1943.
- B. Ghosh, "On random distance between two rectangles," Science and Culture, vol. 8 (11), p. 464, 1943. [3]
- B. Ghosh, "Random distance within a rectangle and between two rectangles," Bulletin of the Calcutta Mathematical Society, vol. 43, pp. 17-24, 1951.
- Ghosh, M. N. "Expected Travel Among Random Points in a Region." Calcutta Statistical Association Bulletin 2.2 (1949): 83-87.

# Motivation

## Video on Computing the Mean Distance

Method 2: Exact answer with integrals



Substitute

$$4 \int_0^{\pi/4} 2 \int_0^{1/\cos \theta} f(r, \theta) r dr d\theta$$

$$8 \int_0^{\pi/4} \int_0^{1/\cos \theta} \sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta} (1 - r \cos \theta)(1 - r \sin \theta) r dr d\theta$$

$$8 \int_0^{\pi/4} \int_0^{1/\cos \theta} r(1 - r \cos \theta)(1 - r \sin \theta) r dr d\theta$$

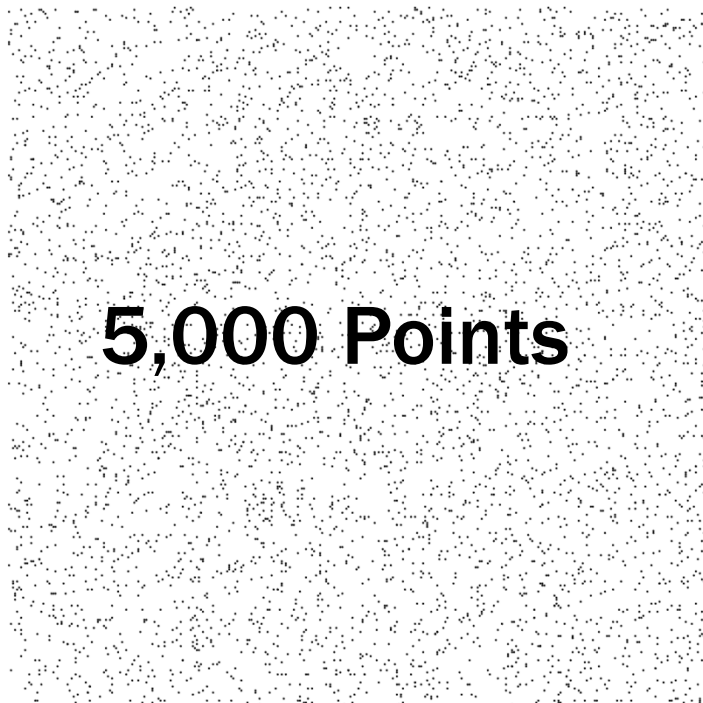
$$8 \int_0^{\pi/4} \int_0^{1/\cos \theta} r^2 - r^3 \cos \theta - r^3 \sin \theta + r^4 \cos \theta \sin \theta dr d\theta$$

$$8 \int_0^{\pi/4} \frac{\sec^3 \theta}{12} - \frac{\sec^3 \theta \tan \theta}{20} d\theta$$

<https://youtu.be/i4VqXRRXi68>

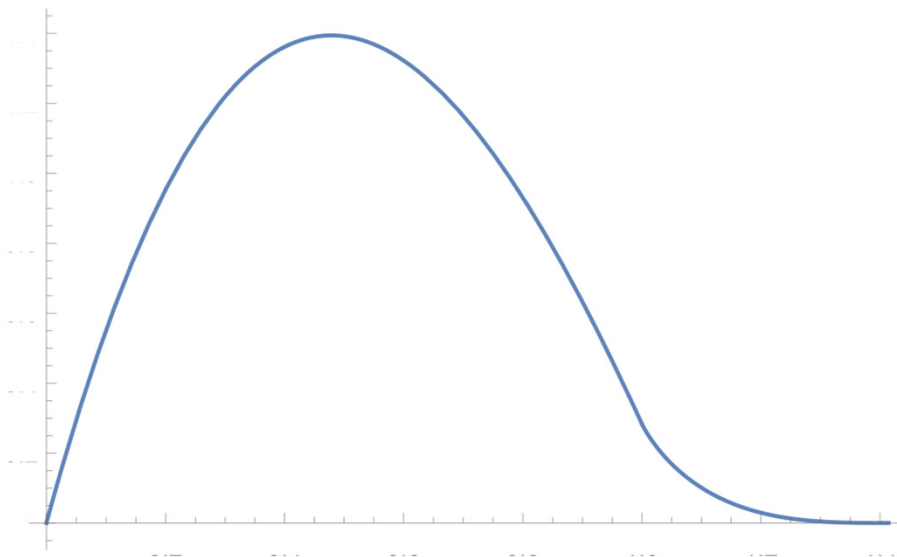
# Motivation

How far apart are these points?



Analytical Solution

$$P(l) = \begin{cases} 2l(l^2 - 4l + \pi) & 0 \leq l \leq 1 \\ 2l\left(-l^2 - 4 \tan^{-1}(\sqrt{l^2 - 1}) + 4\sqrt{l^2 - 1} + \pi - 2\right) & 1 < l \leq \sqrt{2} \end{cases}$$





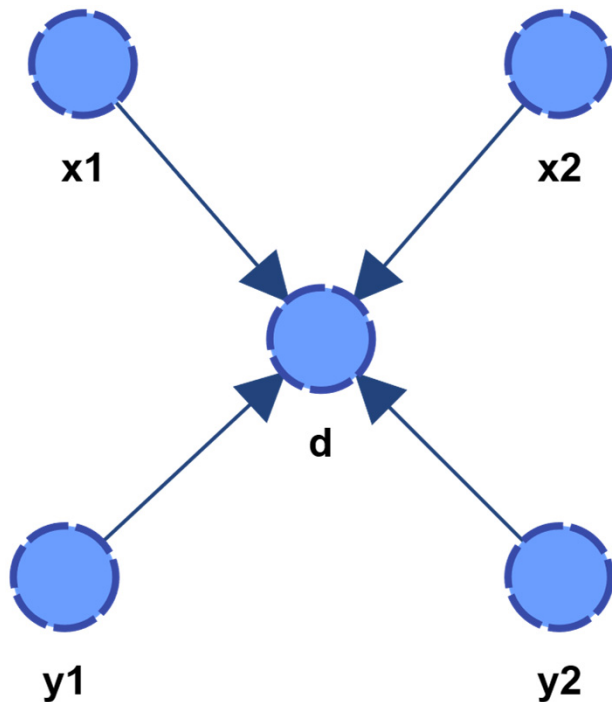
# Modeling Objectives

Simple

Practical

Fast

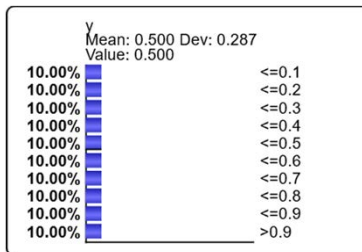
“Reasonable”



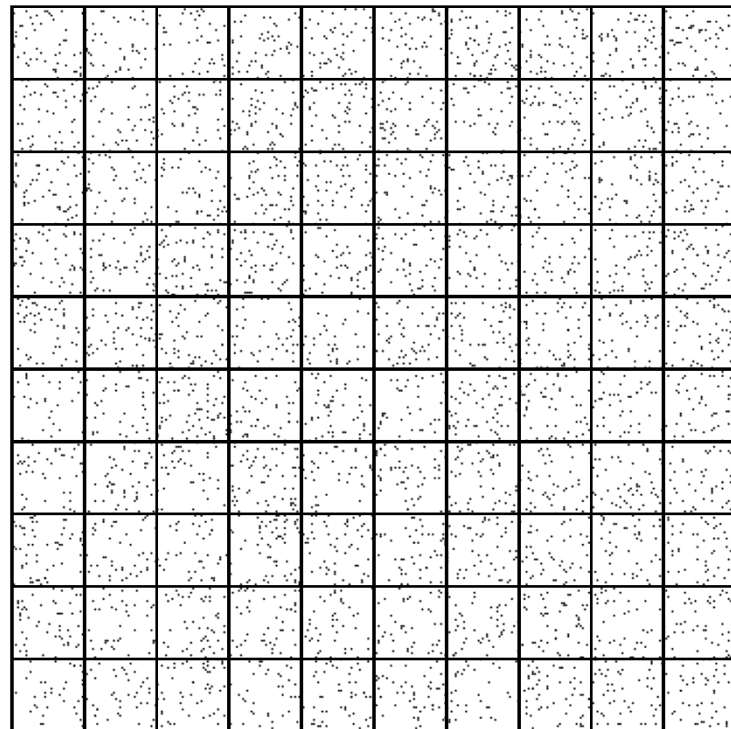
# Bayesian Networks

## Simplification

- Breaking the unit square into a 10×10 grid.

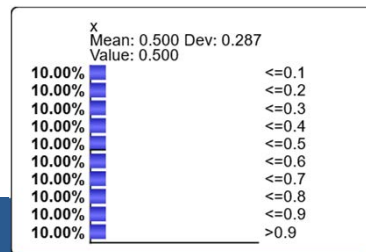


y



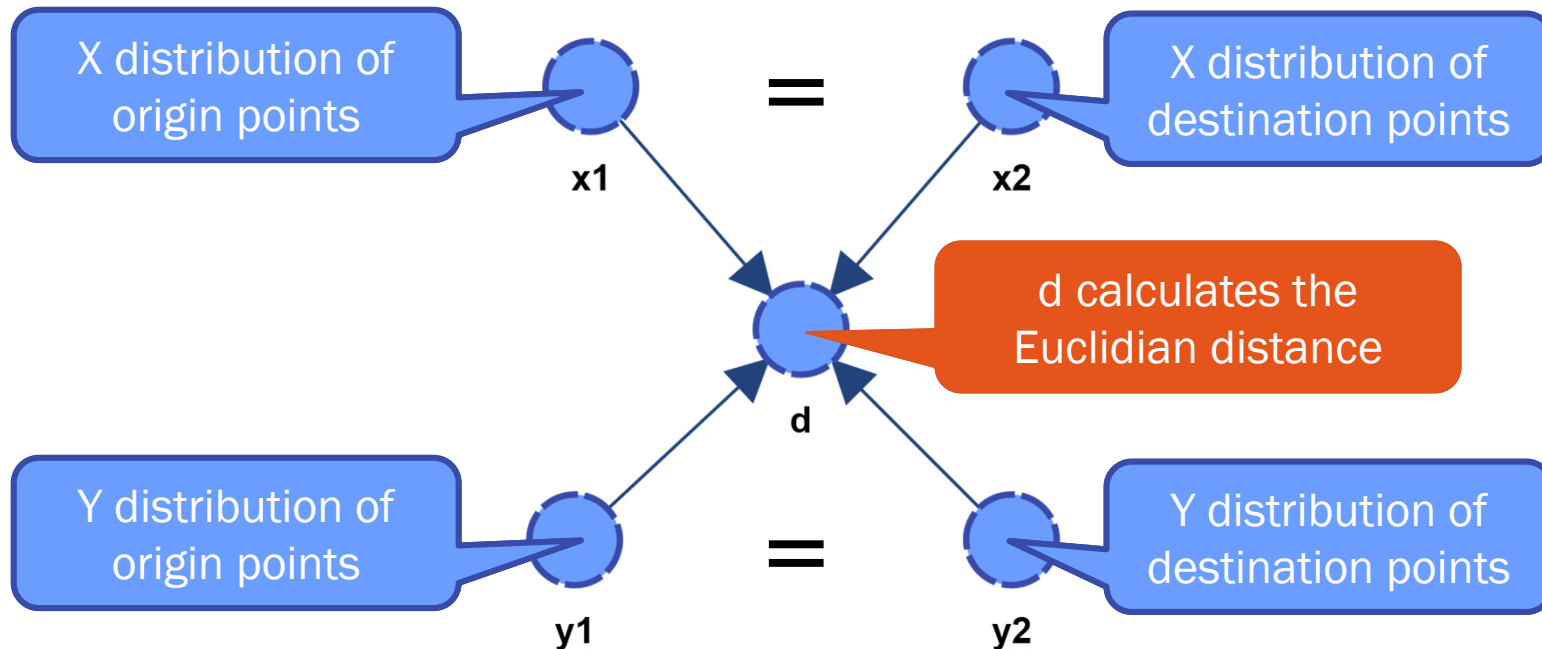
X

Too coarse for you?



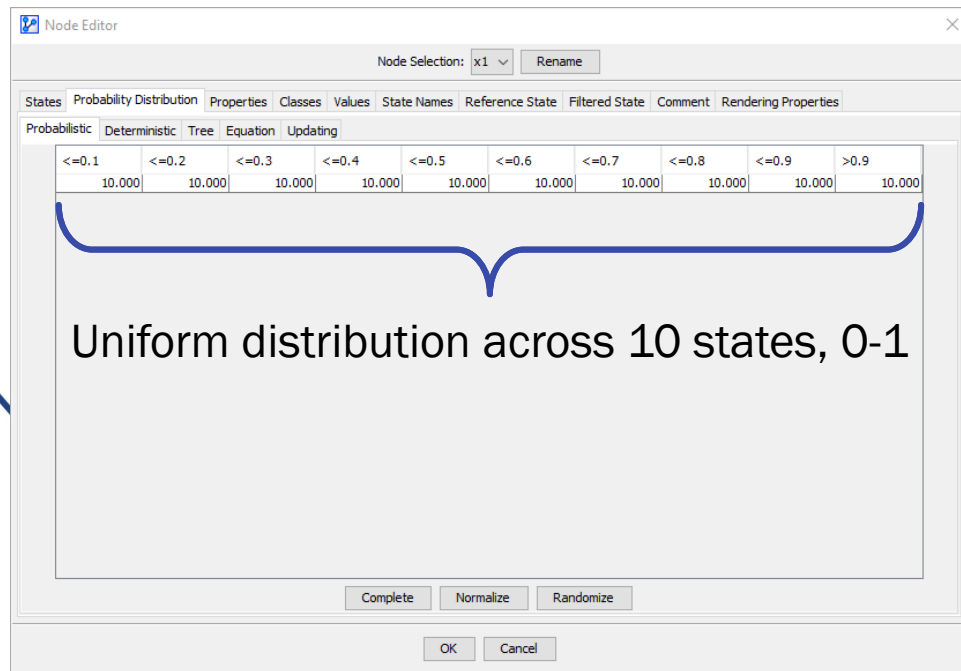
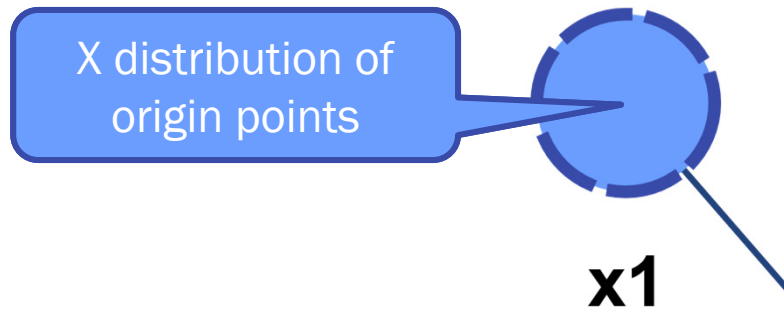
# Bayesian Networks to the Rescue

## A New Math for Social Distancing



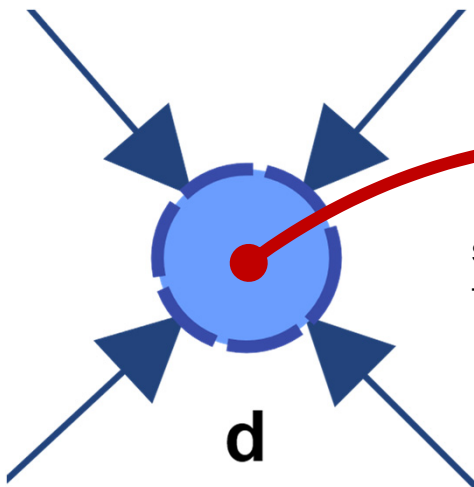
# Bayesian Networks to the Rescue

## A New Math for Social Distancing



# Bayesian Networks to the Rescue

## A New Math for Social Distancing



Node d has 15 states to capture the range of 0 to  $\sqrt{2}$

Node Editor

Node Selection: d Rename

State Names	Reference State	Filtered State	Comment	Rendering Properties
States	Probability Distribution	Properties	Classes	Values

Node type  
Continuous

0 1.5

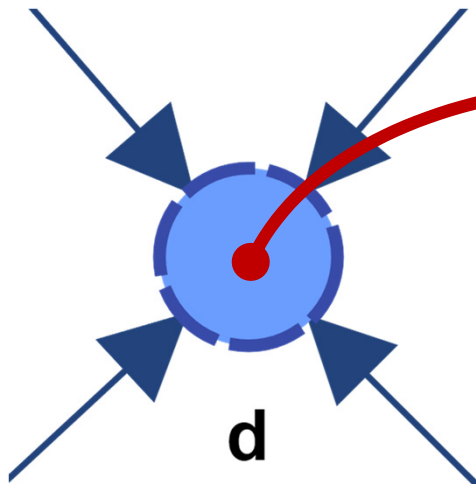
Discrete	Min	Max
<=0.1	0.000	0.100
<=0.2	0.100	0.200
<=0.3	0.200	0.300
<=0.4	0.300	0.400
<=0.5	0.400	0.500
<=0.6	0.500	0.600
<=0.7	0.600	0.700
<=0.8	0.700	0.800
<=0.9	0.800	0.900
<=1	0.900	1.000
<=1.1	1.000	1.100
<=1.2	1.100	1.200
<=1.3	1.200	1.300
<=1.4	1.300	1.400
>1.4	1.400	1.500

OK Cancel

Buttons: Add Before, Add After, Delete, Aggregates, Normalize, Generate Names, Generate Intervals

# Bayesian Networks to the Rescue

## A New Math for Social Distancing



Node Editor

Node Selection: d Rename

States Probability Distribution Properties Classes Values State Names Reference State Filtered State Comment Rendering Properties

Probabilistic Deterministic Tree Equation Updating

x1	y1	x2	y2	<=0.1	<=0.2	<=0.3	<=0.4	<=0.5	<=0.6	<=0.7	<=0.8	<=0.9	<=1	<=1.1	<=1.2	<=1.3	<=1.4	>1.4
<=0.1	<=0.1	<=0.1	<=0.1	97.100	2.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=0.2	<=0.1	<=0.2	41.900	57.800	0.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=0.3	<=0.1	<=0.3	0.000	46.100	53.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=0.4	<=0.1	<=0.4	0.000	0.000	48.000	51.900	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=0.5	<=0.1	<=0.5	0.000	0.000	0.000	47.800	52.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=0.6	<=0.1	<=0.6	0.000	0.000	0.000	0.000	46.100	53.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=0.7	<=0.1	<=0.7	0.000	0.000	0.000	0.000	0.000	48.300	51.600	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=0.8	<=0.1	<=0.8	0.000	0.000	0.000	0.000	0.000	0.000	47.800	52.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=0.9	<=0.1	<=0.9	0.000	0.000	0.000	0.000	0.000	0.000	48.300	51.700	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=1	<=0.1	<=1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	48.800	51.200	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=1.1	<=0.1	<=1.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=1.2	<=0.1	<=1.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=1.3	<=0.1	<=1.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	<=1.4	<=0.1	<=1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.1	>1.4	<=0.1	>1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.1	<=0.2	<=0.1	0.000	47.100	52.700	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.2	<=0.2	<=0.2	0.000	22.200	77.500	3.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.3	<=0.2	<=0.3	0.000	1.600	62.300	36.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.4	<=0.2	<=0.4	0.000	0.000	6.800	73.600	19.600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.5	<=0.2	<=0.5	0.000	0.000	0.000	11.600	76.600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.6	<=0.2	<=0.6	0.000	0.000	0.000	0.000	18.500	7.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.7	<=0.2	<=0.7	0.000	0.000	0.000	0.000	0.000	0.000	71.100	6.400	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.8	<=0.2	<=0.8	0.000	0.000	0.000	0.000	0.000	0.000	23.200	73.300	3.800	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	<=0.9	<=0.2	<=0.9	0.000	0.000	0.000	0.000	0.000	0.000	26.500	70.400	3.100	0.000	0.000	0.000	0.000	0.000	0.000
<=0.2	>0.9	<=0.2	>0.9	0.000	0.000	0.000	0.000	0.000	0.000	26.500	71.200	2.300	0.000	0.000	0.000	0.000	0.000	0.000
<=0.3	<=0.1	<=0.3	<=0.1	0.000	0.000	46.500	53.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.3	<=0.2	<=0.3	<=0.2	0.000	0.000	32.600	65.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.3	<=0.3	<=0.3	<=0.3	0.000	0.000	8.000	73.200	18.800	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.3	<=0.4	<=0.3	<=0.4	0.000	0.000	0.000	24.600	72.700	2.700	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.3	<=0.5	<=0.3	<=0.5	0.000	0.000	0.000	0.000	47.900	51.600	0.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<=0.3	<=0.6	<=0.3	<=0.6	0.000	0.000	0.000	0.000	2.100	63.000	34.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Complete Normalize Randomize

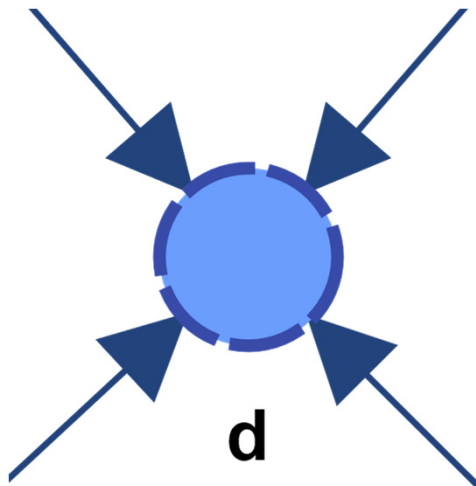
OK Cancel

CPT Size:  $15 \times 10 \times 10 \times 10 \times 10 = 150,000$  cells



# Bayesian Networks to the Rescue

## A New Math for Social Distancing



Euclidian Distance

Node Editor

States | Probability Distribution | Properties | Classes | Values | State Names | Reference State | Filtered State | Comment | Rendering Properties

Probabilistic | Deterministic | Tree | Equation | Updating

Equation Type: ☒ Deterministic ☐ Probabilistic

**?d? =**  
$$\text{SQRT}((?x2?-?x1?)^2+(?y2?-?y1?)^2)$$

Please validate formula!

Samples: 1000 Smoothing: 0 ☒ Fixed Seed: 31

Discrete Proba Distributions  
Continuous Proba Distributions  
Special Functions  
Arithmetic Functions  
Transformation Functions  
Conversion Functions  
Trigonometric Functions  
Relational Operators  
Boolean Operators

OK Cancel

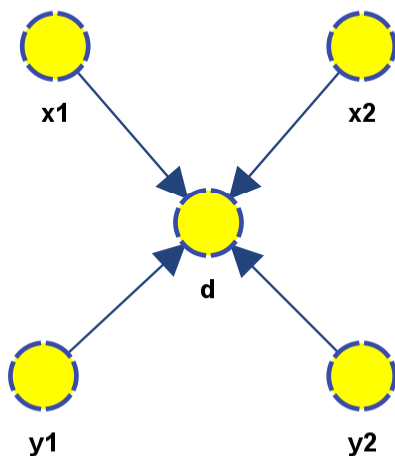
Complete Normalize Randomize

OK Cancel

x1	y1	x2	y2	<=0.1	<=0.2	<=0.3	<=0.4
		<=0.1		97.100	22.900	0.000	
		<=0.2		41.900	57.800	0.300	
		<=0.3		0.000	46.100	53.900	
		<=0.4		0.000	0.000	48.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		39.900	59.900	0.400	
		<=0.2		10.900	78.400	10.700	
		<=0.3		0.000	26.700	69.100	
		<=0.4		0.000	0.000	29.300	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	47.100	52.700	
		<=0.2		0.000	22.200	74.500	
		<=0.3		0.000	1.600	62.300	
		<=0.4		0.000	0.000	6.800	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
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		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
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		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
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		<=0.7		0.000	0.000	0.000	
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		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
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		<=0.3		0.000	0.000	0.000	
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		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
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		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
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		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
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		<=0.3		0.000	0.000	0.000	
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		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
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		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
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		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
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		<=0.7		0.000	0.000	0.000	
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		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4		0.000	0.000	0.000	
		<=0.5		0.000	0.000	0.000	
		<=0.6		0.000	0.000	0.000	
		<=0.7		0.000	0.000	0.000	
		<=0.8		0.000	0.000	0.000	
		<=0.9		0.000	0.000	0.000	
		>0.9		0.000	0.000	0.000	
		<=0.1		0.000	0.000	0.000	
		<=0.2		0.000	0.000	0.000	
		<=0.3		0.000	0.000	0.000	
		<=0.4</					

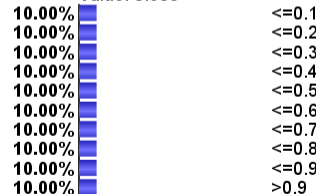


Distances.xbl \*

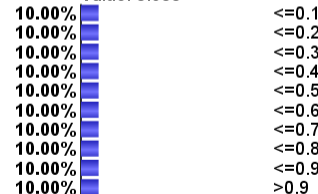


Joint Probability: 100.00%  
Log-Loss: 0  
Total Value: 2.521  
Mean Value: 0.504

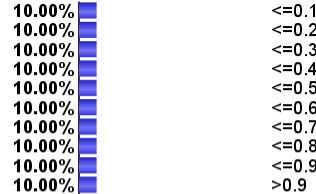
**x1**  
Mean: 0.500 Dev: 0.287  
Value: 0.500



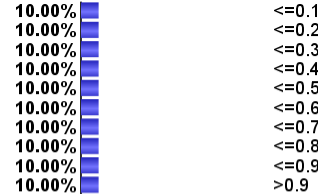
**x2**  
Mean: 0.500 Dev: 0.287  
Value: 0.500



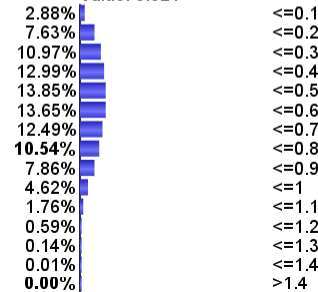
**y1**  
Mean: 0.500 Dev: 0.287  
Value: 0.500



**y2**  
Mean: 0.500 Dev: 0.287  
Value: 0.500



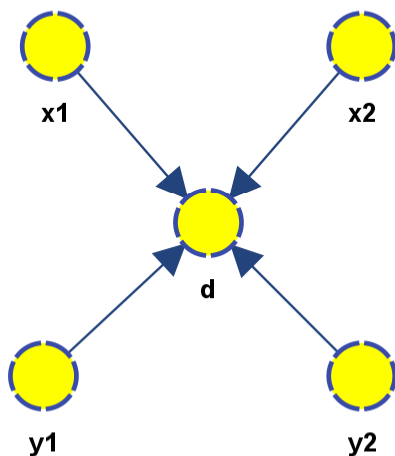
**d**  
Mean: 0.521 Dev: 0.250  
Value: 0.521



*done!*

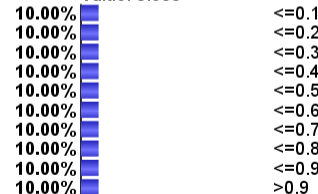


Distances.xbl \*

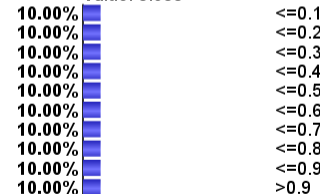


Joint Probability: 100.00%  
Log-Loss: 0  
Total Value: 2.521  
Mean Value: 0.504

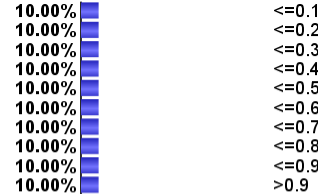
x1  
Mean: 0.500 Dev: 0.287  
Value: 0.500



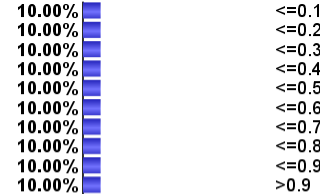
x2  
Mean: 0.500 Dev: 0.287  
Value: 0.500



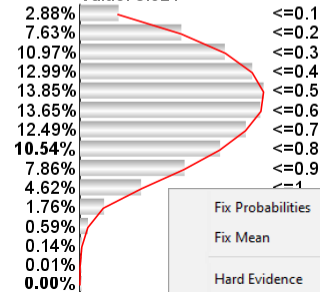
y1  
Mean: 0.500 Dev: 0.287  
Value: 0.500



y2  
Mean: 0.500 Dev: 0.287  
Value: 0.500



d  
Mean: 0.521 Dev: 0.250  
Value: 0.521



Fix Probabilities

Fix Mean

Hard Evidence

Likelihood Evidence

Probabilistic Evidence

Numerical Evidence

• Observation

Intervention

Absolute Bars

Relative Bars

• Relative Curve

Absolute Variations

Relative Variations

• Show Probabilities

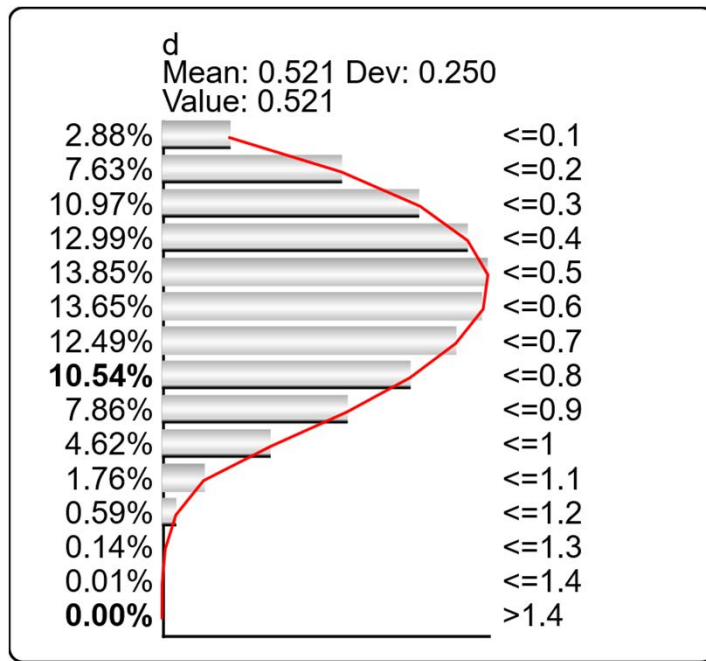
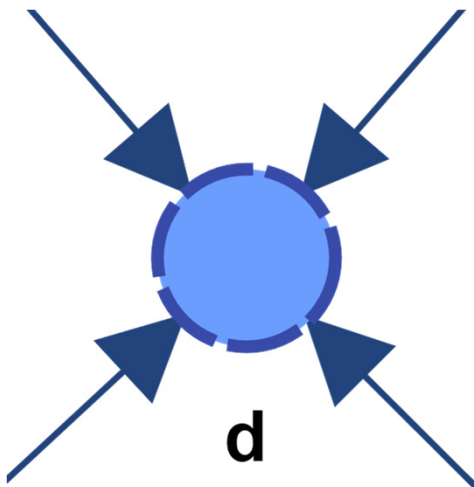
Show Expected Log-Loss

Copy

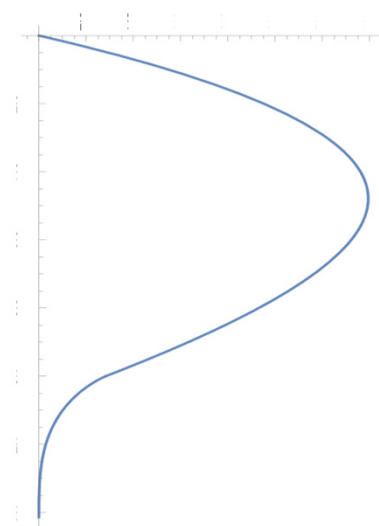
Delete

# Bayesian Networks to the Rescue

## A New Math for Social Distancing

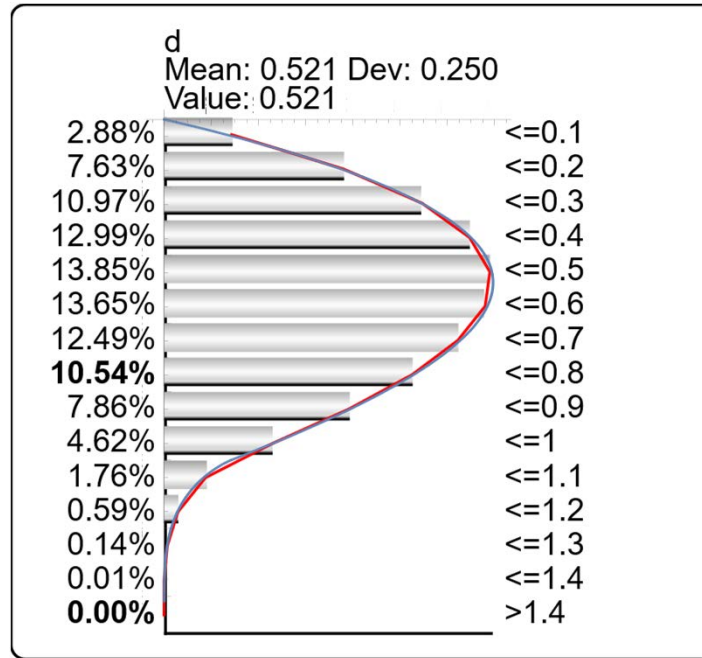
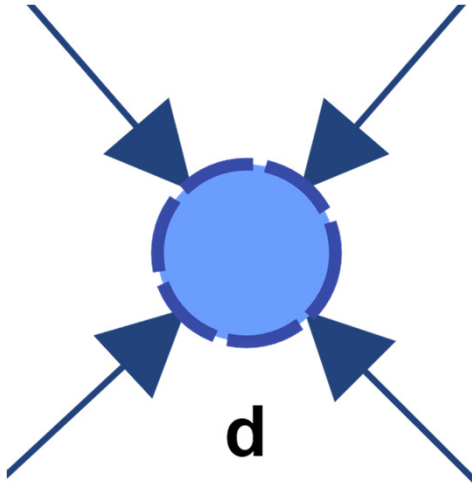


Analytical Solution



# Bayesian Networks to the Rescue

## A New Math for Social Distancing



The only formula needed:  
Euclidian distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

# Motivation

How far apart are these 5,000 points?

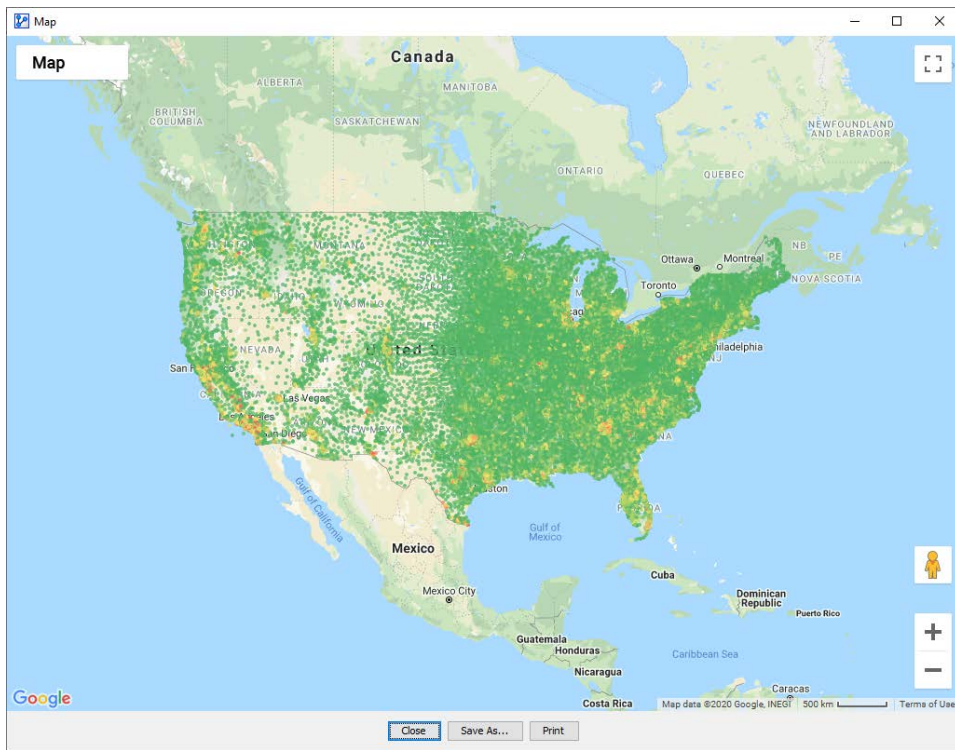


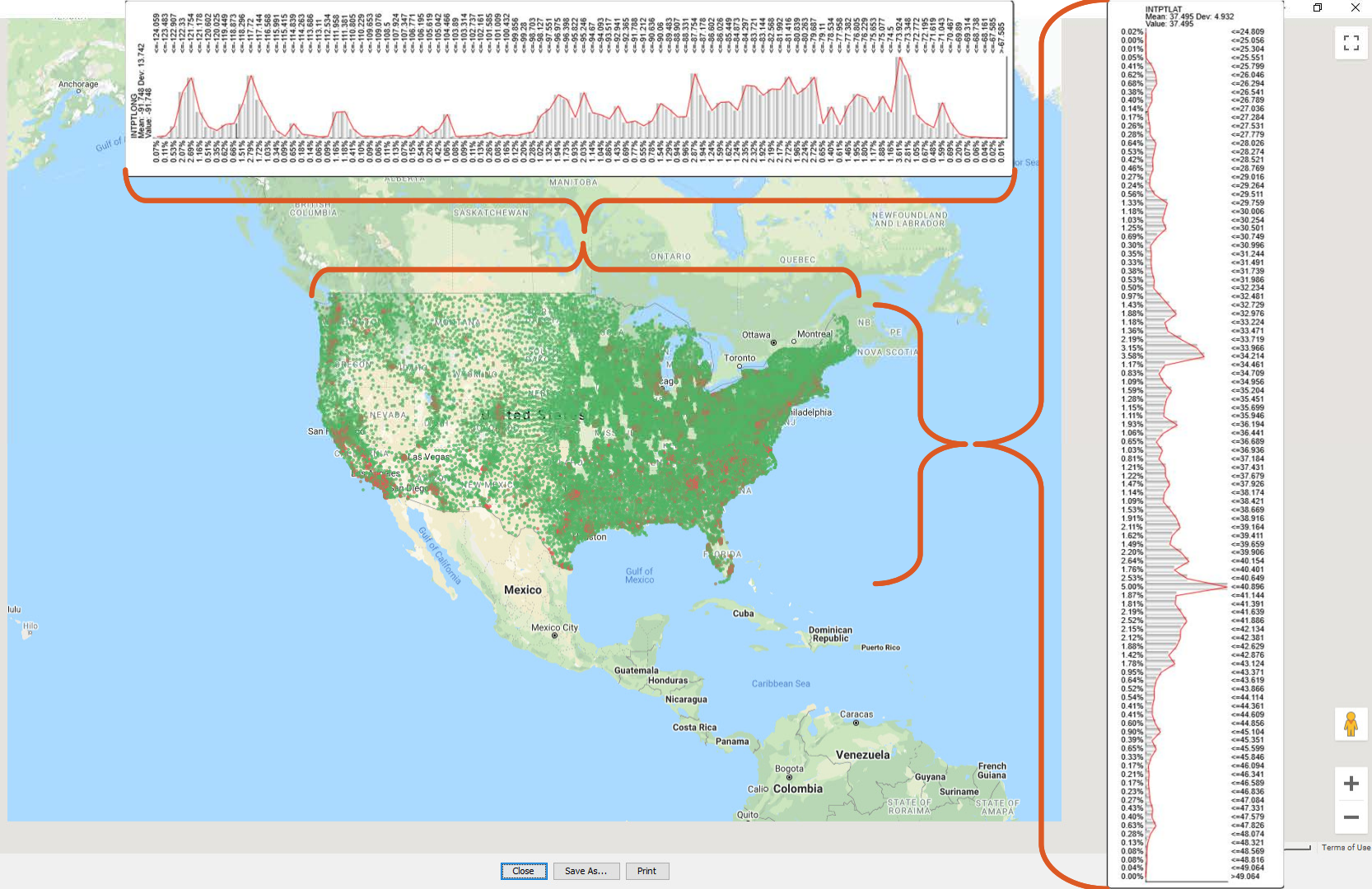
*Now what?*

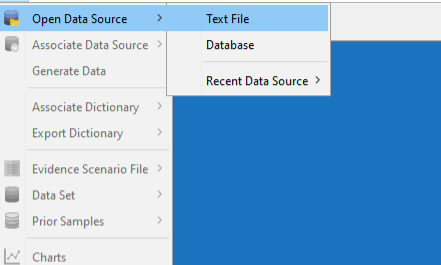


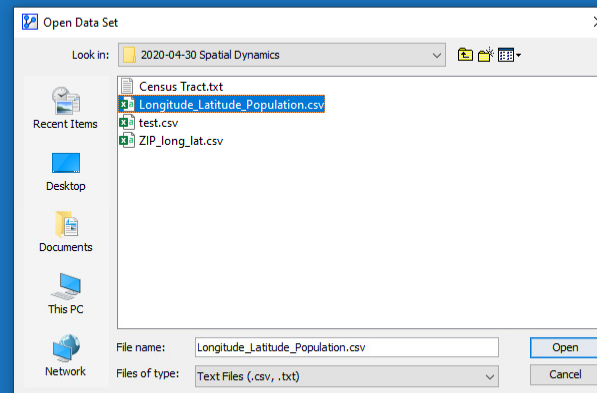
# Motivation

How far apart are these ~~5,000 points?~~ *330 million people*











**Data Import - C:\Users\StefanConrad\OneDrive - Bayesia USA\Presentations\2020-04-30 Spatial Dynamics\Longitude\_Latitude\_Popula...**

**Data Structure Definition**

**Separators**

☐ Tab ☐ Semicolon ☒ Comma

☐ Space ☐ Other ☐

**Encoding**

windows-1252

**Options**

☒ Title Line

☐ End of Line Character ☐

☐ Consider Identical Consecutive separators as a Unique One

☐ Consider Different Consecutive Separators as a Unique One

☐ Double Quotes ☐ Remove ☐ as String Delimiters

☐ Simple Quotes ☐ Remove ☐ as String Delimiters

☐ Transpose

**Missing Values**

N/R NR NC

Add Remove

**Filtered Values**

VF FV N/A

Add Remove

**Sampling**

Define Sample

**Learning/Test**

Define Learning/Test Sets

**Data**

Latitude	Longitude	Population
44.819713	-67.062328	1950
44.910958	-67.008913	1410
44.985316	-67.103562	1600
33.650799	-117.372457	52488
33.692675	-117.302951	19564
33.787292	-117.320673	56727
33.617845	-117.259304	29914
33.658592	-117.173618	40762
33.692805	-117.250024	17935
33.708976	-117.198846	21206
33.903134	-117.398044	54455
33.934335	-117.367319	46360
33.891132	-117.32675	35686

Cancel Previous Next Save Finish



**Data Import** - C:\Users\StefanConrad\OneDrive - Bayesia USA\Presentations\2020-04-30 Spatial Dynamics\Longitude\_Latitude\_Popula... X

Definition of Variable Types

Type

☐ Discrete

☐ Continuous

☒ Weight

☐ Learning/Test

☐ Row Identifier

☐ Unused

Multiple Typing

Set All Discrete

Set All Continuous

Set Missing Values Threshold

Information

Number of Rows	32370	100.00%
Discrete	0	0.00%
Continuous	2	66.67%
Others	1	33.33%
Unused	0	0.00%
Missing Values	0	0.00%
Filtered Values	0	0.00%

Data

Latitude	Longitude	Population
44.619713	-67.062328	1950
44.910958	-67.008913	1410
44.985316	-67.103562	1600
33.650799	-117.372457	52488
33.692675	-117.302951	19564
33.787292	-117.320673	56727
33.617845	-117.259304	29914
33.658592	-117.173618	40762
33.692805	-117.250024	17935
33.708976	-117.198846	21206
33.903134	-117.398044	54455
33.934335	-117.367319	46360
33.891132	-117.52675	35686
34.059722	-117.39104	32893
33.99504	-117.373184	22172

Cancel Previous Next Save Finish





**Data Import - C:\Users\StefanConrad\OneDrive - Bayesia USA\Presentations\2020-04-30 Spatial Dynamics\Longitude\_Latitude\_Popula...**

**Data Selection and Filtering**

**Missing Value Processing**

☐ Filter

☒ OR

☐ AND

☐ Replace by :

☐ Value

☐ Mean/Modal

☐ Infer

☐ Static Imputation

☐ Dynamic Imputation

☒ Structural EM

☐ Entropy-Based Static Imputation

☐ Entropy-Based Dynamic Imputation

**Information**

Number of Rows	32370	100.00%
Discrete	0	0.00%
Continuous	2	66.67%
Others	1	33.33%
Unused	0	0.00%
Missing Values	0	0.00%
Filtered Values	0	0.00%

**Select Values**

☐ OR

☒ AND

**Data**

Latitude	Longit...	Popula...
44.819713	-67.062328	1990
44.910958	-67.008913	1410
44.985316	-67.103562	1600
33.650799	-117.372457	52488
33.692675	-117.302951	19564
33.787292	-117.320673	56727



**Data Import** - C:\Users\StefanConrad\OneDrive - Bayesia USA\Presentations\2020-04-30 Spatial Dynamics\Longitude\_Latitude\_Popula... X

Discretization and Aggregation

Multiple Discretization

Type: Equal Distance

Intervals: 20

☐ Log Transformation

☐ Create a Class for Each Type of Discretization

Load Discretizations

Data

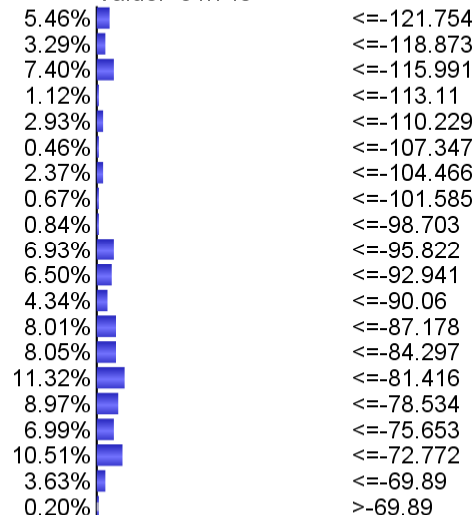
Latitude	Longitude	Population
44.819713	-67.062328	1950
44.910958	-67.008913	1410
44.985316	-67.103562	1600
33.650799	-117.372457	52488
33.692675	-117.302951	19564
33.787292	-117.320673	56727
33.617845	-117.259304	29914
33.658592	-117.173618	40762
33.692805	-117.250024	17935
33.708976	-117.198846	21206
33.903134	-117.398044	54455
33.934335	-117.367319	46360
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34.059722	-117.39104	32893
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Select All Continuous Select All Discrete

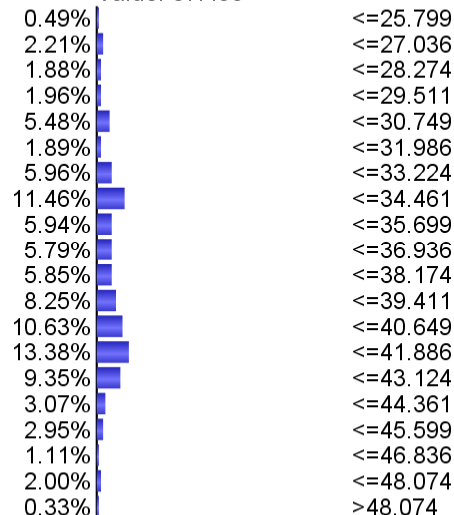
Cancel Previous Next Save Finish

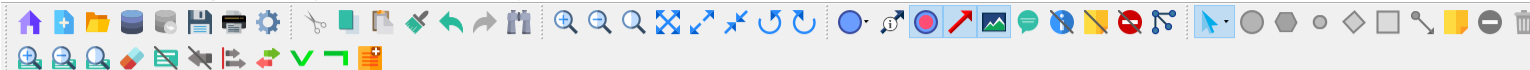


Longitude  
Mean: -91.748 Dev: 13.742  
Value: -91.748



Latitude  
Mean: 37.495 Dev: 4.932  
Value: 37.495





Associated graph 8.xbl \*



Joint Probability: 100.00%  
 Log-Loss: 0  
 Cases: 32,370  
 Total Value: -54.253  
 Mean Value: -27.127

x1  
 Mean: -91.748 Dev: 13.742  
 Value: -91.748

5.46%	<=-121.754
3.29%	<=-118.873
7.40%	<=-115.991
1.12%	<=-113.11
2.93%	<=-110.229
0.46%	<=-107.347
2.37%	<=-104.466
0.67%	<=-101.585
0.84%	<=-98.703
6.93%	<=-95.822
6.50%	<=-92.941
4.34%	<=-90.06
8.01%	<=-87.178
8.05%	<=-84.297
11.32%	<=-81.416
8.97%	<=-78.534
6.99%	<=-75.653
10.51%	<=-72.772
3.63%	<=-69.89
0.20%	>-69.89

y1  
 Mean: 37.495 Dev: 4.932  
 Value: 37.495

0.49%	<=25.799
2.21%	<=27.036
1.88%	<=28.274
1.96%	<=29.511
5.48%	<=30.749
1.89%	<=31.986
5.96%	<=33.224
11.46%	<=34.461
5.94%	<=35.699
5.79%	<=36.936
5.85%	<=38.174
8.25%	<=39.411
10.63%	<=40.649
13.38%	<=41.886
9.35%	<=43.124
3.07%	<=44.361
2.95%	<=45.599
1.11%	<=46.836
2.00%	<=48.074
0.33%	>48.074



Associated graph 8.xbl \*



y1



x1

- Edit
- Rename
- Copy
- Delete
- Exclude
- Set as Target Node
- Monitor
- Charts
- Imputation
- Probabilities >
- Select >
- Connect >
- Alignment >
- Properties >



Associated graph 8.xbl \*



y1



x1

Copy Selection

Paste

Paste Format

Delete Selection

Delete All Arcs

Delete all Unfixed Arcs

Delete all Unconnected Nodes

Exclude all Unconnected Nodes



Edit Structural Coefficient



Edit Costs



Edit Classes



Edit Constants



Edit Forbidden Arcs



Edit Structural Priors



Edit Temporal Indices



Edit Virtual Numbers of States



Edit Reference States



Edit Filtered States



Edit Local Structural Coefficients



Edit Number of Uniform Prior Samples



Edit Experts



Parameter Estimation with Trees



Use Time Variable

Properties &gt;

Save as Image...





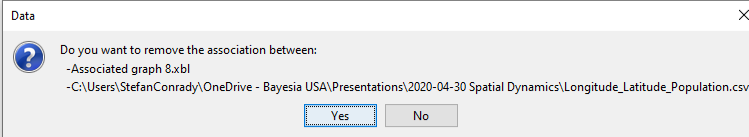
Associated graph 8.xbl \*



y1



x1





Associated graph 8.xbl \*



y1



y1[1]



x1



x1[1]



Associated graph 8.xbl \*



y1



y2



x1



x2



Associated graph 8.xbl \*



y1



y2



N1



x1



x2



Associated graph 8.xbl \*



y1



y2



d



x1



x2



Associated graph 8.xbl \*



y1



d



x1

Node Editor

Node Selection: d Rename

States Probability Distribution Properties Classes Values State Names Reference State Filtered State Comment Rendering Properties

Node type  
Continuous

	Min	Max
Discrete		
False	0.000	0.500
True	0.500	1.000

Add Before  
Add After  
Delete  
Aggregates  
Normalize  
Generate Names  
Generate Intervals

**Generate Intervals**

Number of Intervals: 20

Minimum 0 Maximum 3,500

☒ Automatic Naming  
☐ Numbered Names Prefixed by: L

OK Cancel

OK Cancel



Associated graph 8.xbl \*



y1



d



x1

Node Editor

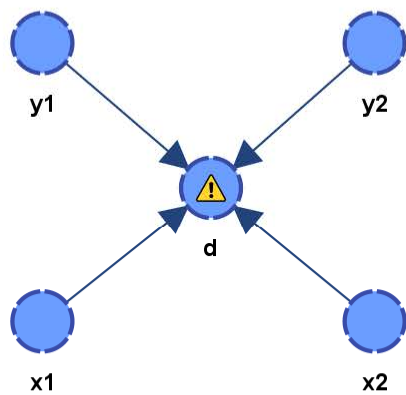
Node Selection: d

States Probability Distribution Properties Classes Values State Names Reference State Filtered State Comment Rendering Properties

Node type  
Continuous

0 3500

Discrete	Min	Max
<=175	0.000	175.000
<=350	175.000	350.000
<=525	350.000	525.000
<=700	525.000	700.000
<=875	700.000	875.000
<=1050	875.000	1050.000
<=1225	1050.000	1225.000
<=1400	1225.000	1400.000
<=1575	1400.000	1575.000
<=1750	1575.000	1750.000
<=1925	1750.000	1925.000
<=2100	1925.000	2100.000
<=2275	2100.000	2275.000
<=2450	2275.000	2450.000
<=2625	2450.000	2625.000
<=2800	2625.000	2800.000
<=2975	2800.000	2975.000
<=3150	2975.000	3150.000
<=3325	3150.000	3325.000
>3325	3325.000	3500.000







Associated graph 8.xbl \*

## Node Editor

Node Selection: d 

States Probability Distribution Properties Classes Values State Names Reference State Filtered State Comment Rendering Properties

Probabilistic Deterministic Tree Equation Updating

y1	y2	x1	x2	<=175	<=350	<=525	<=700	<=875	<=1050	<=1225	<=1400	<=1575	<=1750	<=1925	<=2100	<=2275	<=2450	<=2625	<=2800	<=2975	<=3150	<=3325	>3325
			<=-121.754																				
			<=-118.873																				
			<=-115.991																				
			<=-113.11																				
			<=-110.229																				
			<=-107.347																				
			<=-104.466																				
			<=-101.585																				
			<=-98.703																				
			<=-95.822																				
			<=-92.941																				
			<=-90.06																				
			<=-87.178																				
			<=-84.297																				
			<=-81.416																				
			<=-78.534																				
			<=-75.653																				
			<=-72.772																				
			<=-69.89																				
			>-69.89																				
			<=-121.754																				
			<=-118.873																				



$$d = \sqrt{((x_2 - x_1) * 54.6)^2 + ((y_2 - y_1) * 69)^2}$$

Node Editor

States Probability Distribution Properties Classes Values State Names

Probabilistic Deterministic Tree Equation Updating

Equation Type: ☒ Deterministic ☐ Probabilistic

```
d =
SQRT(((x2-x1)*54.6)^2+((y2-y1)*69)^2)
```

Please validate formula!

Samples: 1000 Smoothing: 0 ☒ Fixed Seed: 31 Validate

- Discrete Proba Distributions
- Continuous Proba Distributions
- Special Functions
- Arithmetic Functions
- Transformation Functions
- Conversion Functions
- Trigonometric Functions
- Relational Operators
- Boolean Operators

In the Continental US:  
 $\Delta 1^\circ$  (Longitude)  $\approx 54.6$  miles

In the Continental US:  
 $\Delta 1^\circ$  (Latitude)  $\approx 69$  miles

Using the great-circle distance  
 formula would be a more precise  
 way to calculate the distance:

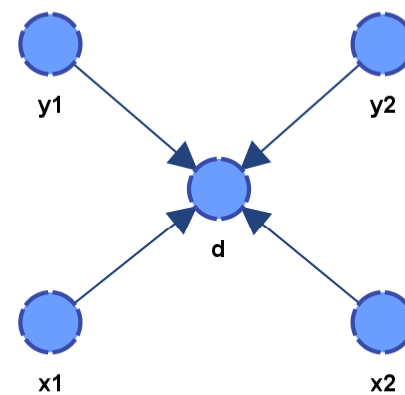
$$d = 2r \arcsin \left( \sqrt{\text{hav}(\varphi_2 - \varphi_1) + \cos(\varphi_1) \cos(\varphi_2) \text{hav}(\lambda_2 - \lambda_1)} \right)$$

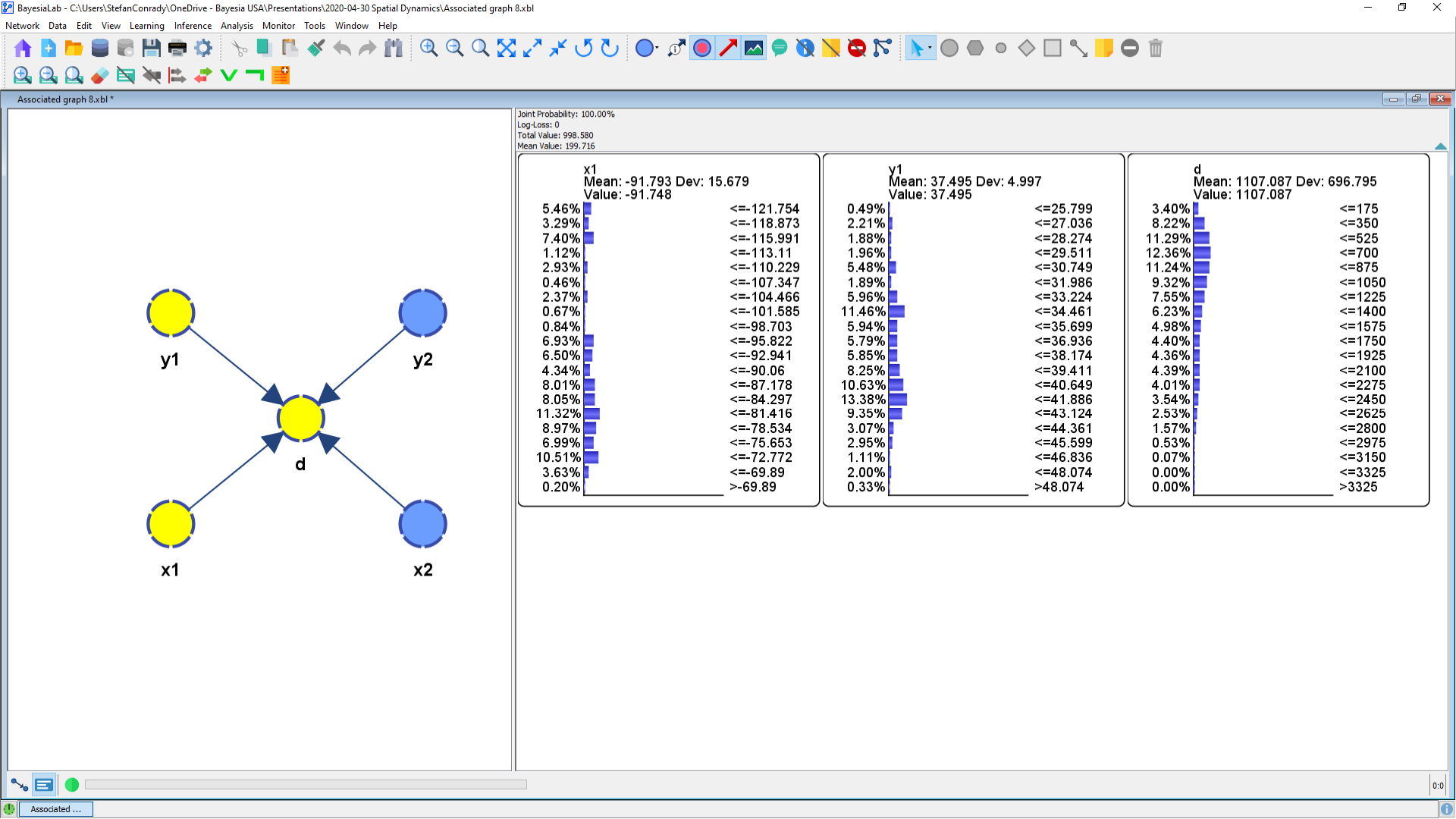
$$= 2r \arcsin \left( \sqrt{\sin^2 \left( \frac{\varphi_2 - \varphi_1}{2} \right) + \cos(\varphi_1) \cos(\varphi_2) \sin^2 \left( \frac{\lambda_2 - \lambda_1}{2} \right)} \right)$$

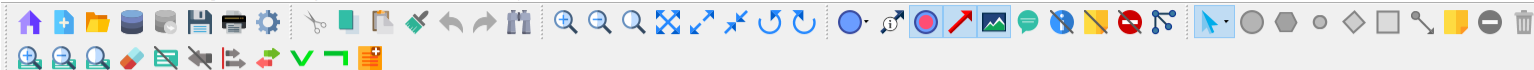




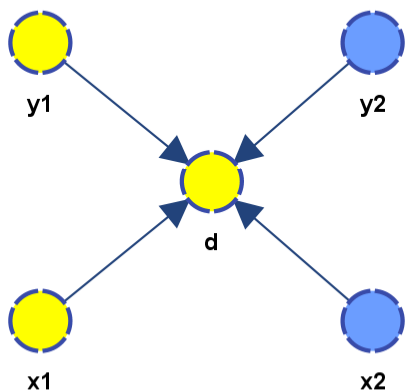
Associated graph 8.xbl





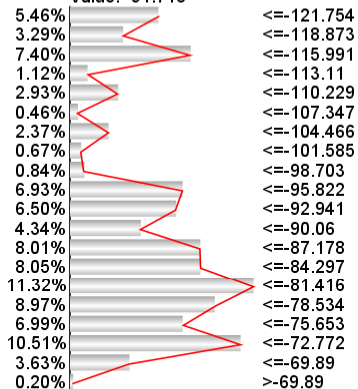


Associated graph 8.xbl \*

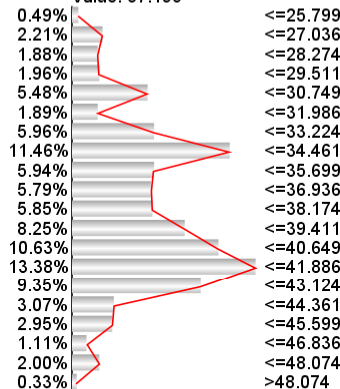


Joint Probability: 100.00%  
Log-Loss: 0  
Total Value: 998.580  
Mean Value: 199.716

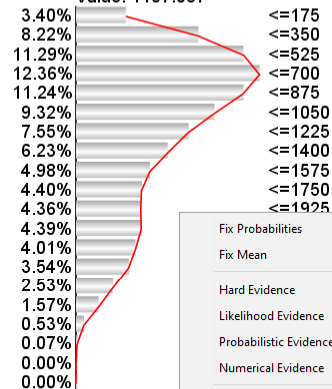
**x1**  
Mean: -91.793 Dev: 15.679  
Value: -91.748



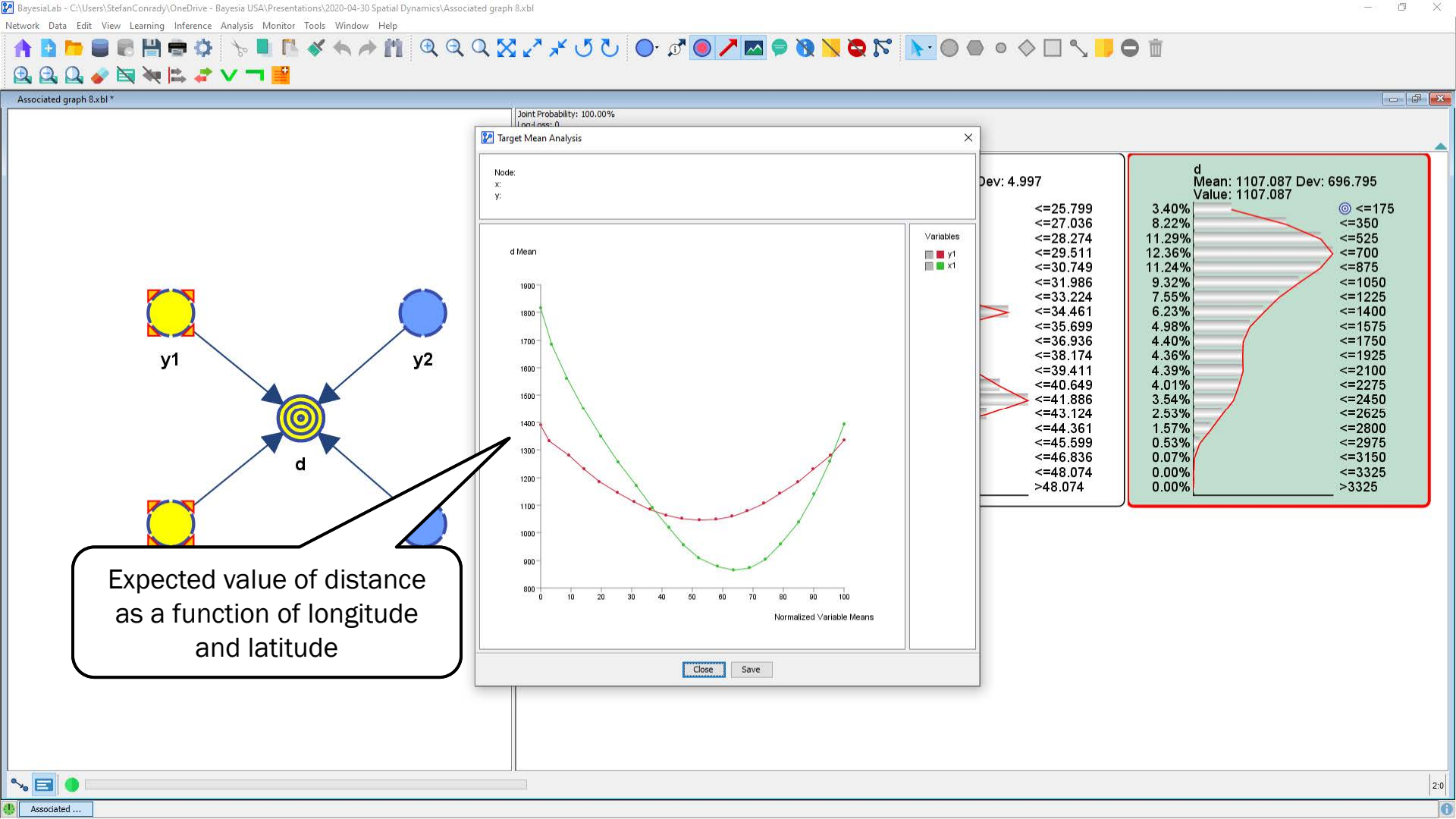
**y1**  
Mean: 37.495 Dev: 4.997  
Value: 37.495

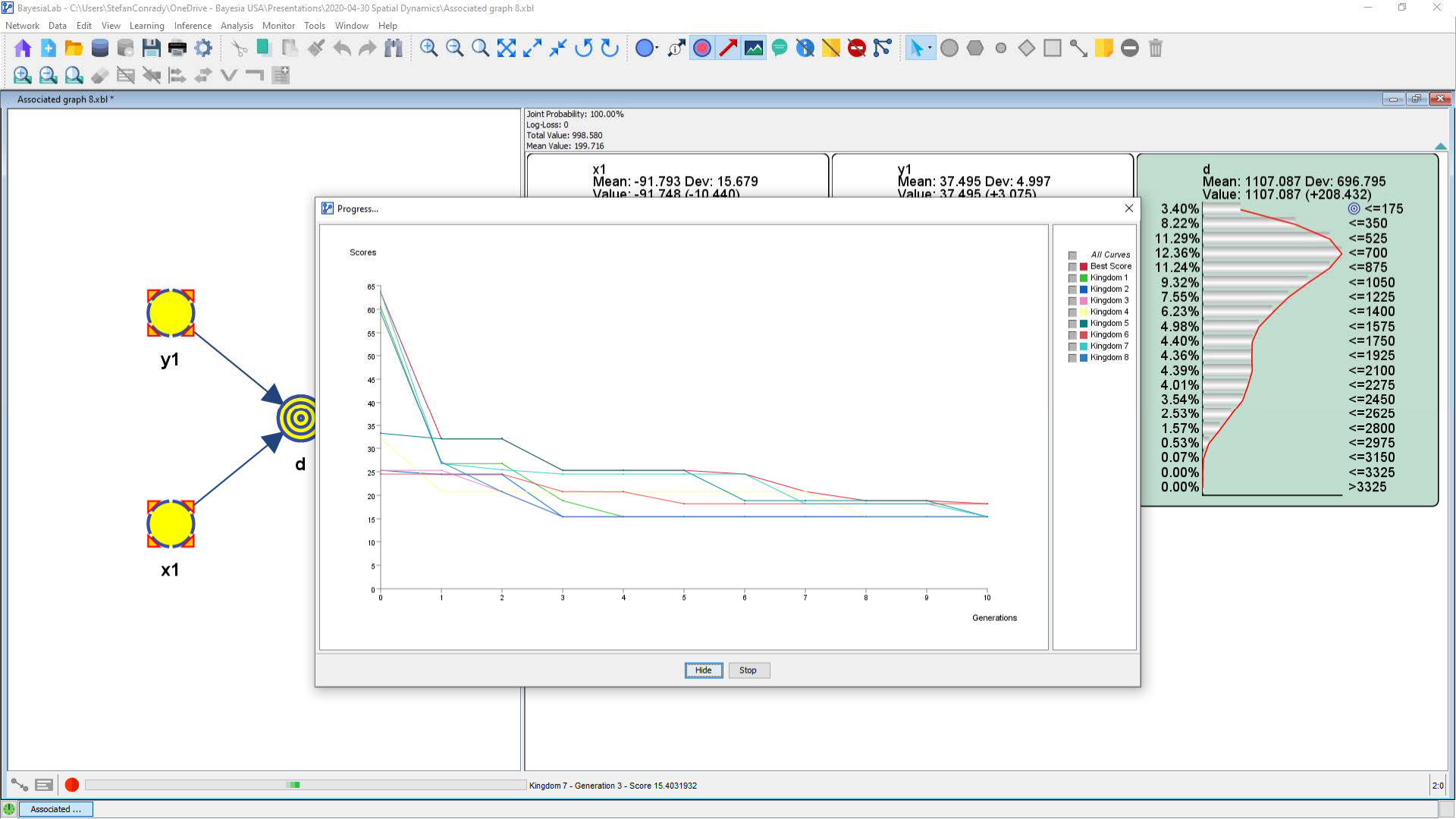


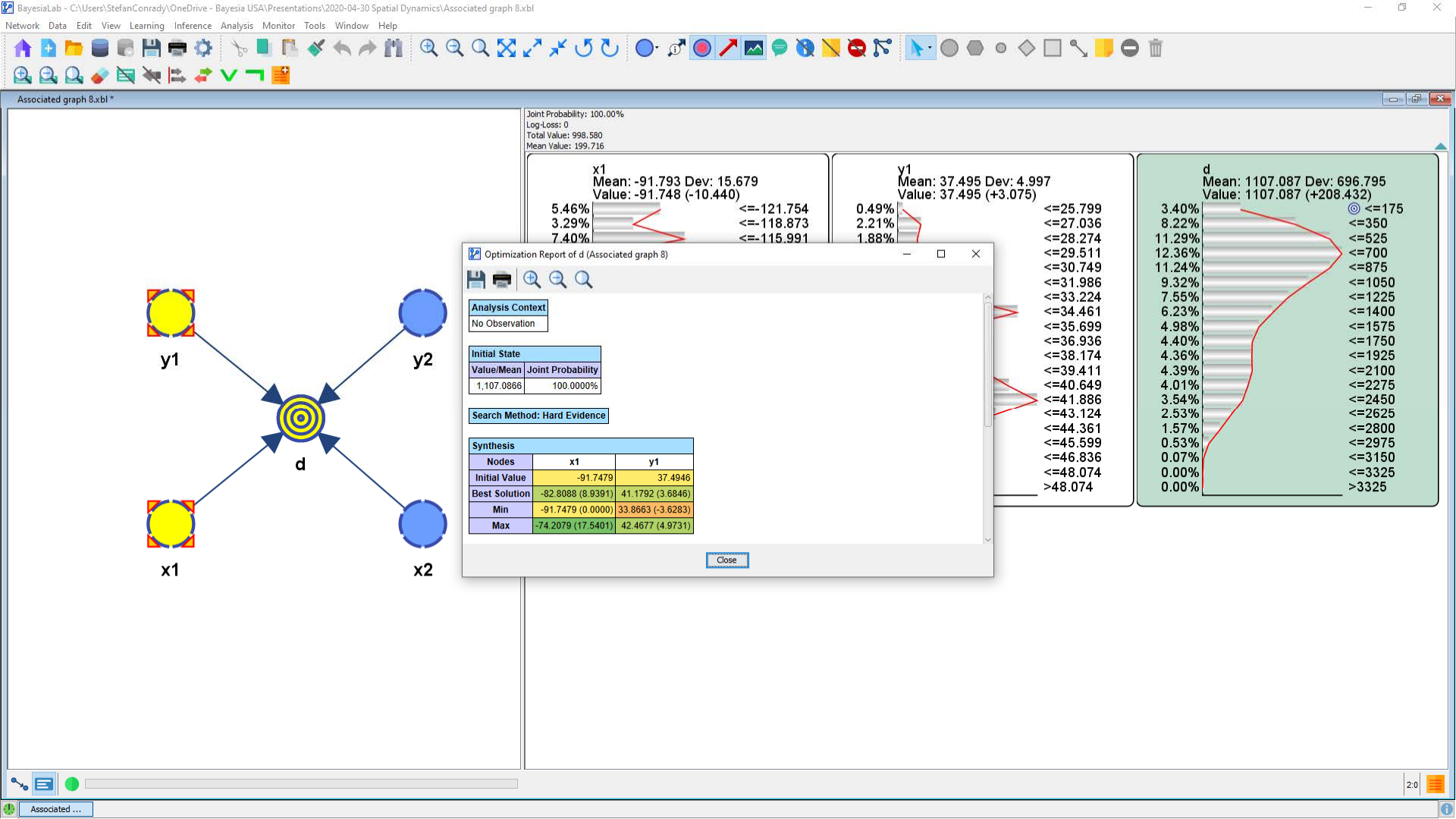
**d**  
Mean: 1107.087 Dev: 696.795  
Value: 1107.087



- Fix Probabilities
- Fix Mean
- Hard Evidence
- Likelihood Evidence
- Probabilistic Evidence
- Numerical Evidence
- ☒ Observation
- Intervention
- Absolute Bars
- Relative Bars
- ☒ Relative Curve
- Absolute Variations
- Relative Variations
- ☒ Show Probabilities
- Show Expected Log-Loss
- Copy
- Delete



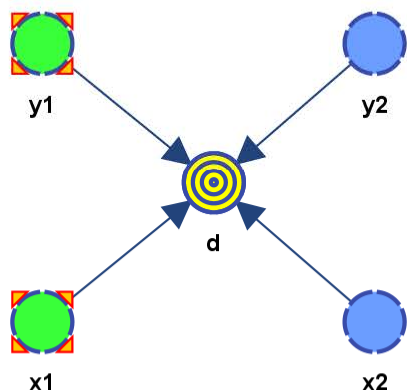








Associated graph 8.xbl \*



Joint Probability: 1.51%  
Log-Loss: 6.05  
Total Value: 773.473  
Mean Value: 154.695

**x1**  
Mean: -82.856 Dev: 0.000  
Value: -82.809 (+8.939)

0.00%	<= -121.754
0.00%	<= -118.873
0.00%	<= -115.991
0.00%	<= -113.11
0.00%	<= -110.229
0.00%	<= -107.347
0.00%	<= -104.466
0.00%	<= -101.585
0.00%	<= -98.703
0.00%	<= -95.822
0.00%	<= -92.941
0.00%	<= -90.06
0.00%	<= -87.178
0.00%	<= -84.297
100.00%	<= -81.416
0.00%	<= -78.534
0.00%	<= -75.653
0.00%	<= -72.772
0.00%	<= -69.89
0.00%	> -69.89

**y1**  
Mean: 41.267 Dev: 0.000  
Value: 41.179 (+3.685)

0.00%	<= 25.799
0.00%	<= 27.036
0.00%	<= 28.274
0.00%	<= 29.511
0.00%	<= 30.749
0.00%	<= 31.986
0.00%	<= 33.224
0.00%	<= 34.461
0.00%	<= 35.699
0.00%	<= 36.936
0.00%	<= 38.174
0.00%	<= 39.411
0.00%	<= 40.649
100.00%	<= 41.886
0.00%	<= 43.124
0.00%	<= 44.361
0.00%	<= 45.599
0.00%	<= 46.836
0.00%	<= 48.074
0.00%	> 48.074

**d**  
Mean: 869.356 Dev: 640.079  
Value: 869.356 (-237.730)

7.38%	<= 175
12.53%	<= 350
16.11%	<= 525
17.18%	<= 700
13.07%	<= 875
6.97%	<= 1050
3.44%	<= 1225
2.21%	<= 1400
1.65%	<= 1575
2.33%	<= 1750
4.51%	<= 1925
5.56%	<= 2100
5.16%	<= 2275
1.82%	<= 2450
0.10%	<= 2625
0.00%	<= 2800
0.00%	<= 2975
0.00%	<= 3150
0.00%	<= 3325
0.00%	> 3325

Select an evidence set

Index	Comment
0	GA Score: 15.4031932
1	GA Score: 18.177061
2	GA Score: 18.8420802
3	GA Score: 20.7291169
4	GA Score: 20.7884891
5	GA Score: 20.8185179
6	GA Score: 21.6674884
7	GA Score: 22.9615237
8	GA Score: 23.8758095
9	GA Score: 24.4250777

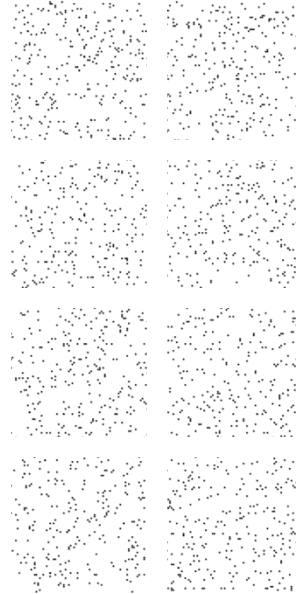
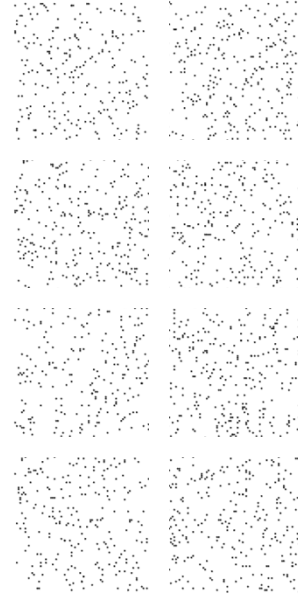
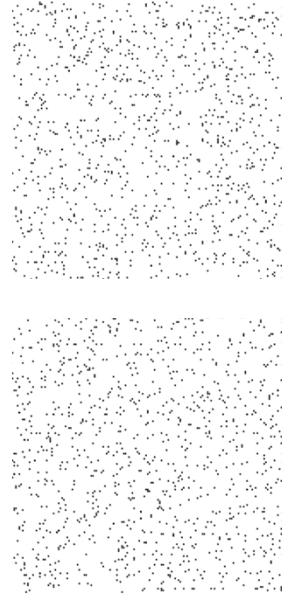
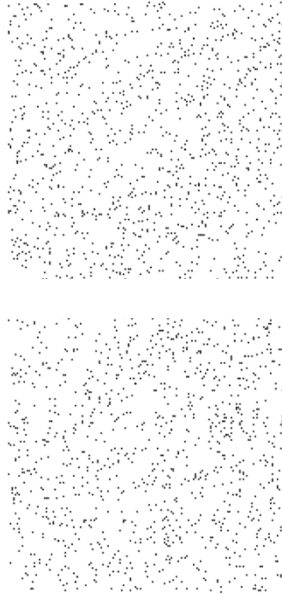
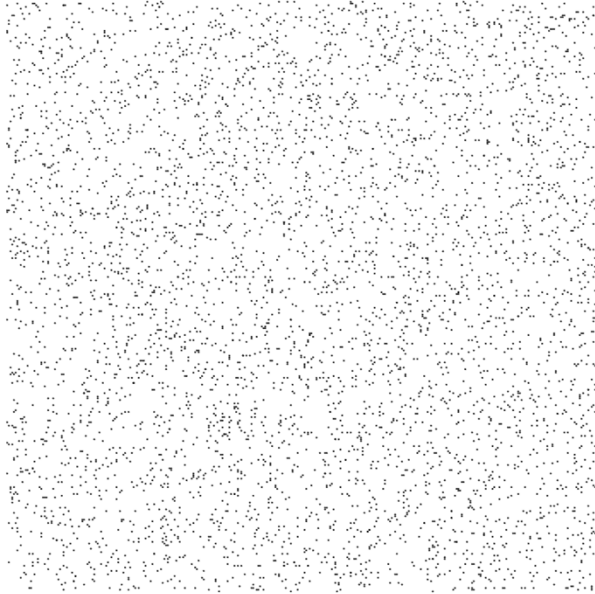
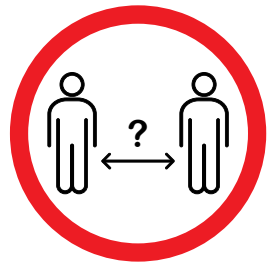
10 Rows ☐ Include Not Observable

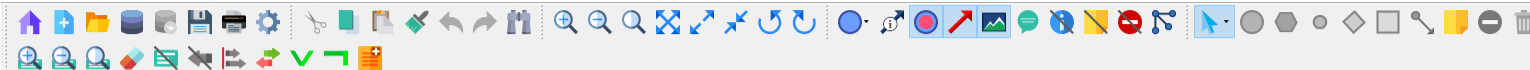
GA Score: 15.4031932

2.0

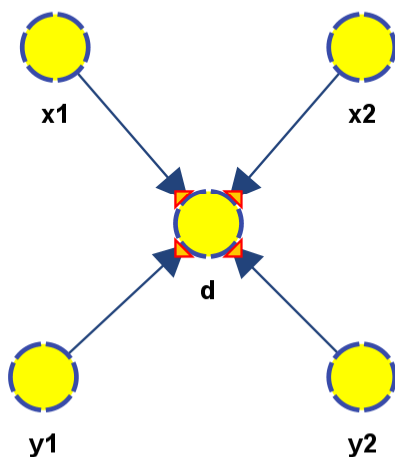
# Bayesian Network

How to distance?



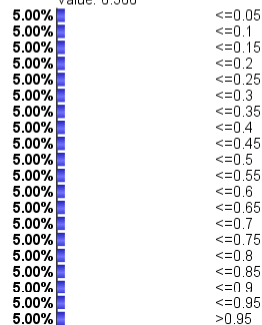


Distances2.xbl

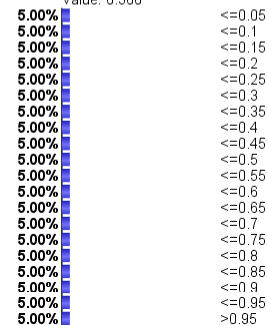


Joint Probability: 100.00%  
Log-Loss: 0  
Total Value: 2.521  
Mean Value: 0.504

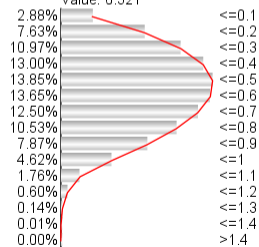
x1  
Mean: 0.500 Dev: 0.288  
Value: 0.500



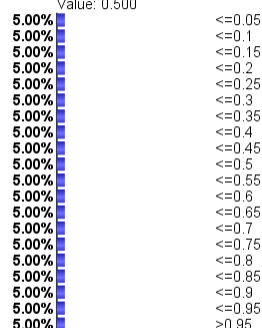
y1  
Mean: 0.500 Dev: 0.288  
Value: 0.500



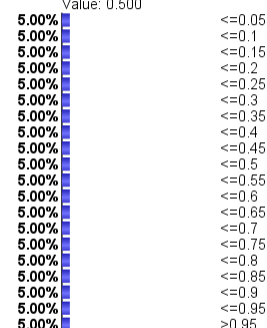
d  
Mean: 0.521 Dev: 0.250  
Value: 0.521

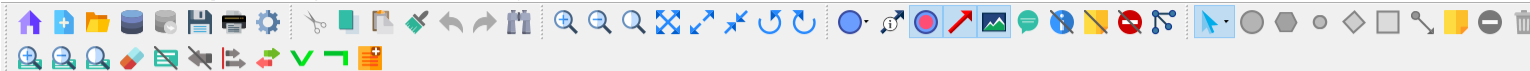


x2  
Mean: 0.500 Dev: 0.288  
Value: 0.500

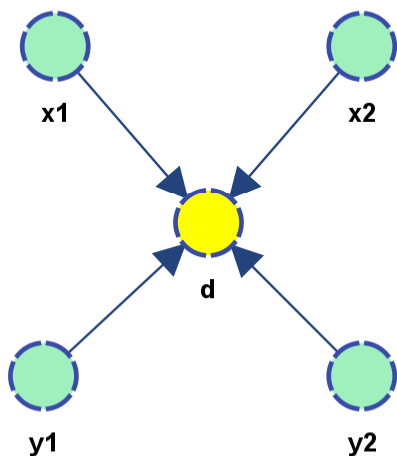


y2  
Mean: 0.500 Dev: 0.288  
Value: 0.500





Distances2.xbl



Joint Probability: 65.61%  
Log-Loss: 0.61  
Total Value: 2.550  
Mean Value: 0.510

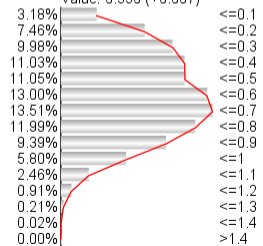
x1  
Mean: 0.500 Dev: 0.304  
Value: 0.500 (+0.000)

5.56%	<=0.05
5.56%	<=0.1
5.56%	<=0.15
5.56%	<=0.2
5.56%	<=0.25
5.56%	<=0.3
5.56%	<=0.35
5.56%	<=0.4
5.56%	<=0.45
0.00%	<=0.5
0.00%	<=0.55
5.56%	<=0.6
5.56%	<=0.65
5.56%	<=0.7
5.56%	<=0.75
5.56%	<=0.8
5.56%	<=0.85
5.56%	<=0.9
5.56%	<=0.95
5.56%	>0.95

y1  
Mean: 0.500 Dev: 0.304  
Value: 0.500 (-0.000)

5.56%	<=0.05
5.56%	<=0.1
5.56%	<=0.15
5.56%	<=0.2
5.56%	<=0.25
5.56%	<=0.3
5.56%	<=0.35
5.56%	<=0.4
5.56%	<=0.45
0.00%	<=0.5
0.00%	<=0.55
5.56%	<=0.6
5.56%	<=0.65
5.56%	<=0.7
5.56%	<=0.75
5.56%	<=0.8
5.56%	<=0.85
5.56%	<=0.9
5.56%	<=0.95
5.56%	>0.95

d  
Mean: 0.550 Dev: 0.262  
Value: 0.550 (+0.007)

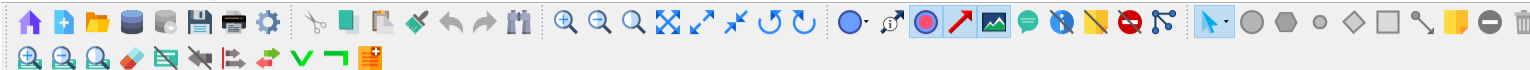


x2  
Mean: 0.500 Dev: 0.304  
Value: 0.500 (+0.000)

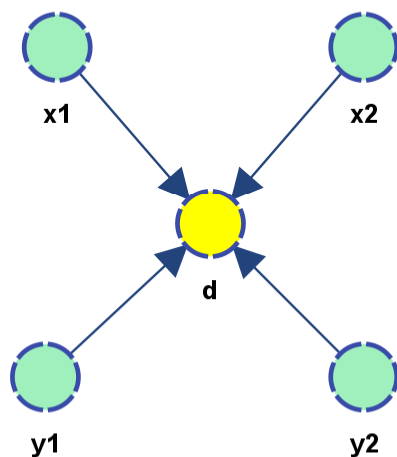
5.56%	<=0.05
5.56%	<=0.1
5.56%	<=0.15
5.56%	<=0.2
5.56%	<=0.25
5.56%	<=0.3
5.56%	<=0.35
5.56%	<=0.4
5.56%	<=0.45
0.00%	<=0.5
0.00%	<=0.55
5.56%	<=0.6
5.56%	<=0.65
5.56%	<=0.7
5.56%	<=0.75
5.56%	<=0.8
5.56%	<=0.85
5.56%	<=0.9
5.56%	<=0.95
5.56%	>0.95

y2  
Mean: 0.500 Dev: 0.304  
Value: 0.500

5.56%	<=0.05
5.56%	<=0.1
5.56%	<=0.15
5.56%	<=0.2
5.56%	<=0.25
5.56%	<=0.3
5.56%	<=0.35
5.56%	<=0.4
5.56%	<=0.45
0.00%	<=0.5
0.00%	<=0.55
5.56%	<=0.6
5.56%	<=0.65
5.56%	<=0.7
5.56%	<=0.75
5.56%	<=0.8
5.56%	<=0.85
5.56%	<=0.9
5.56%	<=0.95
5.56%	>0.95

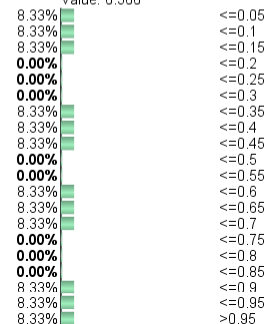


Distances2.xbl \*

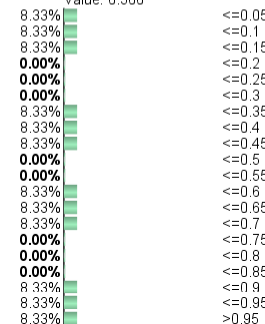


Joint Probability: 12.96%  
Log-Loss: 2.95  
Total Value: 2.571  
Mean Value: 0.514

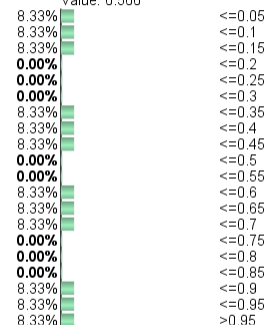
x1  
Mean: 0.500 Dev: 0.316  
Value: 0.500



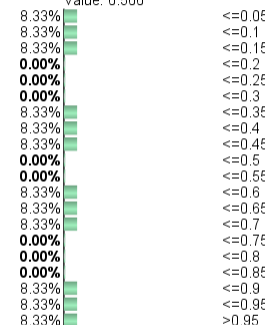
y1  
Mean: 0.500 Dev: 0.316  
Value: 0.500



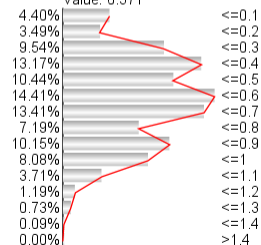
x2  
Mean: 0.500 Dev: 0.316  
Value: 0.500

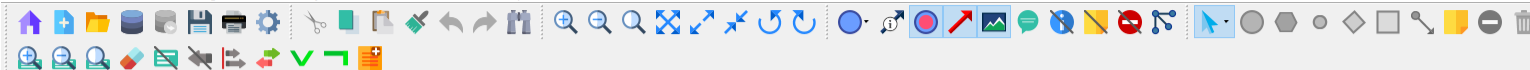


y2  
Mean: 0.500 Dev: 0.316  
Value: 0.500

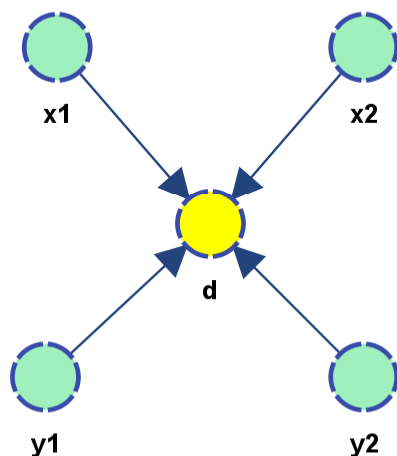


d  
Mean: 0.571 Dev: 0.274  
Value: 0.571

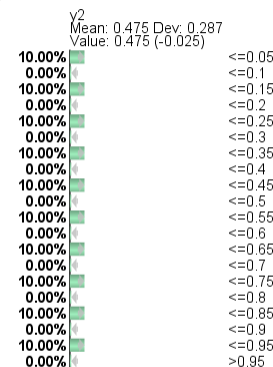
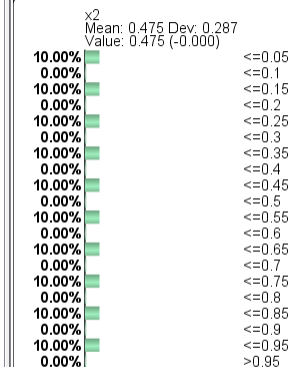
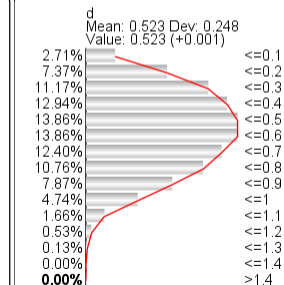
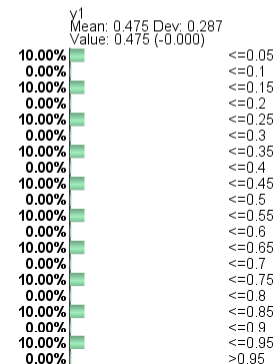
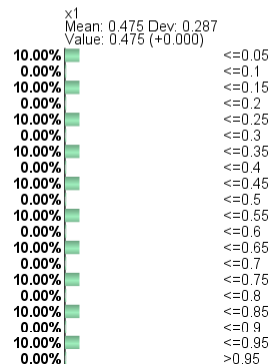


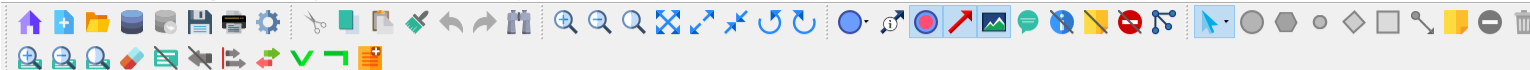


Distances2.xbl

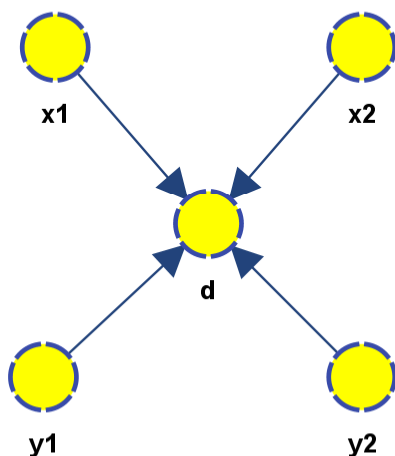


Joint Probability: 6.25%  
Log-Loss: 4  
Total Value: 2.423  
Mean Value: 0.485



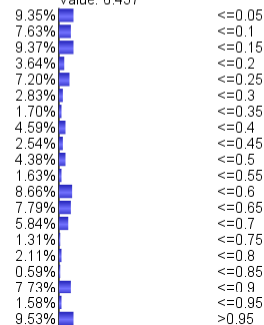


Distances2.xbl \*

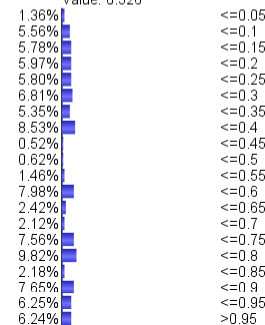


Joint Probability: 100.00%  
Log-Loss: 0  
Total Value: 2.521  
Mean Value: 0.504

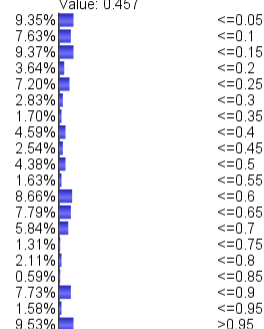
x1  
Mean: 0.457 Dev: 0.316  
Value: 0.457



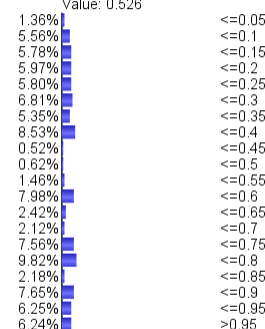
y1  
Mean: 0.526 Dev: 0.298  
Value: 0.526



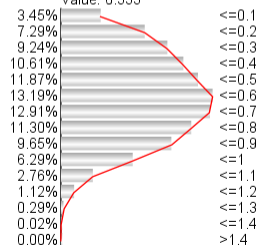
x2  
Mean: 0.457 Dev: 0.316  
Value: 0.457



y2  
Mean: 0.526 Dev: 0.298  
Value: 0.526

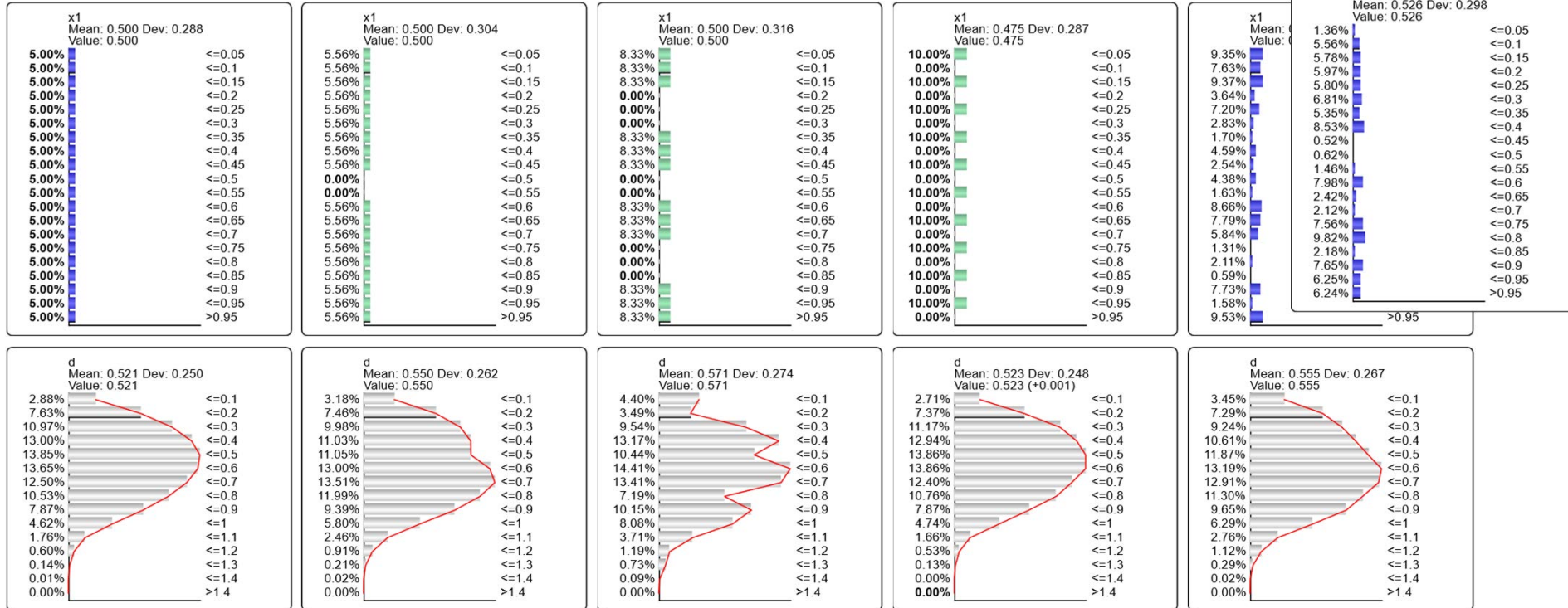


d  
Mean: 0.555 Dev: 0.267  
Value: 0.555

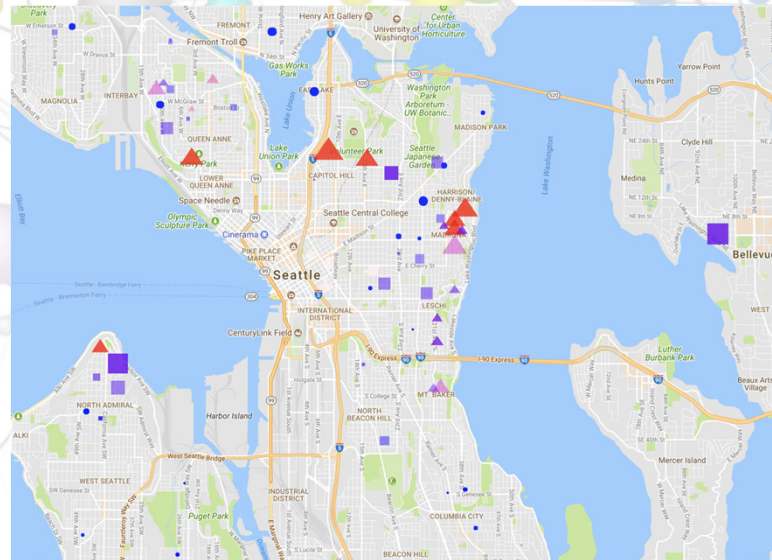


# Simulation of Distancing Scenarios

## Comparison of Patterns

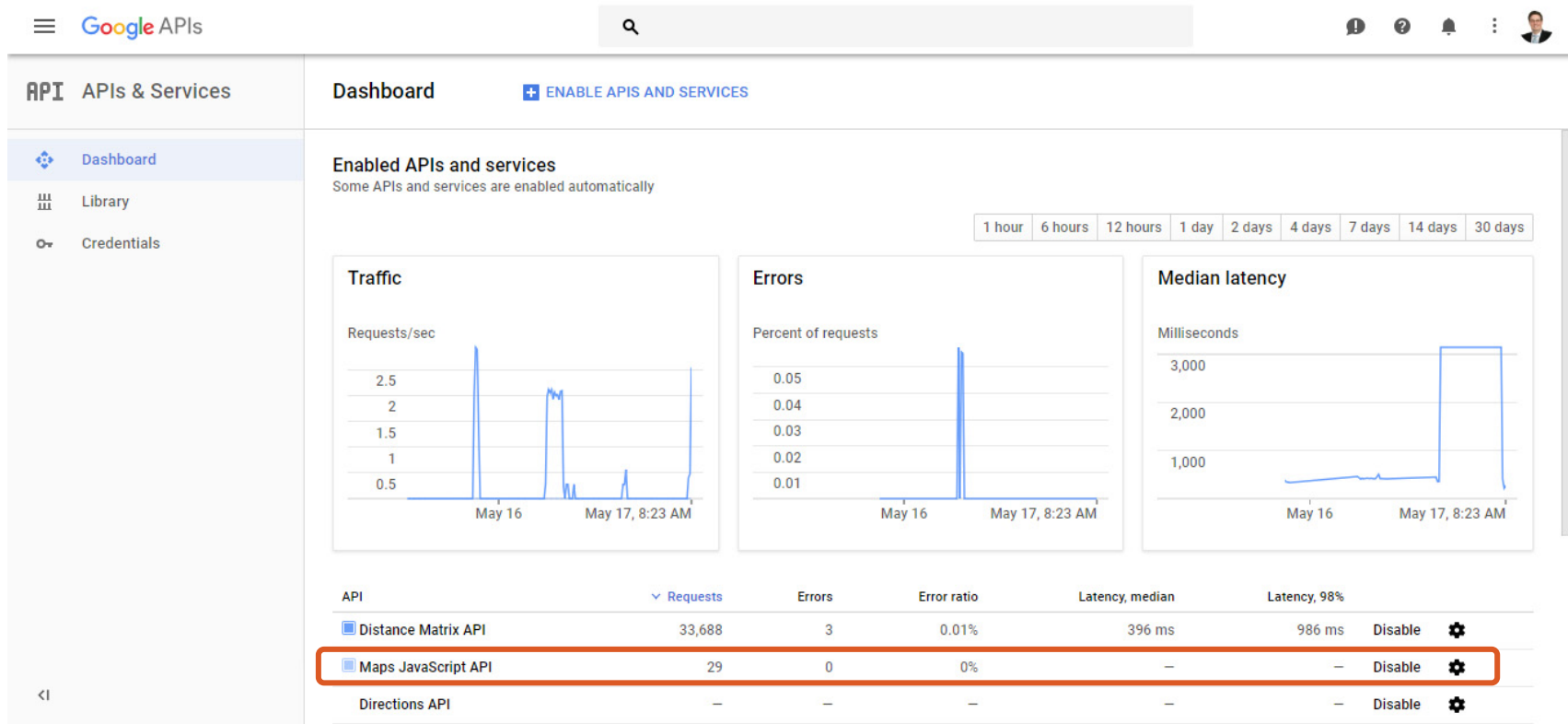




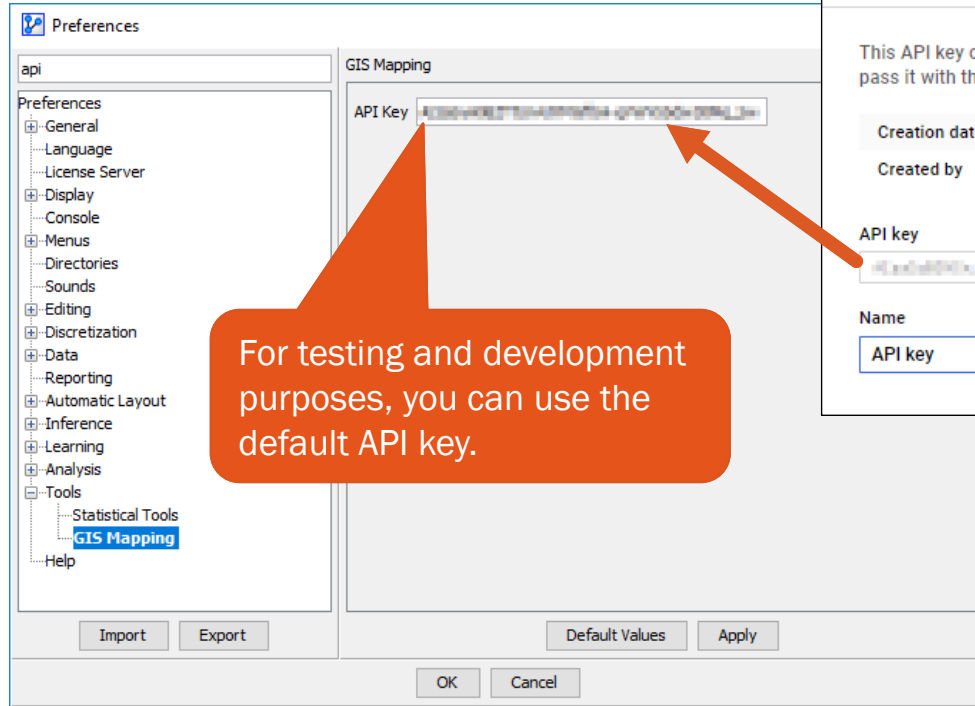


# GIS Mapping with BayesiaLab and the Google API

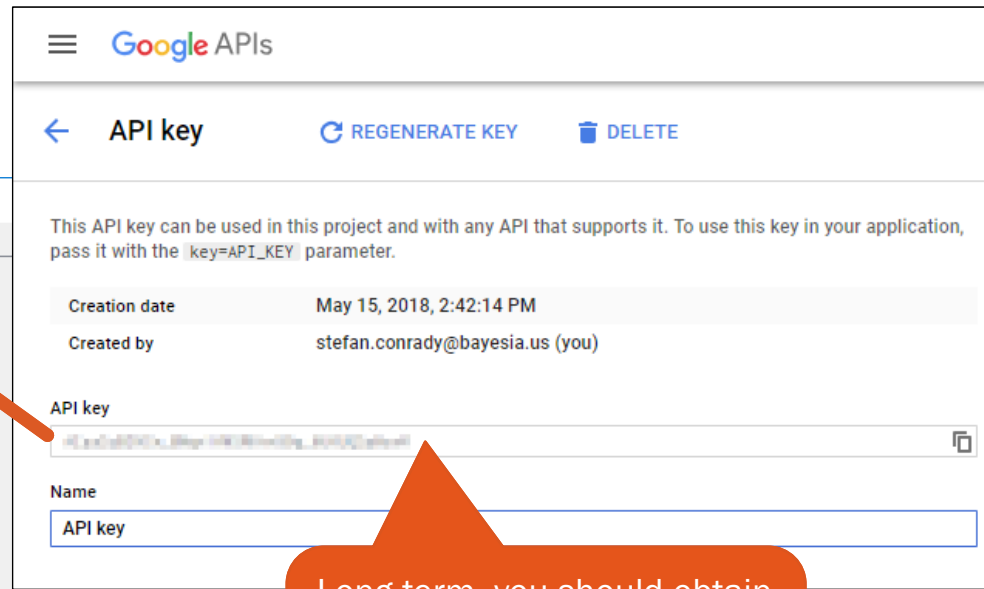
# Google Maps JavaScript API



# BayesiaLab Setup



For testing and development purposes, you can use the default API key.



Long-term, you should obtain your own Google API key so you are not subject to any API call limitations.

# GIS Mapping with BayesiaLab and the Google API

## The Basics

- BayesiaLab can display observed or inferred values with coordinates on a Google map.
- Longitude and latitude are used as coordinates.
- Longitude and latitude must be defined as continuous variables and **discretized** during import, even though they will be used as **undiscretized** values for map display.

**Data Import**

Define Variable Type

Type

- ☐ Not Distributed
- ☐ Discrete
- ☒ Continuous
- ☐ Weight
- ☐ Learning/Test
- ☐ Row Identifier

Action

- Columns with Missing Values
- All not Distributed
- All Discrete
- All Continuous

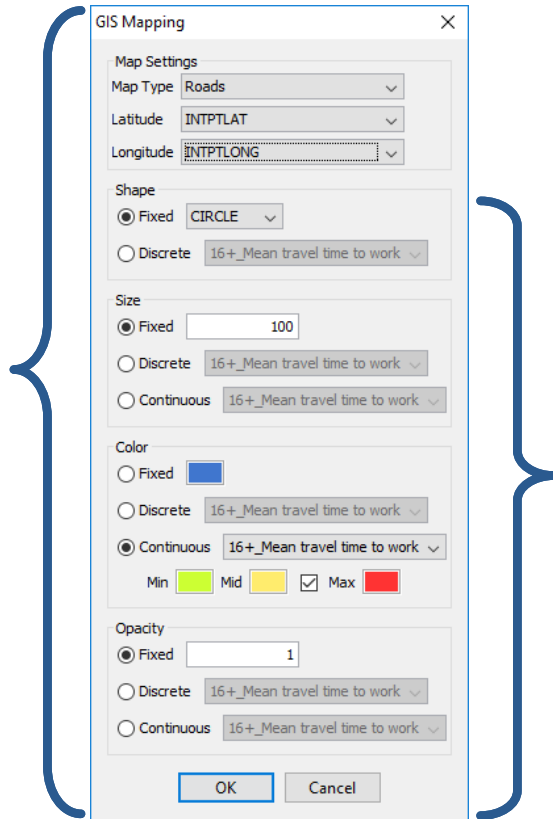
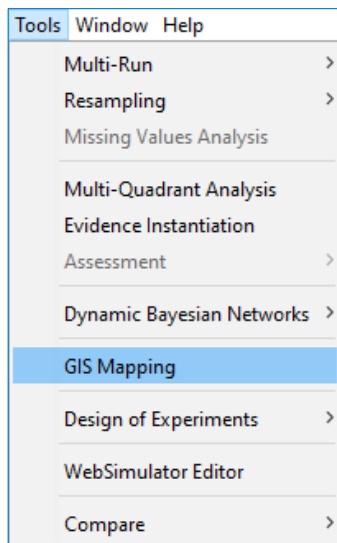
Data

Longitude (2)	Latitude (2)	City (2)	State (2)
-75.57	38.46	Delmar	DE
-88.68	41.52	Sheridan	IL
-98.62	29.62	San Antonio	TX
-111.89	40.76	Salt Lake City	UT
-116.11	44.07	Banks	ID
-78.8	35.8	Cary	NC
-68.49	46.41	Oxbow	ME
-117.82	33.6	Newport Coast	CA
-87.49	36.85	Hopkinsville	KY
-71.09	41.9	Taunton	MA
-91.95	35.55	Tumbling Sh...	AR
-79.16	36.4	Leasburg	TX
-80.35	36.47	Westfield	MA
-92.9	42.77	Bristow	IA

<

Cancel

# GIS Mapping with BayesiaLab and the Google API

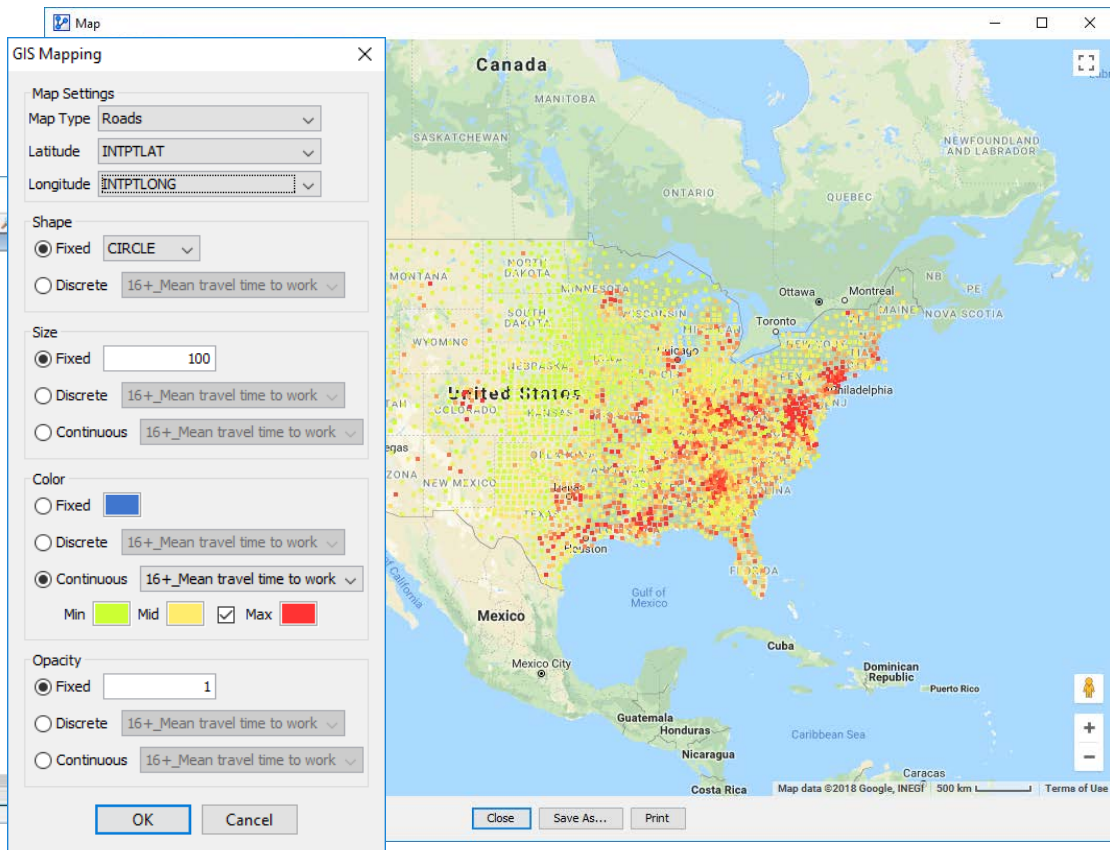
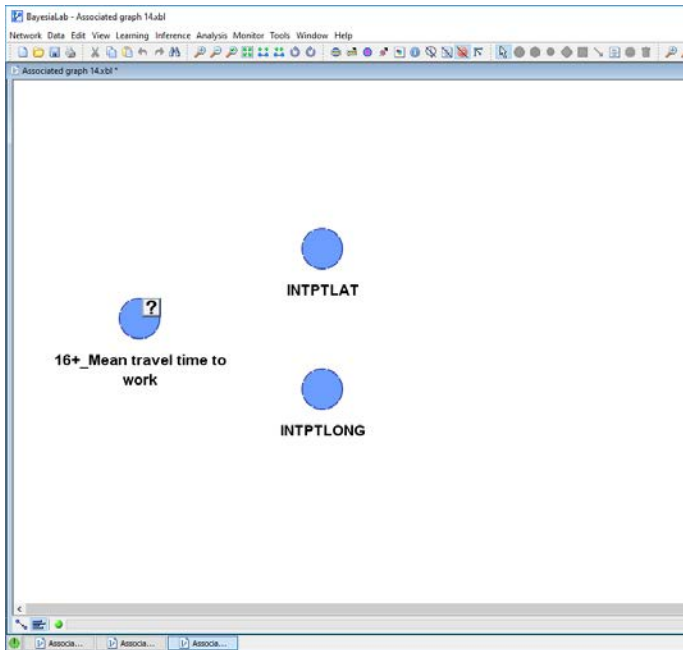


Four attributes can be displayed per observations:

- Shape
- Size
- Color
- Opacity

# Example: Mapping Commuting Time

## Commuting Time by County









# Spatial Learning and Optimization

Bayesian Networks & BayesiaLab

# Spatial Learning and Optimization

## Optimization Problems Under Consideration

- |  |                              |
|--|------------------------------|
| 1. One origin, one destination   | ➔ Shortest Path Problem      |
|  2. One origin, many destinations            | ➔ Drive Time Bands           |
| 3. Many origins, one destination   | ➔ Store Location Problem     |
|  4. Many origins, one hub, many destinations | ➔ Hub Location Problem       |
| 5. Many origins, multiple hubs, many destinations  | ➔ Multi-Hub Location Problem |

## Common Objective

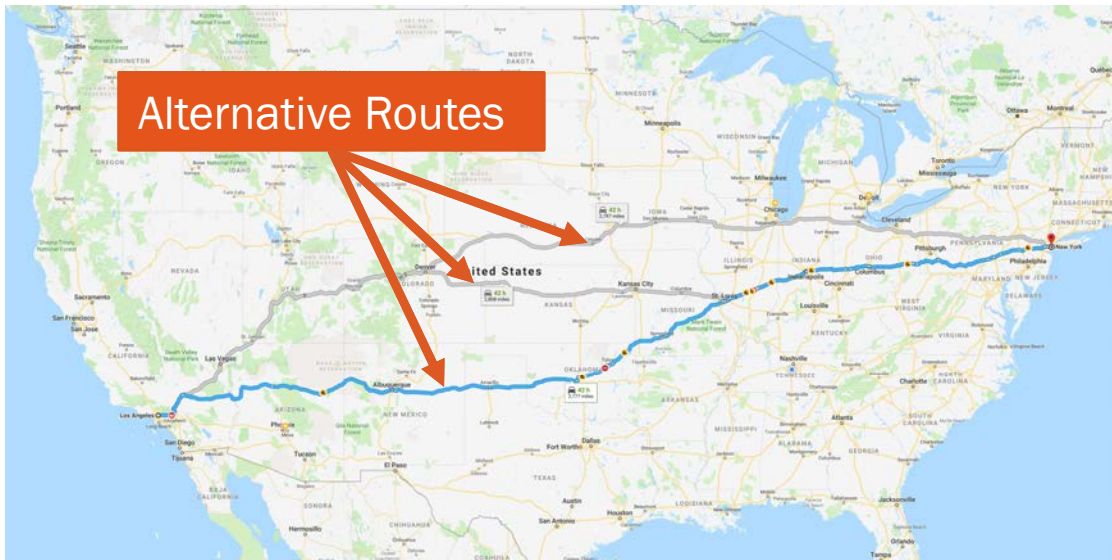
- Minimize “cost function,” e.g., travel time, distance, fuel consumption, number of turns, etc.
- Further assumption: all “participants” have same objective.



# Drive Time Bands

## Computing the “Cost” for One Origin and One Destination

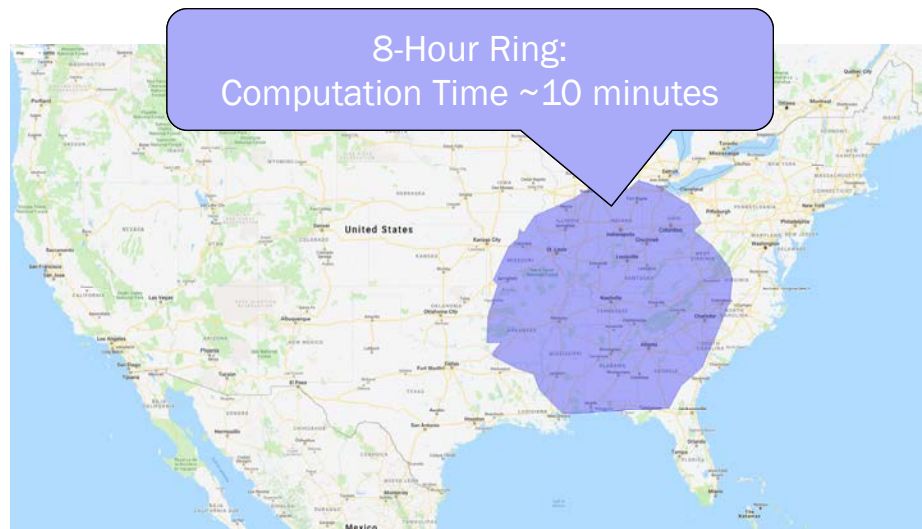
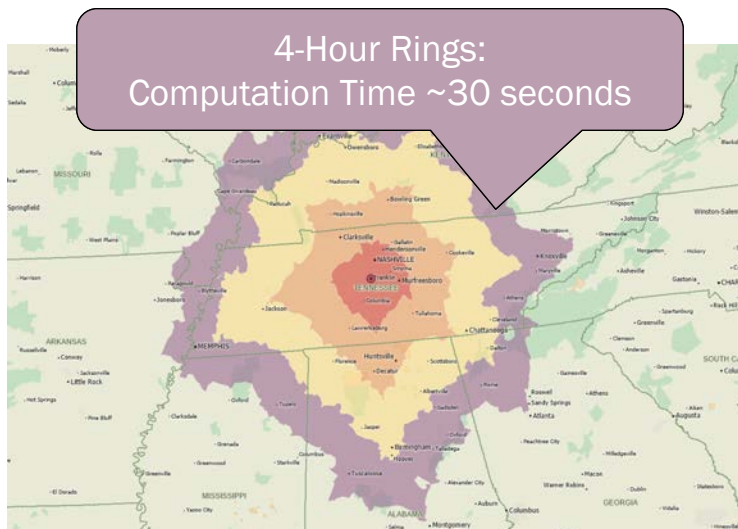
- “Search the Map” ➔ slow, but accurate



# Drive Time Bands

## Computing the Cost for One Origin and Many Destinations

- “Search the Map” ➔ slow, but accurate



# Drive Time Bands

## Computing the Cost

- “Search the Map” → slow, but accurate

- Great-Circle Distance Computation

$$\begin{aligned}d &= 2r \arcsin\left(\sqrt{\text{hav}(\varphi_2 - \varphi_1) + \cos(\varphi_1) \cos(\varphi_2) \text{hav}(\lambda_2 - \lambda_1)}\right) \\&= 2r \arcsin\left(\sqrt{\sin^2\left(\frac{\varphi_2 - \varphi_1}{2}\right) + \cos(\varphi_1) \cos(\varphi_2) \sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)}\right)\end{aligned}$$

- Easy and fast to calculate.
- “As the crow flies” may be an unrealistic assumption.
- Travel time may be more relevant than distance.

# Drive Time Bands

## Idea: “Create a Look-Up Table for All Origin-Destination Pairs”

- 29,788 ZIP Codes in the U.S.
- A complete distance matrix would contain 887,324,944 cells.
- Current computation speed with Google Distance Matrix API: 2 requests/sec.
- Estimated computation time: ~14 years.

*Approximate!*

# Drive Time Bands

## Idea

- Approximation through machine learning.

## Proposed Approach

- Utilize database of actual point-to-point travel data.
- Learn a Bayesian network from this dataset.
- Now we can infer the “cost” as a function of origin and destination.





# Drive Time Bands

## Workflow in Detail

- Take a random sample (~100,000) of origin-destination ZIP code pairs and calculate routes with the Google Distance Matrix API.
- Perform Augmented Naïve Learning.
- Evaluate Target Performance.
- Associate new data set with points to be evaluated.
- Generate map.



  Associa...

**Data Import**

Define Variable Type

Type

☒ Not Distributed

☐ Discrete

☐ Continuous

☐ Weight

☐ Learning/Test

☐ Row Identifier

Action

Columns with Missing Values

All not Distributed

All Discrete

All Continuous

Information



Number of Rows	95710	100.00%
Not Distributed	7	58.33%
Discrete	0	0.00%
Continuous	5	41.67%
Others	0	0.00%
Missing Values	0	0.00%
Filtered Values	0	0.00%



Data

Longitude	Latitude	City	State	Zipcode	Longitude (2)	Latitude (2)	City (2)	State (2)	Zipcode (2)	Time (h)
-89.8	37.44	Millersville	MO	63766	-75.57	38.46	Delmar	DE	19940	15.24694444
-89.8	37.44	Millersville	MO	63766	-88.68	41.52	Sheridan	IL	60551	5.313611111
-89.8	37.44	Millersville	MO	63766	-98.62	29.62	San Antonio	TX	78256	13.18111111
-89.8	37.44	Millersville	MO	63766	-111.89	40.76	Salt Lake City	UT	84133	20.87638889
-89.8	37.44	Millersville	MO	63766	-116.11	44.07	Banks	ID	83602	25.88611111
-89.8	37.44	Millersville	MO	63766	-78.8	35.8	Cary	NC	27513	11.67916667
-89.8	37.44	Millersville	MO	63766	-68.49	46.41	Oxbow	ME	4764	24.59388889
-89.8	37.44	Millersville	MO	63766	-117.82	33.6	Newport Coast	CA	92657	27.59416667
-89.8	37.44	Millersville	MO	63766	-87.49	36.85	Hopkinsville	KY	42240	2.9425
-89.8	37.44	Millersville	MO	63766	-71.09	41.9	Taunton	MA	2780	19.25972222
-89.8	37.44	Millersville	MO	63766	-91.95	35.55	Tumbling Sh...	AR	72581	4.341944444
-89.8	37.44	Millersville	MO	63766	-79.16	36.4	Leasburg	MO	27291	11.54527778
-89.8	37.44	Millersville	MO	63766	-80.35	36.47	Westfield	MA	27053	10.07194444
-89.8	37.44	Millersville	MO	63766	-92.9	42.77	Bristow	IA	50611	7.327777778

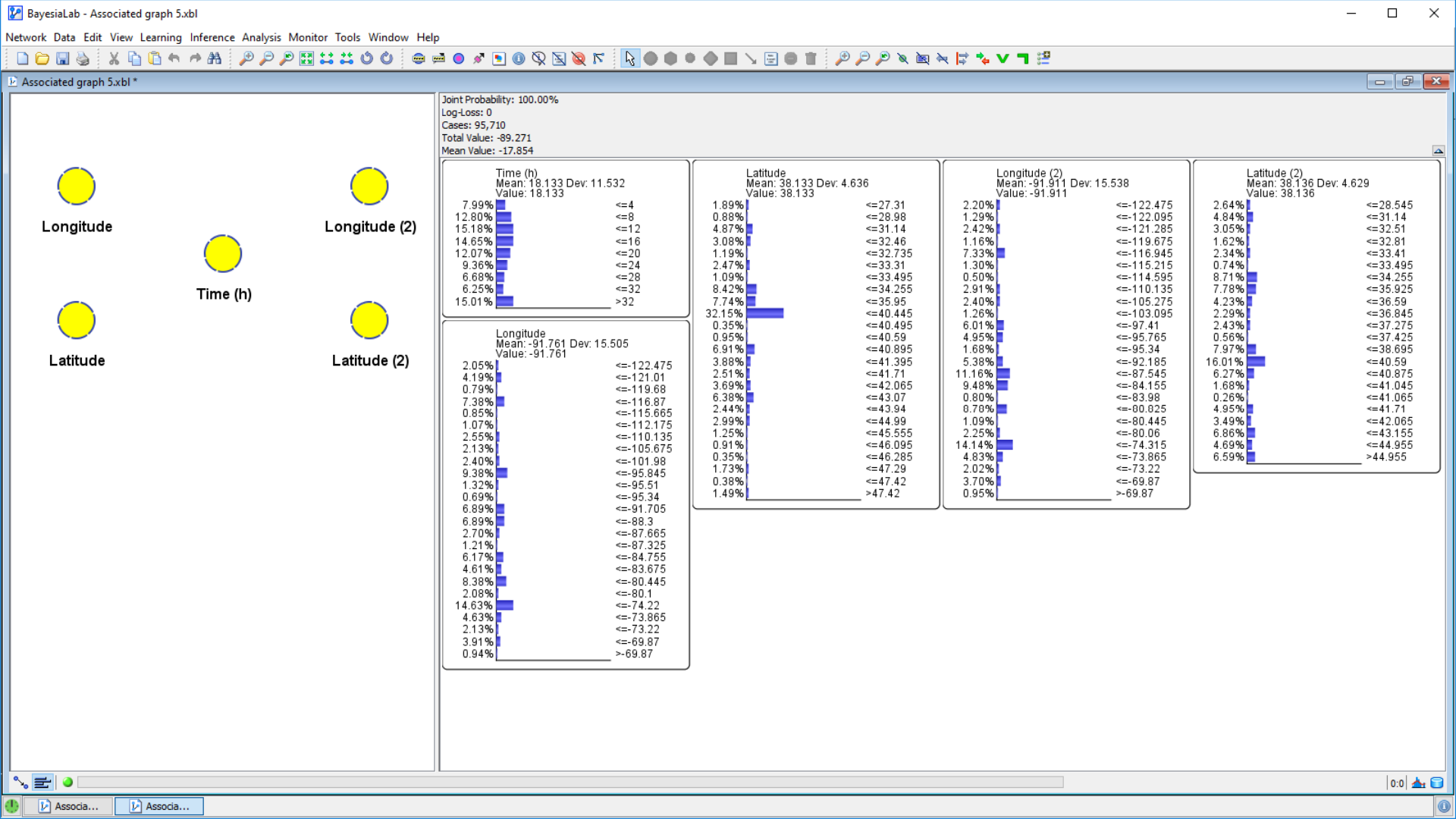
Cancel Previous Next Save Finish

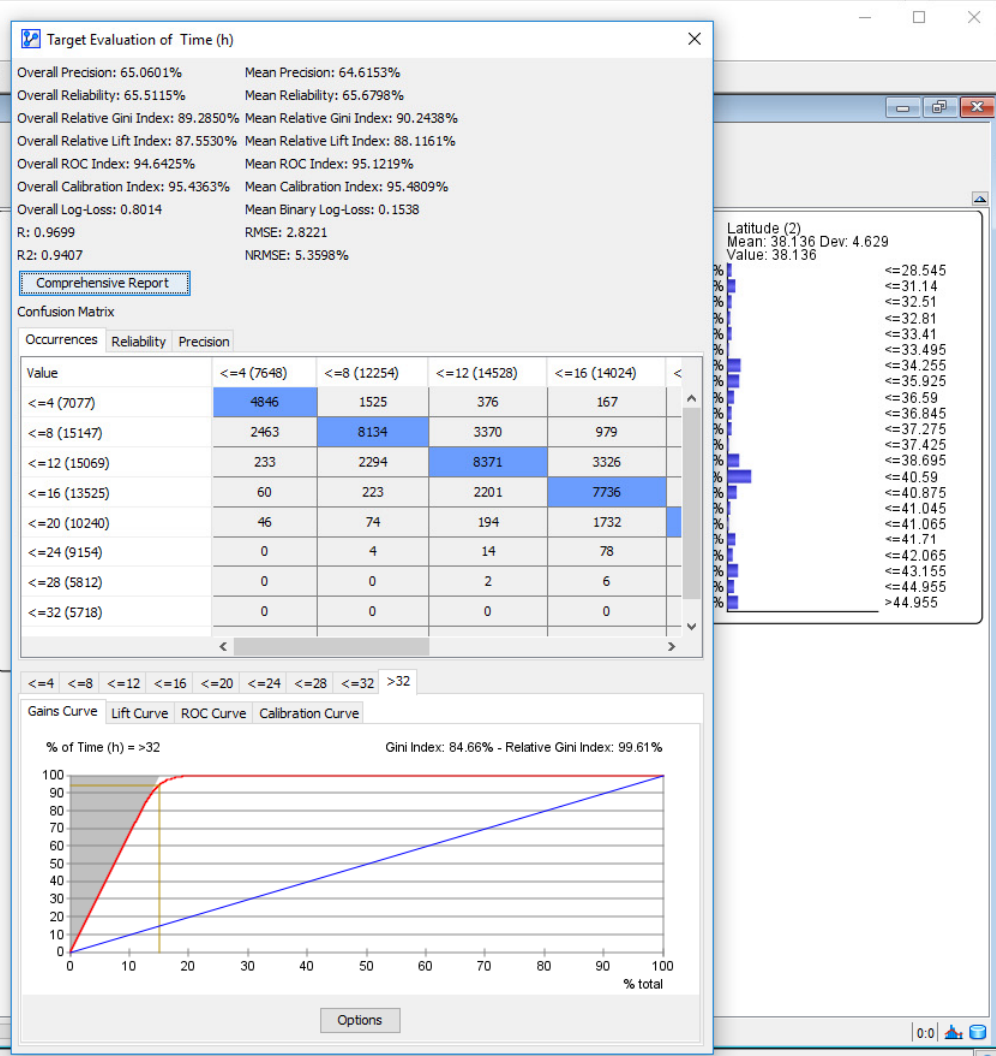
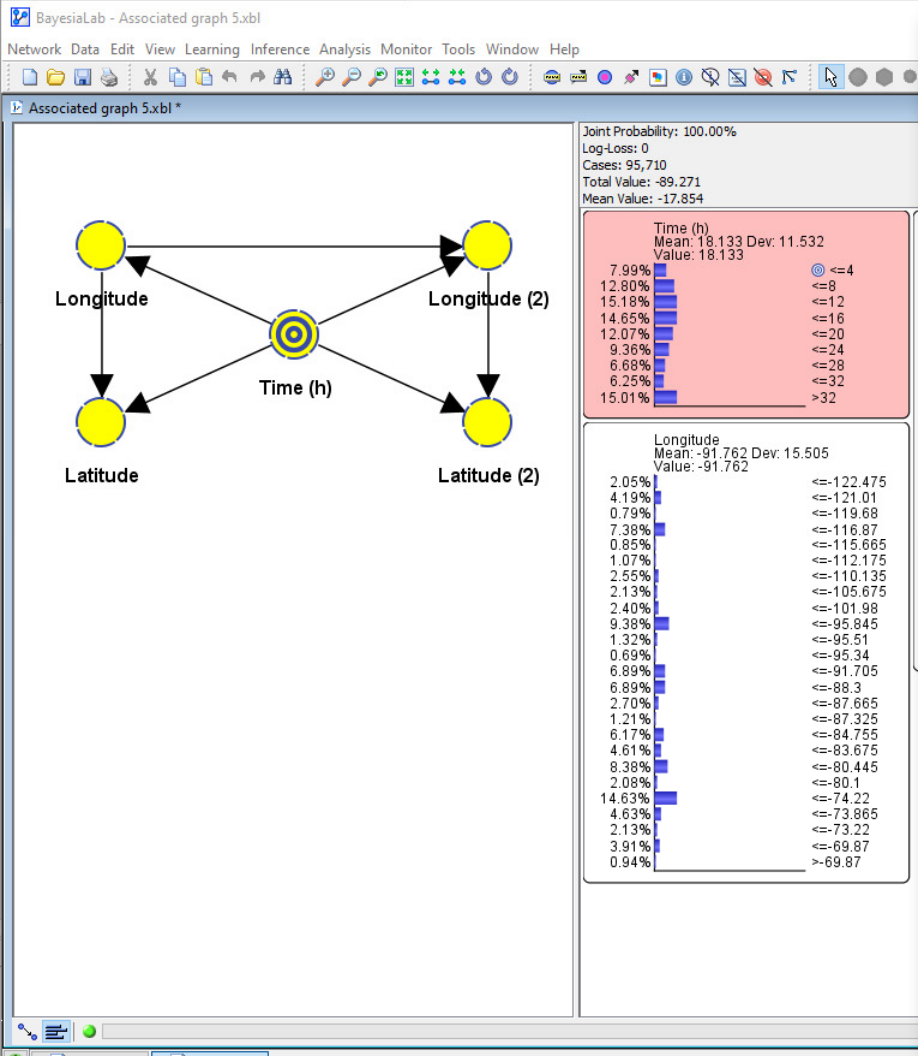


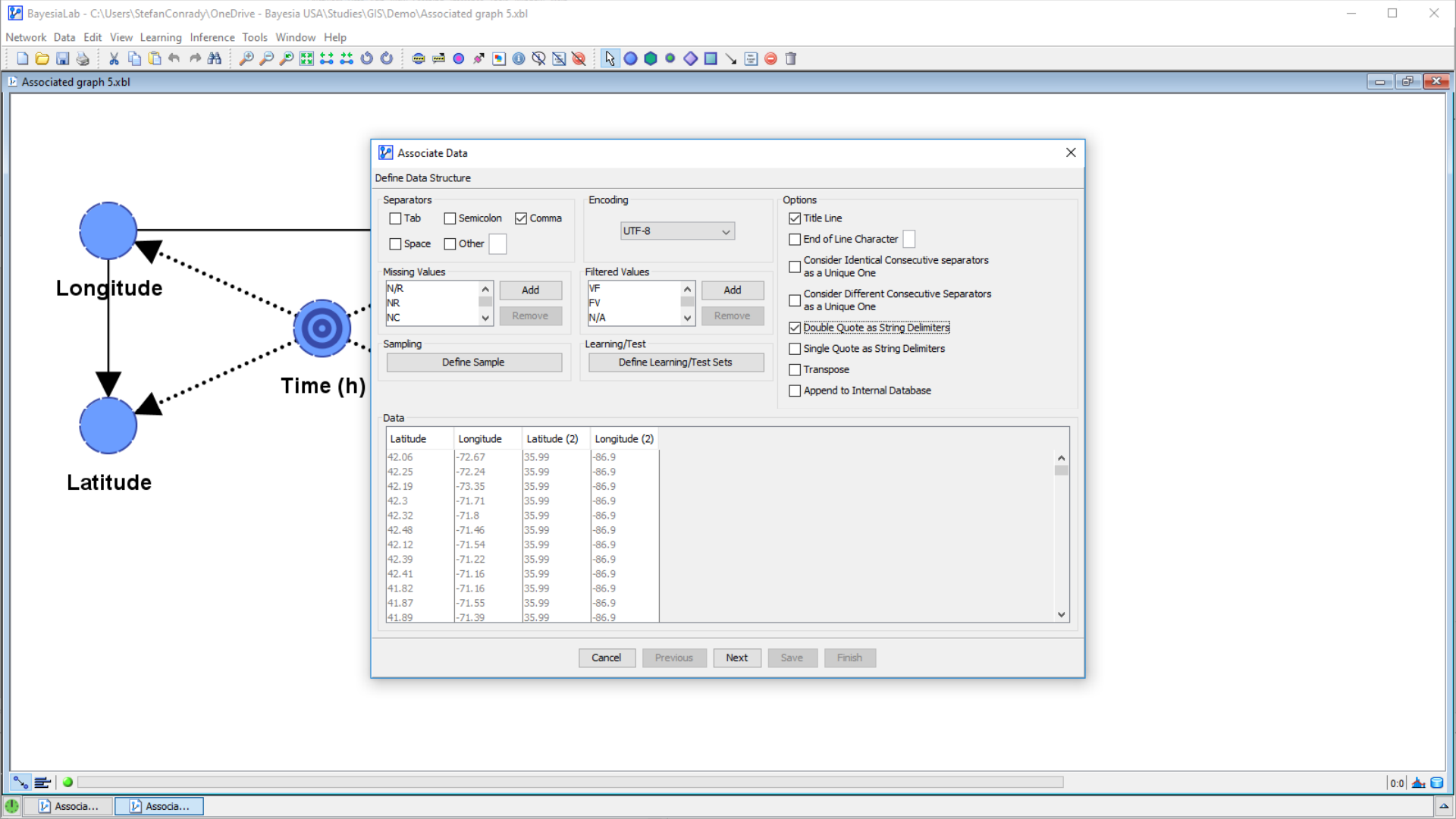
  Associa...

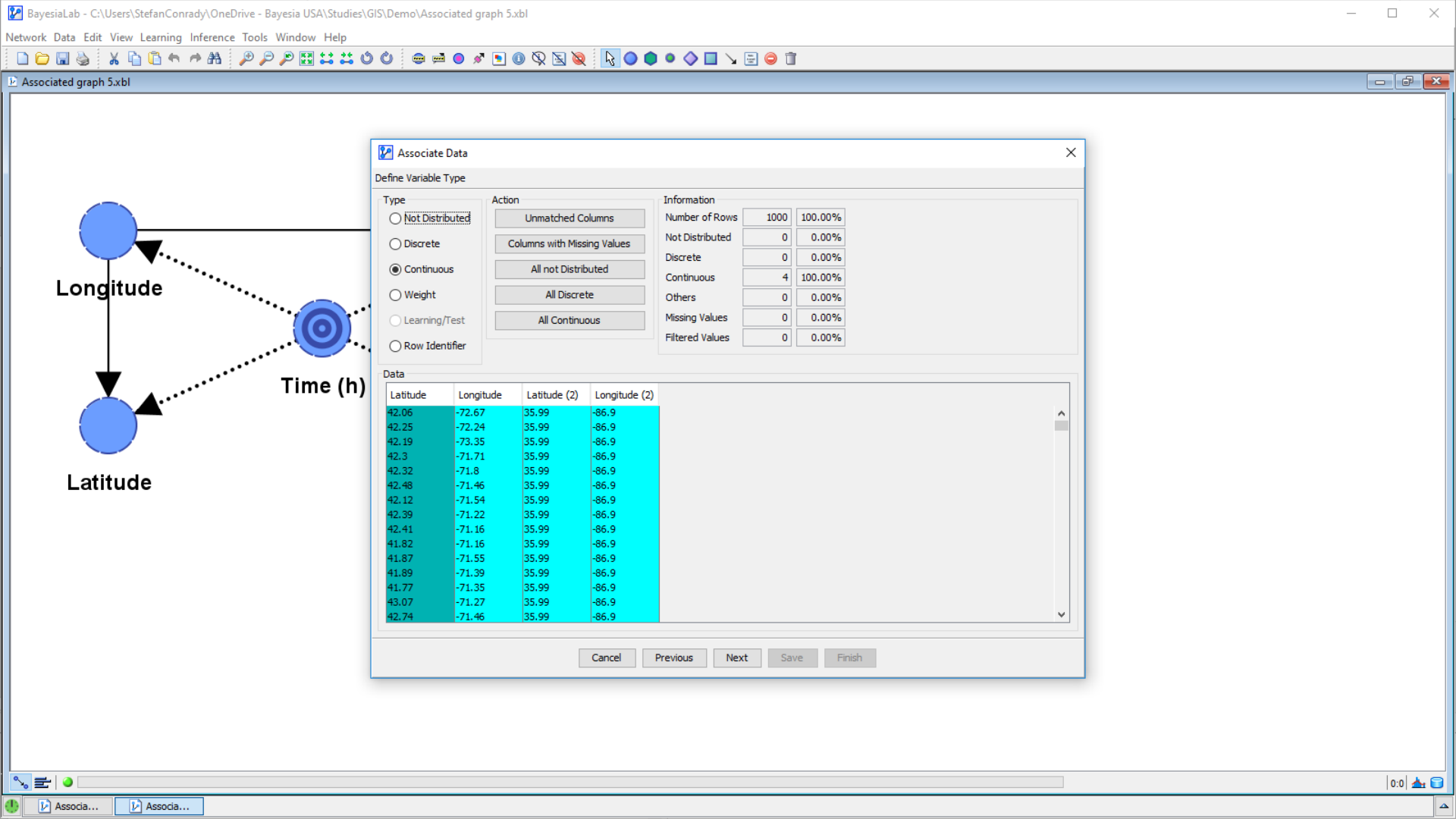
  Associa...

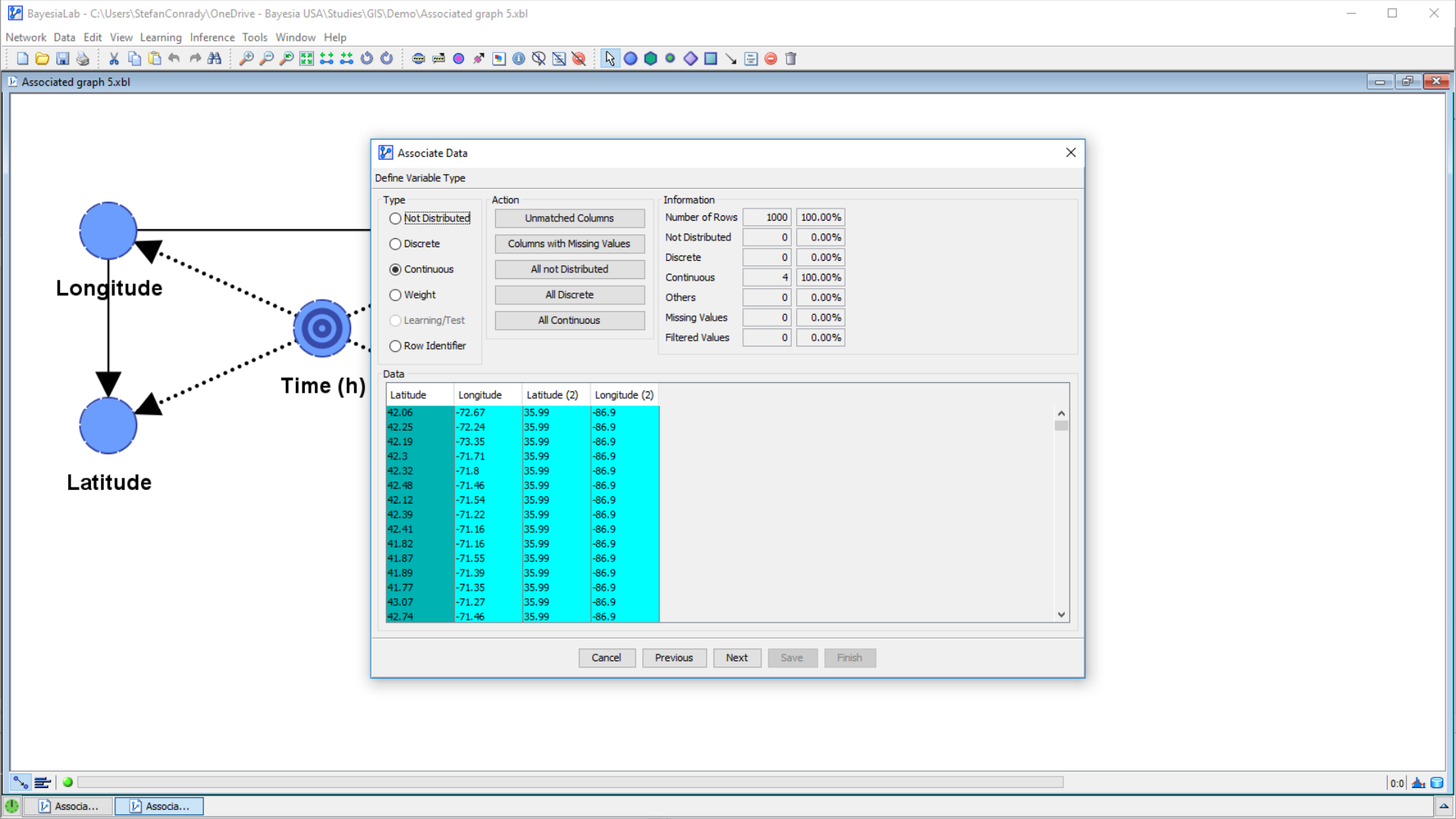
  Associa...



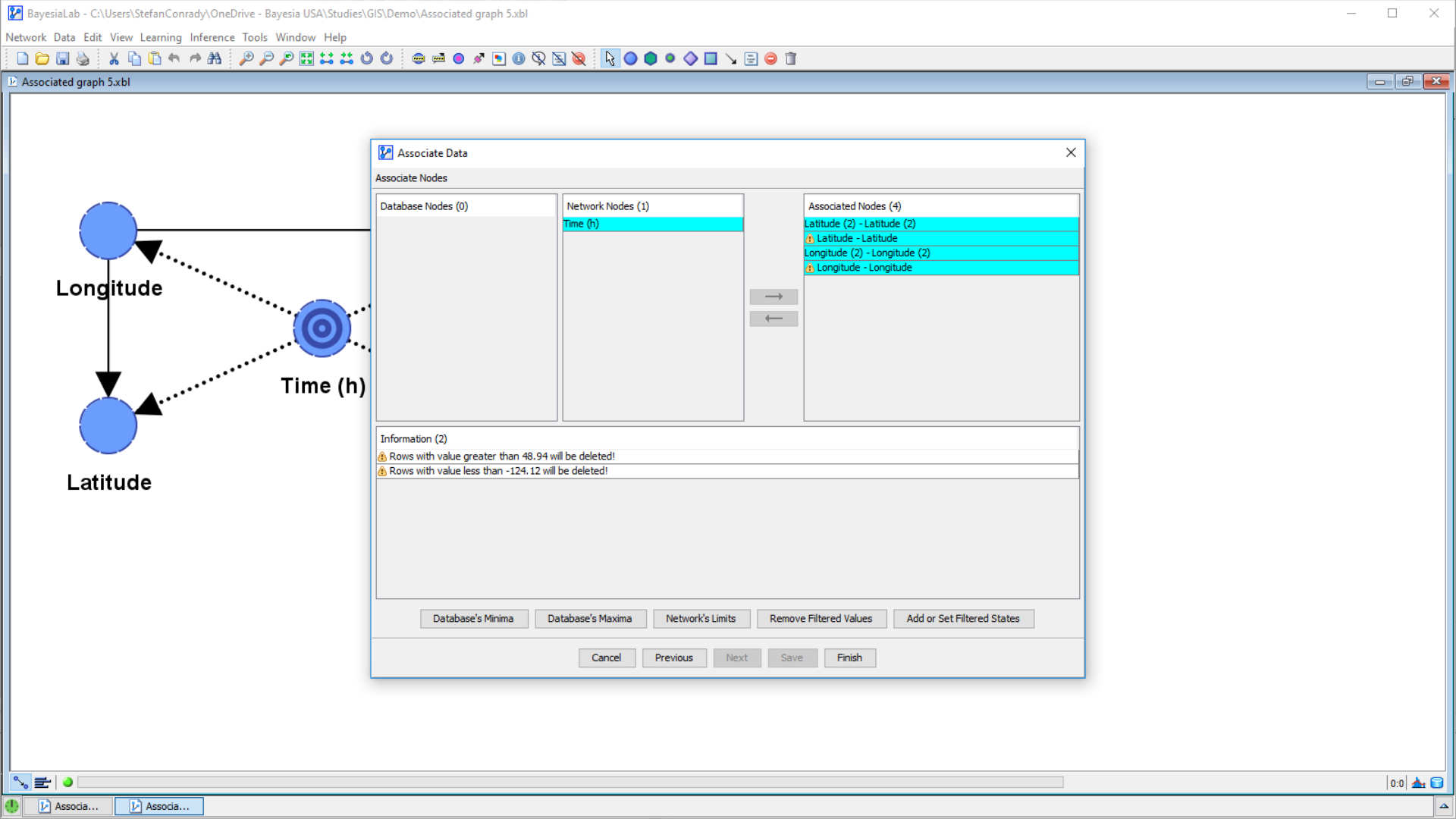


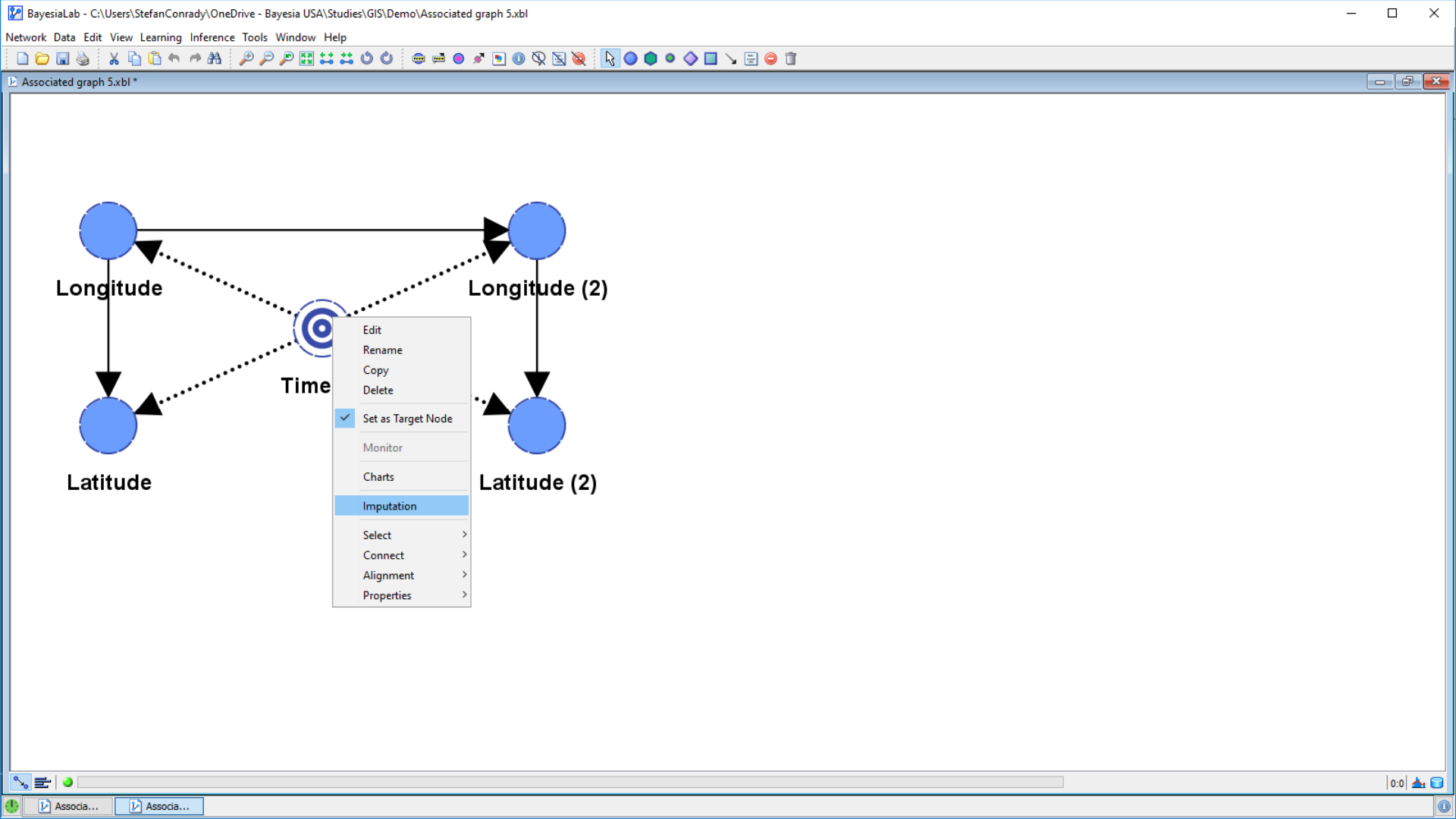


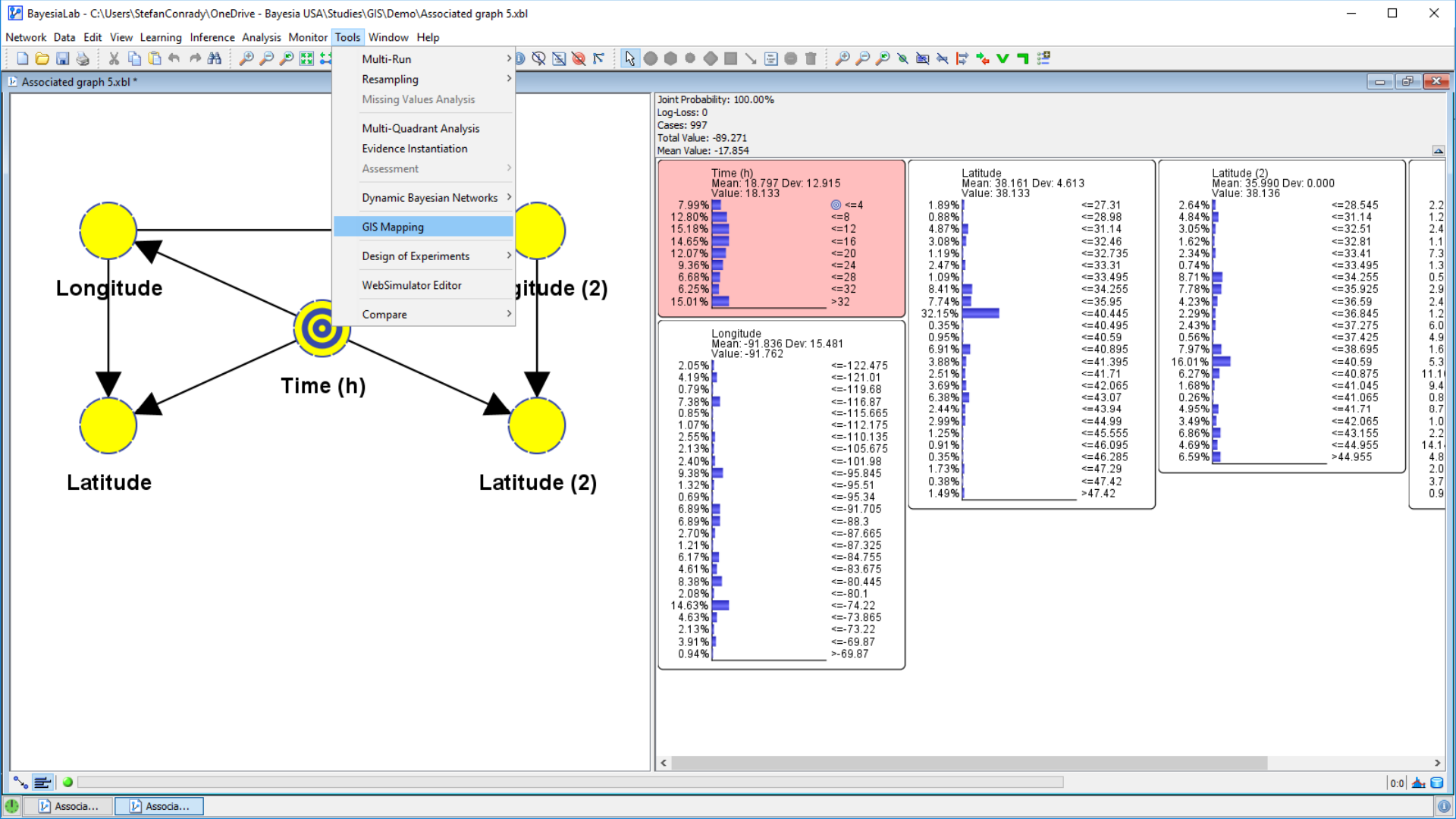






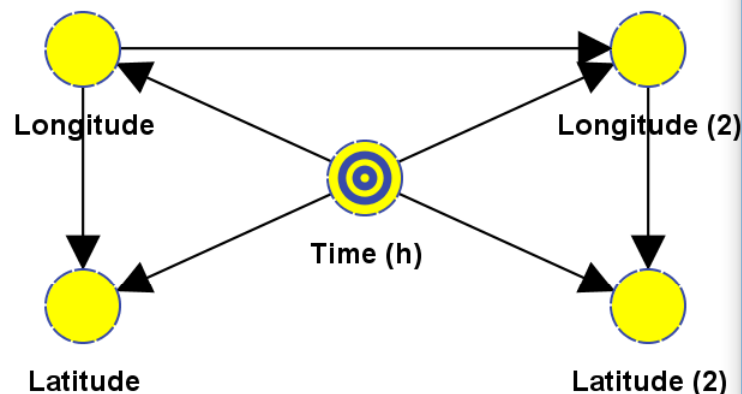








Associated graph 5.xbl \*



Joint Probability: 100.00%  
Log-Loss: 0  
Cases: 997  
Total Value: -89.271

### GIS Mapping

Map Settings

Map Type: Roads

Latitude: Latitude

Longitude: Longitude

Shape

☒ Fixed: CIRCLE

☐ Discrete: Latitude (2)

Size

☒ Fixed: 100

☐ Discrete: Latitude (2)

☐ Continuous: Latitude (2)

Color

☐ Fixed: [Blue]

☐ Discrete: Latitude (2)

☒ Continuous: Time (h)

Min: [Green] Mid: [Yellow] Max: [Red]

Opacity

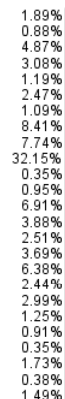
☒ Fixed: 1

☐ Discrete: Latitude (2)

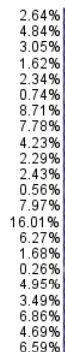
☐ Continuous: Latitude (2)

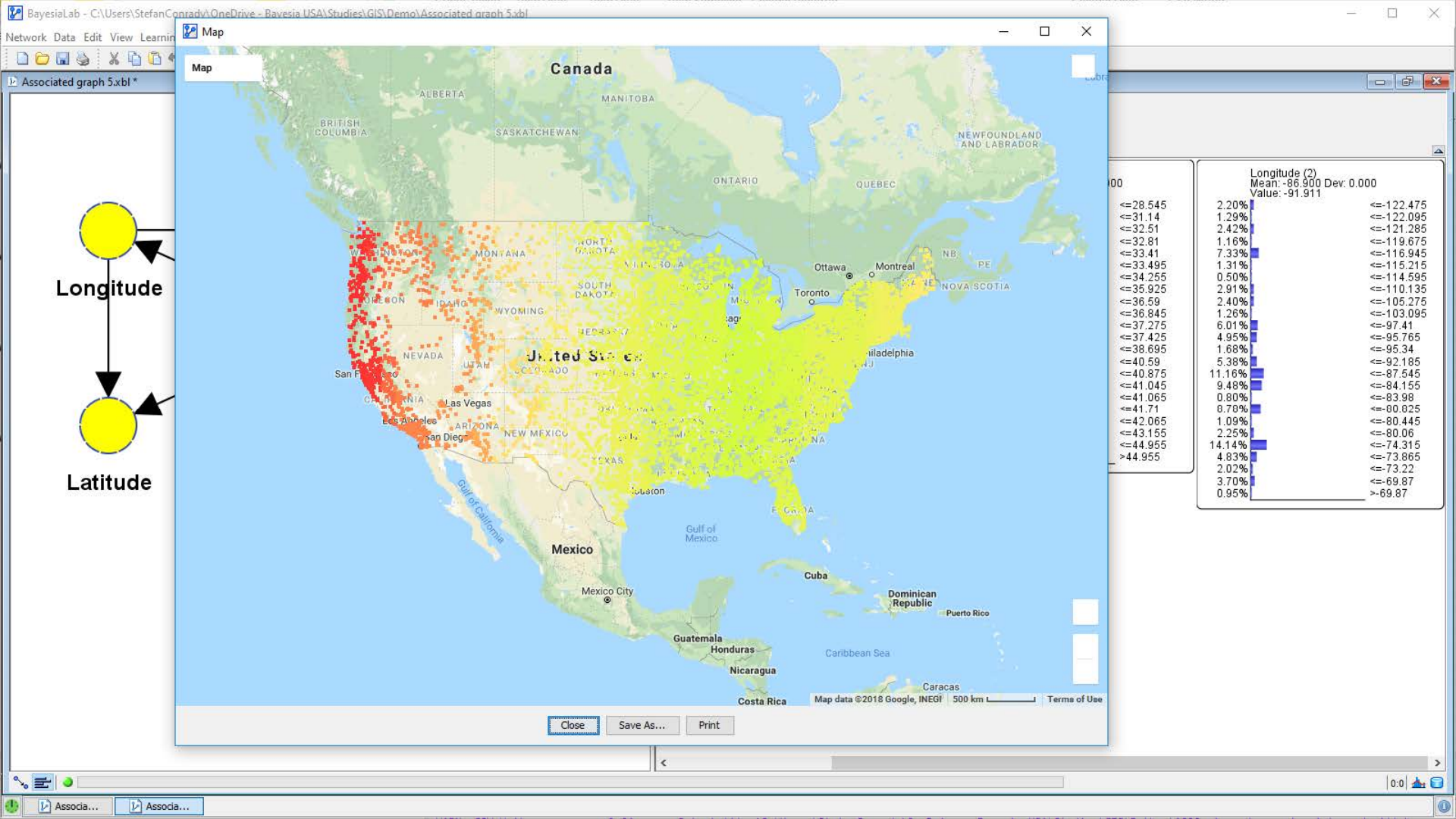
OK Cancel

Latitude  
Mean: 38.161 Dev: 4.613  
Value: 38.133



Latitude (2)  
Mean: 35.990 Dev: 0.000  
Value: 38.136





# Drive Time Bands

## Computing the Cost

- “Search the Map” → slow, but accurate
- Great-Circle Distance Computation → fast, but inaccurate
- Learn & Infer → fast and good approximation




# Spatial Learning and Optimization

Hub Location

# Spatial Learning and Optimization

## Optimization Problems Under Consideration

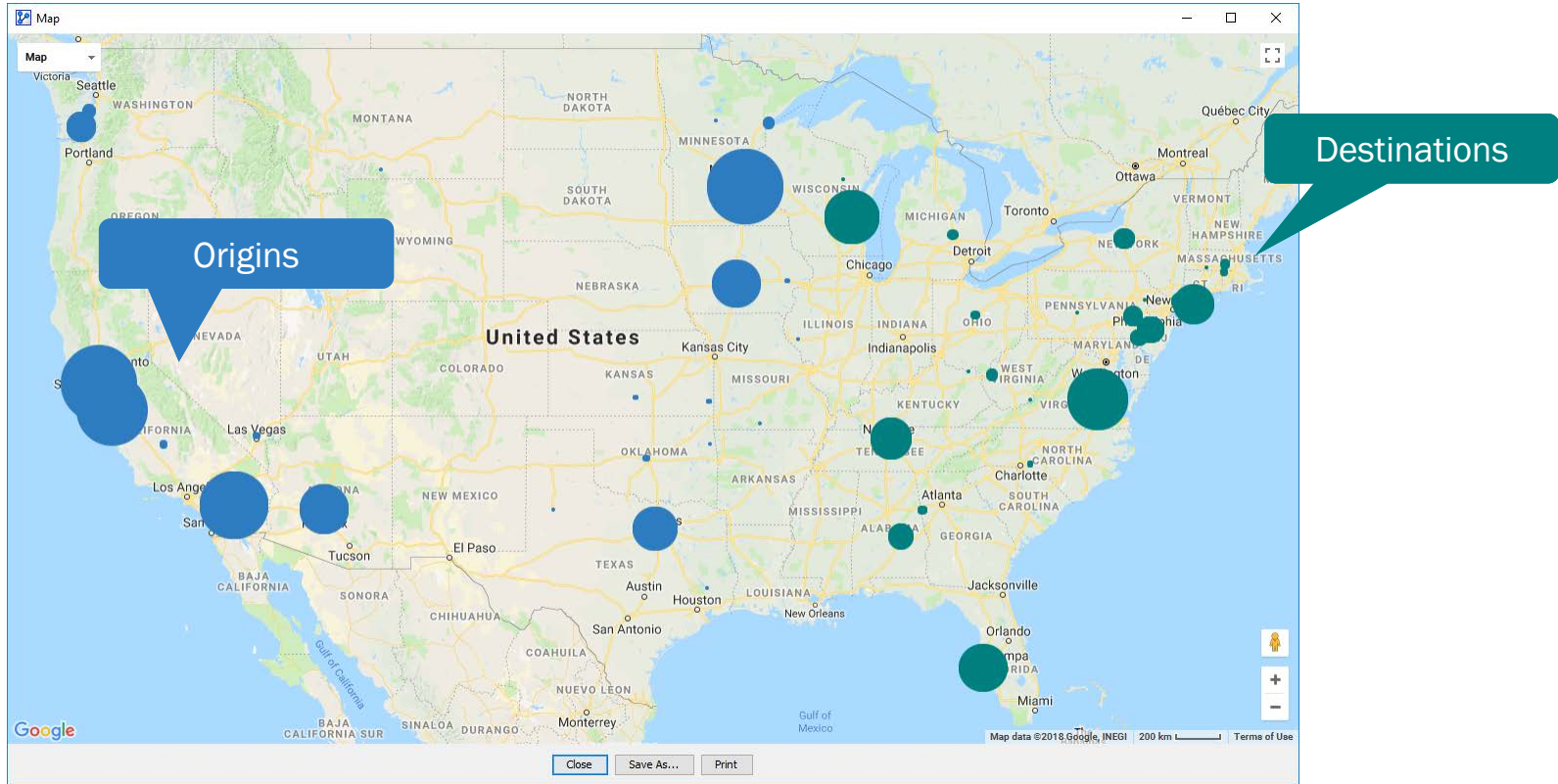
- |  |                              |
|--|------------------------------|
| 1. One origin, one destination   | ➔ Shortest Path Problem      |
| 2. One origin, many destinations   | ➔ Drive Time Bands           |
| 3. Many origins, one destination   | ➔ Store Location Problem     |
|  4. Many origins, one hub, many destinations | ➔ Hub Location Problem       |
| 5. Many origins, multiple hubs, many destinations  | ➔ Multi-Hub Location Problem |

## General Objective

- Minimize “cost function,” e.g., travel time, distance, fuel consumption, number of turns, etc.
- Further assumption: all “participants” have same objective.



# Hub Location Problem

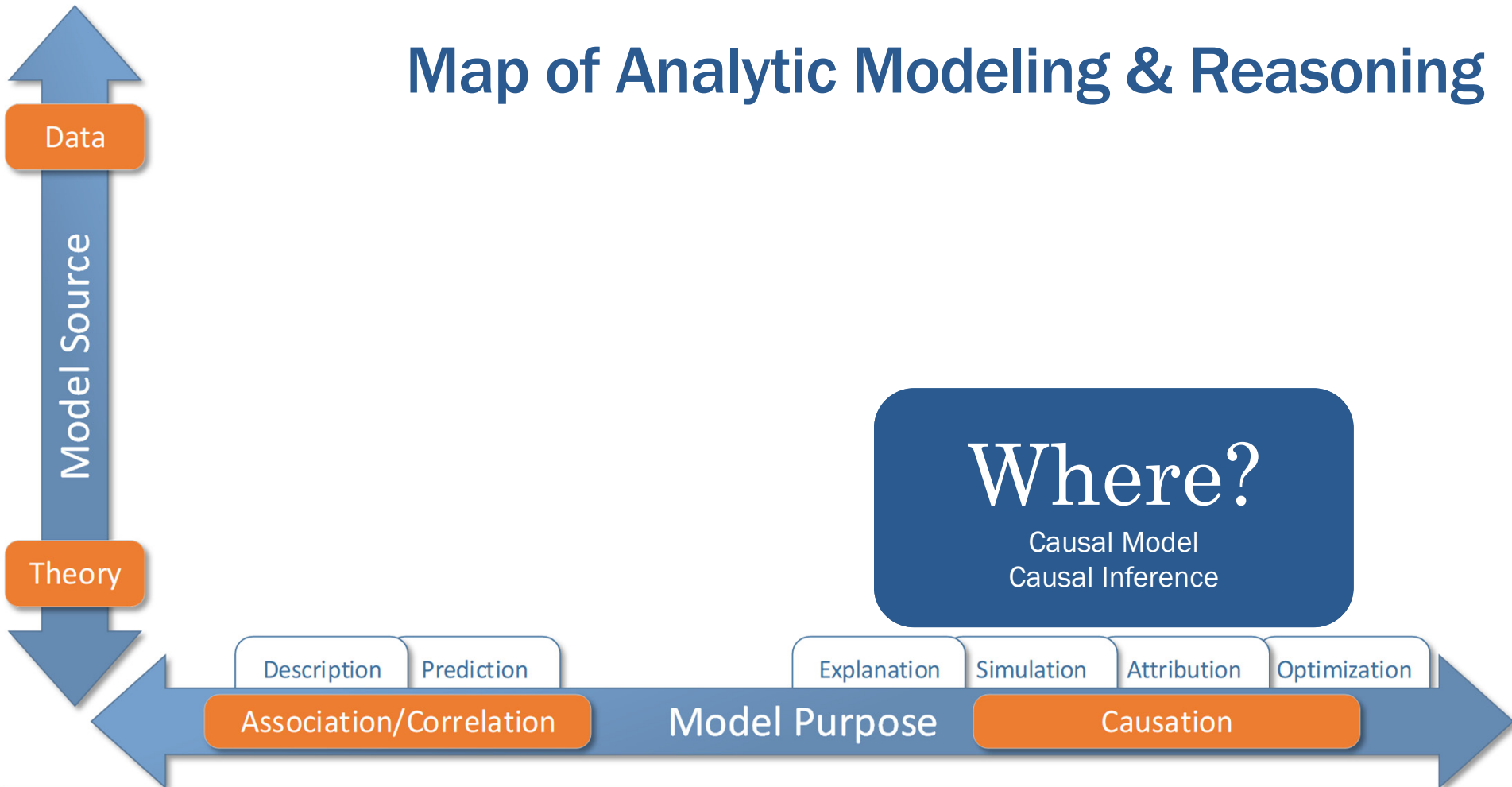


# Hub Location Problem

## Workflow

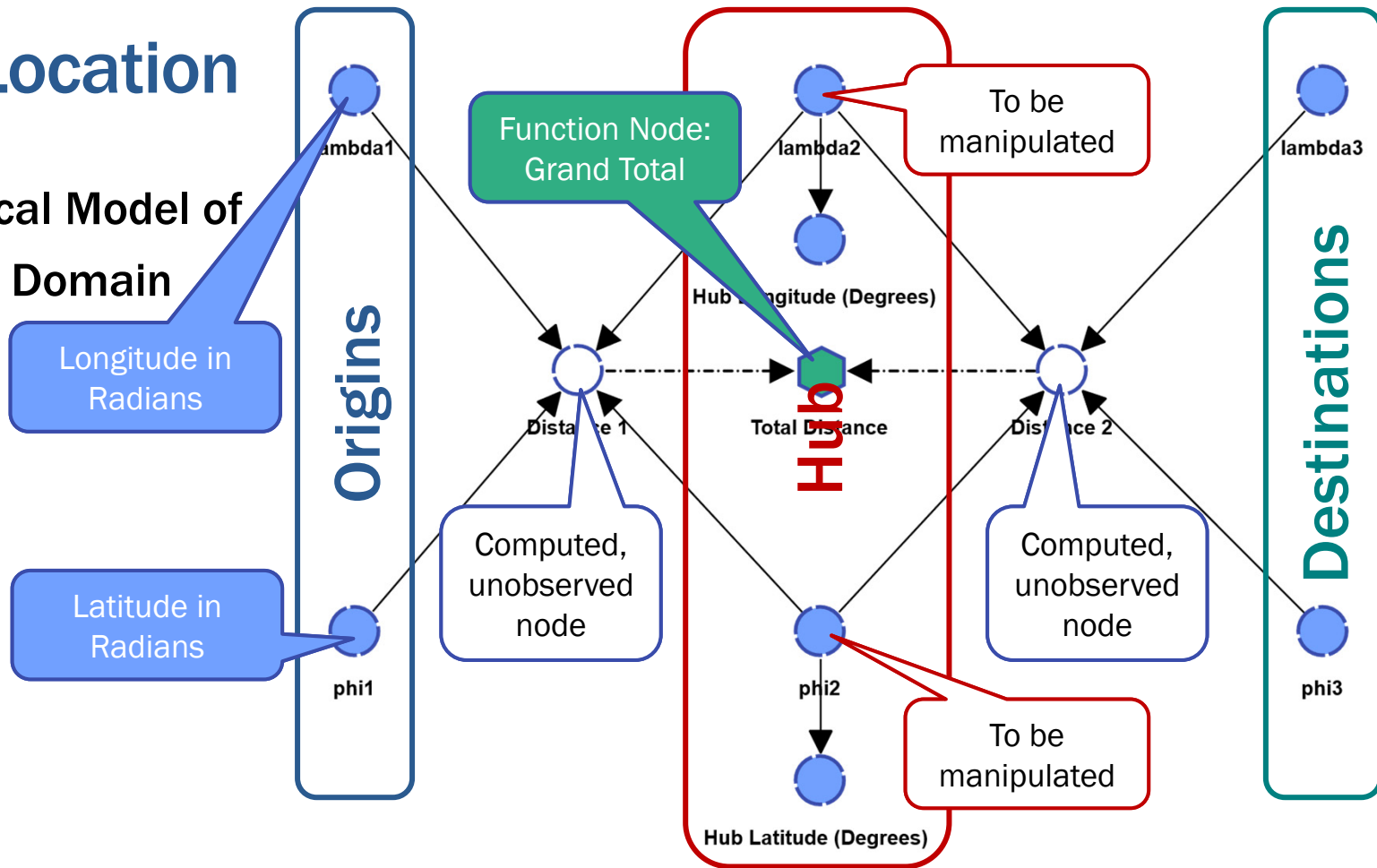
- Encode theoretical model of problem domain.
- Define Nodes
  - Observed
  - Unobserved
  - Functions
- Load data for origins and destinations.
- Perform Function Optimization.

# Map of Analytic Modeling & Reasoning



# Hub Location

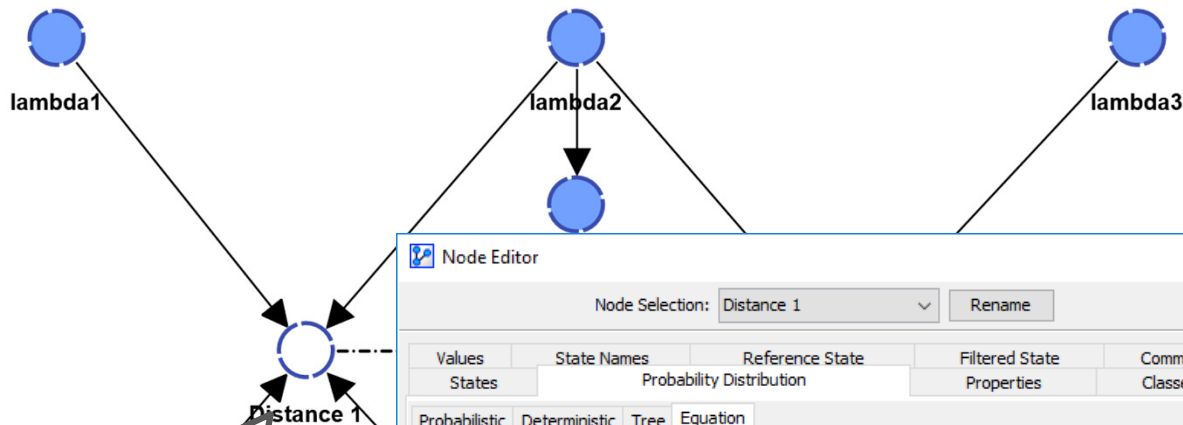
## Theoretical Model of Problem Domain



# Hub Location

## Theoretical Model of Problem Domain

$$d = 2r \arcsin \left( \sqrt{\text{hav}(\varphi_2 - \varphi_1) + \cos(\varphi_1) \cos(\varphi_2) \text{hav}(\lambda_2 - \lambda_1)} \right)$$
$$= 2r \arcsin \left( \sqrt{\sin^2 \left( \frac{\varphi_2 - \varphi_1}{2} \right) + \cos(\varphi_1) \cos(\varphi_2) \sin^2 \left( \frac{\lambda_2 - \lambda_1}{2} \right)} \right)$$



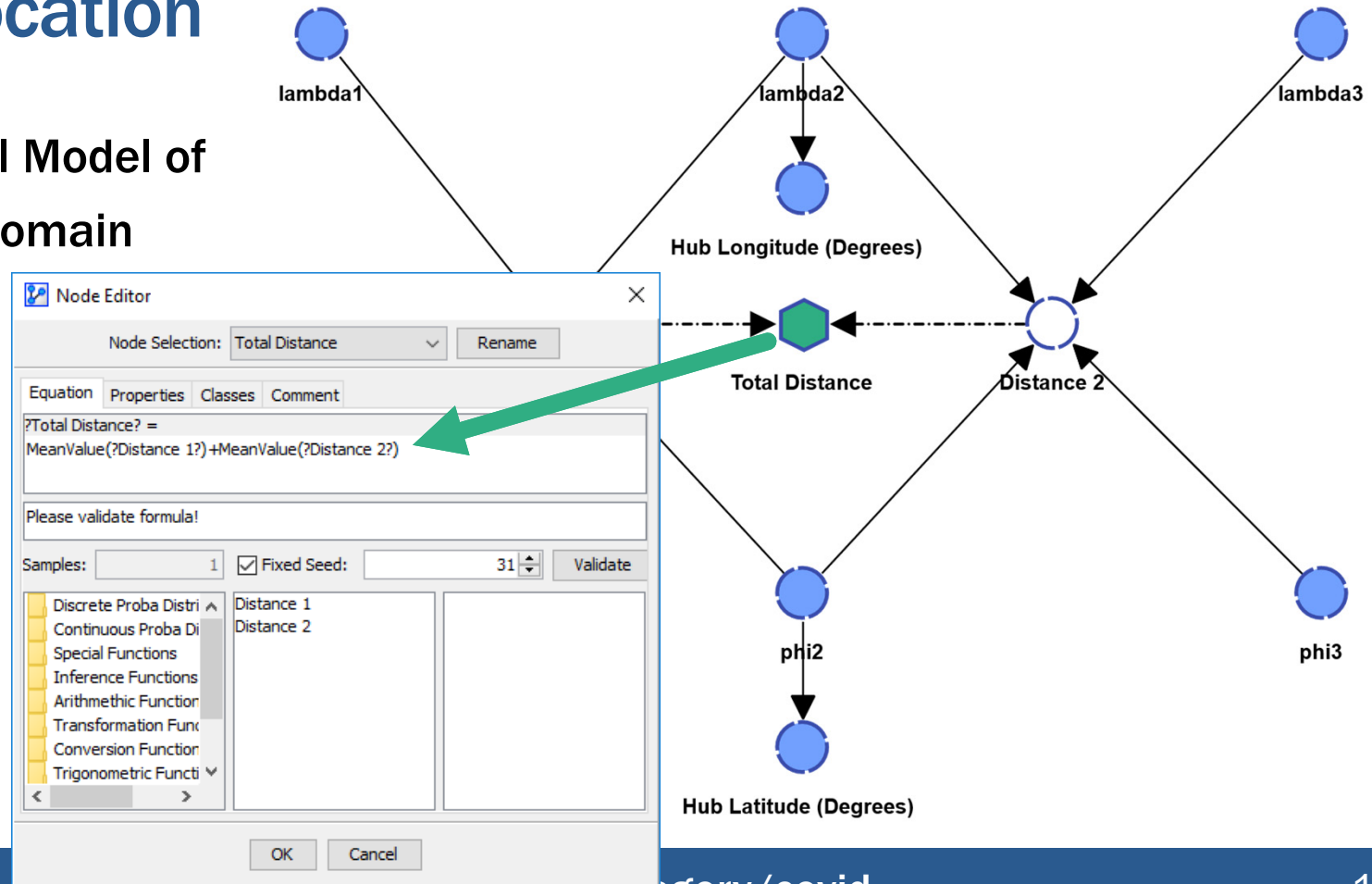
Node Editor

Node Selection: Distance 1 Rename

Values	State Names	Reference State	Filtered State	Comment
States	Probability Distribution		Properties	Classes
<p>Probabilistic <input type="radio"/> Deterministic <input checked="" type="radio"/> Tree <input type="radio"/> Equation</p> <p>Equation Type: <input checked="" type="radio"/> Deterministic <input type="radio"/> Probabilistic</p> <p>?Distance 1? =</p> <p><math>2 * 3959 * \text{Asin}(\sqrt{(\sin((?phi 1? - ?phi 1?)/2))^2 + \cos(?phi 1?) * \cos(?phi 2?) * (\sin((?lambda 2? - ?lambda 1?)/2))^2})</math></p> <p>Table successfully generated!</p> <p>Sample: 1000 Smoothing: 0 <input checked="" type="checkbox"/> Fixed Seed: 31 <span>Valid</span></p> <div><div><div>Discrete Proba Distributions</div><div>Continuous Proba Distribution</div><div>Special Functions</div><div>Arithmetic Functions</div><div>Transformation Functions</div><div>Conversion Functions</div><div>Trigonometric Functions</div><div>Relational Operators</div></div><div><div>Distance 1</div><div>lambda1</div><div>phi1</div><div>lambda2</div><div>phi2</div></div></div> <div><div>OK</div><div>Cancel</div></div>				

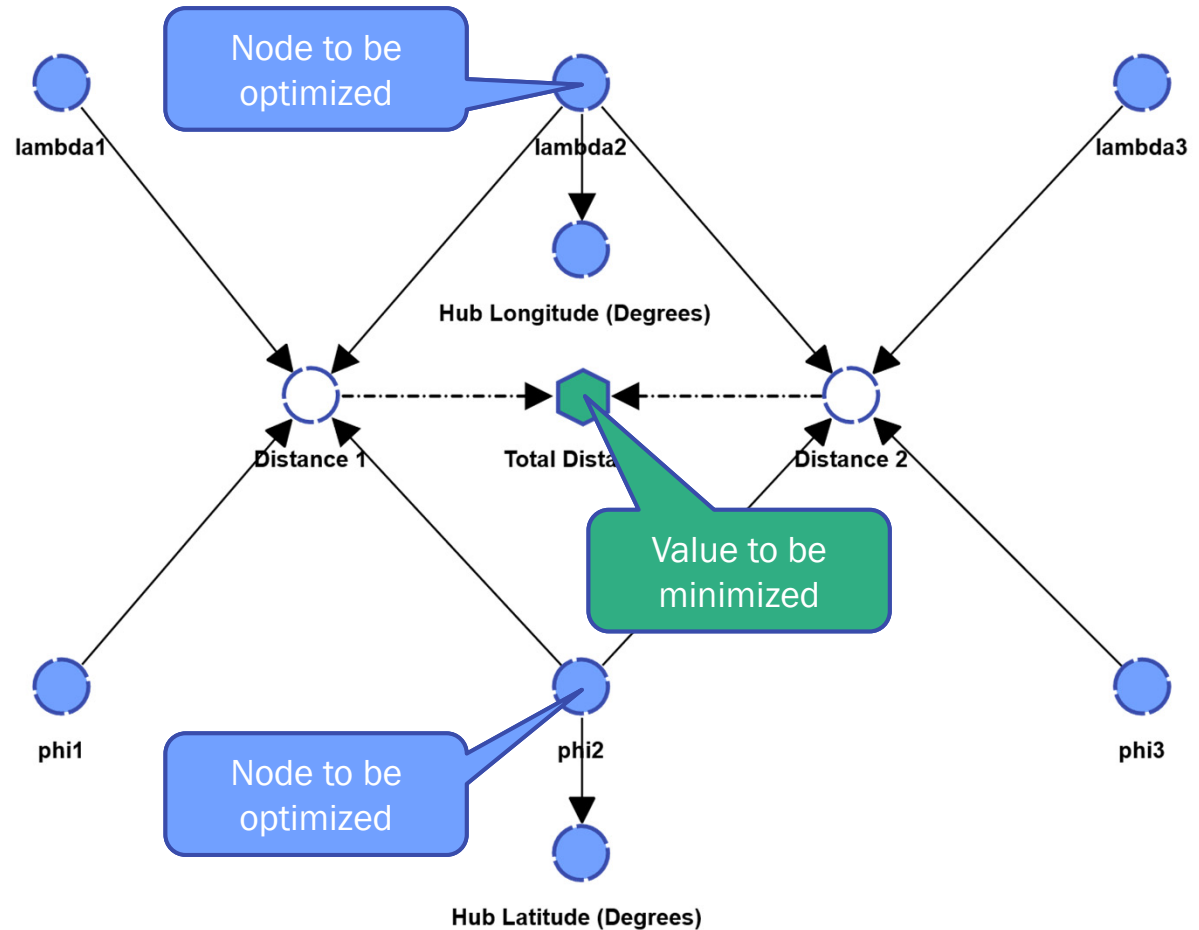
# Hub Location

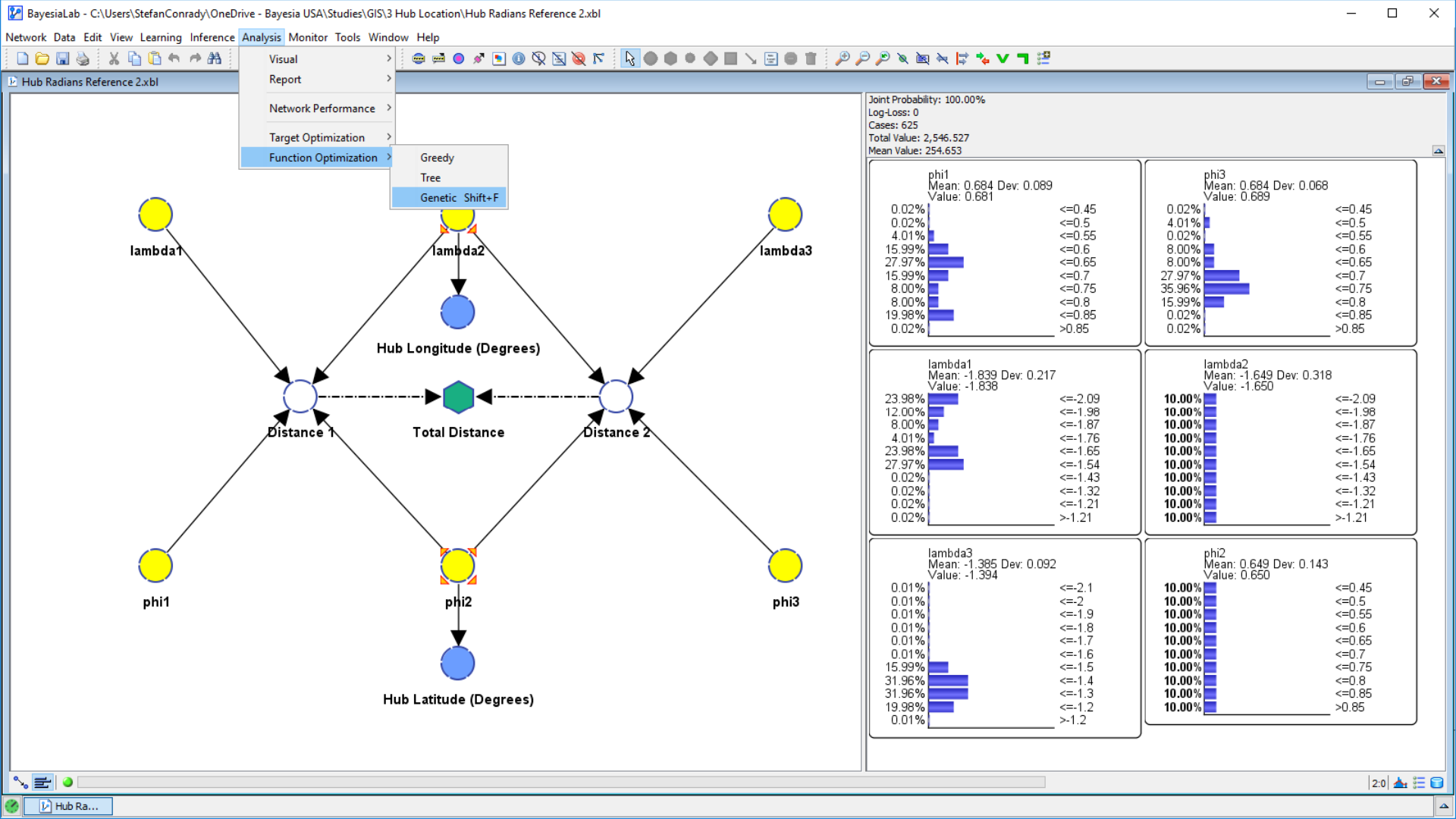
## Theoretical Model of Problem Domain



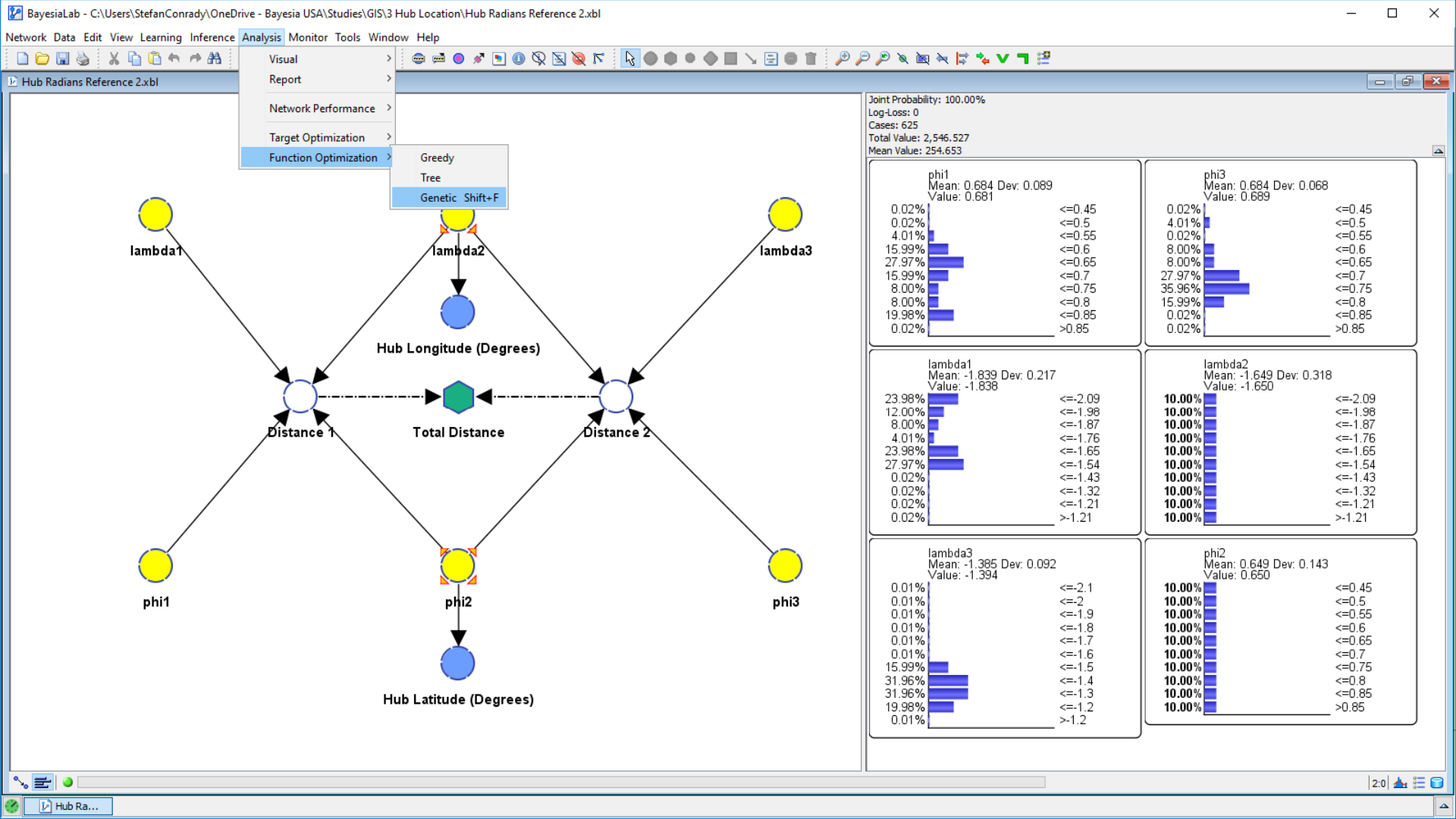
# Hub Location

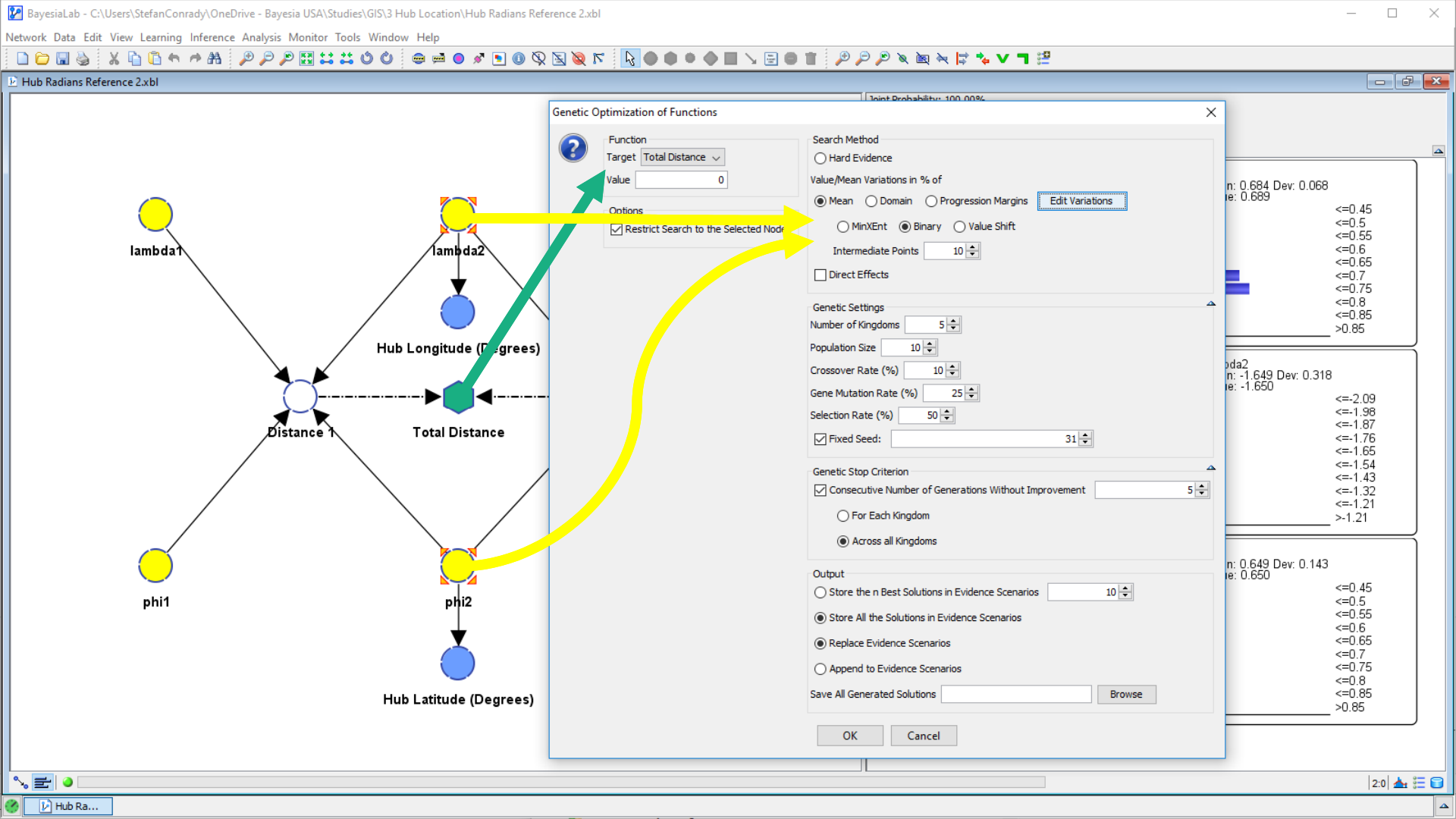
## Theoretical Model of Problem Domain

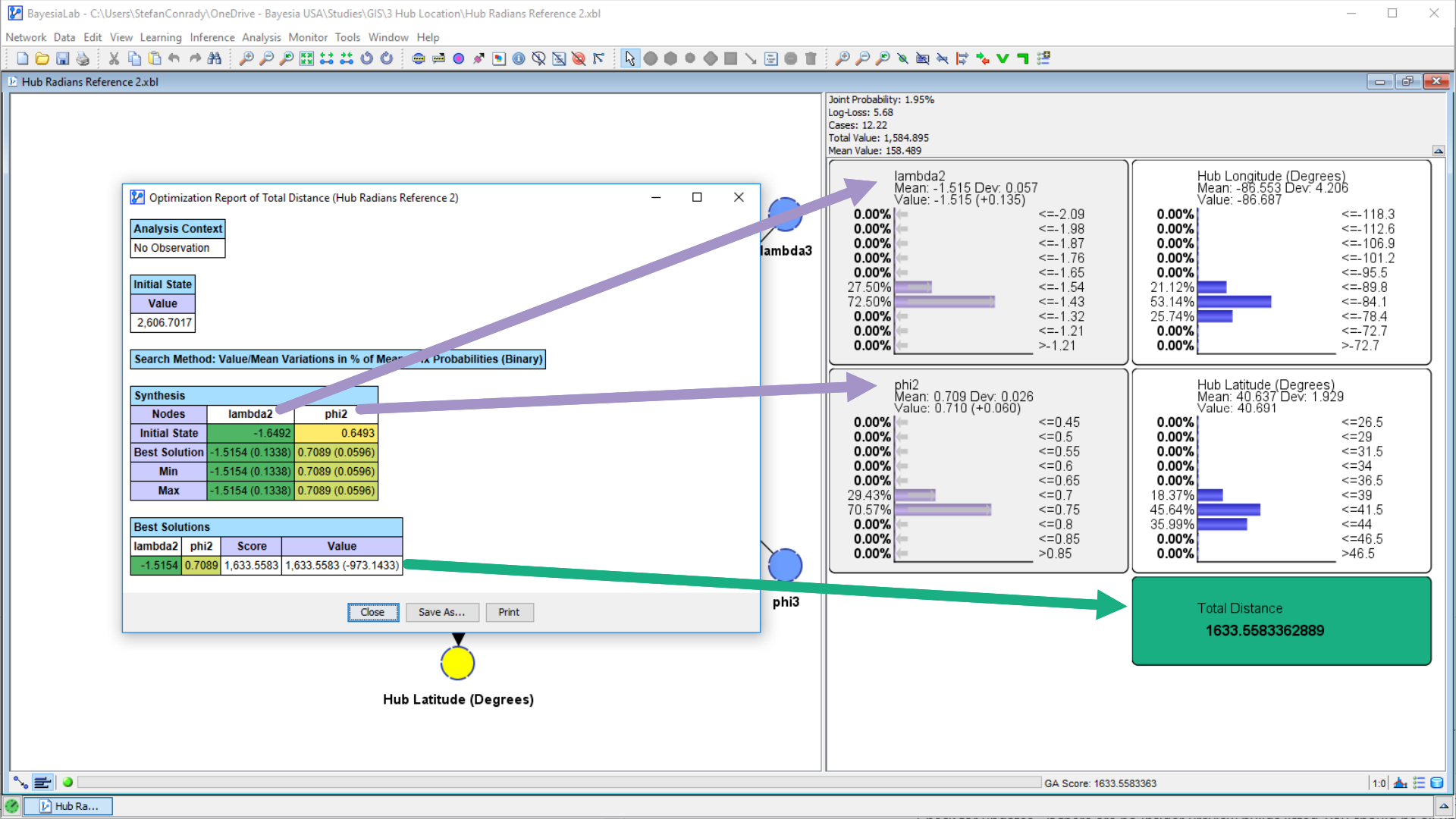




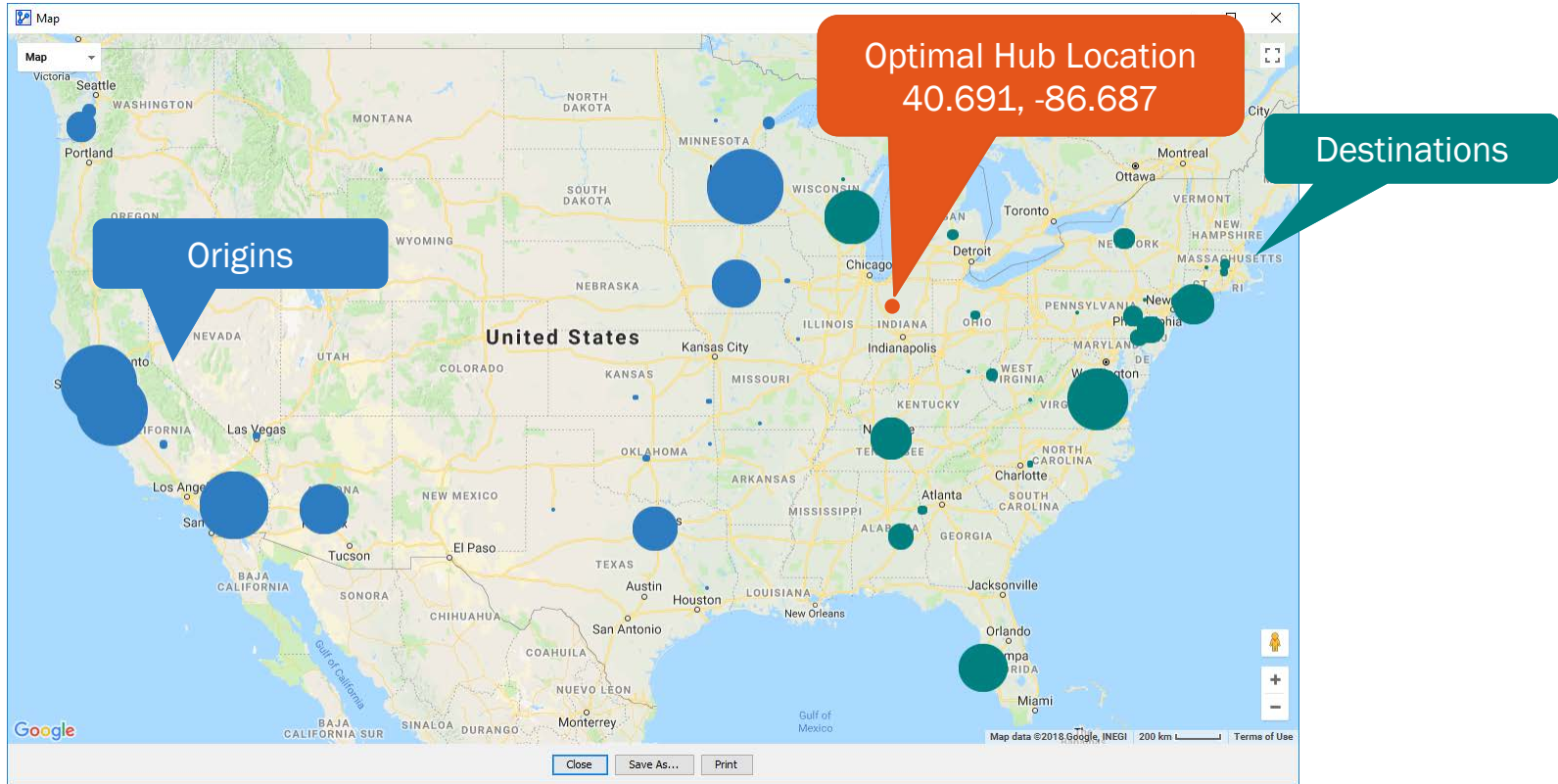








# Hub Location Problem





**In Conclusion...**

## Try BayesiaLab Today!

- 



# Webinar Series

## Reasoning Under Uncertainty

- March 20      Part 1 — Differential Diagnosis of Diseases
- March 26      Part 2 — Pandemic Triage with Bayesian Networks
- April 9        Part 3 — Epidemic Modeling with Temporal Bayesian Networks
- April 30      Part 4 — Representing Spatial and Temporal Dynamics
- t.b.d.         Part 5 — “Test & Treat” vs. Presumptive Treatment Policy

**Program Subject to Change — Check for Updates**





# BayesiaLab Self-Study Course

Introductory Course & Advanced Course Available

60-Day License to BayesiaLab Education Edition

20+ Hours of Screen/Lecture Recordings

300-Page Training Manual

[bayesia.com/2020-intro-course-self-study](https://bayesia.com/2020-intro-course-self-study)

[bayesia.com/2020-advanced-course-self-study](https://bayesia.com/2020-advanced-course-self-study)





# 8th Annual BayesiaLab Conference

ON SCHEDULE



# 8<sup>th</sup> Annual BayesiaLab Conference

**October 8–9, 2020**

The Exchange Tower  
Ivey Donald K. Johnson Centre  
130 King Street West  
Toronto, ON M5X 1K6, Canada

Registration is now open:

[bayesia.com/bayesialab-conference-2020](https://bayesia.com/bayesialab-conference-2020)



# Thank you and be safe!



[stefan.conrady@bayesia.us](mailto:stefan.conrady@bayesia.us)  
[jouffe@bayesia.com](mailto:jouffe@bayesia.com)



[BayesianNetwork](#)



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[linkedin.com/in/lioneljouffe](https://www.linkedin.com/in/lioneljouffe)



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