

Mathematics

1. If H is the harmonic mean between P and Q , then

$\frac{H}{P} + \frac{H}{Q}$ is [2012]

- (a) 2 (b) $\frac{P+Q}{Q}$ (c) $\frac{PQ}{P+Q}$ (d) None of these

Ans. (a)

Sol. Given that, H is the harmonic mean between P and Q

$$\text{i.e. } H = \frac{2PQ}{P+Q} \Rightarrow \frac{H}{2} = \frac{PQ}{P+Q} \dots (i)$$

$$\Rightarrow \frac{2}{H} = \frac{P+Q}{PQ}$$

$$\text{Now, } \frac{H}{P} + \frac{H}{Q} = H \left(\frac{P+Q}{PQ} \right) = H \cdot \frac{2}{H} = 2 \text{ [from Eq. (i)]}$$

2. The number of values of k for which the system of equations $(k+1)x + 8y = 4k$ and $kx + (k+3)y = 3k-1$ has infinitely many solution, is [2012]

- (a) 0 (b) 1 (c) 2 (d) infinite

Ans. (b)

Sol. Given system of equations,

$$(k+1)x + 8y = 4k$$

$$kx + (k+3)y = 3k-1$$

Since, the given system has infinitely many solutions

$$\therefore \frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$$

Taking Ist and IIIrd part,

$$(k+1)(3k-1) = 4k^2$$

$$\Rightarrow 3k^2 + 2k - 1 = 4k^2$$

$$\Rightarrow k^2 - 2k + 1 = 0$$

$$\Rightarrow (k-1)^2 = 0$$

$$\therefore k = 1$$

3. The sum of ${}^{20}C_8 + {}^{20}C_9 + {}^{21}C_{10} + {}^{22}C_{11} - {}^{23}C_{11}$ is [2012]

- (a) ${}^{22}C_{12}$ (b) ${}^{23}C_{12}$ (c) 0 (d) ${}^{21}C_{10}$

Ans. (c)

$$\text{Sol. } {}^{20}C_8 + {}^{20}C_9 + {}^{21}C_{10} + {}^{22}C_{11} - {}^{23}C_{11}$$

$$= ({}^{21}C_9 + {}^{21}C_{10}) + {}^{22}C_{11} - {}^{23}C_{11}$$

$$= ({}^{22}C_{10} + {}^{22}C_{11}) - {}^{23}C_{11} = {}^{23}C_{11} - {}^{23}C_{11} = {}^{n+1}C_{r+1}$$

$$= 0$$

4. The value of $\cot^{-1}(21) + \cot^{-1}(13) + \cot^{-1}(-8)$ is [2012]

- (a) 0 (b) π (c) ∞ (d) $\frac{\pi}{2}$

Ans.

Sol. (b) $\cot^{-1}(21) + \cot^{-1}(13) + \cot^{-1}(-8)$

$$\Rightarrow \tan^{-1}\left(\frac{1}{21}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \cot^{-1}(-8) \left(\because \cot^{-1}x = \tan^{-1}\frac{1}{x} \right)$$

$$\Rightarrow \tan^{-1}\left\{ \frac{\frac{1}{21} + \frac{1}{13}}{1 - \frac{1}{21} \cdot \frac{1}{13}} \right\} + \cot^{-1}(-8)$$

$$\left\{ \because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \right\}$$

$$\Rightarrow \tan^{-1}\left(\frac{34}{272}\right) + \tan^{-1}\left(-\frac{1}{8}\right) = \tan^{-1}\left(\frac{34}{272}\right) + \pi - \tan^{-1}\left(\frac{1}{8}\right)$$

$$\Rightarrow \pi + \tan^{-1}\left\{ \frac{\frac{34}{272} - \frac{1}{8}}{1 + \frac{34}{272} \cdot \frac{1}{8}} \right\} = \tan^{-1}\left\{ \frac{34-34}{2210} \right\} + \pi$$

$$= \pi + \tan^{-1}(0) = 0 + \pi = \pi$$

5. Normal to the curve $y = x^3 - 3x + 2$ at the point $(2, 4)$ is [2012]

- (a) $9x - y - 14 = 0$ (b) $x - 9y + 40 = 0$
(c) $x + 9y - 38 = 0$ (d) $-9x + y + 22 = 0$

Ans. (c)

Sol. Given curve, $y = x^3 - 3x + 2$

Now,

$$\frac{dy}{dx} = 3x^2 - 3$$

$$\Rightarrow \frac{dy}{dx}_{\text{at}(2,4)} = 3(2)^2 - 3 = 12 - 3 = 9$$

$$\therefore \text{Slope of normal} = -\frac{1}{9}$$

Hence, the equation of normal at point $(2, 4)$

$$\Rightarrow (y-4) = -\frac{1}{9}(x-2)$$

$$\Rightarrow 9y-36 = -x+2$$

$$\Rightarrow x+9y = 38$$

$$\Rightarrow x+9y-38 = 0$$

6. The value of

$$\lim_{n \rightarrow \infty} \frac{\pi}{n} \left[\sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{(n-1)\pi}{n} \right] \text{ is [2012]}$$

- (a) 0 (b) π (c) 2 (d) $\frac{\pi}{2}$

Ans. (a)

$$\begin{aligned} \text{Sol. } \lim_{n \rightarrow \infty} \frac{\pi}{n} \left\{ \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \left(\frac{n-1}{n} \right) \pi \right\} \\ = \lim_{n \rightarrow \infty} \frac{\pi}{n} \left\{ \sin \left(\frac{\pi}{n} \right) + \sin \left(\frac{\pi}{n} + \frac{\pi}{n} \right) + \dots + \sin \left(\frac{\pi}{n} + \frac{n\pi}{n} \right) \right\} \end{aligned}$$

$$\because \sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots + \sin(\alpha + n\beta)$$

$$= \frac{\sin \left(\frac{2\alpha + n\beta}{2} \right) \cdot \sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}}$$

$$= \lim_{n \rightarrow \infty} \frac{\pi}{n} \cdot \frac{\sin \left\{ \frac{\pi}{n} \left(\frac{\pi}{n} + \frac{n\pi}{n} \right) \right\} \cdot \sin \frac{n}{2} \cdot \frac{\pi}{n}}{\sin \frac{\pi}{2n}}$$

$$= \lim_{n \rightarrow \infty} \frac{\pi}{n} \cdot \frac{\sin \left(\frac{2n + n\pi}{n} \right) \cdot \sin \frac{\pi}{2}}{\sin \frac{\pi}{2n}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{2 \left(\frac{\sin \frac{\pi}{2n}}{\frac{\pi}{2n}} \right) \cdot \sin \left(\pi + \frac{2\pi}{n} \right) \cdot 1} \left(\because \lim_{\theta \rightarrow \infty} \frac{\sin \frac{1}{\theta}}{\frac{1}{\theta}} = 1 \right)$$

$$= \frac{1}{2 \cdot 1} \cdot \sin(\pi + 0)$$

$$= \frac{1}{2} \cdot 0 = 0$$

7. The point on the curve $y = 6x - x^2$, where the tangent is parallel to X -axis is [2012]

- (a) (0,0) (b) (2,8) (c) (6,0) (d) (3,9)

Ans. (d)

Sol. Given curve, $y = 6x - x^2$

On differentiating w.r.t. x , we get

$$\frac{dy}{dx} = 6 - 2x$$

$\therefore$ Slope of tangent parallel to x -axis is $\frac{dy}{dx} = 0$

$$\therefore 6 - 2x = 0 \Rightarrow x = 3$$

$$y = 6(3) - (3)^2 = 18 - 9$$

$$y = 9$$

[from Eq. (i)]

$\therefore$ Only one point (3,9) at which the tangent is parallel to X -axis.

8. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and

$I_4 = \int_1^2 2^{x^3} dx$, then [2012]

- (a) $I_1 = I_2$ (b) $I_2 > I_1$ (c) $I_3 > I_4$ (d) $I_4 > I_3$

Ans. (d)

Sol. $\because x^2 > x^3 \forall x \in (0,1)$

$$\Rightarrow 2^{x^2} > 2^{x^3} \forall x \in (0,1)$$

$$\Rightarrow \int_0^1 2^{x^2} dx > \int_0^1 2^{x^3} dx$$

Now, $x^2 < x^3; \forall x \in (1,2)$

$$\Rightarrow 2^{x^2} < 2^{x^3}, \forall x \in (1,2)$$

$$\Rightarrow \int_1^2 2^{x^2} dx < \int_1^2 2^{x^3} dx \dots (i)$$

$$\Rightarrow I_3 < I_4 \text{ or } I_4 > I_3$$

9. The value of integral $\int_0^{\pi/2} \log \tan x dx$ is [2012]

- (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) 0

Ans. (d)

Sol. Let $I = \int_0^{\pi/2} \log \tan x dx$

Use definite integral property,

$$I = \int_0^{\pi/2} \log \tan \left(\frac{\pi}{2} - x \right) dx$$

$$= \int_0^{\pi/2} \log \cot x dx \quad (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} (\log \tan x + \log \cot x) dx$$

$$(\because \log m + \log n = \log mn)$$

$$= \int_0^{\pi/2} \log (\tan x \cdot \cot x) dx$$

$$= \int_0^{\pi/2} \log 1 dx = \int_0^{\pi/2} 0 dx$$

$$= 0$$

10. A determinant is chosen at random from the set of all determinants of matrices of order 2 with elements 0 and 1 only. The probability that the determinant chosen is non-zero, is [2012]

- (a) $\frac{3}{16}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) None of these

Ans. (b)

Sol. The total sample events $n(s) = 4 \cdot (2)^2 = 4 \times 4 = 16$ and

total favourable cases $n(E) = 6$

Which is $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{6}{16} = \frac{3}{8}$$

11. If $\sin^2 x = 1 - \sin x$, then $\cos^4 x + \cos^2 x$ is equal to [2012]

- (a) 0 (b) 1 (c) $\frac{2}{3}$ (d) -1

Ans. (b)

Sol. Given $\sin^2 x = 1 - \sin x$

$$\Rightarrow 1 - \cos^2 x = 1 - \sin x$$

$$\Rightarrow \sin x = \cos^2 x$$

Now, $\cos^4 x + \cos^2 x = (\cos^2 x)^2 + \cos^2 x$

$$= (\sin x)^2 + \sin x$$

$$= \sin^2 x + \sin x$$

$$= (1 - \sin x) + \sin x \quad [\text{from Eq. (i)}]$$

$$= 1$$

12. The equation of the plane passing through the point (1, 2, 3) and having the vector $\mathbf{N} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ as its normal, is [2012]

- (a) $2x - y + 3z + 7 = 0$ (b) $3x - y + 2z + 7 = 0$
(c) $3x - y + 2z = 7$ (d) $3x + y + 2z = 7$

Ans. (c)

Sol. The equation of the plane passing through the point (1, 2, 3) and having the vector $\mathbf{N} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ as its normal is $3(x-1) - 1(y-2) + 2(z-3) = 0$

$$\Rightarrow 3x - y + 2z + (-3 + 2 - 6) = 0$$

$$\Rightarrow 3x - y + 2z = 7$$

13. The value of $\int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt$ is

[2012]

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) 1 (d) None of these

Ans. (a)

Sol. Let $f(z) = \int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt$

$$f'(x) = \sin^{-1}(\sin x)(2 \sin x \cos x)$$

$$+ \cos^{-1}(\cos x)(-2 \sin x \cdot \cos x)$$

$$= x \cdot \sin 2x - x \cdot \sin 2x$$

$$= 0$$

$$\Rightarrow f(x) = \text{Constant}$$

Now, we check the constant value of this integration on different value of x .

(i) At $x = \frac{\pi}{4}$

$$f\left(\frac{\pi}{4}\right) = \int_0^{1/2} \sin^{-1} \sqrt{t} dt + \int_0^{1/2} \cos^{-1} \sqrt{t} dt$$

$$= \int_0^{1/2} (\sin^{-1} \sqrt{t} + \cos^{-1} \sqrt{t}) dt$$

$$= \int_0^{1/2} \frac{\pi}{2} dt$$

$$= \frac{\pi}{2} \left(\frac{1}{2} - 0 \right) = \frac{\pi}{4}$$

$$f(0) = 0 + \int_0^1 \cos^{-1} \sqrt{t} dt$$

$$\text{Let } t = \cos^2 \theta, dt = -\sin 2\theta d\theta$$

$$= -\int_{\pi/2}^0 \theta \cdot \sin 2\theta d\theta$$

$$= \int_0^{\pi/2} \theta \cdot \sin 2\theta d\theta$$

$$= \left[-\theta \frac{\cos 2\theta}{2} + \frac{1}{4} \sin 2\theta \right]_0^{\pi/2} f(x) dx = -\int_b^a f(x) dx$$

$$= \left[-\frac{\pi}{2} \cdot \frac{1}{2} (-1) + 0 \right] = \frac{\pi}{4} k$$

(iii) At $x = \frac{\pi}{2}$,

$$f\left(\frac{\pi}{2}\right) = \int_0^1 \sin^{-1} \sqrt{t} dt + 0$$

$$\text{Let } t = \sin^2 \theta, dt = \sin 2\theta d\theta$$

$$= \int_0^{\pi/2} \theta \cdot \sin 2\theta \cdot d\theta$$

$$= \left[-\theta \cdot \frac{\cos 2\theta}{4} + \frac{\sin 2\theta}{4} \right]_0^{\pi/2}$$

$$= \left[-\frac{\pi}{2} \cdot \frac{1}{2} (-1) + 0 \right] = \frac{\pi}{4}$$

14. Coefficients of quadratic equation $ax^2 + bx + c = 0$ are chosen by tossing three fair coins, where 'head' means one and 'tail' means two. Then the probability that roots of the equation are imaginary, is **[2012]**

- (a) $\frac{7}{8}$ (b) $\frac{5}{8}$ (c) $\frac{3}{8}$ (d) $\frac{1}{8}$

Ans. (a)

Sol. Total sample events $n(S) = (2)^3 = 8$

Cases	Value	Condition for imaginary roots] $b^2 - 4ac < 0$
H, T, T	1, 2, 2	$(2)^2 - 4(1)(2) < 0$
H, H, T	1, 1, 2	$(1)^2 - 4(1)(2) < 0$
H, T, H	1, 2, 1	$(2)^2 - 4(1)(1) = 0$
H, H, H	1, 1, 1	$(1)^2 - 4(1)(1) < 0$
T, H, H	2, 1, 1	$(1)^2 - 4(2)(1) < 0$
T, T, H	2, 2, 1	$(2)^2 - 4(2)(1) < 0$
T, H, T	2, 1, 2	$(1)^2 - 4(2)(2) = 0$
T, T, T	2, 2, 2	$(2)^2 - 4(2)(2) < 0$

$\therefore$ Total favourable events $n(E) = 7$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{7}{8}$

15. In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in Physics. Then, the number of students who have passed in Physics only, is **[2012]**

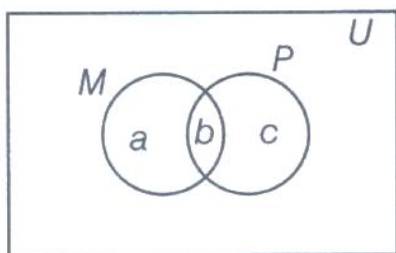
- (a) 22 (b) 33 (c) 10 (d) 45

Ans. (d)

Sol. Given $U = 100$

$$a + b = 55 \dots (i)$$

$$b + c = 67 \dots (ii)$$



and

$$a + b + c = 100 \dots (iii)$$

From Eqs. (i) and (iii), we get

$$(a + b) + c = 100$$

$$\Rightarrow 55 + c = 100$$

$$\Rightarrow c = 100 - 55 = 45$$

Hence, the number of students passed in Physics only is 45.

16. If $(4, -3)$ and $(-9, 7)$ are the two vertices of a triangle and $(1, 4)$ is its centroid, then the area of triangle is **[2012]**

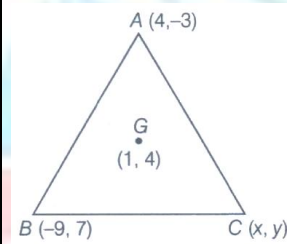
- (a) $\frac{138}{2}$ (b) $\frac{319}{2}$ (c) $\frac{183}{2}$ (d) $\frac{381}{2}$

Ans.

Sol. (c) We know that, Centroid of the triangle,

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) = (1, 4)$$

$$\Rightarrow \left\{ \frac{4 - 9 + x}{3}, \frac{-3 + 7 + y}{3} \right\} = (1, 4)$$



$$\Rightarrow \left(\frac{x - 5}{3}, \frac{y + 4}{3} \right) = (1, 4)$$

$$\Rightarrow x - 5 = 3 \Rightarrow x = 8$$

$$\text{and } y + 4 = 12 \Rightarrow y = 8$$

So, third vertex of a $\triangle ABC$ is $(8, 8)$

$$\text{Now, areas of } \triangle ABC = \frac{1}{2} \begin{vmatrix} 4 & -3 & 1 \\ -9 & 7 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

use $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$,

$$= \frac{1}{2} \begin{vmatrix} 4 & -3 & 1 \\ -13 & 10 & 0 \\ 4 & 11 & 0 \end{vmatrix}$$

Expand with respect C_3

$$= \frac{1}{3} \{-143 - 40\}$$

$$= \frac{1}{2} |-183| = \frac{183}{2}$$

17. The equation of the ellipse with major axis along the X -axis and passing through the points $(4, 3)$ and $(-1, 4)$ is **[2012]**

- (a) $15x^2 + 7y^2 = 247$ (b) $7x^2 + 15y^2 = 247$
 (c) $16x^2 + 9y^2 = 247$ (d) $9x^2 + 16y^2 = 247$

Ans.

Sol. (b) The equation of an ellipse whose major axis along X -axis is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots (i)$$

Eq. (i) passes through the points $(4, 3)$ and $(-1, 4)$,

then
and

$$\frac{16}{a^2} + \frac{9}{b^2} = 1 \dots (ii)$$

$$\frac{1}{a^2} + \frac{16}{b^2} = 1 \dots (iii)$$

From Eqs. (ii) and (iii), we get

$$16 \left(1 - \frac{16}{b^2} \right) + \frac{9}{b^2} = 1$$

$$\Rightarrow \frac{9}{b^2} - \frac{256}{b^2} = 1 - 16$$

$$\Rightarrow \frac{247}{b^2} = 15$$

$$\therefore b^2 = \frac{247}{15}$$

From Eq. (iii),

$$\frac{1}{a^2} = 1 - \frac{16}{b^2} = 1 - \frac{15}{247} \times 16$$

$$\Rightarrow \frac{1}{a^2} = \frac{247 - 240}{247} = \frac{7}{247}$$

$$\Rightarrow \left(a^2 = \frac{247}{7} \right)$$

Now, put the value of a^2 and b^2 in Eq. (i) and get the required equation of an ellipse

$\Rightarrow$

$$\frac{7x^2}{247} + \frac{15y^2}{247} = 1$$

$$7x^2 + 15y^2 = 247$$

18. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is [2012]

- (a) 2 or $-\frac{3}{2}$ (b) -2 or $-\frac{3}{2}$
 (c) 2 or $\frac{3}{2}$ (d) -2 or $\frac{3}{2}$

Ans. (a)

Sol. Let $S_1 \equiv x^2 + y^2 + 2x + 2ky + 6 = 0$

Here $g_1 = 1, f_1 = k, C_1 = 6$, Centre $\rightarrow (-1, -k)$

and

$$S_2 \equiv x^2 + y^2 + 2ky + k = 0$$

Here, $g_2 = 0, f_2 = k$ and $C_2 = k$, Centre $\rightarrow (0, -k)$

If two circles intersect orthogonally, then

(Distance between two centres)²

$$= (\text{Radius of circle } S_1)^2 + (\text{Radius of circle } S_2)^2$$

$$(-1-0)^2 + (-k+k)^2 = (\sqrt{1+k^2-6})^2 + (\sqrt{0+k^2-k})^2$$

$$\Rightarrow 1+0 = (k^2-5) + (k^2-k)$$

$$\Rightarrow 2k^2 - k - 6 = 0$$

$$\Rightarrow 2k^2 - 4k + 3k - 6 = 0$$

$$\Rightarrow 2k(k-2) + 3(k-2) = 0$$

$$\Rightarrow (k-2)(2k+3) = 0$$

$$\therefore k = -\frac{3}{2} \text{ or } 2$$

19. Focus of the parabola

$$x^2 + y^2 - 2xy - 4(x+y-1) = 0 \text{ is } [2012]$$

- (a) (1,1) (b) (1,2) (c) (2,1) (d) (0,2)

Ans. (a)

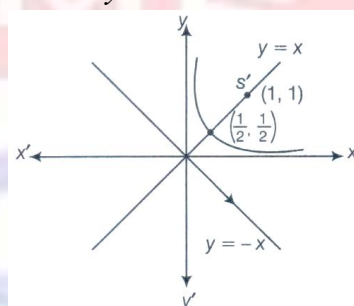
Sol. $x^2 + y^2 - 2xy - 4(x+y-1) = 0$

$\Rightarrow$

$$(x-y)^2 = 4\{(x+y)-1\}$$

Here, $x-y=0$

and $x+y=1$



On solving, we get

$$x = y = \frac{1}{2}$$

$$\therefore \text{Centre of parabola} = \left(\frac{1}{2}, \frac{1}{2} \right)$$

$$\text{Then, its focus, } S' = \left(2 \times \frac{1}{2}, 2 \times \frac{1}{2} \right)$$

$$= (1, 1)$$

20. If **a**, **b** and **c** are unit vectors such that

a + b + c = 0, then the value of **a · b + b · c + c · a** is

[2012]

- (a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

Ans. (d)

Sol. Given **a**, **b** and **c** are unit vectors.

$$\therefore |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$$

Now, we have

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$$

$$\therefore |\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = 0$$

$$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 0$$

$$\Rightarrow 1 + 1 + 1 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 0$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = -\frac{3}{2}$$

21. If two towers of heights h_1 and h_2 subtend angles 60° and 30° respectively at the mid-point of the line joining their feet, then $h_1 : h_2$ is [2012]

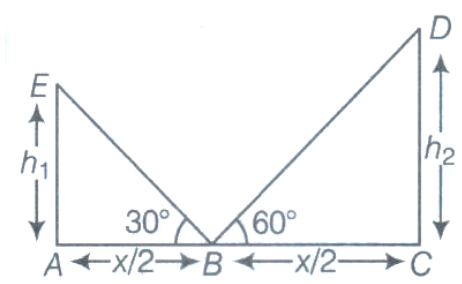
- (a) 1:2 (b) 1:3 (c) 2:1 (d) 3:1

Ans. (b)

Sol. In $\triangle ABE$,

$$\tan 30^\circ = \frac{h_1}{x/2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 2\sqrt{3}h_1 \text{ (i)}$$



and in $\triangle BCD$,

$$\tan 60^\circ = \frac{h_2}{x/2} = \sqrt{3}$$

$$\Rightarrow x = \frac{2h_2}{\sqrt{3}}$$

From Eqs. (i) and (ii), we get

$$2\sqrt{3}h_1 = \frac{2h_2}{\sqrt{3}}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{1}{3} \Rightarrow h_1 : h_2 = 1 : 3$$

22. If the vectors $\mathbf{a} = (1, x, -2)$ and $\mathbf{b} = (x, 3, -4)$ are mutually perpendicular, then the value of x is [2012]

- (a) -2 (b) 2 (c) 4 (d) -4

Ans. (a)

Sol. Given that, the vector $\mathbf{a} = (1, x, -2)$ and $\mathbf{b} = (x, 3, -4)$ are mutually perpendicular.

$$\therefore (1)x + 3(x) + (-4)(-2) = 0$$

$$\Rightarrow x + 3x + 8 = 0$$

$$\Rightarrow 4x = -8$$

$$\therefore x = -2$$

23. What is the value of a for which

$$f(x) = \begin{cases} \sin x, & \text{if } x \leq \frac{\pi}{2} \\ ax, & \text{if } x > \frac{\pi}{2} \end{cases} \text{ is continuous?} \quad [2012]$$

- (a) π (b) $\frac{\pi}{4}$ (c) $\frac{2}{\pi}$ (d) 0

Ans. (c)

$$\text{Sol. Given function, } f(x) = \begin{cases} \sin x, & \text{if } x \leq \frac{\pi}{2} \\ ax, & \text{if } x > \frac{\pi}{2} \end{cases}$$

and the function is continuous at $\frac{\pi}{2}$.

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{h \rightarrow 0} \left(h + \frac{\pi}{2}\right) = \sin \frac{\pi}{2}$$

$$\Rightarrow a\left(0 + \frac{\pi}{2}\right) = 1$$

$$\therefore a = \frac{2}{\pi}$$

24. If the real number x when added to its inverse gives the minimum value of the sum, then the value of x is equal to [2012]

- (a) -2 (b) 2 (c) 1 (d) -1

Ans. (b)

Sol. By given condition, we get
Let

$$f(x) = x + \frac{1}{x} \dots (i)$$

On differentiating w.r.t. x , we get

$$f'(x) = 1 - \frac{1}{x^2}$$

For max or min of $f(x)$,

Put

$$f'(x) = 0$$

$$\Rightarrow 1 - \frac{1}{x^2} = 0$$

$$\Rightarrow \frac{(x^2 - 1)}{x^2} = 0$$

$$\Rightarrow (x-1)(x+1) = 0$$

$$\Rightarrow x = 1 \text{ or } -1$$

Now,

$$f''(x) = \frac{2}{x^3}$$

$$\text{at } x = -1,$$

$$f''(-1) = 2$$

(max)

$$\text{at } x = 1,$$

$$f''(1) = 2$$

(min)

So, $f(x)$ is min at ($x = 1$) and its minimum value at ($x = 1$) is

$$f(1) = 1 = \frac{1}{1} = 2$$

or Let

$$f(x) = x + \frac{1}{x}$$

$\therefore$

$$AM \geq GM$$

$$\Rightarrow \frac{x + \frac{1}{x}}{2} \geq \left(x \cdot \frac{1}{x}\right)^{1/2} \Rightarrow \left(x + \frac{1}{x}\right) \geq 2$$

Min of $f(x)$ is 2.

25. If $\cos(\alpha + \beta) = \frac{4}{5}$ and

$$\sin(\alpha - \beta) = \frac{5}{13}, 0 < \alpha, \beta, \frac{\pi}{4}, \text{ then } \tan(2\alpha) \text{ is equal to}$$

[2012]

$$(a) \frac{56}{33} \quad (b) \frac{63}{65} \quad (c) \frac{16}{63} \quad (d) \frac{33}{56}$$

Ans. (a)

$$\text{Sol. Given, } \cos(\alpha + \beta) = \frac{4}{5}$$

$$\text{and } \sin(\alpha - \beta) = \frac{5}{13} \text{ where, } 0 < \alpha, \beta < \frac{\pi}{4}$$

Using the identity $\sin^2 \theta + \cos^2 \theta = 1$

Now,

$$\sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}}$$

$$\therefore \sin(\alpha + \beta) = \frac{3}{5}$$

$$\text{and } \cos(\alpha - \beta) = \sqrt{1 - \sin^2(\alpha - \beta)}$$

$$= \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}}$$

$$\therefore \cos(\alpha - \beta) = \frac{12}{13}$$

Now,

$$\tan 2\alpha = \tan \{(\alpha + \beta) + (\alpha - \beta)\}$$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)}$$

$$= \frac{\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} + \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}}{1 - \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \cdot \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}}$$

$$= \frac{\frac{3}{5} \times \frac{5}{4} + \frac{5}{13} \times \frac{12}{13}}{1 - \left(\frac{3}{5} \times \frac{5}{4}\right) \left(\frac{5}{13} \times \frac{12}{13}\right)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{15}{4 \cdot 12}}$$

$$= \frac{\frac{9+5}{12}}{1 - \frac{15}{4 \cdot 12}} = \frac{14}{12 - \frac{15}{4}}$$

$$= \frac{14 \times 4}{33} = \frac{56}{33}$$

$$= \frac{14 \times 4}{33} = \frac{56}{33}$$

26. The number of words that can be formed by using the letters of the word 'MATHEMATICS' that start as well as end with T is [2012]

$$(a) 80720 \quad (b) 90720 \quad (c) 20860 \quad (d) 37528$$

Ans. (b)

Sol. $\therefore$ Required number of ways

$$= \frac{9!}{2!2!} = \frac{362880}{2 \cdot 2} = 90720$$

27. If $A - B = \frac{\pi}{4}$, then $(1 + \tan A)(1 - \tan B)$ is equal to [2012]

$$(a) 2 \quad (b) 1 \quad (c) 0 \quad (d) 3$$

Ans. (a)

$$\text{Sol. Given, } A - B = \frac{\pi}{4}$$

$$\Rightarrow \tan(A - B) = \tan \frac{\pi}{4} = 1$$

$$\Rightarrow \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} = 1$$

$$\Rightarrow \tan A - \tan B = 1 + \tan A \cdot \tan B$$

$$\Rightarrow 1 - \tan A + \tan B + \tan A \cdot \tan B = 0$$

$$\Rightarrow 2 = 1 + \tan A - \tan B - \tan A \tan B$$

$$\Rightarrow 2 = (1 - \tan B) + \tan A(1 - \tan B)$$

$$\Rightarrow 2 = (1 - \tan B)(1 + \tan A)$$

28. Let $P(E)$ denote the probability of event E .

Given $P(A) = 1, P(B) = \frac{1}{2}$, the values of $P(A|B)$ and

$P(B|A)$ respectively are [2012]

- (a) $\frac{1}{4}, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{1}{4}$ (c) $\frac{1}{2}, 1$ (d) $1, \frac{1}{2}$

Ans. (d)

Sol. Given, $P(E)$ = Probability of event E

and $P(A) = 1, P(B) = \frac{1}{2}$

$$\text{Now, } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A) = 1$$

$$\text{and } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B) = \frac{1}{2}$$

29. The number of different license plates that can be formed in the format 3 English letters (A ... Z) followed by 4 digits (0, 1, ... 9) with repetitions allowed in letters and digits is equal to [2012]

- (a) $26^3 \times 10^4$ (b) $26^3 + 10^4$ (c) 36 (d) 26^3

Ans. (a)

Sol. The number of arrangements of 3 English letters with repetitions allowed = $26 \cdot 26 \cdot 26 = (26)^3$

The number of arrangements of 4 digits with repetition allowed = $10 \cdot 10 \cdot 10 \cdot 10 = (10)^4$

$\therefore$ Required number of different licence plates = $(26)^3 \times (10)^4$

30. Which of the following is correct? [2012]

- (a) $\sin 1^\circ > \sin 1$ (b) $\sin 1^\circ < \sin 1$
(c) $\sin 1^\circ = \sin 1$ (d) $\sin 1^\circ = \frac{\pi}{180} \sin 1$

Ans. (b)

Sol. $\because 1^\circ < 1 \Rightarrow \sin 1^\circ < \sin 1$

31. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\mathbf{a} + 2\mathbf{b} + 3\mathbf{c}, \lambda\mathbf{b} + 4\mathbf{c}$ and

$(2\lambda - 1)\mathbf{c}$ are non-coplanar for [2012]

- (a) all values of λ
(b) all except one value of λ
(c) all except two values of λ
(d) no value of λ

Ans. (c)

Sol. Let $\mathbf{A} = \mathbf{a} + 2\mathbf{b} + 3\mathbf{c}$

$$\mathbf{B} = \lambda\mathbf{b} + 4\mathbf{c}$$

$$\mathbf{C} = (2\lambda - 1)\mathbf{c}$$

Since, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are non-coplanar vectors.

$$\therefore [\mathbf{A} \ \mathbf{B} \ \mathbf{C}] \neq 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & (2\lambda - 1) \end{vmatrix} \neq 0$$

$$1 \cdot \lambda \cdot (2\lambda - 1) \neq 0$$

$$\Rightarrow \lambda \neq 0, \frac{1}{2}$$

Hence, all except two values of λ .

32. Suppose values taken by a random variable X are such that $a \leq x_i \leq b$, where x_i denotes the value of X in the i th case for $i = 1, 2, 3, \dots, n$, then [2012]

- (a) $(b - a)^2 \geq \text{Var}(X)$ (b) $\frac{a^2}{4} \leq \text{Var}(X)$
(c) $a^2 \leq \text{Var}(X) \leq b^2$ (d) $a \leq \text{Var}(X) \leq b$

Ans. (a)

Sol. Since, standard deviation (SD) < Range

$$\Rightarrow \sigma \leq (b - a)$$

$$\Rightarrow \sigma^2 \leq (b - a)^2$$

$$\Rightarrow (b - a)^2 \geq \sigma^2$$

$$\text{or } (b - a)^2 \geq \text{Var}(X)$$

33. If ω is the cube root of unity, then the system of equations $x + \omega^2 y + \omega z = 0$, $\omega x + y + \omega^2 z = 0$ and $\omega^2 x + \omega y + z = 0$ is [2012]

- (a) consistent and has unique solution
(b) consistent and has more than one solution
(c) inconsistent
(d) None of the above

Ans. (b)

Sol. Given system of homogeneous linear equation are

$$x + \omega^2 y + \omega z = 0$$

$$\omega x + y + \omega^2 z = 0$$

$$\omega^2 x + \omega y + z = 0$$

Let coefficient matrix

$$A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega & 1 & \omega^2 \\ \omega^2 & \omega & 1 \end{bmatrix} \begin{cases} \because \omega^3 = 1 \\ 1 + \omega + \omega^2 = 0 \end{cases}$$

Use operation,

$$R_2 \rightarrow R_2 - \omega R_1, R_3 \rightarrow R_3 - \omega^2 R_1$$

$$A \sim \begin{bmatrix} 1 & \omega^2 & \omega \\ 0 & 1 - \omega^3 & \omega^2 - \omega^2 \\ 0 & \omega - \omega^4 & 1 - \omega^3 \end{bmatrix} \sim \begin{bmatrix} 1 & \omega^2 & \omega \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, $f(A) = r = 1$

and number of unknowns, $n = 3$

Since, $r < n$, so the system of equations is consistent and has more than one solution.

34. If $x = \log_a bc$, $y = \log_b ca$ and $z = \log_c ab$, then

$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ is equal to [2012]

- (a) abc (b) $\sqrt{ab} + \sqrt{bc} + \sqrt{ca}$
(c) 1 (d) None of these

Ans. (c)

Sol. Given that, $x = \log_a bc = \frac{\log bc}{\log a}$

and

$$y = \log_b ca = \frac{\log ca}{\log b}$$

$$z = \log_c ab = \frac{\log ab}{\log c}$$

$$\begin{aligned} \therefore \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} &= \frac{1}{1 + \frac{\log bc}{\log a}} + \frac{1}{1 + \frac{\log ca}{\log b}} + \frac{1}{1 + \frac{\log ab}{\log c}} \\ &= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc} = \frac{\log abc}{\log abc} = 1 \end{aligned}$$

35. If $2^a = 3^b = 6^{-c}$, then $ab + bc + ca$ is equal to

[2012]

- (a) 1 (b) 2 (c) 0 (d) None of these

Ans. (c)

Sol. Given, $2^a = 3^b = 6^{-c} = K$

$$\Rightarrow a = \log_2 K, b = \log_3 K, c = -\log_6 K$$

$$\Rightarrow a = \frac{\log K}{\log 2}, b = \frac{\log K}{\log 3}, c = -\frac{\log K}{\log 6}$$

$$\Rightarrow \log 2 + \log 3 = -\frac{\log K}{c} \quad (\because \log 6 = \log 2 + \log 3)$$

$$\Rightarrow \frac{\log K}{a} + \frac{\log K}{b} = -\frac{\log K}{c} \quad (\because \log K \neq 0)$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

$$\Rightarrow \frac{bc + ca + ab}{abc} = 0 \quad (\because abc \neq 0)$$

$$\Rightarrow ab + bc + ca = 0$$

36. If e and e' be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e^2} + \frac{1}{e'^2}$ is equal to [2012]

- (a) 0 (b) 1 (c) 2 (d) None of these

Ans. (b)

Sol. We know that, the eccentricity of hyperbola is

$$\Rightarrow \frac{b^2 = a^2(e^2 - 1)}{a^2 = e^2 - 1}$$

$$\Rightarrow e^2 = \frac{a^2 + b^2}{a^2}$$

$$\Rightarrow \frac{1}{e^2} = \frac{a^2}{a^2 + b^2}$$

and the eccentricity of its conjugate

$$a'^2 = b^2(e'^2 - 1)$$

$$\Rightarrow \frac{a'^2}{b^2} = e'^2 - 1$$

$$\Rightarrow e'^2 = \frac{a^2 + b^2}{b^2} \dots (ii)$$

$$\Rightarrow \frac{1}{e'^2} = \frac{b^2}{a^2 + b^2}$$

On adding Eqs. (i) and (ii), we get

$$\frac{1}{e^2} + \frac{1}{e'^2} = \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2} = \frac{a^2 + b^2}{a^2 + b^2}$$

$$\Rightarrow \frac{1}{e^2} + \frac{1}{e'^2} = 1$$

37. If a fair coin is tossed n times, then the probability that the head comes odd number of times is [2012]

- (a) $\frac{1}{2}$ (b) $\frac{1}{2^n}$ (c) $\frac{1}{2^{n-1}}$ (d) None of these

Ans. (a)

Sol. Here, $p = \frac{1}{2}$ and $q = \frac{1}{2}$

Now, by binomial distribution,

$$\begin{aligned} &= {}^nC_1(p)^1(q)^{n-1} + {}^nC_3(p)^3(q)^{n-3} + {}^nC_5(p)^5(q)^{n-5} + \dots \\ &= {}^nC_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1} + {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3} + {}^nC_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{n-5} + \dots \\ &= {}^nC_1\left(\frac{1}{2}\right)^n + {}^nC_3\left(\frac{1}{2}\right)^n + {}^nC_5\left(\frac{1}{2}\right)^n + \dots \\ &= \left(\frac{1}{2}\right)^n \{ {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots \} \\ &= \frac{1}{2^n} \cdot (2^{n-1}) = \frac{1}{2} \end{aligned}$$

38. If $\sin(\pi \cos \theta) = \cos(\pi \sin \theta)$, then $\sin 2\theta$ is equal to [2012]

- (a) $\pm \frac{3}{4}$ (b) $\pm \frac{1}{3}$ (c) $\pm \frac{1}{4}$ (d) $\pm \frac{4}{3}$

Ans. (a)

Sol. Given, $\sin(\pi \cos \theta) = \cos(\pi \sin \theta)$

$$\Rightarrow \cos(\pi \sin \theta) = \cos\left\{\frac{\pi}{2} - (\pi \cos \theta)\right\}$$

$$\Rightarrow \pi \sin \theta = \pm \left[\frac{\pi}{2} - \pi \cos \theta \right]$$

$$\Rightarrow \sin \theta + \cos \theta = \frac{1}{2}$$

$$\Rightarrow (\sin \theta + \cos \theta)^2 = \left(\frac{1}{2} \right)^2$$

$$\Rightarrow (\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta \cdot \cos \theta = \frac{1}{4}$$

$$\Rightarrow 1 + \sin 2\theta = \frac{1}{4}$$

$$\Rightarrow \sin 2\theta = -\frac{3}{4} \text{ (i)}$$

$$\Rightarrow \sin \theta = -\frac{1}{2} + \cos \theta \text{ (taking +ve sign) (i)}$$

$$\Rightarrow \cos \theta - \sin \theta = \frac{1}{2} \text{ (i)}$$

On squaring both sides,

$$(\cos \theta - \sin \theta)^2 = \left(\frac{1}{2} \right)^2$$

$$\Rightarrow \cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cdot \cos \theta = \frac{1}{4}$$

$$\Rightarrow 1 - \sin 2\theta = \frac{1}{4}$$

$$\Rightarrow \sin 2\theta = \frac{3}{4} \dots \text{(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\sin 2\theta = \pm \frac{3}{4}$$

39. In which of the following regular polygons, the number of diagonals is equal to number of sides?

[2012]

(a) Pentagon (b) Square (c) Octagon (d) Hexagon

Ans. (a)

Sol. For pentagon,

Number of sides, $n = 5$

$$\begin{aligned} \text{Number of diagonals} &= {}^5C_2 - 5 = \frac{5 \cdot 4}{2} - 5 \\ &= 10 - 5 = 5 \end{aligned}$$

Hence, number of sides is equal to number of diagonal orf pentagon.

40. One hundred identical coins each with probability P of showing up heads are tossed. If $0 < P < 1$ and the probability of heads showing on 50 coins is equal to that of heads on 51 coins, then the value of P is

[2012]

(a) $\frac{1}{2}$ (b) $\frac{49}{101}$ (c) $\frac{50}{101}$ (d) $\frac{51}{101}$

Ans. (d)

Sol. By condition, using binomial distribution,

$${}^{100}C_{50} P^{50} (1-P)^{50} = {}^{100}C_{51} P^{51} (1-P)^{49}$$

$$\Rightarrow \frac{100!}{50!50!} (1-P) = \frac{100!}{51!49!} \cdot P$$

$$\Rightarrow \frac{1}{50} (1-P) = \frac{P}{51}$$

$$\Rightarrow 51 - 51P = 50P$$

$$\Rightarrow 101P = 51$$

$$\therefore P = \frac{51}{101}$$

41. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, where x is a variable has real roots. Then, the interval of p is [2012]

(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

Ans. (d)

Sol. Given equation is

$$(\cos P - 1)x^2 + \cos P \cdot x + \sin P = 0$$

Since, the equation has real roots.

$$\text{So, } \Delta = B^2 - 4AC \geq 0$$

$$\Rightarrow \cos^2 P - 4(\cos P - 1)\sin P \geq 0$$

$$\Rightarrow \cos^2 P - 4\sin P \cdot \cos P + 4\sin P \geq 0$$

$$\Rightarrow \text{For real value of } P$$

$$(-4\sin P)^2 - 4 \cdot 1 \cdot (4\sin P) > 0$$

$$\Rightarrow 16\sin^2 - 16\sin P > 0$$

$$\Rightarrow \sin P (\sin P - 1) > 0$$

$$\Rightarrow \sin P > \sin 0 \text{ or } \sin P > \sin \frac{\pi}{2}$$

$$\Rightarrow P > n\pi + (-1)^n \cdot 0 \text{ or } P > n\pi + (-1)^n \frac{\pi}{2}$$

$$\Rightarrow p \in (0, \pi) \text{ or (no possible)}$$

42. Number of real roots of $3x^5 + 15x - 8 = 0$ is [2012]

(a) 3 (b) 5 (c) 1 (d) 0

Ans. (c)

$$\text{Sol. Let } f(x) \equiv 3x^5 + 15x - 8 = 0$$

For positive roots,

$$f(x) = + \underbrace{+ -}_{1 \text{ change}} = 1$$

$$\therefore \text{Real roots} = \text{Number of positive roots} - \text{Number of negative roots} = 1 - 0 = 1$$

43. The value of k for which the set of equations $3x + ky - 2z = 0$, $x + ky + 3z = 0$ and $2x + 3y - 4z = 0$ has a non-trivial solution, is [2012]

(a) $\frac{15}{2}$ (b) $\frac{17}{2}$ (c) $\frac{31}{2}$ (d) $\frac{33}{2}$

Ans.

Sol. The given system of homogeneous equation

$$3x + Ky - 2z = 0$$

$$x + Ky + 3z = 0$$

$$2x + 3y - 4z = 0$$

For non-trivial solution,

$$\begin{vmatrix} 3 & K & -2 \\ 1 & K & 3 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow$

$$\Rightarrow 3(-4K - 9) - K(-4 - 6) + 2(-3 + 2K) = 0$$

$$\Rightarrow -12K - 27 + 10K - (-6) + 4K = 0$$

$$\therefore +2K - 33 = 0$$

$$\therefore K = +\frac{33}{2}$$

44. If $x = \log_3 5$, $y = \log_{17} 25$, then which one of the following is correct? [2012]

- (a) $x > y$ (b) $x < y$ (c) $x \leq y$ (d) $x = y$

Ans. (a)

Sol. Given, $x = \log_3 5$, $y = \log_{17} 25$

$$x = \frac{\log 10 - \log 2}{\log 3}, y = \frac{2 \log 10 - 2 \log 2}{\log 17}$$

$$x = \frac{0.6990}{0.4771}, y = \frac{1.3980}{1.2296}$$

$$\Rightarrow x = 1.465, y = 1.136 (\therefore x > y)$$

45. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then A^n for any natural number n is [2012]

- (a) $\begin{bmatrix} n & n \\ 0 & n \end{bmatrix}$ (b) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) None of these

Ans. (b)

Sol. Given, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$\text{Now, } A^2 = A \cdot A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

46. A problem in Mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$, respectively. If they all try to solve the problem, what is the probability that the problem will be solved? [2012]

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

Ans. (d)

Sol. $\therefore$ Required probability

$$= 1 - \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{4}\right)$$
$$= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

47. The function x^x decreases in the interval [2012]

- (a) $(0, e)$ (b) $(0, 1)$ (c) $\left(0, \frac{1}{e}\right)$ (d) None of these

Ans. (c)

Sol. Let $y = x^x$

Taking log on both sides, we get

$$\log y = x \log x$$

On differentiating,

$$\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \quad \dots (i)$$

$$\frac{dy}{dx} = y(1 + \log x) = x^x \cdot (1 + \log x)$$

For decreasing of y ,

$$\text{Here, } \frac{dy}{dx} < 0$$

$$x^x \cdot (1 + \log x) < 0 \quad (\text{but } x^x \neq 0 \text{ and } x > 0)$$

$$\Rightarrow 1 + \log x < 0$$

$$\Rightarrow \log x < -1$$

$$\Rightarrow \log x < \log e^{-1}$$

$$\Rightarrow x < \frac{1}{e} \text{ and } x > 0$$

$$\therefore x \in \left(0, \frac{1}{e}\right)$$

48. If $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$, $|\mathbf{a}| = 3$, $|\mathbf{b}| = -5$, $|\mathbf{c}| = 7$, then angle between the vectors $\mathbf{a}$ and $\mathbf{b}$ is [2012]

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Ans. (b)

Sol. Given expression

$$a + b + c = 0$$

$$\Rightarrow a + b = -c$$

On squaring both sides,

$$\begin{aligned}
&\Rightarrow (a+b)^2 = (-c)^2 \\
&\Rightarrow (a+b) \cdot (a+b) = (-c) \cdot (-c) \\
&\Rightarrow (\mathbf{a} \cdot \mathbf{a}) + (\mathbf{b} \cdot \mathbf{a}) + (\mathbf{a} \cdot \mathbf{b}) + (\mathbf{b} \cdot \mathbf{b}) = (\mathbf{c} \cdot \mathbf{c}) \\
&\Rightarrow a^2 + 2\mathbf{a} \cdot \mathbf{b} + b^2 = c^2 \because (\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}) \\
&\Rightarrow |\mathbf{a}|^2 + 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 = |\mathbf{c}|^2 \because (\mathbf{a}^2 = |\mathbf{a}|^2) \\
&\Rightarrow (3)^2 + 2\mathbf{a} \cdot \mathbf{b} + (5)^2 = (7)^2 \\
&\because |\mathbf{a}| = 3, |\mathbf{b}| = 5 \text{ and } |\mathbf{c}| = 7 \\
&\Rightarrow 2 \cdot |\mathbf{a}| |\mathbf{b}| \cos \theta = 15
\end{aligned}$$

Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$.

$$\begin{aligned}
&\Rightarrow 2 \cdot 3 \cdot 5 \cos \theta = 15 \\
&\Rightarrow \cos \theta = \frac{1}{2} = \cos 60^\circ \\
&\therefore \theta = \frac{\pi}{3}
\end{aligned}$$

49. If $\theta (0 \leq \theta \leq \pi)$ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then $\frac{|\mathbf{a} \times \mathbf{b}|}{\mathbf{a} \cdot \mathbf{b}}$ equals to [2012]

(a) $-\cot \theta$ (b) $\tan \theta$ (c) $-\tan \theta$ (d) $\cot \theta$

Ans. (c)

Sol. $\because \theta \in [0, \pi]$

Now,

$$\begin{aligned}
\frac{|\mathbf{a} \times \mathbf{b}|}{\mathbf{a} \cdot \mathbf{b}} &= \frac{|\mathbf{a}| |\mathbf{b}| \sin \theta}{|\mathbf{a}| |\mathbf{b}| (-\cos \theta)} \\
&= \frac{|\mathbf{a}| |\mathbf{b}| \sin \theta}{|\mathbf{a}| |\mathbf{b}| (-\cos \theta)} \\
&= \frac{\sin \theta \cdot 1}{-\cos \theta} = -\tan \theta
\end{aligned}$$

($\cos \theta$ in second quadrant is negative)

50. If $f(a+b) = f(a) \times f(b)$ for all a and b and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is equal to [2012]

(a) 2 (b) 4 (c) 6 (d) 8

Ans. (c)

Sol. Given that,

$$f(a+b) = f(a) \times f(b) \dots (i)$$

$$\text{and } f(5) = 2, f'(0) = 3$$

By definition,

$$f'(5) = \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h}$$

$$f'(5) = \lim_{h \rightarrow 0} \frac{f(5) \times f(h) - f(5)}{h}$$

$$f'(5) = f(5) \cdot \lim_{h \rightarrow 0} \frac{f(h) - 1}{h}$$

By 'L' hospital rule,

$$f'(5) = f(5) \cdot f'(0)$$

$$\Rightarrow f'(5) = 2 \times 3 = 6$$

Analytical Ability and Logical Reasoning

51. If a man walks at the rate of 4 km/h, he misses a train by only 6 min. However, if he walks at the rate of 5 km/h he reaches the station 6 min before the arrival of the train. The distance covered by him to reach the station is

[2012]

(a) 4 km (b) 7 km (c) 9 km (d) 5 km

Ans. (a)

Sol. Let the distance covered by him is x km, then by condition,

$$\begin{aligned}
\frac{x}{4} - \frac{x}{5} &= \frac{12}{60} \\
\Rightarrow \frac{x}{20} &= \frac{1}{5} \\
\therefore x &= 4 \text{ km}
\end{aligned}$$

52. The missing number in the given series

3, 6, 6, 12, 9, ..., 12 is

[2012]

(a) 15 (b) 18 (c) 11 (d) 13

Ans. (b)

Sol. Given series 3, 6, 6, 12, 9, ..., 12

Split the given series into two parts

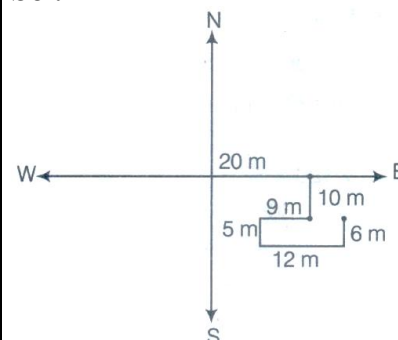


53. A man runs 20 m towards east and turns right, runs 10 m and turns right, runs 9 m and turns left, runs 5 m and turns left, runs 12 m and finally turns left and runs 6 m. Which direction is the man facing? [2012]

(a) North (b) South (c) East (d) West

Ans. (a)

Sol.



Hence, North direction is the man facing.

54. In a club, there are certain number of males and females. If 15 females are absent, then number of males will be half of females. If 45 males are absent, then female strength will be 5 times that of males. Number of males actually present is [2012]
 (a) 45 (b) 80 (c) 105 (d) 175

Ans. (b)

Sol. Let x and y be the certain number of males and females.

Then, by condition,

$$x = \frac{1}{2}(y - 15)$$

$$\Rightarrow 2x = y - 15$$

$$\Rightarrow 2x - y = -15 \text{ (i)}$$

$$\text{and } 5(x - 45) = y \text{ (ii)}$$

$$\Rightarrow 5x - y = 225 \text{ (ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$3x = 240 \Rightarrow x = 80$$

$\therefore$ Number of males = 80

55. The missing number in the following series 6, 12, 21, ..., 48 is [2012]

(a) 40 (b) 33 (c) 38 (d) 45

Ans. (b)

Sol.



Directions (Q.Nos. 56-58) Read the following passage carefully and answer the questions.

Six boys A, B, C, D, E and F are marching in a line.

They are arranged according to their heights, the tallest being at the back and the shortest in the front. F is between B and A. E is shorter than D but taller than C who is taller than A. E and F have two boys between them. A is not the shortest among them.

56. Where is E ? [2012]

(a) Between A and B (b) Between C and A
 (c) Between D and C (d) In front of C

Ans. (c)

Sol. Between D and C

Solution (Q. Nos. 56-58)

By condition D E C A F B (Shortest)

↓
(Longest)

57. If we start counting from the shortest, which boy is fourth in the line? [2012]

(a) E (b) A (c) D (d) C

Ans. (d)

Sol. C

58. Who is next to the shortest? [2012]

(a) C (b) B (c) E (d) F

Ans. (d)

Sol. F

59. Let x, y and z be distinct integers. x and y are odd and positive and z is even and positive. Which one of the following statements cannot be true? [2012]

(a) $(x - z)^2 y$ is even (b) $(x - z)y^2$ is odd

(c) $(x - z)y$ is odd (d) $(x - y)^2 z$ is even

Ans. (a)

Sol. x, y, z are distinct integers.

and x and y are odd positive integers and z is even positive integers.

Then, $(x - z) = \text{Odd number}$

$$(x - z)^2 = \text{Odd positive number}$$

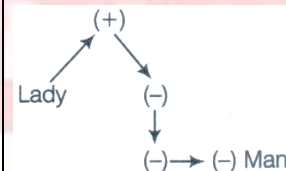
$$\text{and } (x - z)^2 \cdot y = \text{Odd} \times \text{Even} = \text{Even number}$$

60. Pointing to a man in the photograph a lady said, "The father of his brother is the only son of my mother." How is this man in photograph related to the lady? [2012]

(a) Brother (b) Son (c) Grandson (d) Nephew

Ans. (d)

Sol.



So, man is nephew of the lady.

61. Find the odd number in the following series. [2012]
 2, 9, 28, 65, 126, 216, 344, ...

(a) 28 (b) 65 (c) 126 (d) 216

Ans. (d)

Sol.

$$\begin{array}{ccccccc} 2 & 9 & 28 & 65 & 126 & 216 & 344 \\ \hline 2^3+1 & 3^3+1 & 4^3+1 & 5^3+1 & 6^3+1 & 7^3+1 & \dots \end{array}$$

or,

$$2 = 1^3 + 1, 9 = 2^3 + 1, 28 = 3^3 + 1, 65 = 4^3 + 1, 126 = 5^3 + 1$$

$$\text{and } 344 = 7^3 + 1$$

But $216 = 6^3 + 0$ which is odd number among them.

62. Average age of students of an adult school is 40 yr. 120 new students whose average age is 32 yr joined the school. As a result the average age is decreased by 4 yr. The number of students of the school after joining of the new students is [2012]

(a) 1200 (b) 120 (c) 360 (d) 240

Ans. (d)

Sol. Let the total number of students = $x + 120$

$\Rightarrow$ After joining new 120 students = $x + 120$

Now, by condition,

$$\begin{aligned} x \times 40 + 120 \times 32 &= (x + 120) \times 36 \\ \Rightarrow 40x + 3840 &= 36x + 4320 \\ \Rightarrow 4x &= 480 &= x + 120 = 120 + 120 = 240 \\ x &= 120 \\ \text{Total} & \quad \text{number of students} \end{aligned}$$

63. The letters P, Q, R, S, T, U and V not necessarily in that order represent seven consecutive integers from 22 to 33 and

1. U is as much less than Q as R is greater than S.

2. V is greater than U.

3. Q is the middle term.

4. P is greater than S.

Then, the sequence of letters from the lowest value to the highest value, is **[2012]**

- (a) TVPQRSU (b) TRSQUPV
(c) TUSQRPV (d) TVPQSRU

Ans. (c)

Sol. By given condition, we get the required order (sequence) of letters from the lowest value to the highest value is

$$T < U < S < Q < R < P < V$$

i.e. TUSQRPV

64. The minimum number of tiles of size 16 by 24 required to form a square by placing them adjacent to one another is **[2012]**

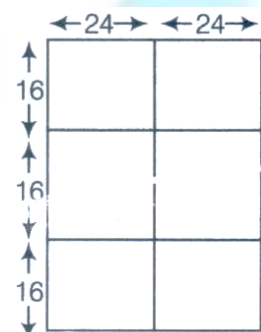
- (a) 6 (b) 8 (c) 11 (d) 16

Ans. (a)

Sol. From option (a),
Let the number of tiles = 6

$\therefore$ Total length = 48

and total breadth = 48



Since, length = breadth

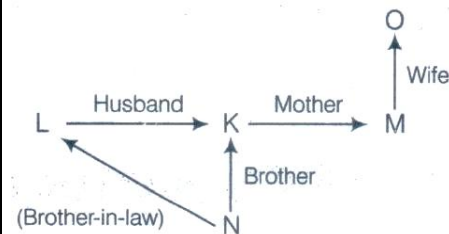
$\therefore$ Number of tiles form a square = 6

65. Five persons K, L, M, N and O are sitting around a dining table. K is the mother of M, M is actually the wife of O, N is the brother of K and L is the husband of K. How is N related to L? **[2012]**

- (a) Son (b) Cousin (c) Brother (d) Brother-in-law

Ans. (d)

Sol.



66. Three men A, B and C play cards. If one loses the game he has to give ₹3. If he wins the game he will gain ₹3 each from the other two losers. If A has won 3 games, B loses ₹3, C wins ₹12, then the total number of games played is **[2012]**

- (a) 12 (b) 21 (c) 20 (d) 6

Ans. (a)

Sol. Required total number of games played is 12.

Directions (Q. Nos. 67-69) Read the following passage carefully and answer the questions.

A causes B or C but not both.

F occurs only if B occurs.

D occurs, if B or C occurs.

E occurs only if C occurs.

J occurs only if E or F occurs.

D causes G or H or both.

H occurs, if E occurs.

G occurs, if F occurs.

67. If A occurs, which may occur? **[2012]**

I. F and G

II. E and H

III. D

(a) Only I

(b) Only II

(c) I and III or II and III, but not both

(d) I, II and III

Ans. (c)

Sol. From Statement (i), A causes B or C but not both. From Statement (ii), F occurs only if B occurs and from Statement (iii), D occurs if B or C occur. It means I and II may occur. From Statement (vi) and (vii), II and III are may occur. So, we conclude that I and III or II and III may occur but not both occur.

Solutions (Q. Nos. 67-69)

(i) A causes B or C but not both.

(ii) F occurs only if B occurs.

(iii) D occurs if B or C occurs.

(iv) E occurs only if C occurs.

(v) C occurs only if E or F occurs

(vi) D causes Gor H or both

(vii) H occurs if E occurs.

(viii) G occurs if F occurs.

68. If B occurs, which must occur? **[2012]**

- (a) D (b) G (c) H (d) J

Ans. (b)

Sol. From Statement (ii) that F occurs only if B occurs and from Statement (viii) that if G occurs if F occurs it means if B occurs G must occur.

69. If J occurs, which must have occurred? [2012]

- (a) Both E and F (b) Either B or C
(c) Both B and C (d) None of these

Ans. (b)

Sol. From Statement (v), that J occurs only if E or F occurs. From Statement (ii), F occurs only if B occurs and from Statement (iv), E occurs only if C occurs it means it J occurs either B or C must have occurs.

70. If 'ROAST' is coded as 'PQYUR' in a certain language, then 'SLOPPY' is coded in that language as [2012]

- (a) MRNAQN (b) NRMNQA
(c) QNMRNA (d) RANNMO

Ans. (c)

Sol. $R \xrightarrow{-2} P$ $S \xrightarrow{-2} Q$

$O \xrightarrow{O+2} P$ $L \xrightarrow{L+2} N$

$A \xrightarrow{-2} U$ $O \xrightarrow{-2} M$
 $S \xrightarrow{+2} U$

$T \xrightarrow{P+2} R$ $P \xrightarrow{-2} N$

$T \xrightarrow{+2} A$ $Y \xrightarrow{+2} A$

71. If 'lelibroon' means 'yellow hat', 'plekafroti' means 'flower garden' and 'frotimix' means 'garden salad', then which word could mean 'yellow flower'? [2012]

- (a) Lelifroti (b) Lelipleka
(c) Plekabroon (d) Frotibroon

Ans. (b)

Sol. Ielibroon → yellow hat

pleka → flower garden

froti mix → garden salad

∴ Pleka → flower

yellow → leli or broon

By option,

yellow flower → lelipleka

72. If + is *, - is, +* is / and / is -, then

$6 - 9 + 8 * \frac{3}{20}$ is equal to

- (a) -2 (b) 6 (c) 10 (d) 12

Ans. (c)

Sol. $E = 6 - 9 + 8 * \frac{3}{20}$

By given condition,

$$E = 6 + 9 * \frac{8}{3} - 20$$

$$E = 6 + 3 * 8 - 20$$

$$E = 6 + 24 - 20$$

$$E = 6 + 4 = 10$$

73. In a certain year, there were exactly four Fridays and four Mondays in January. On what day of the week did the 20th of January fall that year?

[2012]

- (a) Saturday (b) Sunday
(c) Thursday (d) Tuesday

Ans. (b)

Sol. Let in a month of January.

(4 times) Friday → 25, 18, 11, 4 (dates)

(4 times) Monday → 28, 21, 14, 7 (dates)

Then, required dates of Sunday,

Sunday → 27, 20, 13, 6

So, Sunday of the week did the 20th of January fall that year.

74. Krishna said, "This girl is the wife of grandson of my mother". How is Krishna related to girl? [2012]

- (a) Father (b) Father-in-law
(c) Husband (d) Grandfather

Ans. (b)

Sol. Krishna is "father-in-law" of that girl.

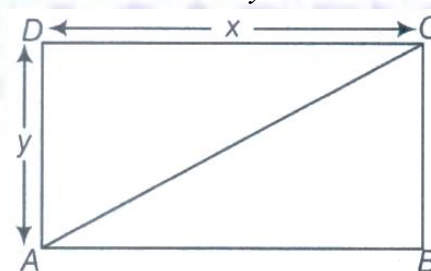
75. Instead of walking along two adjacent sides of a rectangular field, a boy took a shortcut along the diagonal of the field and saved a distance equal to half the longer side. The ratio of the shorter side of the rectangle to the longer side is [2012]

- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Ans. (d)

Sol. Let longer side = $x = DC$

and shorter side = $y = AD$



Now, by condition,

$$AC = y + \frac{x}{2}$$

Now, In $\triangle ACD$,

$$AC^2 = AD^2 + CD^2 \text{ (by Pythagoras theorem)}$$

$$\Rightarrow \left(y + \frac{x}{2}\right)^2 = y^2 + x^2$$

$$\Rightarrow y^2 + \frac{x^2}{4} + xy = x^2 + y^2$$

$$\Rightarrow \frac{x^2}{4} + xy - x^2 = 0$$

$$\Rightarrow x \left\{ \frac{x}{4} + y - x \right\} = 0$$

$$\Rightarrow x \left(y - \frac{3x}{4} \right) = 0$$

$$\Rightarrow \frac{y}{x} = \frac{3}{4}$$

76. Each word in parenthesis below is formed in a method. This method is used in all four examples.

SNIP (NICE) PACE
TEAR (EAST) FAST
TRAY (RARE) FIRE
POUT (OURS) CARS

Based on this method, the word in the parenthesis of CANE (?) BATS is [2012]

(a) NEAT (b) CATS (c) ANTS (d) NETS

Ans. (c)

Sol. SNIP (NICE) PACE

TEAR (EAST) FAST

TRAY (RARE) FIRE

POUT (OURS) CARS

∴ CANE (AN + TS) BATS

ANTS

77. A study of native born residents in an area of Adivasis found that two-third of the children developed considerable levels of nearsightedness after starting school, while their illiterate parents and grandparents, who had no opportunity for formal schooling, showed no signs of this disability.

[2012]

If the above statements are true, which of the following conclusions is most strongly supported by them?

- (a) Only people who have the opportunity for formal schooling develop nearsightedness
(b) People who are illiterate do not suffer from nearsightedness
(c) The nearsightedness in the children is caused by the visual stress required by reading and other class work
(d) Only literate people are nearsighted

Ans. (c)

Sol. From the statements, we clearly say that the reason behind the nearsightedness of the children is caused by the visual stress required by reading and other class work.

Directions (Q. Nos. 78-80) Read the following passage carefully and answer the questions.

Five roommates Randy, Sally, Terry, Uma and Vernon each do one housekeeping task mopping, sweeping, laundry, vacuuming or dusting one day a week, Monday through Friday.

Vernon does not vacuum and does not do his task on Tuesday.

Sally does the dusting and does not do it on Monday of Friday.

The mopping is done on Thursday.

Terry does his task, which is not vacuuming, on Wednesday.

The laundry is done on Friday and not by Uma.

Randy does his task on Monday.

78. The task done by Terry on Wednesday is [2012]

(a) vacuuming (b) dusting (c) mopping (d) sweeping

Ans. (d)

Sol. Sweeping

Solutions (Q. Nos. 78-80)

Randy	Vacuuming	Monday
Sally	Dusting	Tuesday
Terry	Sweeping	Wednesday
Uma	Mopping	Thursday
Vernon	Laundry	Friday

79. The day on which the vacuuming is done, is

[2012]

(a) Friday (b) Monday (c) Tuesday (d) Wednesday

Ans. (b)

Sol. Monday

80. Sally does dusting on

[2012]

(a) Friday (b) Monday (c) Tuesday (d) Wednesday

Ans. (c)

Sol. Tuesday

Directions (Q.Nos. 81-82) Read the following passage carefully and answer the questions.

P, Q, R, S, T, U, V and W are sitting round the circle and are facing the centre. P is second to the right of T, T is the neighbour of R and V. S is not the neighbour of P, V is the neighbour of U, Q is not between S and W and W is not between U and S.

81. Which two of the following are not neighbours?

[2012]

(a) RV (b) UV (c) RP (d) QW

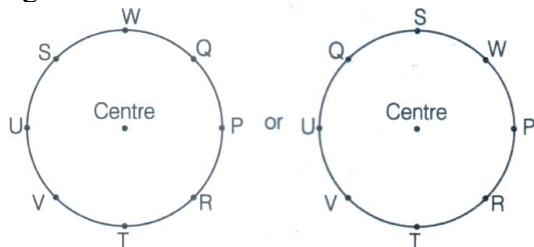
Ans. (a)

Sol. R and V are not neighbours.

Solution (Q. Nos. 81-82)

According to the given data, we get the following

figure.



82. What is the position of S ?

[2012]

- (a) Between U and V
 (b) Second to the right of P
 (c) To the immediate right of W
 (d) Data inadequate

Ans. (d)

Sol. The position of S is not fixed. So, data inadequate.

83. The ratio between a two-digit number and the sum of the digits of that number is $4:1$. If the digit in the unit's place is 3 more than the digit in ten's place, then the number is

[2012]

- (a) 24 (b) 63 (c) 36 (d) 42

Ans. (c)

Sol. Let the ten's place digit = x , then

By condition the unit placed digit = $x + 3$

Now, according to question,

$$\frac{10x + (x + 3)}{x + x + 3} = \frac{4}{1} \Rightarrow \frac{10x + x + 3}{2x + 3} = \frac{4}{1}$$

$$\Rightarrow 11x + 3 = 8x + 12$$

$$\Rightarrow 3x = 9 \Rightarrow x = 3$$

$$\therefore \text{Required number} = x(x + 3) = 3(3 + 3) = 36$$

84. Two positions of a dice are shown below. When number 1 is on the top, what number will be at the bottom?

[2012]



(I)



(II)

- (a) 2 (b) 3 (c) 5 (d) Cannot be determined

Ans. (c)

Sol. After observation of given two dice, we get the number 5 is at the bottom of the dice, when number 1 is on the top.

85. A, B, C, D, E, F and G are sitting in a line facing East. C is immediate to the right of D . B is at one of the extreme ends and has E as his neighbour. G is between E and F . D is sitting third from the South end.

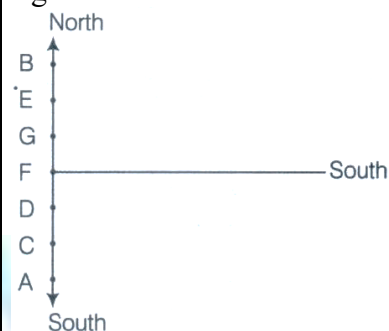
Who is sitting third from North?

[2012]

- (a) A (b) E (c) F (d) G

Ans. (d)

Sol. According to given data, we get the following figure



So, G is sitting third from North.

86. There is a family party consisting of two fathers, two mothers, two sons, one father-in-law, one mother-in-law, one daughter-in-law, one grandfather, one grandmother and one grandson.

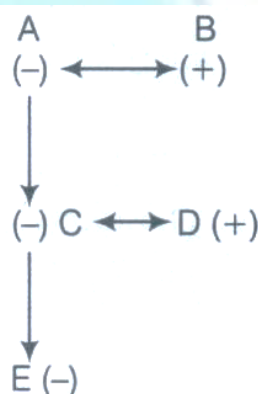
What is the minimum number of persons required, so that this is possible?

[2012]

- (a) 5 (b) 6 (c) 7 (d) 8

Ans. (a)

Sol. Let '-' means 'male' and '+' means 'female'.



Two father (A, C)

Two mothers (B, D)

Two sons (C, E)

One father-in-law (A)

One mother-in-law (B)

One daughter-in-law (D)

One grandfather (A)

One grandmother (B)

One grandson (E)

So, the minimum number of person can be 5.

87. If A is brother of B , C is brother of B and A is brother of D , then which of the following must be true?

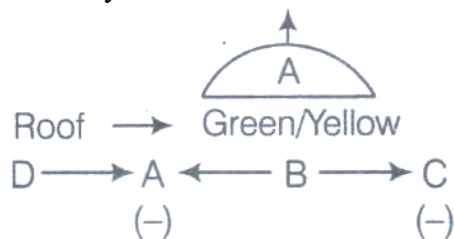
[2012]

- (a) A is brother of C (b) B is brother of C
(c) D is brother of C (d) B is brother of D

Ans. (a)

Sol. According to the direction, the relation can be solved as

Chimney → White/Red/Black



So, A is brother of C.

Directions (Q. Nos. 88-90) Read the following passage carefully and answer the questions.

Five houses lettered A, B, C, D and E are built in a row next to each other. The houses are lined up in the order A, B, C, D and E. Each of the five houses have coloured roofs and chimneys. The roof and chimney of each house must be painted as follows.

1. The roof must be painted either green, red or yellow.
2. The chimney must be painted either white, black or red.
3. No house may have the same colour chimney as the colour of roof.
4. No house may use any of the same colours that adjacent house uses.

5. House E has a green roof.

6. House B has a red roof and a black chimney.

88. Which of the following is true? [2012]

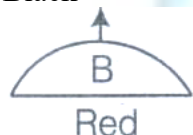
- (a) Atleast two houses have black chimney
(b) Atleast two houses have red roofs
(c) Atleast two houses have white chimneys
(d) Atleast two houses have green roofs

Ans. (b)

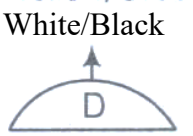
Sol. From the above diagram, it is clear that atleast two house have red roots.

Solution (Q. Nos. 88-90)

Black



Red/Black/White



Red/Black/White



89. If house C has a yellow roof, then which of the following must be true? [2012]

- (a) House E has a white chimney
(b) House E has a black chimney
(c) House E has a red chimney
(d) House D has a red chimney

Ans. (a)

Sol. If house C has a yellow roof it means house D has red roof. So, house D never has a red chimney. So, chimney of D will be of black colour, so colour of chimney of house E will be white.

90. What is the maximum number of green roofs? [2012]

- (a) 1 (b) 2 (c) 3 (d) 4

Ans. (c)

Sol. The maximum number of green roofs are 3.

General English

91. For a word, four spellings are given. Choose the correct one. [2012]

- (a) Ceiling (b) Cealing (c) Ceiling (d) Ceeling

Ans. (c)

Sol. Ceiling is the correct word.

92. Choose the wrongly spelt word. [2012]

- (a) Believe (b) Relieve (c) Grieve (d) Decieve

Ans. (d)

Sol. Decieve is the wrongly spelt word, the correct spell is deceive.

93. Choose the word or phrase that is most similar in meaning to the word POLEMIC. [2012]

- (a) Black (b) Magnetic
(c) Grimace (d) Controversial

Ans. (d)

Sol. Controversial is most similar in meaning to the word 'Polemic'.

94. The sentence below has 2 blanks. Fill in the blanks picking the appropriate pair of words from the ones given below that best completes the meaning of the sentences.

The most technologically advanced societies have been responsible for the greatest _____ indeed, savagery seems to be in direct proportion to _____ [2012]

- (a) wars; viciousness (b) catastrophes; ill-will
(c) atrocities; development (d) triumphs; civilisation

Ans. (c)

Sol. Atrocities; development.

95. Fill in the blank with the correct form of tense.

The thief ... before the police came. [2012]

- (a) escaped (b) had escaped
(c) will escape (d) has been escaped

Ans. (b)

Sol. The thief had escaped before the police came.

96. Fill in the blank with appropriate words given.

Anne had to pay for everything because as usual, Peter ... his wallet at home. [2012]

- (a) had left (b) was leaving
(c) left (d) leave

Ans. (c)

Sol. Anne had to pay for everything because as usual, Peter left his wallet at home.

97. Pick the synonym of the word 'Meagre'. [2012]

- (a) Helpful (b) Abundant
(c) Essential (d) Limited

Ans. (d)

Sol. Synonym of the word 'Meagre' is limited.

98. Choose the words that best express the meaning of the given idiom-Mud slinging. [2012]

- (a) Giving pain (b) Abusing someone
(c) Laying blame (d) Damaging the reputation

Ans. (d)

Sol. Damaging the reputation.

99. Pick the antonym of the word 'Timid'. [2012]

- (a) Bold (b) Lazy (c) Calm
(d) Slow

Ans. (a)

Sol. Antonym of word 'Timid' is bold.

100. Pick the part of the sentence that has an error. If you would have come to me, I would have helped you. [2012]

- (a) If you would have (b) Come to me
(c) I would have (d) Helped you

Ans. (a)

Sol. "If you would have" sentence has an error.

101. Choose the word or phrase that is most nearly opposite in meaning to the word 'Extrinsic'. [2012]

- (a) Reputable (b) Inherent (c) Ambitious (d) Cursory

Ans.

Sol. (b) Opposite in meaning to the word EXTRINSIC is Inherent.

102. Select the alternative giving the closest meaning of the idiom - To eat a humble pie. [2012]

(a) To become a vegetarian

(b) Disinfecting everything

(c) To fill one's belly

(d) To say you are sorry for a mistake that you made

Ans.

Sol. (d) Idiom-To eat a humble pie.

Meaning-To say you are sorry for a mistake that you made.

103. Pick the antonym of the word 'Fabricate'. [2012]

- (a) Construct (b) Weaken (c) Dismantle (d) Evolve

Ans.

Sol. (a) Word → Fabricate

Antonym → Construct

Directions (Q.Nos. 104-110) Fill in the blank with correct option to make a proper sentence.

104. The people ... you socialise are called friends.

[2012]

- (a) with whom (b) who (c) with who (d) whom

Ans.

Sol. (a) The people with whom you socialise are called friends.

105. ... to school yesterday?

[2012]

- (a) Did you walk (b) Did you walked
(c) Do you walk (d) Have you walked

Ans.

Sol. (a) Did you walk to school yesterday?

106. There was no ... in the railway compartment for additional passengers. [2012]

- (a) space (b) place (c) seat (d) room

Ans.

Sol. (d) There was no room in the railway compartment for additional passengers.

107. And now for this evening's main headline; Britain ... another olympic gold medal. [2012]

- (a) had won (b) wins (c) won (d) has won

Ans.

Sol. (b) And now for this evening's main headline; Britain wins another olympic gold medal.

108. If she about his financial situation, she would have helped him out. [2012]

- (a) knew (b) had been knowing
(c) had known (d) have known

Ans.

Sol. (d) If she have known about his financial situation, she would helped him out.

109. I am sure she can teach computers as well. She's not new to the subject. [2012]

