Bayesian Networks for Recommender Systems:
Going Beyond Ratings Prediction with "Most Relevant Explanation"

Michael L. Thompson & Jeevisha Anandani

University of Cincinnati
Carl H. Lindner College of Business

Abstract

Recommender systems are some of the most useful business applications built using Machine Learning. In our talk, we demonstrate how to build a recommender system for movies using Bayesian Machine Learning. The unique features of BayesiaLab, like "Most Relevant Explanation" and "Evidence Instantiation", allow us to extend the recommender system so we can gain insights into the audiences of each movie. Yet, we ask for more! We suggest extensions to BayesiaLab's already powerful feature set.

Outline

1. Background
2. Bayesian Network Ensemble Recommender System
3. Ratings Prediction
4. Audience Analysis
5. Potential Extensions for BayesiaLab
6. Lessons Learned
7. Questions?

Background

Case Profile

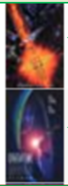
Viewer

Female, 33-44, doctor

Movie

Sci-Fi, Action

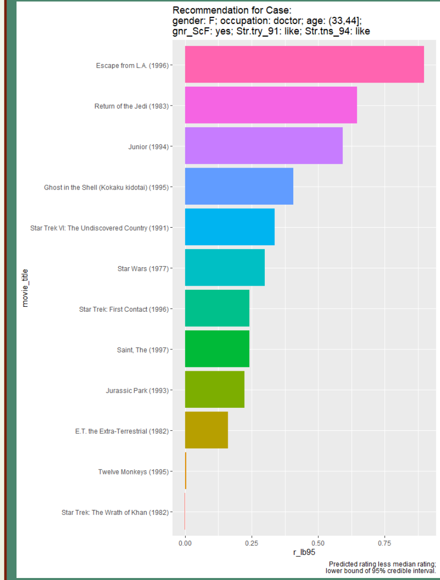
Past Ratings



Star Trek VI: The Undiscovered Country (1991)

Star Trek: Generations (1994)

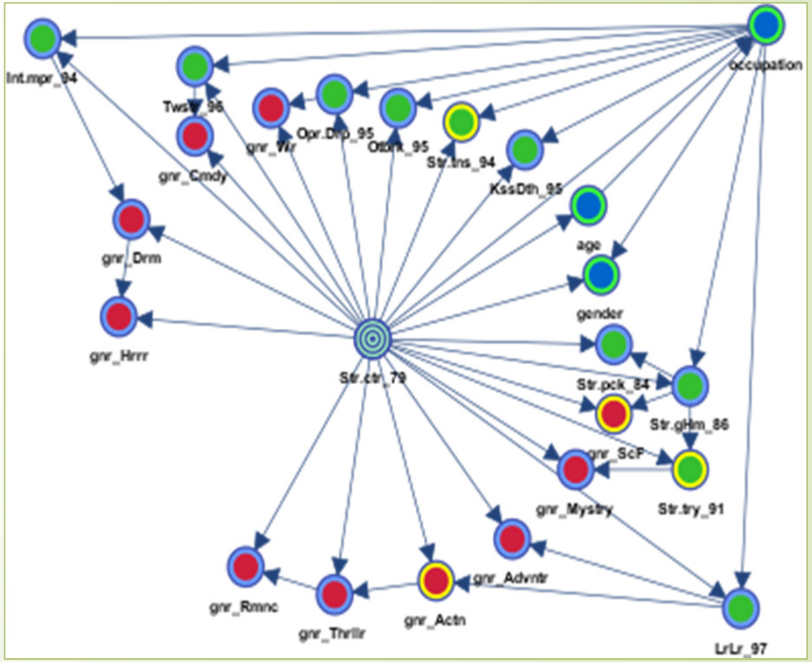
Recommendation



Approach: Build Ensemble of Bayesian Networks

- **Build BBN for each movie, m_i**
  - Tree-Augmented Naïve Bayes (TANB): **Highly confirming/refuting other movies**, **Viewer** & **Movie** Features
    - Avoids giant BBN containing all movies with either (a) limited connection to Viewer & Movie features – limiting their predictive value – or (b) excessive connections to Viewer & Movie features – resulting in intractable inference
  - All movie nodes, including target, have states equal to Viewer Ratings (5-star scale) **centered on each Viewer's median rating**
- **Exploit parallel processing**

Bayesian Belief Network (BBN)
for "Star Trek: The Motion Picture (1979)"



Approach: Selecting Nodes for Each BBN

Generalized Bayes Factor & Weight of Evidence

- **Generalized Bayes Factor, GBF(H:E)**

  Rank order candidate movies as Evidence E given Hypotheses H*=Like Target Movie

  - Find E to Maximize:

$$\boxed{\mathrm{GBF}(H^* : E) = \frac{Odds(H = Like\ Target\ Movie \mid E = Like\ Candidate\ Movie)}{Odds(H = Like\ Target\ Movie)}}$$

$$= \frac{P(E = Like\ Candidate\ Movie \mid H = Like\ Target\ Movie)}{P(E = Like\ Candidate\ Movie \mid H' \neq Like\ Target\ Movie)}$$

  - **Weight of Evidence is the logarithm of GBF**

    $W(H^* : E) = \log_2 \mathrm{GBF}(H^*: E)$;

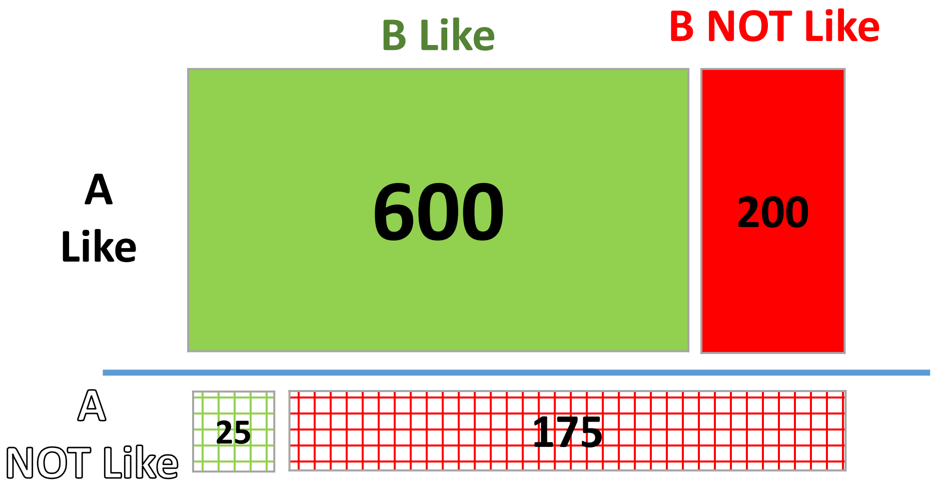
    in decibans: $\mathbf{W(H^* : E) = 10 \times \log_{10} GBF(H^*: E)}$

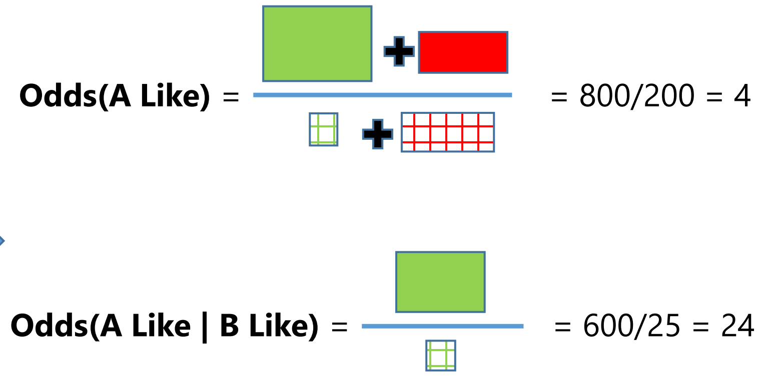
    - Kass & Raftery: evidence provides substantial support if **W(H:E) > 5 decibans = 1.66 bits**
    - I.J. Good: a person can only discern **ΔW > 1 deciban = 0.33 bits**

- **Build TANB: nodes for candidate movies w/top 10 |W(H:E)|**

Finds movies either disproportionately liked or disliked

Example: Total = 1000 viewers, Movie A (pattern), **Movie B** (color)





The odds of you being an A Liker increase by a factor of 6 if we know you liked B vs. us not knowing whether you liked B or not.
So we say,
“The observation 'Like B' is strong confirmatory evidence for the hypothesis 'Like A'."

Hypothesis : H = You are an "A Liker”
Evidence : E = You are a "B Liker”
GBF(H:E) = Odds(H | E) / Odds(H) = 24/4 = **6**
W(H:E) = **2.6 bits = 7.8 decibans**.

Approach: Recommend Movies

- **Apply Bayesian Inference**
  - Compute posterior:
    P(Rating m_i | Case Profile, m_i Seen) $\forall\ m_i$
  - Rank movies by largest to smallest
    Score(m_i) = Lower-Bound-of-95%-Credible-Interval
- Exploit parallel processing

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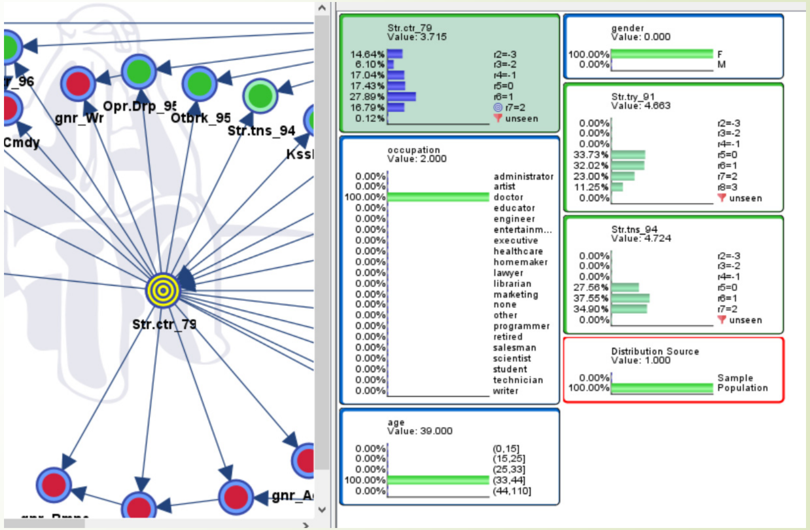

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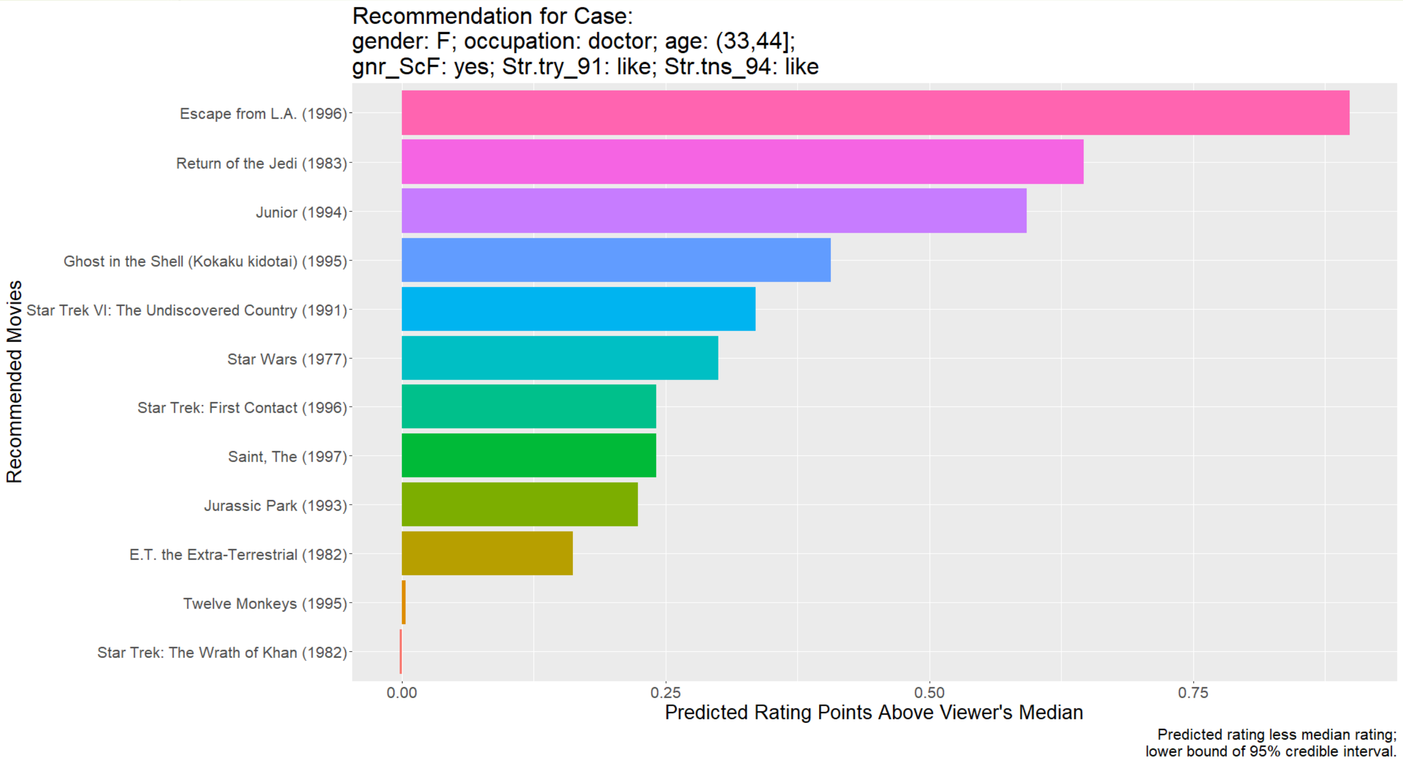
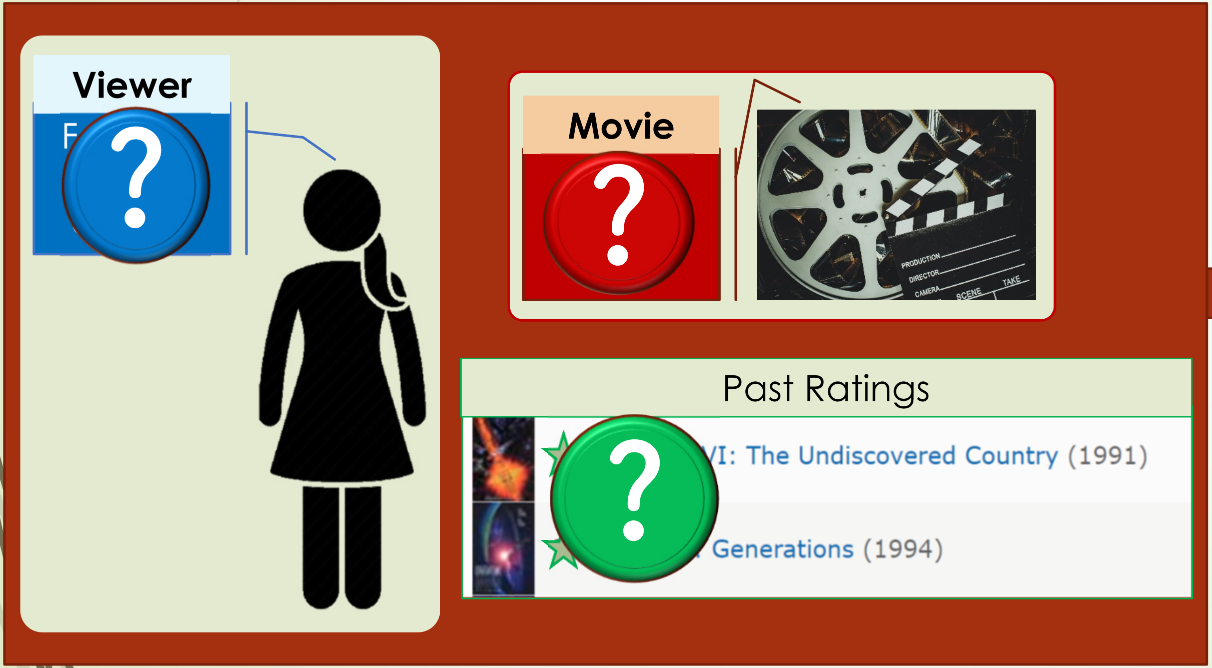


Ratings Prediction under Incomplete Information

Case Profile



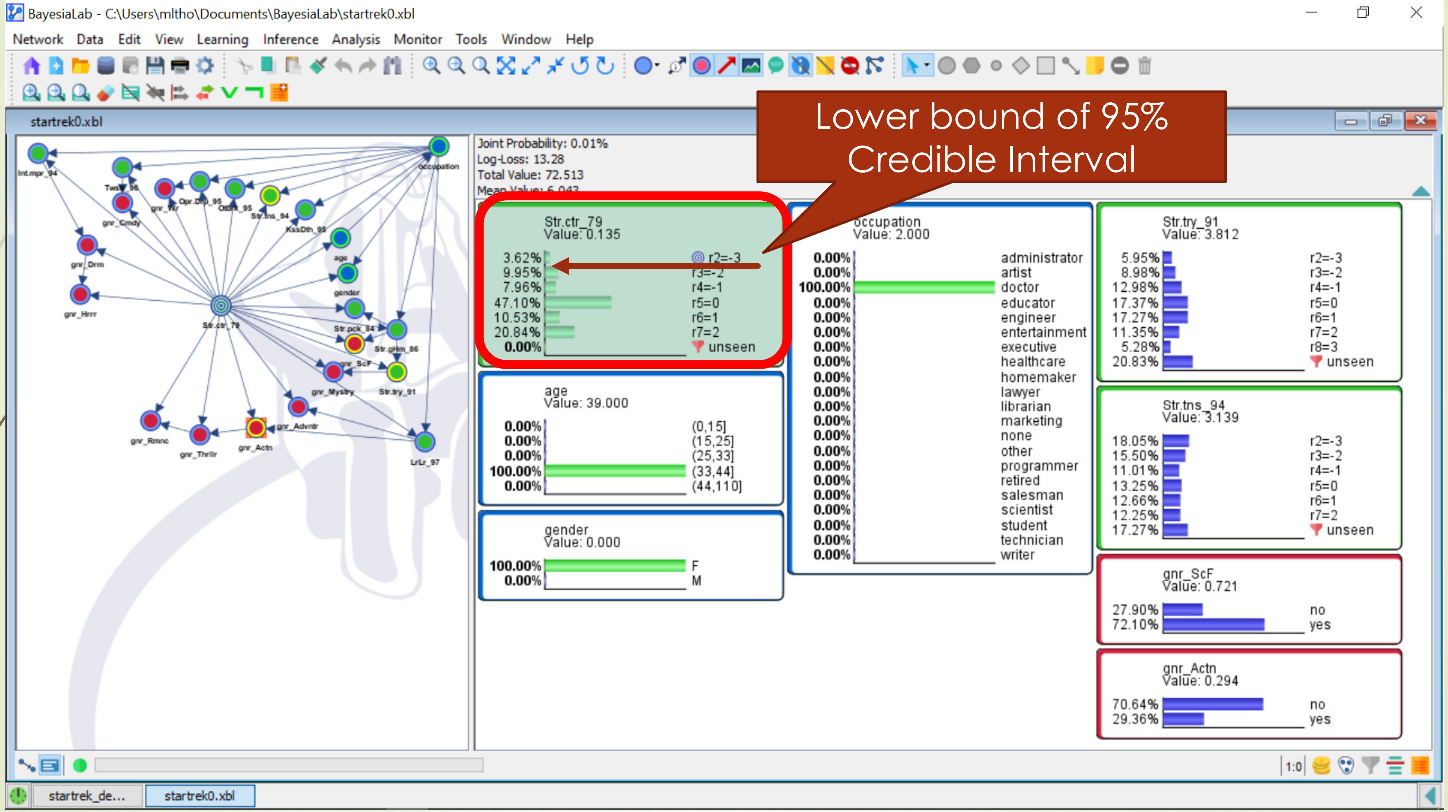
Recommendation

CAVEATS:
Database sample is sparse & biased
–
it is not representative of the US population w.r.t. gender, age, occupation.

Issues: Sparsity & Bias

- **Sparsity – Sample does not capture enough people within many of the gender-age-occupation cohorts**
  - Account for uncertainty by **leveraging posterior distribution** in forming recommendation rankings → Use **Lower-Bound-of-95%-Credible-Interval** as metric for ranking movies
  - Also: **Aggregation** of states; **Prior distributions** on conditional probability tables (CPTs)
- **Bias – Sample proportions of gender-age-occupation cohorts differ greatly from those in the target population to which we wish to apply our models**
  - Account for non-representativeness by **applying post-stratification** to aggregate predictions marginalized over the user features → Use **Evidence Instantiation** to transfer learned preferences within each gender-age-occupation cohort and marginalize over the joint distribution of gender, age, and occupation

Mitigating Issue of Sparsity: Quantify Uncertainty with Full Posterior



Mitigating Issue of Bias: Post-Stratify Outcomes with Population Distn.

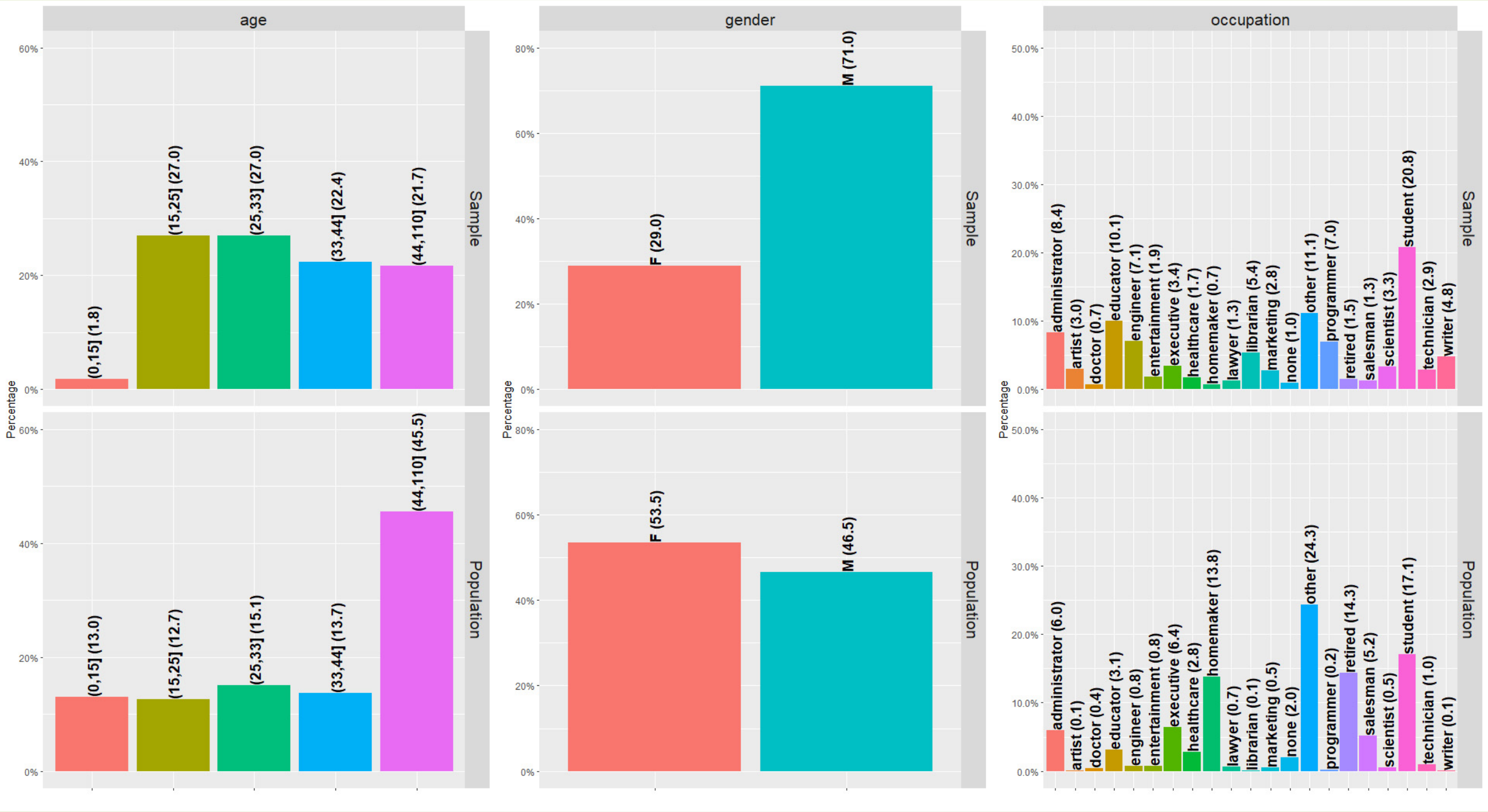
- Each TANB BBN captures the joint distribution
  P(Rating m_i, {Other Movie Ratings}, {Movie features}, {Viewer features})
- Factors into conditional & marginal
  P(Rating m_i, {Other Movie Ratings }, {Movie features} | {Viewer features}) X
  P({Viewer features})

Captures Viewer Preferences: Unbiased

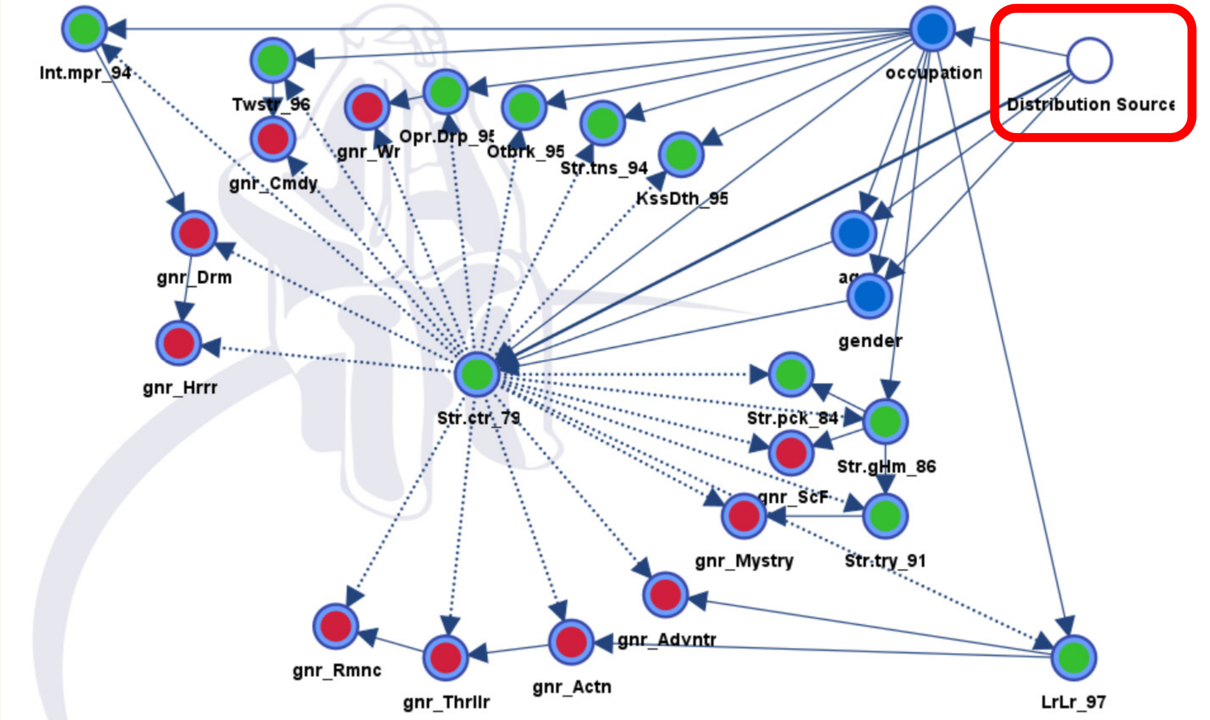
Captures Viewer Feature Distribution: Biased

- Impose Representative Viewer-Feature Distribution P({Viewer features}*)
  - Supply distribution on Gender-Age-Occupation cohorts from U.S. Bureau of Labor Statistics
  - Augment TANB with node "Distribution Source" ∊ {Sample, Population} and arcs **P({Viewer features}|Distribution Source)**
  - Assert evidence "Distribution Source" = Population
  - Use BayesiaLab's "Evidence Instantiation" to create new TANB conditional probability tables consistent with
    P(Rating m_i, {Other Movie Ratings}, {Movie features}, {Viewer features}*)

Marginal Distributions of Viewer Features



Post-Stratification: BayesiaLab's "Evidence Instantiation"



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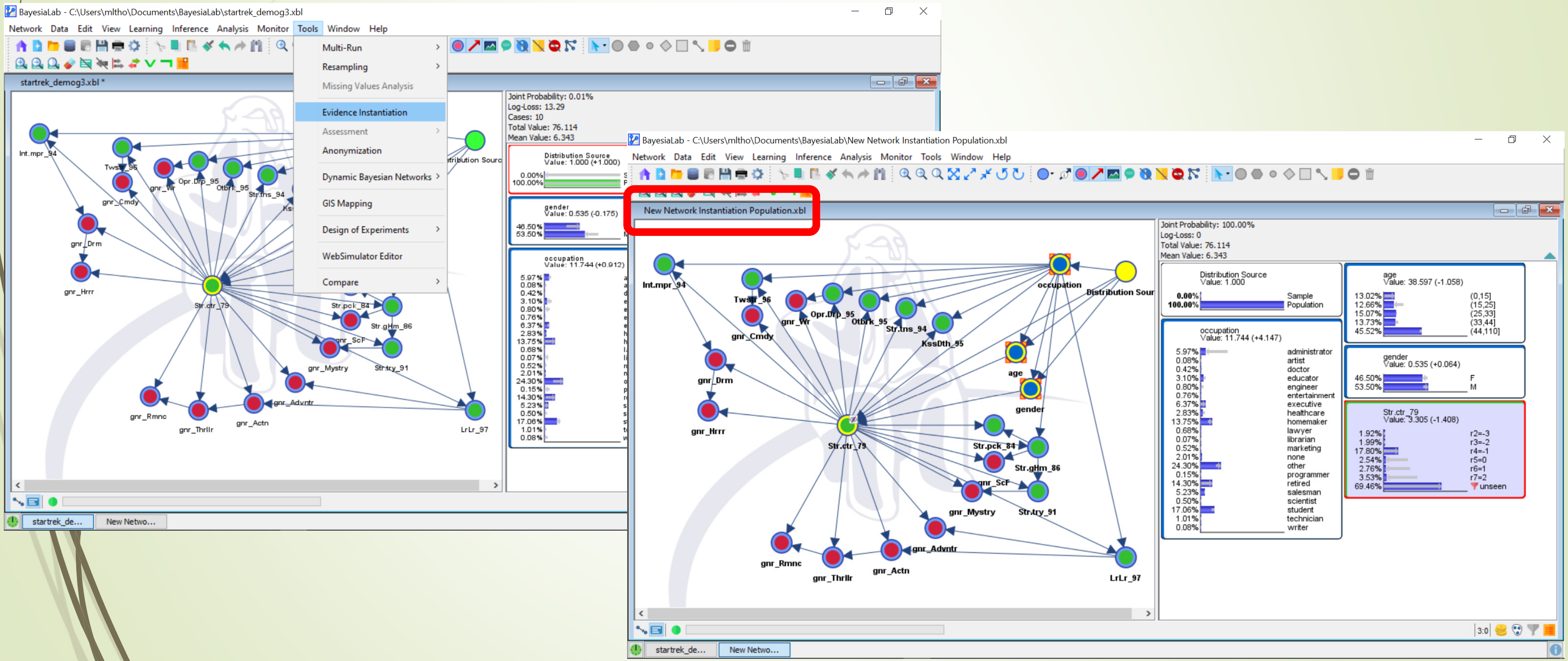


Audience Analysis

Finding Folks who are Likely to Love the Film

- Most Relevant Explanation (**MRE**)

  Fix Evidence E=E*, search over candidate Hypotheses H

  - Find H to Maximize:

$$\mathrm{GBF}(H : E^*) = \frac{P(E = E^* = \textit{Like Target Movie} \mid H = \{\textit{Viewer Features}\})}{P(E = E^* = \textit{Like Target Movie} \mid H \neq \{\textit{Viewer Features}\})} = \frac{\mathrm{Odds}(H|E^*)}{\mathrm{Odds}(H)}$$

Example: Observing someone likes "Star Trek: The Motion Picture (1979)" strongly confirms that person is an engineer **if Likers are far more prevalent among engineers than they are among Non-engineers.**

Which type of Viewers have a higher prevalence of people who Like the movie than exists among people different than that type of Viewer?

- Most Confirmatory Clues (**MCC**)

  Fix Hypothesis H=H *, search over candidate Evidence sets E

  - Find E to Maximize:

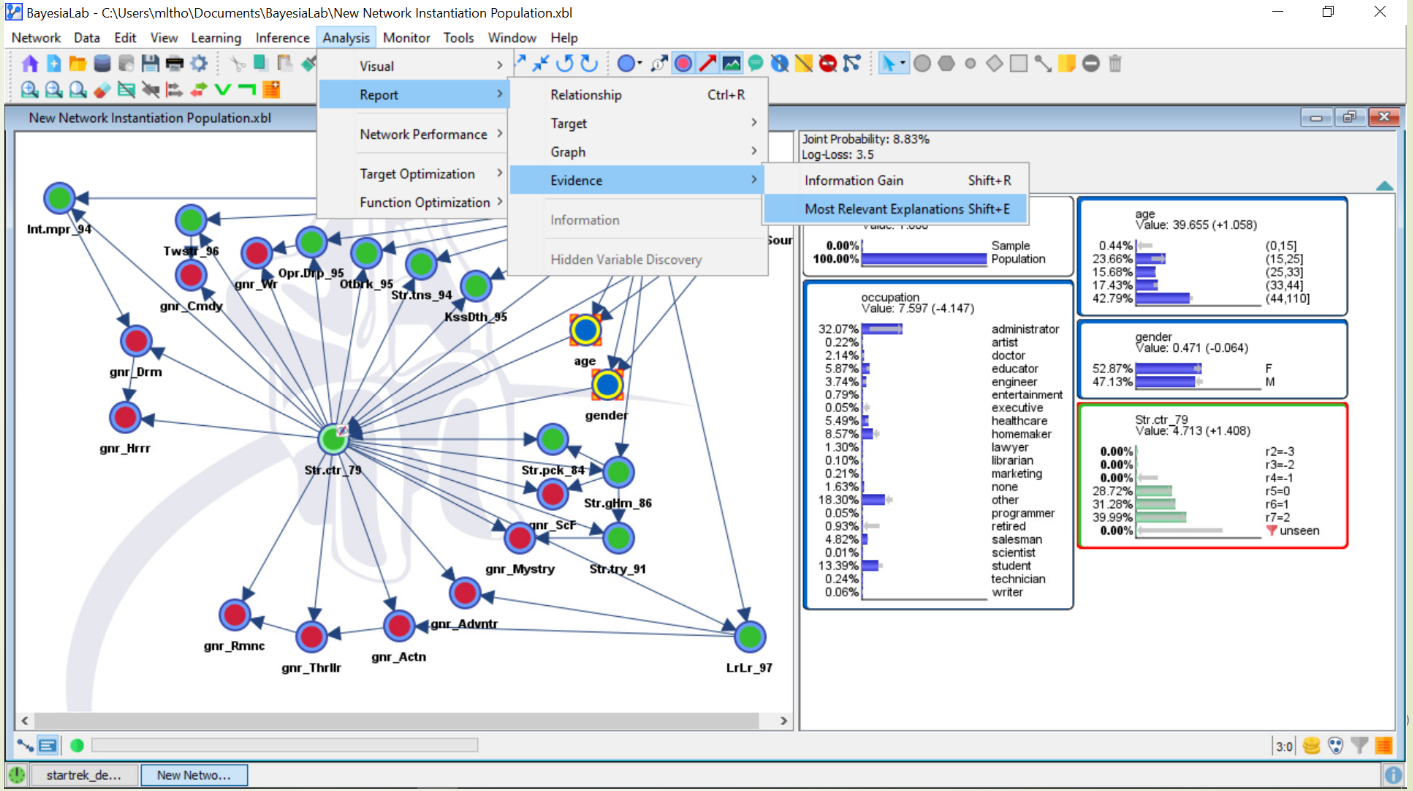
$$\mathrm{GBF}(H^* : E) = \frac{P(E = \{\textit{Viewer Features}\} \mid H = H^* = \textit{Like Target Movie})}{P(E = \{\textit{Viewer Features}\} \mid H \neq \textit{Like Target Movie})} = \frac{\mathrm{Odds}(H^*|E)}{\mathrm{Odds}(H^*)}$$

Gives same order for E as does P(H* | E).

Example: Observing someone is an engineer strongly confirms that person will like "Star Trek: The Motion Picture (1979)" **if engineers are far more prevalent among Likers than they are among Non-Likers.**

Which type of Viewers are far more prevalent among the people who Like the movie than they are among the people who dislike or didn't see the movie?

Audience Analysis:
BayesiaLab's "Most Relevant Explanation"



Most Relevant Explanation: Three Key Issues

Most Relevant Explanations Report (New Network Instantiation Population)

1. Must exceed threshold of ~1.6 to matter

Analysis Context

Joint Probability: 8.83%

...iewer's median rating).

Str.ctr_79 | {r2=-3: 0.00%, r... =1: 100.00%, r7=2: 100.00%, unseen: 0.00%}

epsilon: 0.32%

Must exceed epsilon %.

Out of N=943

	age	gender	occupation	MRE Size	Weight of Evidence [bits]	Generalize Bayes Factor	Likeliho P(E|H)	Posterior Odds O(H|E)	Posterior Probabili P(H|E)	Round W(H:E)	Round Rank	Prio P(H)	Join P(H,E)	US Pop. Cases	Sample Cases	KEEP	P(LIKE|seen)	r | LIKE	r_LB95% | LIKE
8	(44,110]	M	engineer	3	3.5	11.6	99.6%	0.036	3.5%	3.5	1	0.3%	0.3%	908,397	13	FALSE	100%	0.3	0
9	(44,110]		engineer	2	3.4	10.9	93.2%	0.039	3.7%	3.5	1	0.4%	0.3%	976,549	13	TRUE	93%	0.5	0
10	(44,110]	F	administrator	3	3.2	9.5	72.3%	0.182	15.4%	3.2	3	1.9%	1.4%	4,057,869	9	TRUE	74%	0.3	2
11	(44,110]		administrator	2	3.2	9.3	67.4%	0.254	20.3%	3.2	3	2.7%	1.8%	5,336,367	18	TRUE	70%	0.3	0
12	(15,25]	M	student	3	2.9	7.6	59.3%	0.148	12.9%	2.9	5	1.9%	1.1%	3,398,925	58	TRUE	99%	0.2	2
13			administrator	1	2.9	7.4	47.4%	0.472	32.1%	2.9	5	6.0%	2.8%	8,438,290	35	TRUE	53%	0.3	0
14	(44,110]	M	lawyer	3	2.7	6.6	58.1%	0.012	1.2%	2.9	5	0.2%	0.1%	305,185	2	FALSE	60%	0.6	0
15	(25,33]	M	entertainment	3	2.7	6.6	58.1%	0.008	0.8%	2.9	5	0.1%	0.1%	198,457	3	FALSE	60%	0.6	0
16	(25,33]	F	doctor	3	2.7	6.4	56.5%	0.002	0.2%	2.5	9	0.0%	0.0%	63,574	1	FALSE	67%	0.4	0
17	(15,25]	F	educator	3	2.6	6.1	53.3%	0.012	1.2%	2.5	9	0.2%	0.1%	311,974	1	FALSE	59%	0.6	0
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19	(15,25]		educator	2	2.5	5.5	48.2%	0.015	1.5%	2.5	9	0.3%	0.1%	382,863	2	FALSE	55%	0.5	0
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Most Relevant Explanations Repo

Most Relevant Explanation: Three Key Issues

Most Relevant Explanations Report (New Network Instantiation Population)

2. Must differ by ~0.33 to distinguish from each other

Analysis Context

Joint Probability	8.83%	See movie and LIKE (viewer rating >= viewer's median rating
Str.ctr_79		l {r2=-3: 0.00%, r3=-2: 0.00%, r4=-1: 0.00%, r5=0: 100.00%, r6=1: 100.00%, r7=2: 1

Must exceed 1.6 bits; must differ by more than 1 deciban ~ 1/3 bits.

P(LIKE | Profile)/ P(LIKE | ~Profile)

Out of N=943

Best Solutions

	age	gender	occupation	MRE Size	Weight of Evidence [bits]	Generalized Bayes Factor	Likelihood P(E|H)	Posterior Odds O(H|E)	Posterior Probability P(H|E)	Round W(H:E)	Round Rank	Prior P(H)	Joint P(H,E)	US Pop. Cases	Sample Cases	KEEP	P(LIKE|seen)	r | LIKE	r_LB95% | LIKE
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Most Relevant Explanation: Three Key Issues

AutoSave Off — Most Relevant Explanations Report (New Network Instantiation Population) — Thompson, Michael (thomp4mc)

File Home Insert Draw Page Layout Formulas Data Review View Help — Share — Comments

S80

3. Must exceed threshold of ~0.3% to exceed noise

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		administrator	1	2.9	7.4	47.4%	0.472	32.1%	2.9	5	6.0%	2.8%	8,438,290	35	TRUE	53%	0.3	0
(44,110]	M	lawyer	3	2.7	6.6	58.1%	0.012	1.2%	2.9	5	0.2%	0.1%	305,185	2	FALSE	60%	0.6	0
(25,33]	M	entertainment	3	2.7	6.6	58.1%	0.008	0.8%	2.9	5	0.1%	0.1%	198,457	3	FALSE	60%	0.6	0
(25,33]	F	doctor	3	2.7	6.4	56.5%	0.002	0.2%	2.5	9	0.0%	0.0%	63,574	1	FALSE	67%	0.4	0
(15,25]	F	educator	3	2.6	6.1	53.3%	0.012	1.2%	2.5	9	0.2%	0.1%	311,974	1	FALSE	59%	0.6	0
(25,33]		doctor	2	2.6	6.1	53.2%	0.006	0.6%	2.5	9	0.1%	0.1%	150,093	2	FALSE	64%	0.4	0
(15,25]		educator	2	2.5	5.5	48.2%	0.015	1.5%	2.5	9	0.3%	0.1%	382,863	2	FALSE	55%	0.5	0
(44,110]	F	doctor	3	2.5	5.5	48.3%	0.005	0.5%	2.5	9	0.1%	0.0%	128,331	1	FALSE	55%	0.5	0
(44,110]		doctor	2	2.5	5.5	48.0%	0.012	1.2%	2.5	9	0.2%	0.1%	319,052	2	FALSE	52%	0.5	0
	F	doctor	2	2.4	5.4	47.5%	0.009	0.9%	2.5	9	0.2%	0.1%	238,585	1	FALSE	56%	0.4	0
(25,33]	M	artist	3	2.4	5.4	47.4%	0.001	0.1%	2.5	9	0.0%	0.0%	13,946	3	FALSE	50%	0.6	1
(15,25]	M	artist	3	2.4	5.2	46.2%	0.000	0.0%	2.5	9	0.0%	0.0%	3,158	3	FALSE	50%	0.5	0
		doctor	1	2.4	5.2	44.8%	0.022	2.1%	2.2	18	0.4%	0.2%	564,374	4	FALSE	52%	0.4	0
	M	engineer	2	2.4	5.1	43.9%	0.036	3.5%	2.2	18	0.7%	0.3%	911,318	13	FALSE	99%	0.0	0
(25,33]		educator	2	2.3	5.0	42.7%	0.036	3.4%	2.2	18	0.7%	0.3%	907,345	10	FALSE	50%	0.4	0
(15,25]		artist	2	2.3	4.9	43.6%	0.000	0.0%	2.2	18	0.0%	0.0%	6,473	4	FALSE	50%	0.5	0
		engineer	1	2.3	4.8	41.2%	0.039	3.7%	2.2	18	0.8%	0.3%	984,707	13	TRUE	91%	0.0	0
(33,44]	F	other	3	2.2	4.7	38.7%	0.086	7.9%	2.2	18	1.8%	0.7%	2,074,326	4	TRUE	50%	0.6	1

Most Relevant Explanations Repo

80%

Modified MRE

File | Home | Insert | Draw | Page Layout | Formulas | Data | Review | View | Help

A2 | Joint Probability

Analysis Context

Joint Probability	8.83%	See movie and LIKE (viewer rating >= viewer's median rating).
Str.ctr_79		I {r2=-3: 0.00%, r3=-2: 0.00%, r4=-1: 0.00%, r5=0: 100.00%, r6=1: 100.00%, r7=2: 100.00%, unseen: 0.00%}

epsilon: 0.32%

Must exceed 1.6 bits; must differ by more than 1 deciban ~ 1/3 bits.

P(LIKE | Profile)/ P(LIKE | ~Profile)

Must exceed epsilon %.

Out of N=943

Best Solutions

	age	gender	occupation	MRE Size	Weight of Evidence [bits]	Generalize Bayes Factor	Likeliho P(E|H)	Posterior Odds O(H|E)	Posterior Probabili P(H|E)	Round W(H:E)	W(H:E) [decibans]	Round Rank	Prio P(H)	Join P(H,E)	US Pop. Cases	Sample Cases	KEEP
9	(44,110]		engineer	2	3.4	10.9	93.2%	0.039	3.7%	3.5	10.4	1	0.4%	0.3%	976,549	13	TRUE
11	(44,110]		administrator	2	3.2	9.3	67.4%	0.254	20.3%	3.2	9.7	3	2.7%	1.8%	5,336,367	18	TRUE
12	(15,25]	M	student	3	2.9	7.6	59.3%	0.148	12.9%	2.9	8.8	5	1.9%	1.1%	3,398,925	58	TRUE
13			administrator	1	2.9	7.4	47.4%	0.472	32.1%	2.9	8.7	5	6.0%	2.8%	8,438,290	35	TRUE
29			engineer	1	2.3	4.8	41.2%	0.039	3.7%	2.2	6.8	18	0.8%	0.3%	984,707	13	TRUE
30	(33,44]	F	other	3	2.2	4.7	38.7%	0.086	7.9%	2.2	6.7	18	1.8%	0.7%	2,074,326	4	TRUE
36	(15,25]		student	2	1.9	3.6	29.2%	0.148	12.9%	1.9	5.6	26	3.9%	1.1%	3,398,925	58	TRUE
41	(15,25]	M		2	1.7	3.3	25.2%	0.220	18.0%	1.5	5.1	34	6.3%	1.6%	4,745,496	66	TRUE

Potential Extensions for BayesiaLab: Generalize MRE feature

- Allow "Most Confirmatory Clues", MCC
  - argmax E: GBF(H* : E) – currently, "Most Relevant Explanation", MRE, is argmax H: GBF(H : E*) ; generalizes Target Optimization P(H*|E) : H* can involve multiple nodes (compound hypothesis)
  - Checkbox to signal fixing Hypothesis and searching over Evidence combos
- Allow threshold on solutions as well as number of solutions
  - Entry field to accept minimum acceptable GBF (or W)
- Allow threshold on joint P(E,H) to avoid returning solutions that are just noise
  - Entry field to accept minimum acceptable P(E,H) for a solution, whether MRE or MCC; default equal 0, thus no imposition of threshold
- Allow tolerance in comparing GBF to account for human discernibility & noise
  - Entry field to accept minimum acceptable difference in GBF for two solutions to be considered different; default equal to 1 deciban per I.J. Good – W(H:E) in decibans is 10 X log10(GBF(H:E)).
- Allow minimization of GBF for "LRE", Least Relevant Explanation, & "LCC", Least Confirmatory Clues
  - Checkbox to signal searching for strongest Refutation rather than Confirmation

Lessons Learned

- **Analysis over entire Joint Probability Distribution is a powerful feature of BBN**
  - ***Caveat***: Be wary of chasing noise – analysis in the tails is much less robust than analysis of conditional expectations in the body of the distribution
- **Bayesian methods allow principled post-stratification & uncertainty quantification**
  - ***Caveat***: "Garbage In, Garbage Out" – No amount of reweighting can compensate for extreme sparsity and/or selection bias, esp. if unobserved context changes behavior of sample cohorts relative to the same population cohorts
- **BayesiaLab offers state-of-the-art capabilities for Bayesian Analysis**
  - ***Caveat***: Even BayesiaLab can be made more powerful!

Questions?