Modelling epistemic and aleatory uncertainty in Bayesian Network for dependability analysis

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Outlines

- Definitions of studied problems
- Forms and Sources of uncertainties
- Theoretical Frameworks for modeling
- Encoding in Bayesian Network
- Applications to dependability assessment

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- Forms and Sources of uncertainties
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Definitions

- **Dependability** :
  - Reliability is the ability of an entity to accomplish its required function in some given conditions for a time period. (Binary hypothesis)
  - Availability is the ability of an entity to accomplish its required function in some given conditions, at an instant or during a time period given that any means needed are provided (Binary hypothesis).
- Satisfiability
  - It’s the ability of an entity to satisfy an expected accomplishment level of its function (Non Binary hypothesis).

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Sources of uncertainty(ies)

Two levels problems

- **Data**:
  - The instant of a basic event (failure) is unknown (randomness)
  - Few amount of data (Problem of exhaustiveness)
  - Experts judgments given with different expressions (Imprecision and doubt)
  - Sparse data (Incompleteness)
  - Variations of operational conditions (different from labs)
  - Validity of data provided
- Models
  - Partial models
  - Incomplete models
  - Concurrent models

Forms of uncertainty(ies)

Two main forms of uncertainty(ies)

- Aleatory uncertainty
  - Due to the random character or natural variability of physical phenomenon. The value are precise but different due to natural variations).
  - We are talking about stochastic uncertainty or variability.
  - It is usually associated to observable quantities and considered as not reducible.
- Epistemic uncertainty
  - Due to the imprecise character of the information or to a lack of knowledge.
  - It is usually associated to non observable quantities, or observable quantities with a doubt.
  - It is considered reducible.

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Some possible frameworks

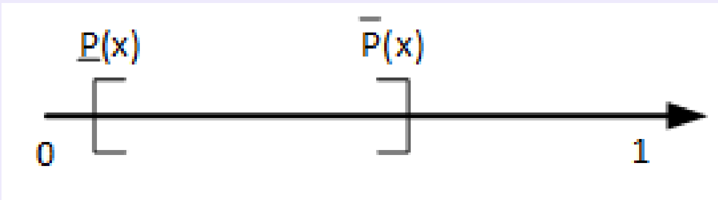
Probability and other languages

- Probability theory
  - Well suited for modeling the natural variability
  - Two possible interpretations: Objective, Subjective
- Non additive Theories
  - Well suited for modeling epistemic uncertainty
  - Interval Theory (Imprecision)
  - Theory of possibility (Imprecision, Certainty)
  - Evidence Theory (Incompleteness, Ignorance)
  - Imprecise Probabilities (Sets of probability functions and intervals)
  - ...

Remark: No universal framework but different languages of uncertainty(ies)

Modeling uncertainty(ies)

Interval Valued Probability

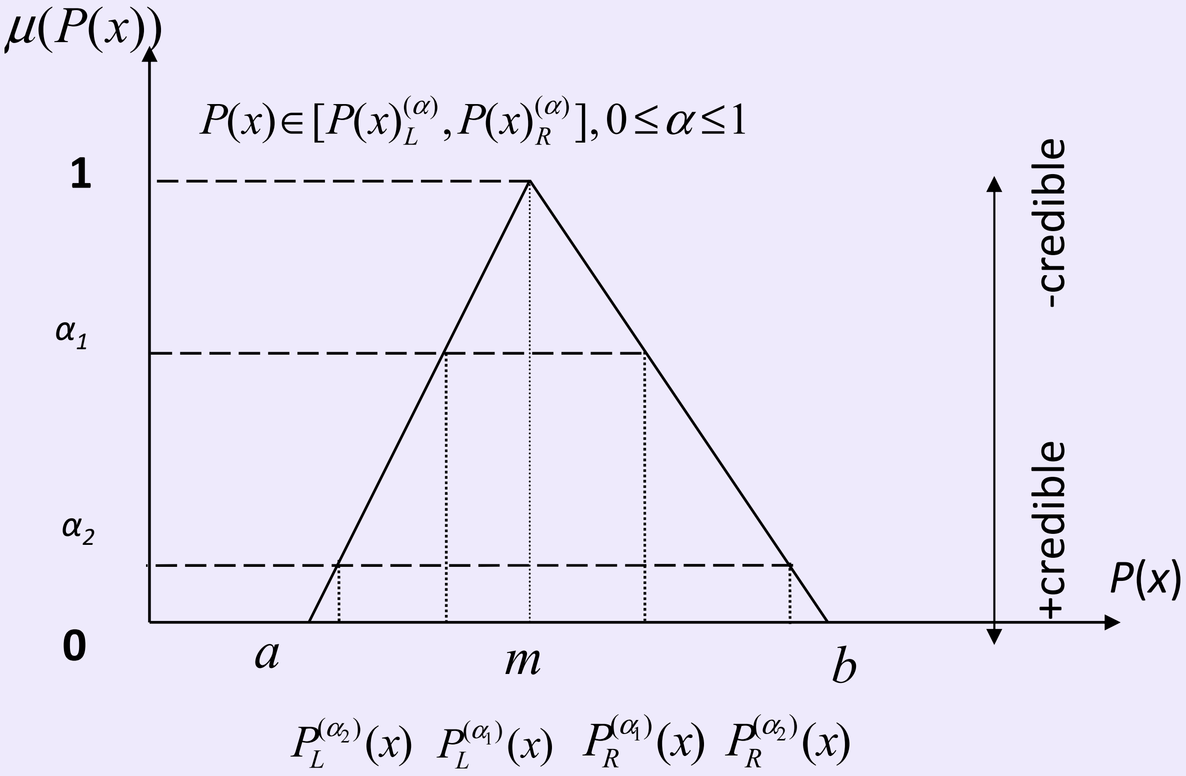


- The real value of p(x) is unknown but lies between the two bounds $\underline{p}(x)$ and $\overline{p}(x)$
- The intervals are convex
- The results of computation should be convex intervals

Modeling uncertainty(ies)

Fuzzy valued Probability

- Aleatory uncertainty+Imprecision+credibility with probability+Interval+α cut

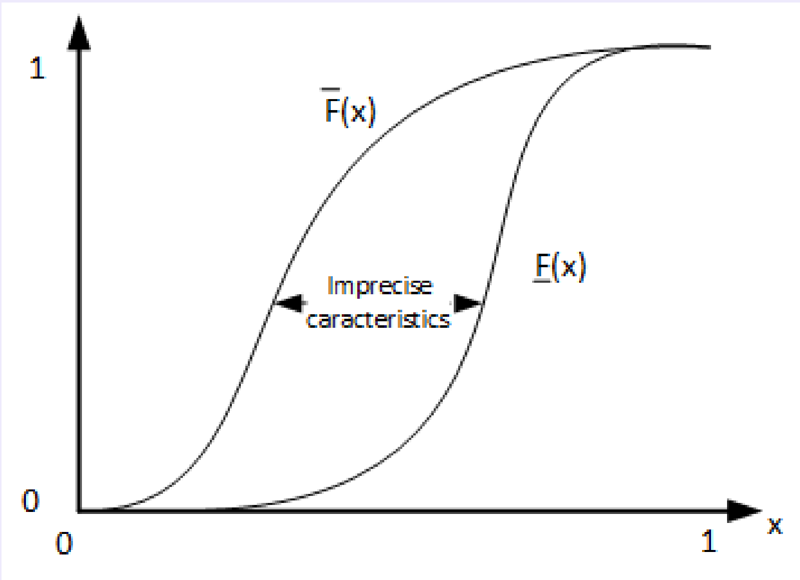


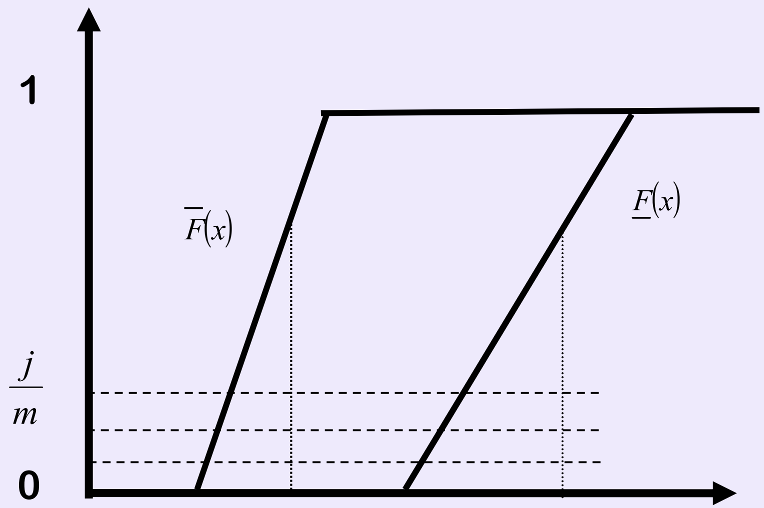
Fuzzy valued probabilities are sets of embodied interval valued probabilities defined by α-cuts

Modeling uncertainty(ies)

Probability boxes

- P-Box : Family of probability distributions bounded by upper and lower cumulative distributions





$$[\underline{F}_X, \overline{F}_X] = \{ ([\underline{X}_1, \overline{X}_1], p_i); ([\underline{X}_2, \overline{X}_2], p_i); \ldots\ldots; ([\underline{X}_j, \overline{X}_j], p_i); \ldots\ldots; ([\underline{X}_m, \overline{X}_m], p_i) \}$$

Remark: Sets of interval valued probabilities with weight

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Encoding interval valued probabilities in BN

Dempster Shafer Theory

- Frame of discernment $\Omega = \{H_1, H_2, \dots, H_q\}$
- Dempster-Shafer Structure (power set of Ω)

$$A_i \in 2^{\Omega} : \{\varnothing, A_1 = \{H_1\}, \dots, A_q = \{H_q\}, A_{q+1} = \{H_1, H_2\}, \dots, A_{2^q-1} = \{H_1, \dots, H_q\}\}$$

- Basic Mass Assignment

$$m : 2^{\Omega} \rightarrow [0,1] \qquad m(\varnothing) = 0 \qquad \sum_{A_i \in 2^{\Omega}} m(A_i) = 1$$

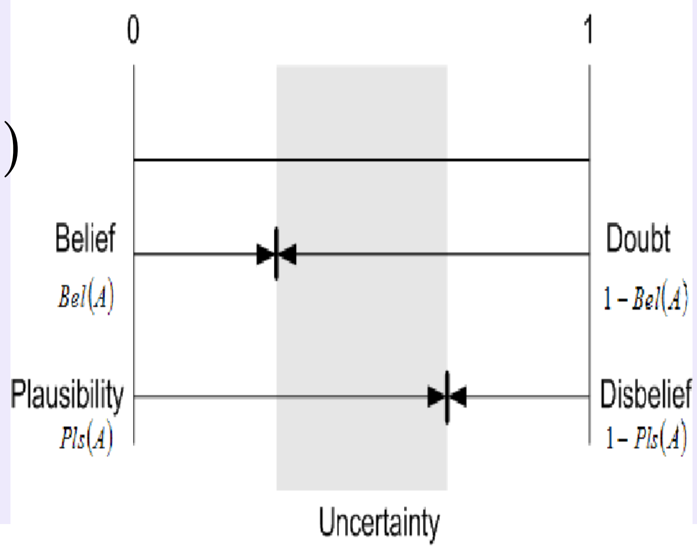
- Measures (bounds)

$$Bel(A_i) \le P(A_i) \le Pls(A_i)$$

$$Bel(A_i) = \sum_{B|B \subseteq A_i} m(B) \qquad Pls(A_i) = \sum_{B|A_i \cap B \neq \varnothing} m(B)$$

- Möbius Transform

$$m(A_i) = \sum_{B|B \subseteq A_i} (-1)^{|A_i| - |B|} Bel(B)$$



Encoding interval valued probabilities in BN

Dempster Shafer Theory

- All nodes encode the Dempster-Shafer Structure

$$A_i \in 2^{\Omega} : \{\varnothing, A_1 = \{H_1\}, \ldots, A_q = \{H_q\}, A_{q+1} = \{H_1, H_2\}, \ldots, A_{2^q - 1} = \{H_1, \ldots, H_q\}\}$$

- Allocation of masses in each set under the constraint

$$\sum m(A_i) = 1$$

- Prior masses are computed from interval valued probabilities and the Möbius transform (root nodes)

$$[\underline{P}(H_i), \overline{P}(H_i)] = [Bel(H_i), Pls(H_i)] \qquad M_X = \left[0 \ \underline{P}(A_1) \cdots \sum_{B | B \subseteq A_i} (-1)^{|A_i| - |B|} \underline{P}(B) \cdots \right]$$

- Child nodes integrate conditonal belief mass on the cartesian product of each variables

$$M_{X_i} \mid M_{pa(X_i)}$$

- Computation of masses through the junction tree and the bayesian inference extended to belief masses

Encoding interval valued probabilities in BN

Binary case for dependability analysis

- Each node has the following frame of discernment

$$2^{\Omega} = \{Up, Down, \{Up, Down\}\}$$

- Interval valued probability to mass distribution through Möbius transform

$$M_{X_i} = \left[\underline{P}(X_i = \{Up\}) \quad \left(1 - \overline{P}(X_i = \{Up\})\right) \quad \left(\overline{P}(X_i = \{Up\}) - \underline{P}(X_i = \{Up\})\right)\right]$$

- In dependability analysis, conditional probability tables are mainly based on binary logic but extended to the frame of discernment (AND, OR, KooN ...)

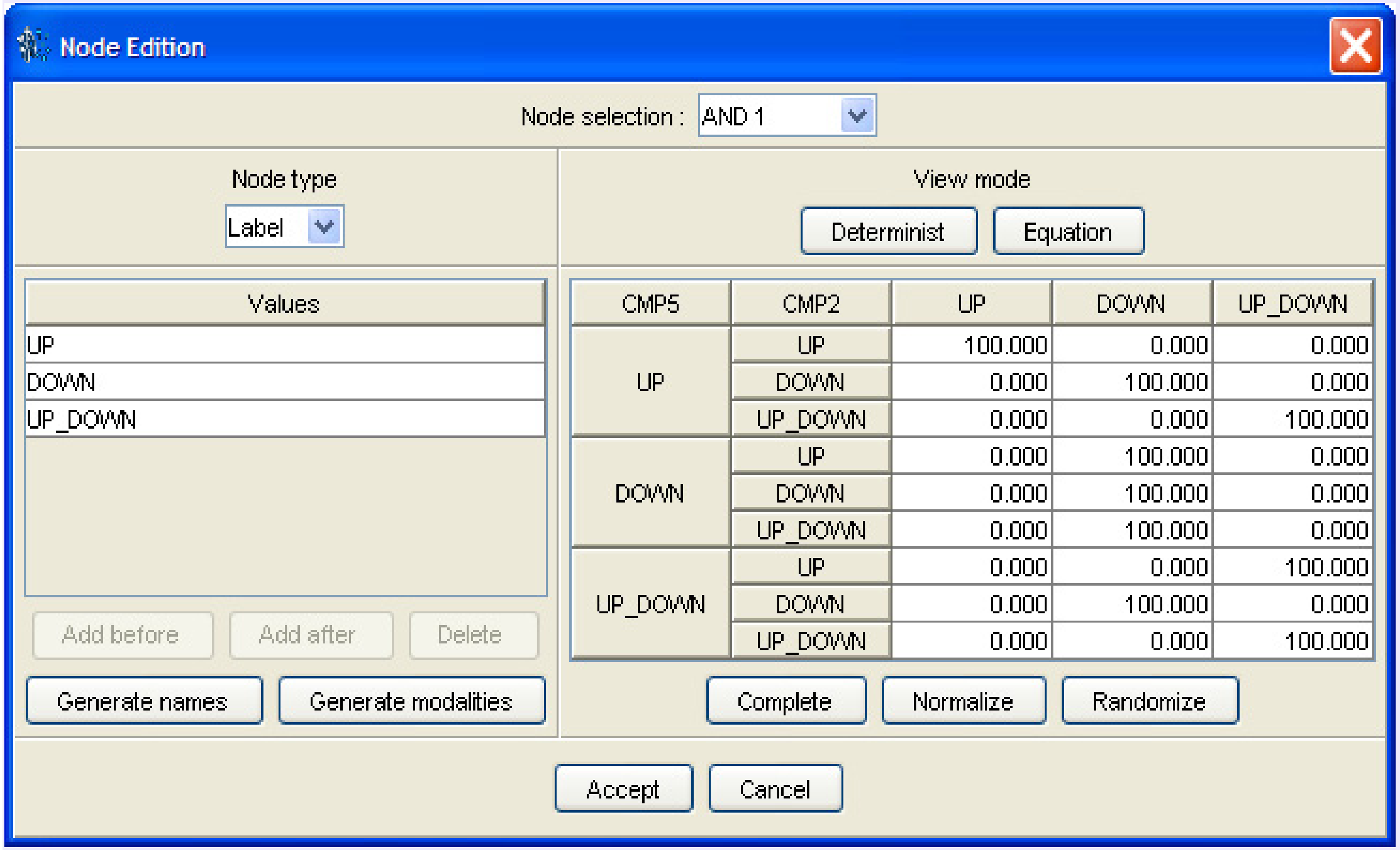
- Finally, from any node, the belief and plausibility measures give the bounds of probability

$$Bel(X_i = Up) = \sum_{state|state \subseteq Up} m(X_i = state) \rightarrow \underline{P}(X_i = Up) \qquad Pls(X_i = Up) = \sum_{state|state \cap Up \neq \varnothing} m(X_i = state) \rightarrow \overline{P}(X_i = Up)$$

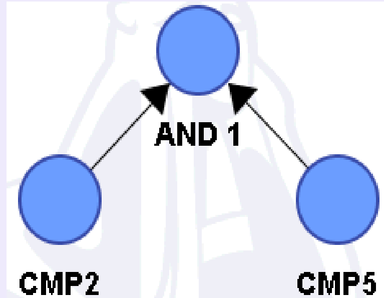
$$\left[Bel(X_i = Up), Pls(X_i = Up)\right] = \left[\underline{P}(X_i = Up), \overline{P}(X_i = Up)\right]$$

Conditional probability table for series systems

AND Logic with 3 states

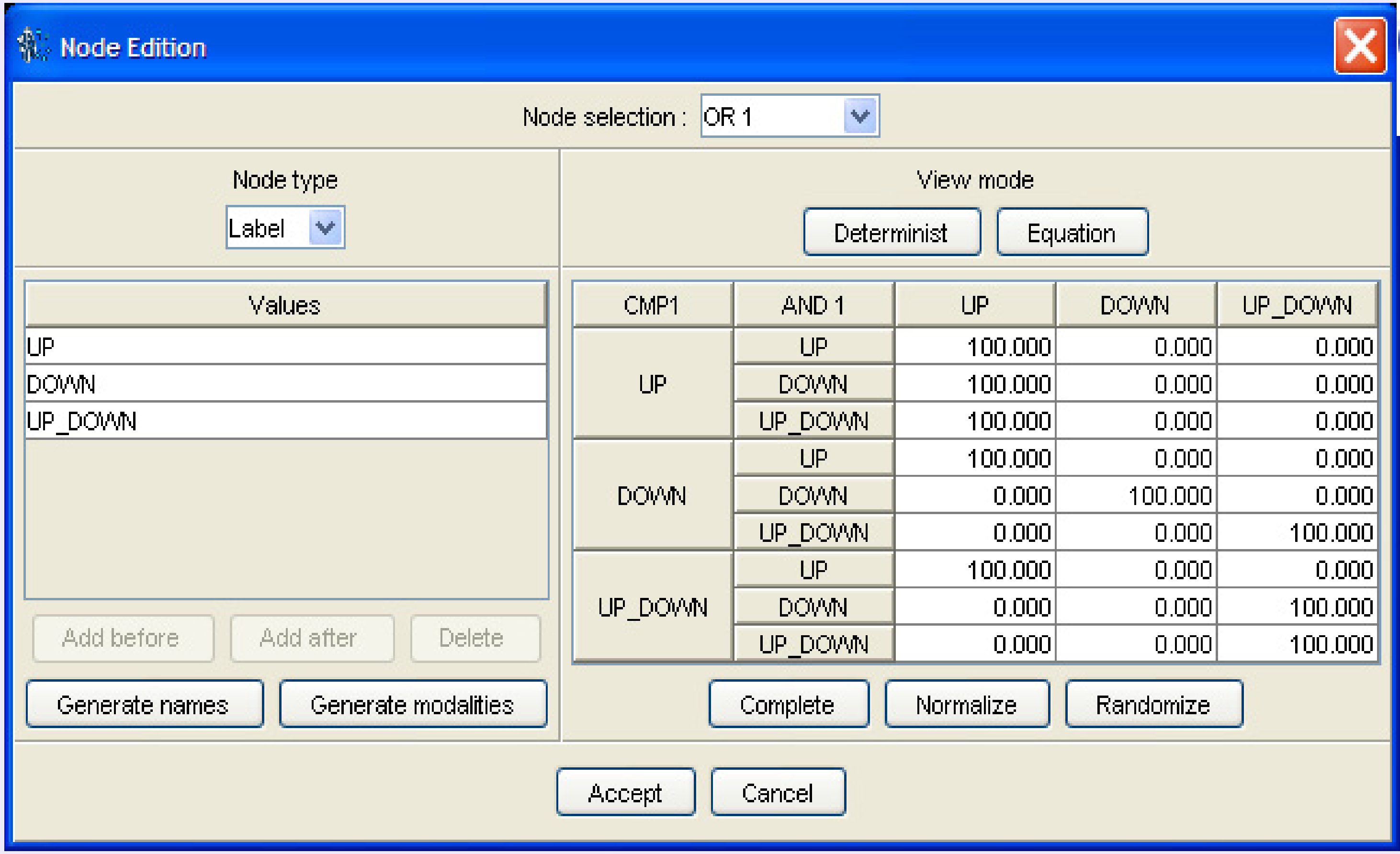


CMP5	CMP2	UP	DOWN	UP_DOWN
UP	UP	100.000	0.000	0.000
	DOWN	0.000	100.000	0.000
	UP_DOWN	0.000	0.000	100.000
DOWN	UP	0.000	100.000	0.000
	DOWN	0.000	100.000	0.000
	UP_DOWN	0.000	100.000	0.000
UP_DOWN	UP	0.000	0.000	100.000
	DOWN	0.000	100.000	0.000
	UP_DOWN	0.000	0.000	100.000

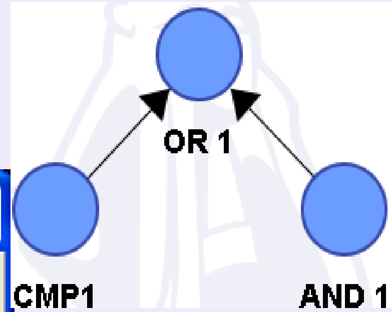


Conditional probability table for // systems

OR Logic with 3 states



CMP1	AND 1	UP	DOWN	UP_DOWN
UP	UP	100.000	0.000	0.000
	DOWN	100.000	0.000	0.000
	UP_DOWN	100.000	0.000	0.000
DOWN	UP	100.000	0.000	0.000
	DOWN	0.000	100.000	0.000
	UP_DOWN	0.000	0.000	100.000
UP_DOWN	UP	100.000	0.000	0.000
	DOWN	0.000	0.000	100.000
	UP_DOWN	0.000	0.000	100.000



Conditional probability table for KooN

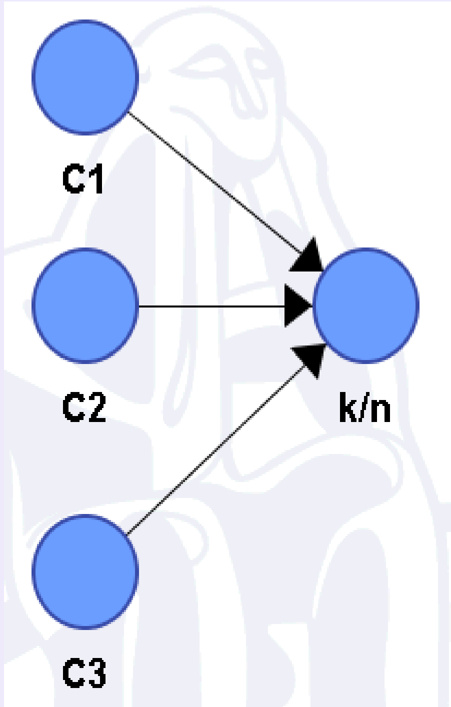
Logic with 3 states

Mode de visualisation

Déterministe | Equation

C1	C2	C3	Up	Down	Up,Down
Up	Up	Up	100,000	0,000	0,000
		Down	100,000	0,000	0,000
		Up,Down	100,000	0,000	0,000
	Down	Up	100,000	0,000	0,000
		Down	0,000	100,000	0,000
		Up,Down	0,000	0,000	100,000
	Up,Down	Up	100,000	0,000	0,000
		Down	0,000	0,000	100,000
		Up,Down	0,000	0,000	100,000
Down	Up	Up	100,000	0,000	0,000
		Down	0,000	100,000	0,000
		Up,Down	0,000	0,000	100,000
	Down	Up	0,000	100,000	0,000
		Down	0,000	100,000	0,000
		Up,Down	0,000	100,000	0,000
	Up,Down	Up	0,000	0,000	100,000
		Down	0,000	100,000	0,000
		Up,Down	0,000	0,000	100,000
Up,Down	Up	Up	100,000	0,000	0,000
		Down	0,000	0,000	100,000
		Up,Down	0,000	0,000	100,000
	Down	Up	0,000	0,000	100,000
		Down	0,000	100,000	0,000
		Up,Down	0,000	0,000	100,000
	Up,Down	Up	0,000	0,000	100,000
		Down	0,000	0,000	100,000
		Up,Down	0,000	0,000	100,000

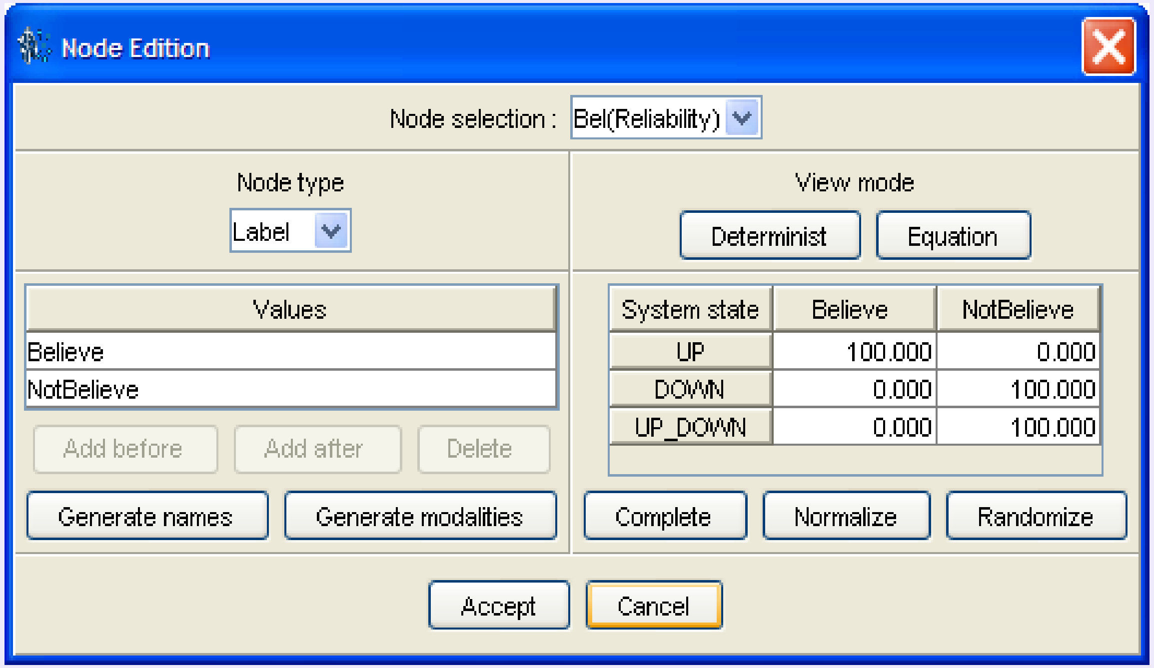
Compléter | Normaliser | Aléatoire

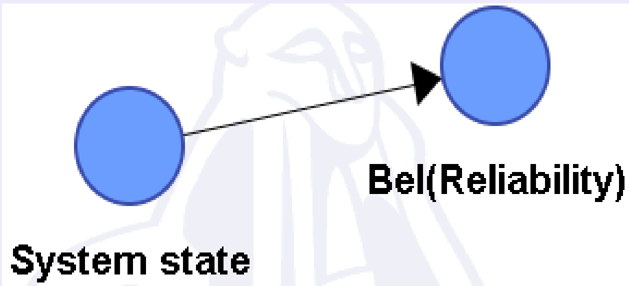


Conditional probability table for Belief measure

Logic formulation of belief measure

$$Bel(X = Up) = \sum_{state \mid state \subseteq Work} m(state)$$

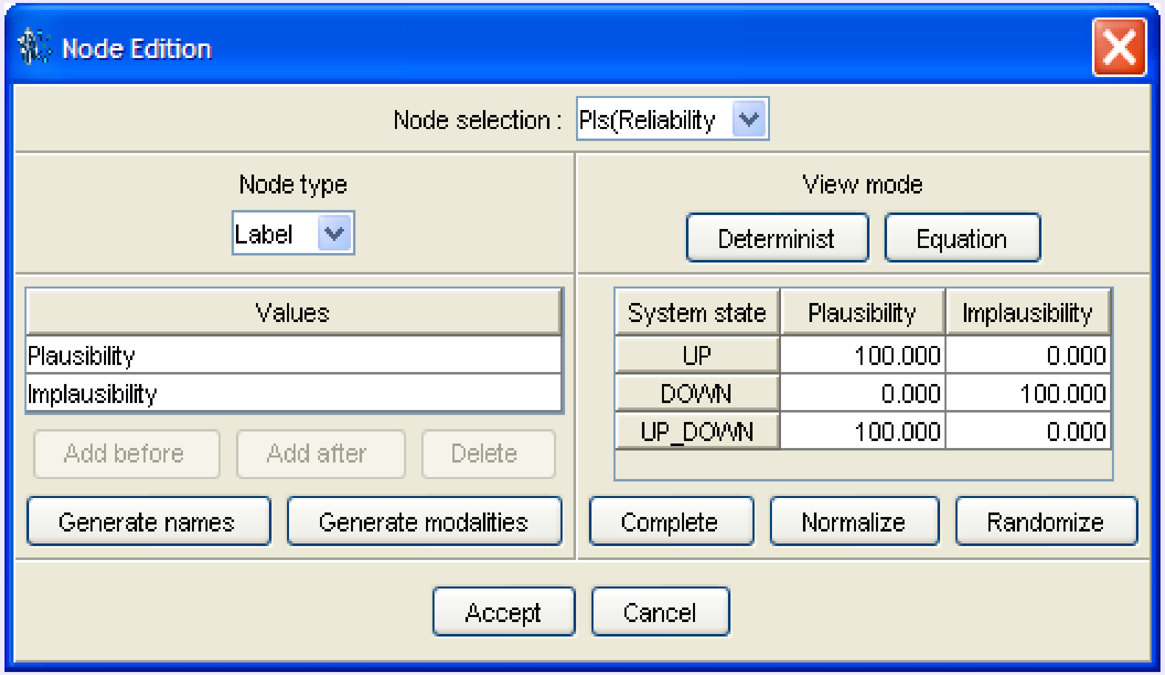


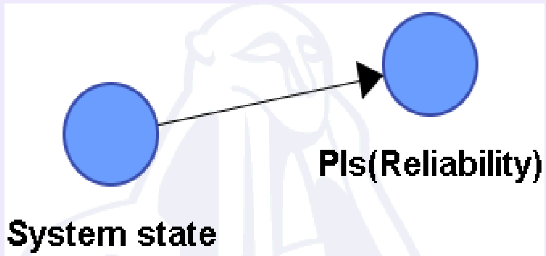


Conditional probability table for Plausibility measure

Logic formulation of plausibility measure

$$Pls(X=Up)=\sum_{state|Up\cap state\neq\emptyset} m(X=state)$$





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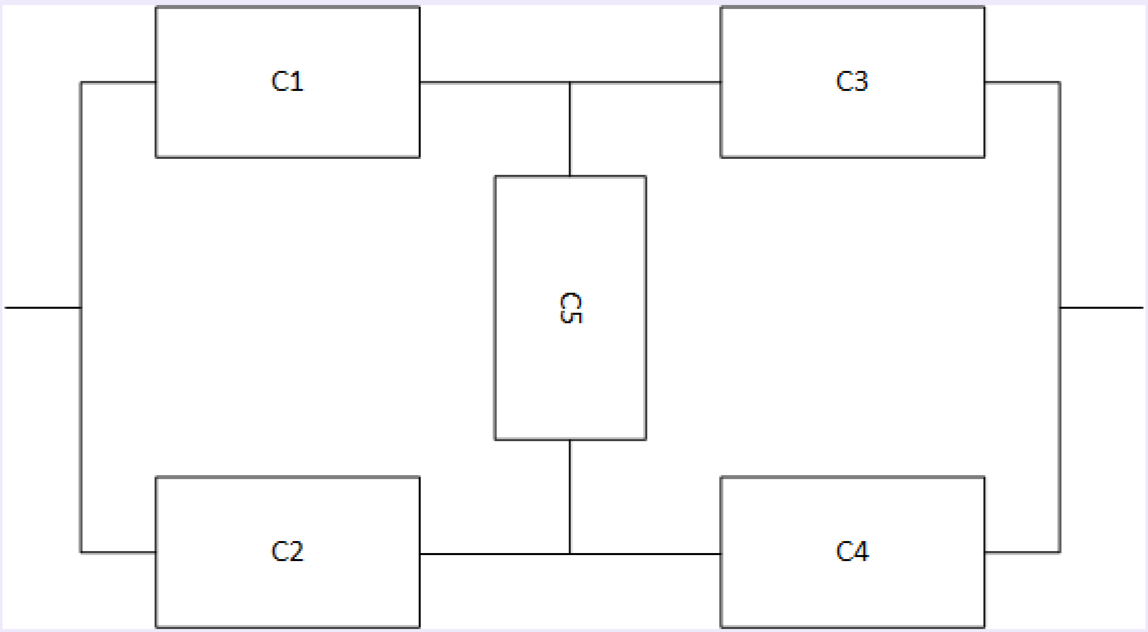
Application to system reliability

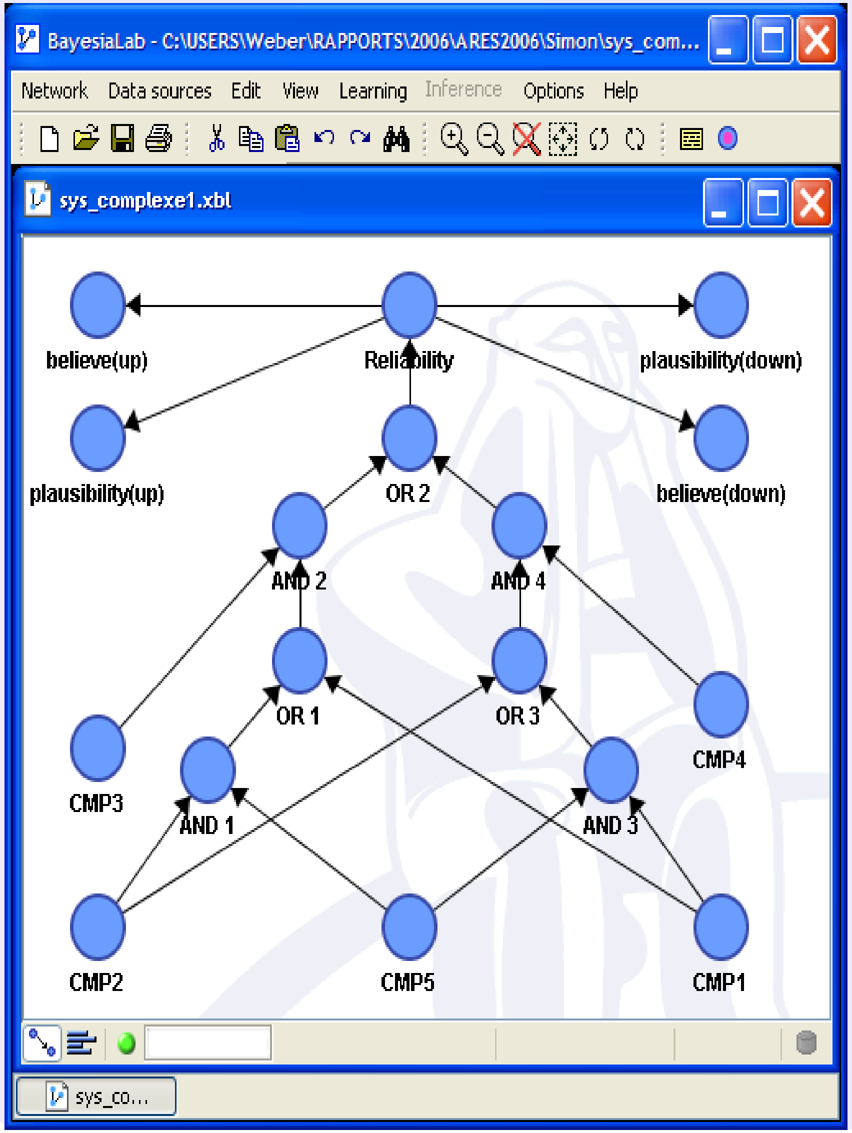
Reliability block diagram of a simple complex system

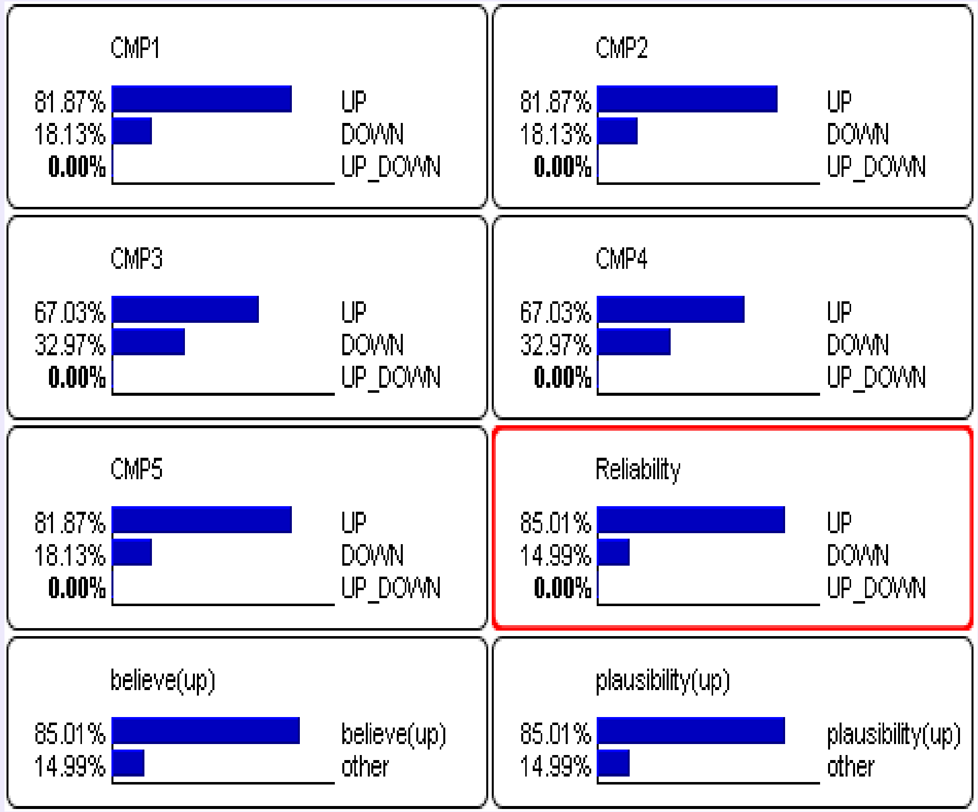
- There is a set of paths from left to right
- A path is defined by a set of working components



System reliability

Without precise probability

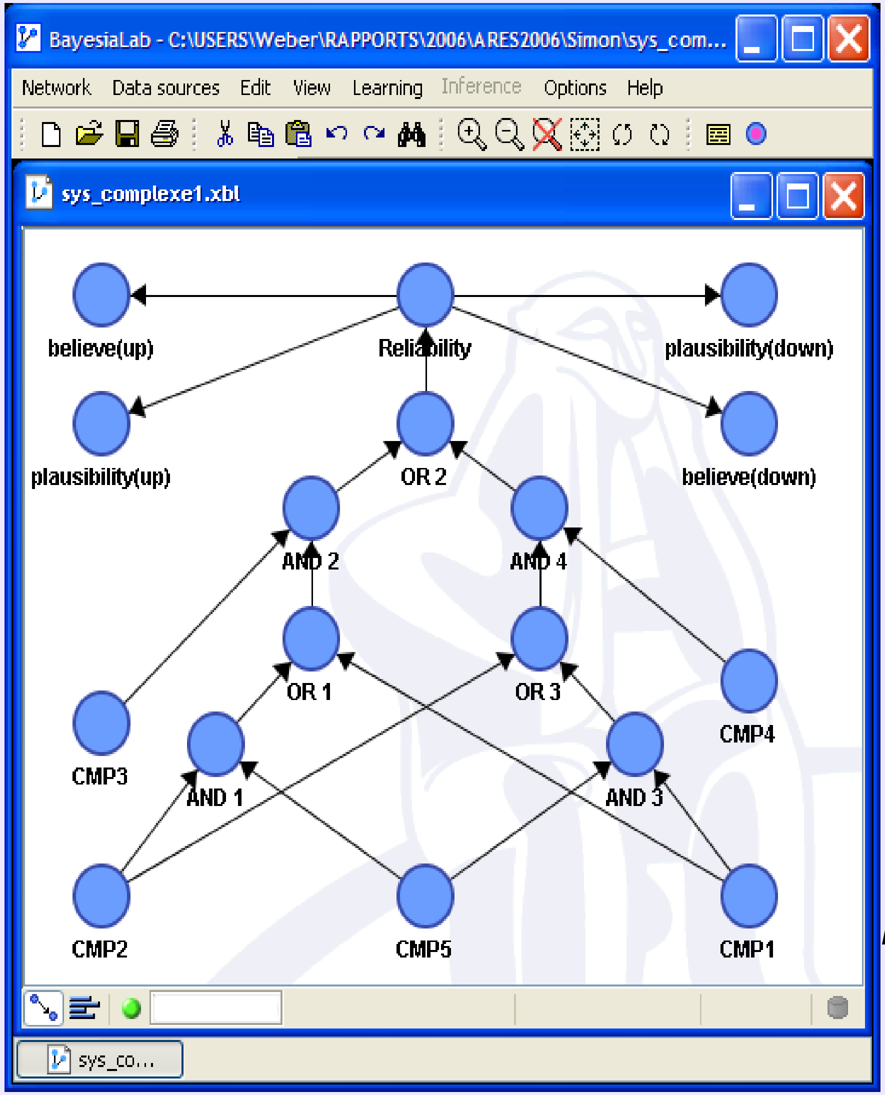


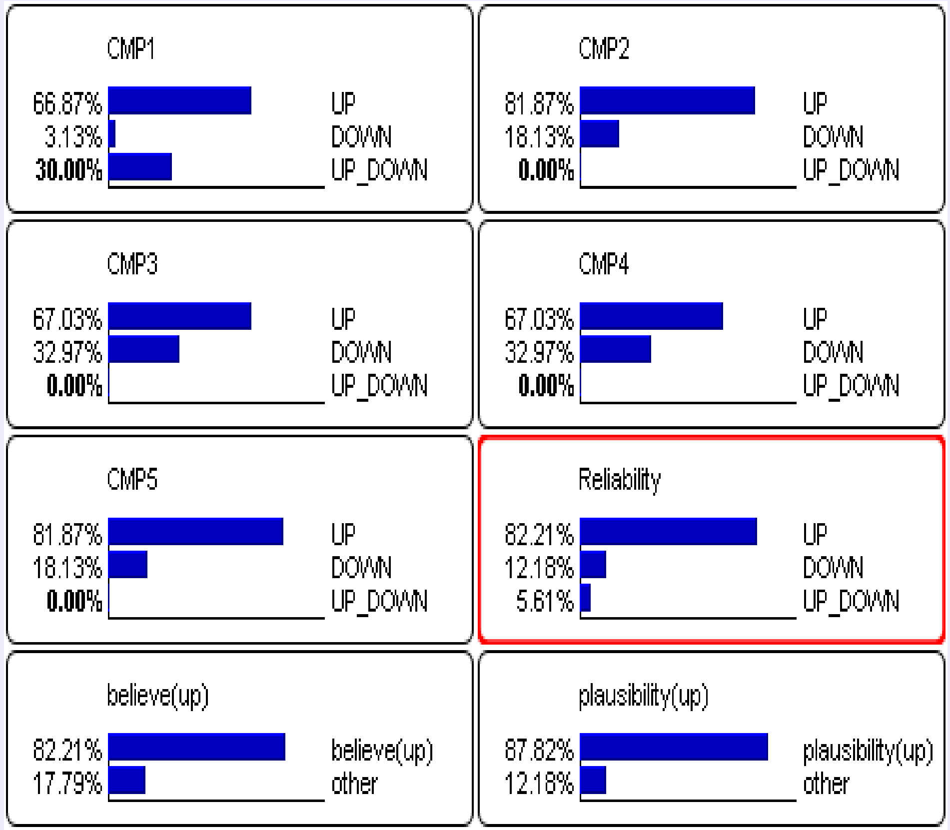


$$Bel(Up) = Pls(Up) = R_S(T_F) = 0,85$$

System reliability with precise probabilities

Without precise probability

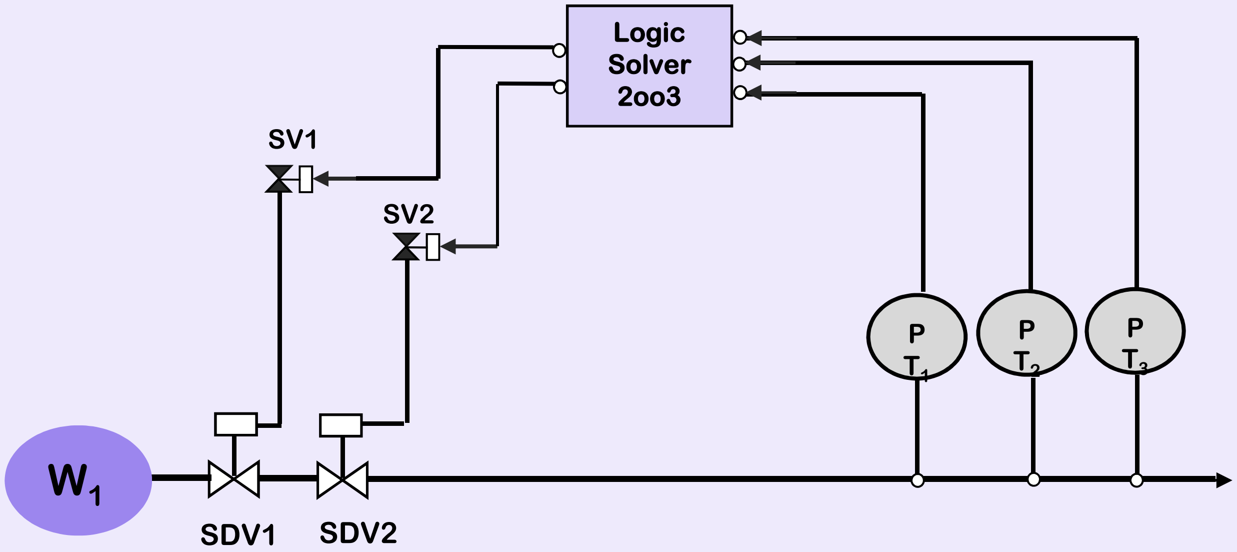




$$Bel(Up) = 0{,}82 < R_S(T_F) = 0{,}85 < Pls(Up) = 0{,}87$$

Study of a High Integrity Safety System (Fuzzy Probabilities)

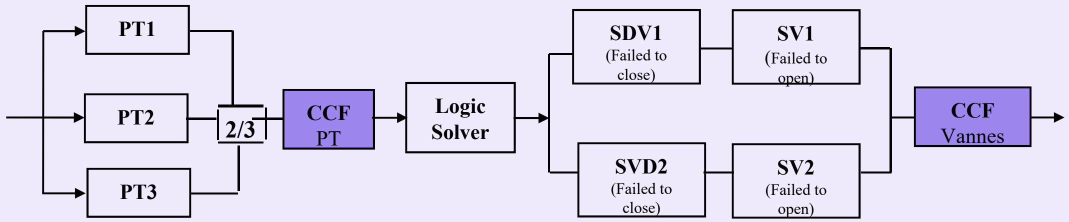
Assessing the probability of failure on demand



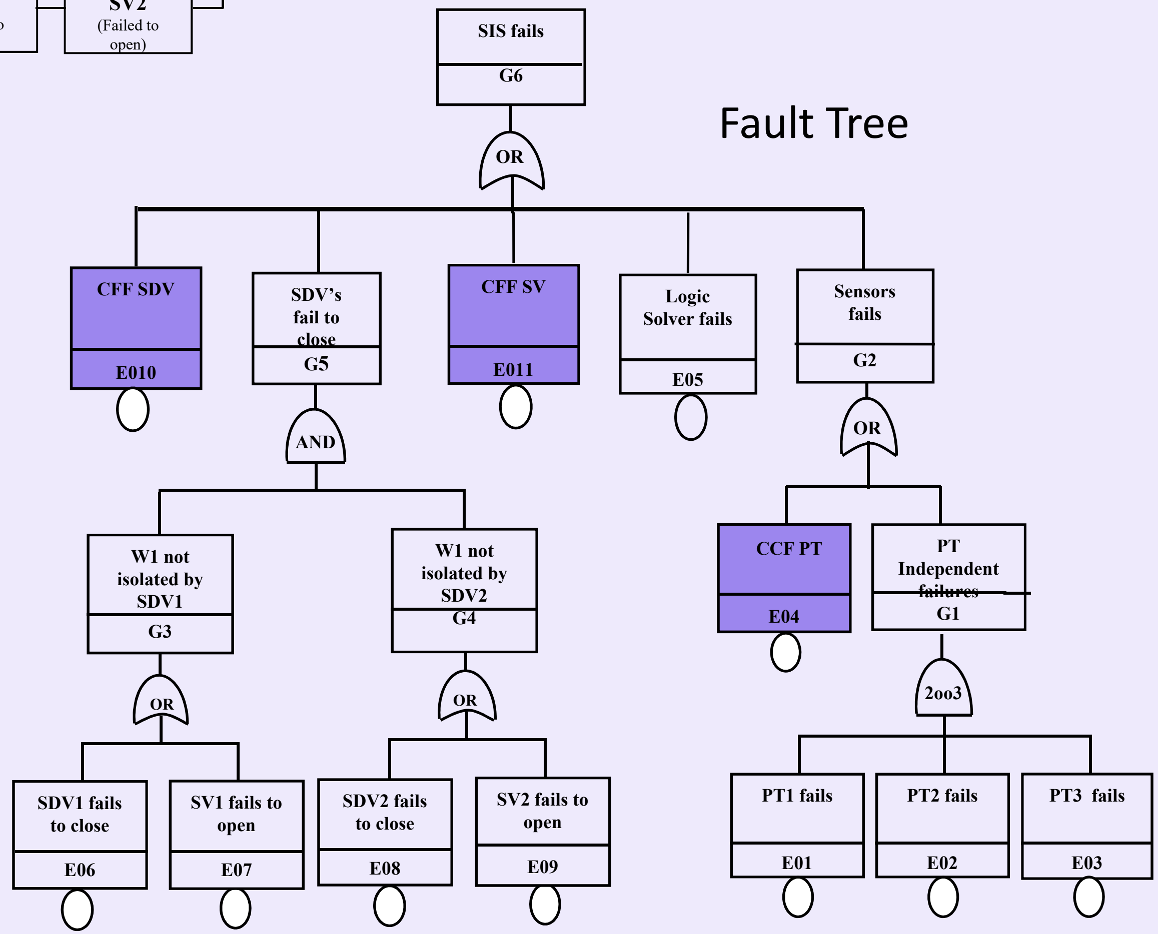
Composants du SIS	λ_D (h^{-1})	DC	β (%) CCF	MTTR	Ti (h)
PT	2.70E- 7	0	<3, 5, 8>	0	T1 = 1460
SDV	5.70E- 6	0	<8, 10,13>	0	T2 = 1460
SV	5.70E- 6	0	<8, 10,13>	0	T2 = 1460
Logic Solver	5.00E- 6	1	-	10	-

Study of a High Integrity Safety System

Reliability Bloc Diagramme or Faul Tree Modeling of the System



Reliability Block Diagram



Fault Tree

Computing fuzzy probability into the network

Algorithm for inference with fuzzy probability

For α from 0 to 1

Define $I^{(\alpha)}$, the set of α-cut of the inputs

ForAll inputs

Define the corresponding basic probability assignment

EndFor

Compute the basic mass assignment of each output in O by inference

ForAll ouputs,

Build the α-cuts of $O^{(\alpha)}$ from the output basic mass assignments

EndFor

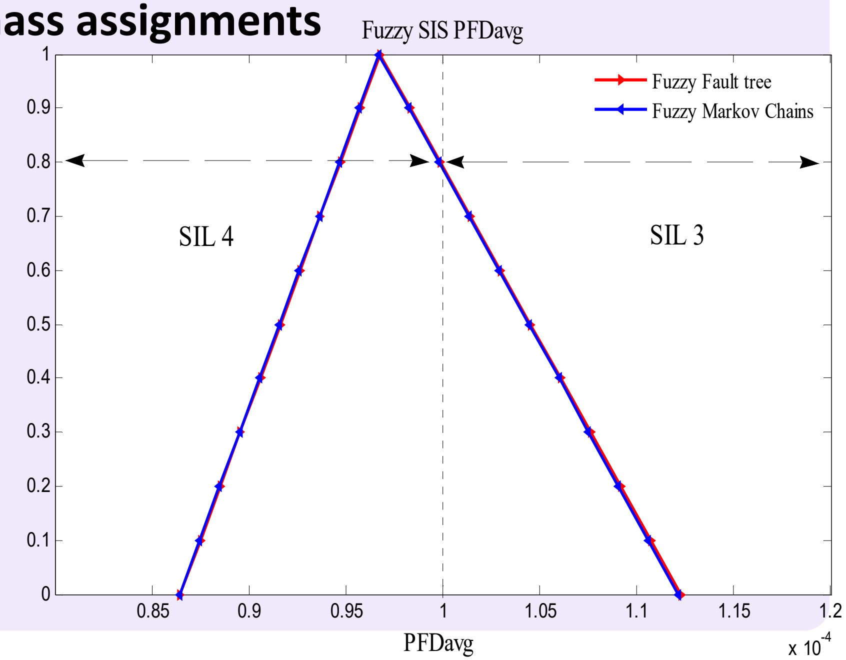
EndFor

ForAll outputs

Embody the nested intervals

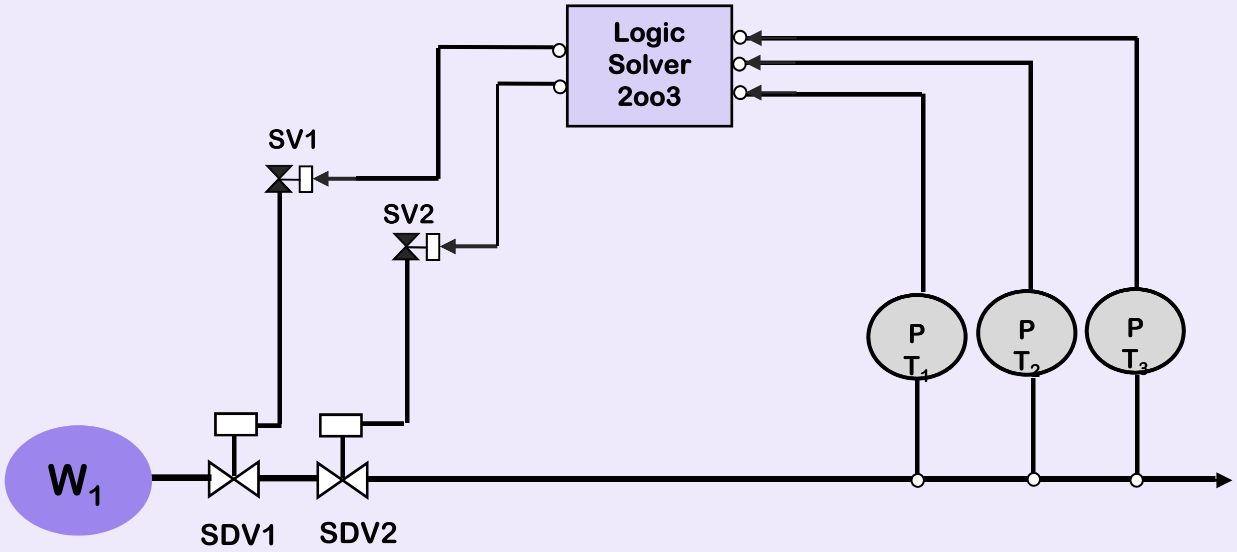
EndFor

where I is the set of input fuzzy numbers and O is the set of output fuzzy numbers.



Study of a High Integrity Safety System (p-boxes)

Assessing the probability of failure on demand

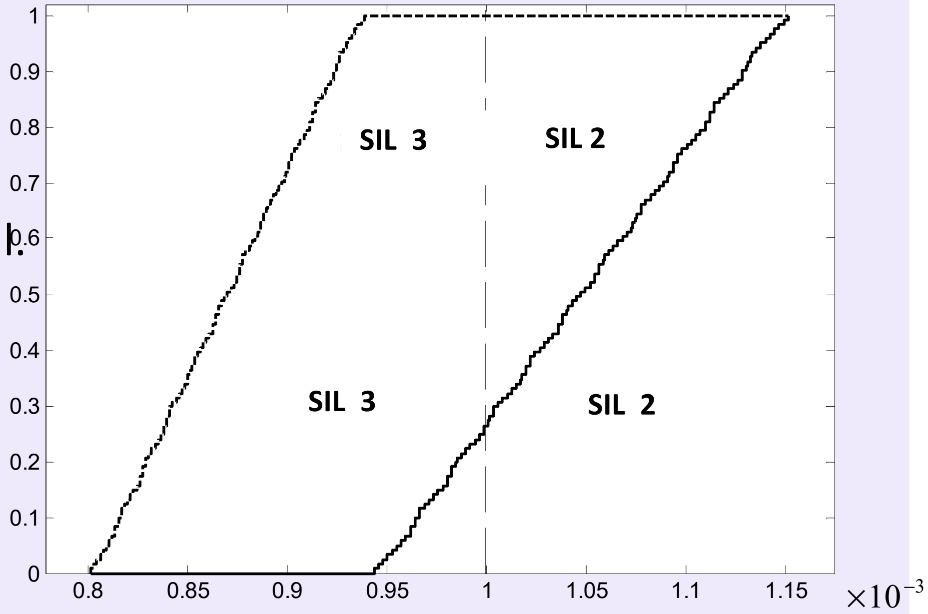


Composants du SIS	λ (h^{-1})	β (%) CCF	Ti (h) Cas 1
PTi	7.00E- 7	U ([3, 5];[5, 8])	T1 = 2190
SDV	3.10E- 6	U ([8, 10];[10,13])	T2 = 2190
SV	2.60E- 6	U ([8, 10];[10,13])	T2 = 2190
Logic Solver	2.15E- 7	-	T3 = 2190

Study of a High Integrity Safety System

Results :

For an arbitrarily large value
    For each input
        Choose a value N in [0,1]
        Define the interval from the p-box corresponding to N.
        Define the weight at 1/N
        Transform the interval into a bpa.
    EndFor
    Compute the outputs by EN inference.
    For each output
        Transform each resulting bpa in an interval.
        Transform the set of intervals and
        weights in a p-box.
    EndFor
EndFor



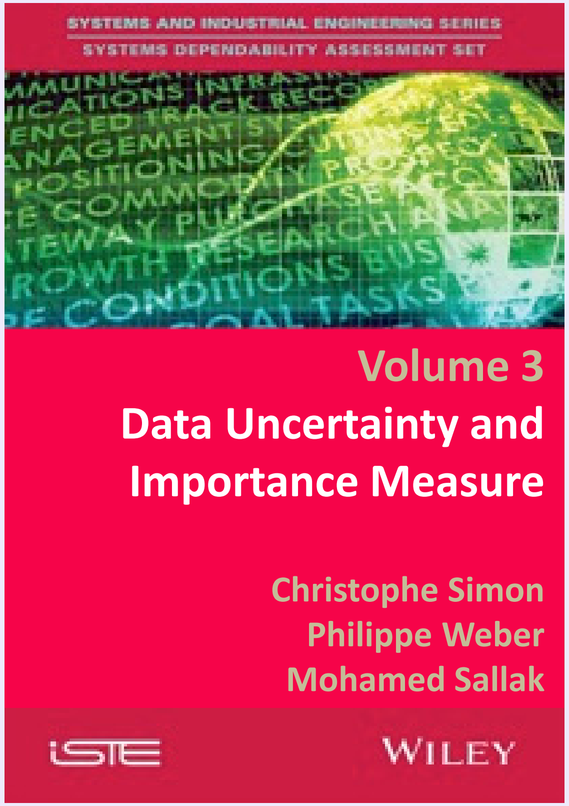
Conclusion

- Bayesian networks can model
  - dependability analysis problems (binary hypothesis)
  - Satisfiabilty problems(non binary problems)
  - Risk analysis
- Bayesian network handle several forms of uncertainty
  - Interval valued probability
  - Fuzzy valued probability
  - Family of probability
- Results are as best case/worst case computation
- Bayesialab is our polyvalent tool for modeling

Open problems

- Complexity of Models regarding the complexity of Systems (managing complexity)
- Model Uncertainty of large systems
- Model validation to increase the confidence
- Dependencies between events
- Sequence of Events
- Incoherent Systems
- Modeling Human behavior
- Integrating new information (according to the theoretical framework used)

Thank you for your attention



Simon C., Weber P., Sallak M.
Data uncertainty and Important measure.
Wiley ISTE, 2018 (172p)