Mathematics

1. If $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{bmatrix}$ and $6A^{-1}=A^2+cA+dI$, where A^{-1} is the inverse of A, I is the identity matrix, then (c,d) is **[2013]**

(a) $(-6,11)$ (b) $(6,-11)$

(c) $(11,-6)$ (d) $(6,11)$

Ans. (a)

Sol. Given,

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{bmatrix}$$

and $6A^{-1}=A^2+cA+dI\ldots(i)$

Now, multiplying Eq. (i) by A, we get

$$6I=A^3+cA^2+dA$$

$$\Rightarrow A^3+cA^2+dA-6I=0\ \ldots.(ii)$$

Applying Cayley Hamilton rule

$$\begin{vmatrix}1-\lambda & 0 & 0\\ 0 & 1-\lambda & 1\\ 0 & -2 & 4-\lambda\end{vmatrix}=0$$

$$\Rightarrow(1-\lambda)\{(1-\lambda)(4-\lambda)+2\}=0$$

$$\Rightarrow\lambda^3-6\lambda^2+11\lambda-6=0\ldots.(iii)$$

On comparing Eqs. (ii) and (iii), we get

$c=-6$ and $d=11$

2. If $\mathbf{a}=\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\mathbf{c}=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$, then vector $\mathbf{b}$ satisfying $(\mathbf{a}\times\mathbf{b})+\mathbf{c}=0$ and $\mathbf{a}\cdot\mathbf{b}=3$ is **[2013]**

(a) $-\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ (b) $2\hat{i}-\hat{j}+2\hat{k}$

(c) $\hat{\mathbf{i}}-\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

Ans. (a)

Sol. Given, $\mathbf{a}\cdot\mathbf{b}=3$

and $(a\times b)+c=0\ldots.(i)$

Multiplying Eq. (i) by $\mathbf{b}$,

$\mathbf{b}\cdot(\mathbf{a}\times\mathbf{b})+\mathbf{b}\cdot\mathbf{c}=0$

$\Rightarrow\mathbf{b}\cdot\mathbf{c}=0$

Let $\mathbf{b}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$, we get

$y-z=3$

and $x-y-z=0$

From these equations,

$x=2z+3, y=z+3$

$\Rightarrow\mathbf{b}=(2z+3)\hat{\mathbf{i}}+(z+3)\hat{\mathbf{j}}+z\hat{\mathbf{k}}$

$\because\ (\mathbf{a}\times\mathbf{b})+\mathbf{c}=0$

So, $z=-2$

Thus, $b=-\hat{i}+\hat{j}-2\hat{k}$

3. Find the number of elements in the union of four sets A, B, C and D having $150, 180, 210$ and 240 elements respectively, given that each pair of sets has 15 elements in common. Each triple of sets has 3 elements in common and $A\cap B\cap C\cap D=\phi$. **[2013]**

(a) 616 (b) 512 (c) 111 (d) 702

Ans. (d)

Sol. As per the given condition,

$$\begin{aligned}A\cup B\cup C\cup D&=A+B+C+D-(A\cap B)-(A\cap C)\\&\quad-(A\cap D)-(B\cap C)-(B\cap D)-(C\cap D)\\&\quad+(A\cap B\cap C)+(A\cap B\cap D)+(A\cap C\cap D)\\&\quad+(B\cap C\cap D)-(A\cap B\cap C\cap D)\\&=150+180+210+240-15-15-15\\&\quad-15-15-15+3+3+3+3-0\\&=780-90+15-3=795-90-3\\&=705-3=702\end{aligned}$$

4. If the straight line $ax+by+c=0$ always passes through $(1,-2)$, then a,b and c are in **[2013]**

(a) AP (b) HP

(c) GP (d) None of these

Ans. (a)

Sol. Given, $ax+by+c=0$ passes through $(1,-2)$.

So, point will satisfy the given equation,

$a\times1+b\times(-2)+c=0$

$\Rightarrow a-2b+c=0$

$\Rightarrow a+c=2b$

Hence, a,b and c are in AP.

5. A six faced die is a biased one. It is thrice more likely to show an odd number than to show an even number. It is thrown twice. The probability that the sum of the numbers in the two throws is even, is **[2013]**

(a) $\frac{4}{8}$ (b) $\frac{5}{8}$ (c) $\frac{6}{8}$ (d) $\frac{7}{8}$

Ans. (b)

Sol. Let the probability to show an even number be P.

As per the question,

Probability to show an odd number will be $3P$.

$\therefore \quad 3P+P=1$

$\Rightarrow \quad 4P=1$

$\Rightarrow \quad P=\frac{1}{4}$

So, $\quad 3P=\frac{3}{4}$

Required probability

$$=\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{9}{16}+\frac{1}{16}=\frac{10}{16}=\frac{5}{8}$$

6. If $I_n=\int_0^{\pi/4}\tan^n\theta d\theta$, then I_8+I_6 is equal to **[2013]**

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

Ans. (d)

Sol. Given, $I_n=\int_0^{\pi/4}\tan^n\theta d\theta$

$$\Rightarrow I_n=\int_0^{\pi/4}\tan^{n-2}\theta\tan^2\theta d\theta$$

$$\Rightarrow I_n=\int_0^{\pi/4}\tan^{n-2}\theta\left(\sec^2\theta-1\right)d\theta$$

$$\left[\because \tan^2\theta=\left(\sec^2\theta-1\right)\right]$$

$$\Rightarrow I_n=\int_0^{\pi/4}\tan^{n-2}\theta\sec^2\theta-\int_0^{\pi/4}\tan^{n-2}\theta d\theta$$

$$\because I_n+I_{n-2}=\int_0^1 t^{n-2}dt=\left[\frac{t^{n-1}}{n-1}\right]_0^1 \quad [\because \text{ let } \tan\theta=t]$$

$$\Rightarrow I_n+I_{n-2}=\frac{1}{n-1}$$

Now, putting $n=8$, we get

$$I_8+I_{8-2}=\frac{1}{8-1}$$

$$\Rightarrow I_8+I_6=\frac{1}{7}$$

7. If $\triangle ABC$ is a triangle whose area is $10\sqrt{3}$ units with side lengths $|AB|=8$ units and $|AC|=5$ units. Find the possible values of $\angle A$. **[2013]**

(a) 60° or 120° (b) 45° or 135°

(c) Only 30° (d) Only 90°

Ans. (a)

Sol. Given, area of triangle $=10\sqrt{3}$

$$\Rightarrow 10\sqrt{3}=\frac{1}{2}\times ab\times\sin\theta$$

$$[\because \text{area of triangle}=\frac{1}{2}ab\sin\theta]$$

$$\Rightarrow 10\sqrt{3}=\frac{1}{2}\times 8\times 5\sin\theta$$

$$\Rightarrow \sin\theta=\frac{10\sqrt{3}\times 2}{8\times 5}$$

$$\Rightarrow \sin\theta=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}\Rightarrow\theta=60^\circ \text{ or} 120^\circ$$

8. Person A can hit a target 4 times in 5 attempts. Person B can hit a target 3 times in 4 attempts. Person C can hit a target 2 times in 3 attempts. They fire a volley. The probability that the target is hit atleast two times, is **[2013]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{5}{6}$ (d) 1

Ans. (c)

Sol. A can hit the target 4 times in 5 attempts.

So, P (hit the target by A) $=\frac{4}{5}$

Similarly, P (hit the target by B) $=\frac{3}{4}$

and P (hit the target by C) $=\frac{2}{3}$

Now,

Required probability $=P$ (target hit atleast two times)

$$=\frac{4}{5}\times\frac{3}{4}\times\frac{1}{3}+\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}+\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}+\frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}$$

$$=\frac{1}{5}+\frac{2}{15}+\frac{1}{10}+\frac{2}{5}$$

$$\left[\because AB\bar{C}+A\bar{B}C+\bar{A}BC+ABC \text{ (atleast two times)}\right]$$

$$=\frac{50}{60}=\frac{5}{6}$$

9. The value of the integral $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is **[2013]**

(a) 0 (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Ans. (d)

Sol. Given,

$$I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx....(i)$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}dx$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx....(ii)$$

Now, adding Eqs. (i) and (ii), we get

$$2I=\int_0^{\pi/2}1dx\Rightarrow 2I=[x]_0^{\pi/2}$$

$$\Rightarrow 2I=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

10. If ω is a cube root of unity, then find the value of

$$\begin{vmatrix}1+\omega & \omega^2 & -\omega\\ 1+\omega^2 & \omega & -\omega^2\\ \omega^2+\omega & \omega & -\omega^2\end{vmatrix}$$ **[2013]**

(a) 3ω (b) -3ω (c) $3\omega^2$ (d) $-3\omega^2$

Ans. (d)

Sol.

$$\begin{vmatrix}1+\omega & \omega^2 & -\omega\\ 1+\omega^2 & \omega & -\omega^2\\ \omega^2+\omega & \omega & -\omega^3\end{vmatrix}=\begin{vmatrix}-\omega^2 & \omega^2 & \omega\\ \omega & \omega & -\omega^2\\ 1 & \omega & -\omega^2\end{vmatrix}$$

$$\left[\because 1+\omega+\omega^2=0\right]$$

$$=\omega^2\begin{vmatrix}-\omega & \omega & -1\\ -1 & 1 & -\omega\\ -1 & \omega & \omega^2\end{vmatrix}=\omega^2\begin{vmatrix}0 & \omega & -1\\ 0 & 1 & -1\\ \omega-1 & \omega & \omega^2\end{vmatrix}=\omega^2 x-3=-3\omega^2$$

11. If the vectors $2\hat{\mathbf{i}}-3\hat{\mathbf{j}},\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}-\hat{\mathbf{k}}$ form three coterminus edges of a parallelopiped, then the volume of parallelopiped is **[2013]**

(a) 8 (b) 10 (c) 4 (d) 14

Ans. (c)

Sol. Given vectors are $2\hat{i}-3\hat{j}+0\hat{k}$,

$\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}+0\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$$\text{Volume of parallelopiped}=\begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}$$

$$=2(-1)+3(2)=-2+6=4$$

12. In a GP consisting of positive terms, each term equals the sum of the next two terms. Then, the common ratio of the GP is **[2013]**

(a) $\frac{(1-\sqrt{5})}{2}$ (b) $\frac{(\sqrt{5})}{2}$

(c) $\sqrt{5}$ (d) $\frac{(\sqrt{5}-1)}{2}$

Ans. (d)

Sol. Let the term of GP be

$a,ar,ar^2,ar^3,ar^4,\ldots$

As per the question,

$$a=ar+ar^2\Rightarrow a(1)=a(r+r^2)\Rightarrow 1=r+r^2$$

$$\Rightarrow 1-r-r^2=0\Rightarrow r^2+r-1=0$$

Here, $a=1,b=1$ and $c=-1$

$$\therefore\quad r=\frac{-b\pm\sqrt{b^2-4ac}}{2\times a}=\frac{-1\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-1\pm\sqrt{(1)^2-4\times1\times(-1)}}{2\times1}=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

Since, r is positive.

Hence, $r=\frac{-1+\sqrt{5}}{2}$

13. If $f(x)=\tan^{-1}\left[\frac{\sin x}{1+\cos x}\right]$, then what is the first derivative of $f(x)$? **[2013]**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2

Ans. (a)

Sol. Given, $f(x)=\tan^{-1}\left[\frac{\sin x}{1+\cos x}\right]$

$$\Rightarrow f(x)=\tan^{-1}\left[\frac{2\sin x/2\cos x/2}{2\cos^2 x/2}\right]$$

$$\Rightarrow f(x)=\tan^{-1}\left[\frac{\sin x/2}{\cos x/2}\right]$$

$$\Rightarrow f(x)=\tan^{-1}[\tan x/2]\Rightarrow f(x)=x/2$$

$$\therefore\quad f'(x)=1/2$$

14. The solution of $\sin x+1=\cos x$, such that $0\le x\le 2\pi$, is **[2013]**

(a) $0,\pi$ (b) $0,\frac{\pi}{2}$

(c) $\frac{\pi}{2},\frac{3\pi}{2}$ (d) $0,\frac{3\pi}{2}$

Ans. (d)

Sol. Given, $\sin x+1=\cos x$

$\Rightarrow \cos x-\sin x=1$

Dividing by $\sqrt{2}$ on both sides, we get

$$\frac{\cos x}{\sqrt{2}}-\frac{\sin x}{\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x+\frac{\pi}{4}\right)=\cos\frac{\pi}{4}$$

$$\therefore \quad x+\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}$$

On putting $n=0$,

$$x+\frac{\pi}{4}=0\pm\frac{\pi}{4}$$

$\Rightarrow x=0 \quad [\because taking + \pi/4]$

Now, putting $n=1$,

$$x+\frac{\pi}{4}=2\times1\times\pi\pm\frac{\pi}{4}\Rightarrow x=2\pi-\frac{\pi}{4}-\frac{\pi}{4}$$

$\Rightarrow x=\frac{3\pi}{2} \quad [\because \text{ taking } -\pi/4]$

15. If T_n denotes the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1}-T_n=21$, then n equals **[2013]**

(a) 5 (b) 7 (c) 6 (d) 4

Ans. (b)

Sol. As per the given condition,

T_n triangle can be formed by using the vertices of a regular polygon of n

$\therefore \quad {}^{n+1}C_3-{}^{n}C_3=21\Rightarrow {}^{7+1}C_3-{}^{7}C_3=21$

Clearly, $n=7$

16. If $\bar{x}_1$ and $\bar{x}_2$ are the means of two distributions such that $\bar{x}_1<\bar{x}_2$ and $\bar{x}$ is the mean of the combined distribution, then **[2013]**

(a) $\bar{x}<\bar{x}_1$ (b) $\bar{x}>\bar{x}_2$

(c) $\bar{x}=\frac{\bar{x}_1+\bar{x}_2}{2}$ (d) $\bar{x}_1<\bar{x}<\bar{x}_2$

Ans. (d)

Sol. Since, $\bar{x}_1<\bar{x}_2$

Also, $\bar{x}$ is the mean of combined distribution.

So, from options, option (d) is correct.

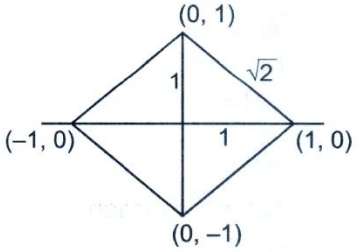
17. The area enclosed within the curve $|x|+|y|=1$ (in sq units) is **[2013]**

(a) $\sqrt{2}$ (b) 1 (c) $\sqrt{3}$ (d) 2

Ans. (d)

Sol. Given, curve is $|x|+|y|=1$

The graph of curve $|x|+|y|=1$ is given below:



This is a square of side $\sqrt{2}$.

$\therefore$ Area $=$ (Side)$^2=(\sqrt{2})^2$

$=2$ sq units

18. Let $f(x)$ be a polynomial function of second degree and $f(1)=f(-1)$. If a,b and c are in AP, then $f'(a), f'(b)$ and $f'(c)$ are in **[2013]**

(a) GP (b) HP (c) AGP (d) AP

Ans. (d)

Sol. Let the 2 degree polynomial be

$f(x)=px^2+qx+r$

$\therefore \quad f(1)=f(-1)$

$\Rightarrow p+q+r=p-q+r\Rightarrow q=0$

$\because \quad f'(x)=2px$

So, $f'(a), f'(b), f'(c)$ will be $2pa, 2pb$ and $2pc$.

Hence, it is an AP.

19. Find the point at which the tangent to the curve $y=\sqrt{4x-3}-1$ has its slope $\frac{2}{3}$ **[2013]**

(a) $(3,3)$ (b) $(3,2)$ (c) $(2,3)$ (d) $(2,2)$

Ans. (b)

Sol. $\therefore$ Slope $=\frac{dy}{dx}$

$$\Rightarrow \frac{4}{2\sqrt{4x-3}}=\frac{2}{3}$$

$$\left[\because y=\sqrt{4x-3}-1\Rightarrow \frac{dy}{dx}=\frac{4}{2\sqrt{4x-3}}\right]$$

$\Rightarrow \sqrt{4x-3}=3$

On squaring both sides, we get

$4x-3=9$

$\Rightarrow \quad 4x=12\Rightarrow x=3$

and $y=2$

Hence, required point is $(3,2)$.

20. Atal speaks truth in 70% and George speaks the truth in 60% cases. In what percentage of cases they are likely to contradict each other in stating the same

fact? **[2013]**

(a) $\frac{13}{50}$ (b) $\frac{11}{50}$ (c) $\frac{23}{50}$ (d) $\frac{33}{50}$

Ans. (c)

Sol. Atal speak truth in 70% cases.

So, probability of Atal speak truth is $\frac{70}{100}$ i.e. $\frac{7}{10}$.

Similarly,

Probability of George speak truth is $\frac{40}{100}$ i.e. $\frac{4}{10}$.

As per the given condition, they are likely to contradict each other in stating the same fact only when one will speak truth.

$\therefore$ Required probability $= \frac{7}{10}\times\frac{4}{10}+\frac{3}{10}\times\frac{6}{10}$

$= \frac{28}{100}+\frac{18}{100}=\frac{28+18}{100}=\frac{46}{100}=\frac{23}{50}$

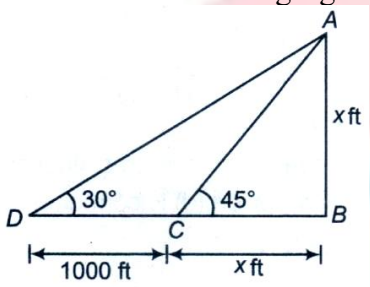
21. A man observes the angle of elevation of the top of a mountain to be 30°. He walks 1000 ft nearer and finds the angle of elevation to be 45°. What is the distance of the first point of observation from the foot of the mountain? **[2013]**

(a) $500\sqrt{3}\left(\sqrt{3}+1\right)$ft (b) $500\left(\sqrt{3}+1\right)$ft

(c) $500\left(\sqrt{3}-1\right)$ft (d) $500\sqrt{3}\left(\sqrt{3}-1\right)$ft

Ans. (a)

Sol. As per the given information in the question, we can draw the following figure.



where,

$AB = x$ft

$DC = 1000$ft

$CB = x$ft

In $\triangle ABD$,

$\Rightarrow \quad \frac{AB}{BD} = \tan\theta$

$\Rightarrow \quad \tan 30^\circ = \frac{x}{x+1000}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{x+1000}$

$\Rightarrow \quad \sqrt{3}x = x+1000$

$\Rightarrow \quad \sqrt{3}x - x = 1000$

$\Rightarrow \quad x = \frac{1000}{\sqrt{3}-1}$

$\therefore BD = 1000+\frac{1000}{\sqrt{3}-1} = 500\sqrt{3}\left(\sqrt{3}+1\right)$ft

22. The sum of n terms of an arithmetic series is 216. The value of the first term is n and the value of the nth term is $2n$. The common difference d is **[2013]**

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{12}{11}$

Ans. (d)

Sol. $\because S_n = 216$

$\because \quad \frac{n}{2}\left[2a+(n-1)d\right]=216$

where,

$T_n = 2n$

$a = n$

$2n = n+(n-1)d$

$n = (n-1)d$

and $d = \frac{n}{(n-1)}$

$\therefore \quad \frac{n}{2}\left[2n+(n-1)\times\frac{n}{(n-1)}\right]=216$

$\Rightarrow \quad \frac{n}{2}[3n]=216 \Rightarrow \frac{3n^2}{2}=216$

$\Rightarrow \quad n^2 = \frac{216\times 2}{3} = 72\times 2$

$\Rightarrow \quad n^2 = 144 \Rightarrow n = 12$

$\therefore \quad d = \frac{12}{11}$

23. Force $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ are acting on a particle and displace it from the point $2\hat{\mathbf{i}}-\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ to the point $4\hat{\mathbf{i}}-2\hat{\mathbf{j}}+7\hat{\mathbf{k}}$, then the work done by the force is **[2013]**

(a) 18 units (b) 30 units

(c) 24 units (d) 36 units

Ans. (c)

Sol. Work done = Force × Displacement

$$=\left(5\hat{i}+3\hat{j}+2\hat{k}\right)\times\left(2\hat{i}-2\hat{j}+10\hat{k}\right)$$

$$\left[\because \text{ force} = 5\hat{i}+3\hat{j}+2\hat{k} \text{ and displacement} = 2\hat{i}-2\hat{j}+10\hat{k}\right]$$

$$=10-6+20=24$$

24. The value of $9^{\frac{1}{3}}\cdot 9^{\frac{1}{9}}\cdot 9^{\frac{1}{27}}\ldots$ is **[2013]**

(a) 3 (b) 6

(c) 9 (d) None of these

Ans. (a)

Sol. $9^{\frac{1}{3}}\cdot 9^{\frac{1}{9}}\cdot 9^{\frac{1}{27}}\ldots$

$$=9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots}$$

$$=9^{\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)} \quad \left[\because a+ar+ar^2+\ldots\infty=\frac{a}{1-r}\right]$$

$$=9^{\left(\frac{\frac{1}{3}}{3-1}\right)}=9^{\frac{1}{3}/\frac{2}{3}}=9^{\frac{1}{3}\times\frac{3}{2}}=9^{1/2}=3$$

25. The minimum value of the function $y=2x^3-21x^2+36x-20$ is **[2013]**

(a) -120 (b) -126 (c) -128 (d) None of these

Ans. (d)

Sol. As they can increase upto any value and decrease upto any level. So, there is no maxima and minima for any odd degree function.

26. In how many different ways can the letters of the word 'CORPORATION' be arranged, so that all the vowels always come together? **[2013]**

(a) 810 (b) 1440 (c) 2880 (d) 50400

Ans. (d)

Sol. In word 'CORPORATION',

R repeated 2 times

O repeated 3 times

$\therefore$ Required probability

$$=\frac{7!5!}{2!3!}=7!\times 2\times 5$$

$$=7\times 6\times 5\times 4\times 3\times 2\times 1\times 2\times 5=50400$$

27. If $\log_x y=100$ and $\log_2 x=10$, then the value of y is **[2013]**

(a) 2^{10} (b) 2^{100} (c) 2^{1000} (d) 2^{10000}

Ans. (c)

Sol. Given, $\log_2 x=10$

$$\Rightarrow x=2^{10}$$

$$\therefore \log_{2^{10}} y=100$$

$$\Rightarrow y=\left(2^{10}\right)^{100}\Rightarrow y=2^{1000}$$

28. The equation of the line parallel to the line $2x-3y=7$ and passing through the mid-point of the line segment joining the points $(1,3)$ and $(1,-7)$ is **[2013]**

(a) $2x-3y-4=0$ (b) $2x-3y+4=0$

(c) $2x-3y-8=0$ (d) $2x-3y+8=0$

Ans. (c)

Sol. Slope of line $2x-3y=7$ is $\frac{2}{3}$.

Since, required line is parallel to this line, so slope of the line will be 2 / 3.

So, the mid-point of line segment joining the points $(1,3)$ and $(1,-7)$ is $(1,-2)$.

$\therefore$ Required line is $y+2=\frac{2}{3}(x-1)$

$$\Rightarrow 3y+6=2x-2$$

$$\Rightarrow 2x-3y-8=0$$

29. In a $\triangle ABC$, if $(c+a+b)(a+b-c)=ab$, then measure of $\angle C$ is **[2013]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) None of these

Ans. (c)

Sol. $\because (a+b)^2-c^2=ab$

$$\Rightarrow a^2+b^2+2ab-c^2=ab$$

$$\Rightarrow a^2+b^2+ab-c^2=0$$

$$\Rightarrow a^2+b^2-c^2=-ab$$

$$\therefore \cos c=\frac{a^2+b^2-c^2}{2ab}$$

$$\Rightarrow \cos c=\frac{-ab}{2ab}=\frac{-1}{2}\Rightarrow c=2\pi/3$$

30. The number of non-negative integers less than 1000 that contain the digit 1 are **[2013]**

(a) 9^2 (b) 9^3 (c) 10^2-9^2 (d) 10^3-9^3

Ans. (d)

Sol. $\because 1000=10^3$

So, the numbers containing 1 is 10^3-9^3.

31. The lines $3x-4y+4=0$ and $6x-8y-7=0$ are tangent to the same circle. The radius of this circle is **[2013]**

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{7}{10}$

Ans. (b)

Sol. Since, lines are parallel.

So, distance between them is the diameter of the circle.

Lines are $3x-4y+4=0$

and $3x-4y-\frac{7}{2}=0$

$\therefore$ Distance between them

$$=\frac{4-(-7/2)}{\sqrt{(3)^2+(4)^2}}=\frac{15}{10}=\frac{3}{2}$$

So, radius $=\frac{3}{4}$

32. The area of the parallelogram whose diagonals are $\mathbf{a}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ is **[2013]**

(a) $10\sqrt{3}$ (b) $5\sqrt{3}$ (c) $10\sqrt{2}$ (d) $5\sqrt{2}$

Ans. (a)

Sol. If the sides of parallelogram are p and q. Then, their area is $|p\times q|=$ Magnitude of

$$\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & 1 & -2\\ 1 & -3 & 4\end{vmatrix}$$

$=$ Magnitude of $-2\hat{\mathbf{i}}-14\hat{\mathbf{j}}-10\hat{\mathbf{k}}$

$$=\sqrt{300}=10\sqrt{3}$$

33. If $\sin x+a\cos x=b$, then what is the expression for $|a\sin x-\cos x|$ in terms of a and b ? **[2013]**

(a) $\sqrt{a^2-b^2-1}$ (b) $\sqrt{a^2+b^2-1}$

(c) $\sqrt{a^2+b^2+1}$ (d) $\sqrt{a^2-b^2+1}$

Ans. (d)

Sol. Given,

$$\sin x+a\cos x=b\ldots(i)$$

$$a\sin x-\cos x=p\ldots(ii)$$

Squaring and adding Eqs. (i) and (ii), we get

$$\sin^2x+a^2\cos^2x+a^2\sin^2x+\cos^2x=b^2+p^2$$

$$\Rightarrow\ p^2=1+a^2-b^2$$

Hence, $|p|=|a\sin x-\cos x|=\sqrt{1+a^2-b^2}$

34. If A and B are two events such that $P(A\cup B)=\frac{5}{6}$, $P(A\cap B)=\frac{1}{3}$ and $P(\bar{B})=\frac{1}{2}$, then the events A and B are **[2013]**

(a) dependent (b) independent

(c) mutually exclusive (d) None of these

Ans. (b)

Sol. Given, $P(A\cup B)=5/6$

$$P(A\cap B)=\frac{1}{3}$$

and $P(B)=\frac{1}{2}$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow\frac{5}{6}=P(A)+\frac{1}{2}-\frac{1}{3}$$

$$\Rightarrow P(A)=\frac{4}{6}=\frac{2}{3}$$

So, $P(A)\cdot P(B)=P(A\cap B)$

$\therefore$ Events are independent.

35. If the three vectors $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}},\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}+\lambda\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ are coplanar, then λ is **[2013]**

(a) -1 (b) -2 (c) -3 (d) -4

Ans. (d)

Sol. Since, vectors are coplanar.

$$\therefore\quad\begin{vmatrix}2 & -1 & 1\\ 1 & 2 & -3\\ 3 & \lambda & 5\end{vmatrix}=0$$

$$\Rightarrow\lambda=-4$$

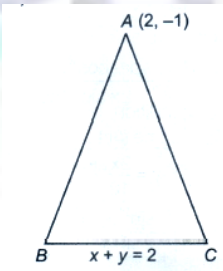
36. The equation of the base of an equilateral triangle is $x+y=2$ and the vertex is $(2,-1)$. The length of the side of the triangle is **[2013]**

(a) $\sqrt{\frac{3}{2}}$ (b) $\sqrt{2}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{20}{3}}$

Ans. (c)

Sol. Equation of base is $x+y=2$.

Vertex is $(2,-1)$.



Length of perpendicular $=\left|\frac{2-1-2}{\sqrt{2}}\right|=\frac{1}{\sqrt{2}}$

Side of the triangle is $\frac{h}{\sin B}=\frac{1/\sqrt{2}}{\sqrt{3}/2}=\sqrt{\frac{2}{3}}$

37. The total number of numbers that can be formed using the digits 3,5 and 7 only, if no repetitions are allowed, is **[2013]**

(a) 39 (b) 105 (c) 15 (d) 27

Ans. (c)
Sol. Single digit number $=3$
Two digit numbers are $3\times2=6$
Three digit numbers are $3\times2\times1=6$
Hence, total numbers are 15.

38. If $x=a\cos t, y=b\sin t$, then $\frac{d^2y}{dx^2}$ is **[2013]**

(a) $-\frac{b^4}{a^2y^3}$ (b) $\frac{b^4}{a^2x^3}$

(c) $\frac{b}{ay^4}$ (d) $-\frac{a^4}{bx^3}$

Ans. (a)
Sol. Given, $x=a\cos t$
and $y=b\sin t$

$$\therefore \quad \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{-b}{a}\cot t$$

Let $u=\frac{dy}{dx}=\cot t$

$$\therefore \quad \frac{d^2y}{dx^2}=\frac{du}{dx}=\frac{du}{dt}\times\frac{dt}{dx}=\left(\frac{b}{a}\operatorname{cosec}^2t\right)\times\left(\frac{-1}{a\sin t}\right)$$

$$=\frac{-b}{a^2\sin^3t}=\frac{-b^4}{a^2y^3}$$

39. A random variable x has the distribution law as given below: **[2013]**

x	1	2	3
$P(X=x)$	0.3	0.4	0.3

The variance of the distribution is
(a) 0.4 (b) 0.6 (c) 2 (d) None of these
Ans. (b)
Sol. Variance $=\bar{z}\left(x_i-\bar{x}\right)^2 p_i$

$\bar{x}=\bar{z}\,p_ix_i=1\times0.3+2\times0.4+3\times0.3=2$
$\therefore$ Variance
$=(1-2)^2\times0.3+(2-2)^2\times0.4+(3-2)^2\times0.3=0.6$

40. The value of $\tan\theta+2\tan2\theta+4\tan4\theta+8\cot8\theta$ is **[2013]**

(a) $\cot\theta$ (b) $\tan\theta$ (c) $\sin\theta$ (d) $\cos\theta$
Ans. (a)
Sol. $\because \tan A-\cot A=\frac{\sin^2A-\cos^2A}{\cos A\sin A}$

$$=\frac{-2\cos2A}{\sin2A}=-2\cot2A$$

Adding and subtracting $\cot\theta$ in the equation, we get
$\tan\theta+2\tan2\theta+4\tan4\theta+8\cot8\theta$
$=\tan\theta-\cot\theta+2\tan2\theta+4\tan4\theta+8\cot8\theta+\cot\theta$
$=-2\cot2\theta+2\tan2\theta+4\tan4\theta+8\cot8\theta+\cot\theta$

41. The sum of integers between 200 and 400 , that are multiples of 7 is **[2013]**
(a) 8729 (b) 8700 (c) 8972 (d) 8279
Ans. (a)
Sol. First term multiplied by 7 in between 200 and 400 , is 203 .
$\therefore 203+210+\ldots+399[\because$ last term $=399]$
There are $\frac{399-203}{7}+1=29$ terms in the series.

$$\therefore \text{ Sum of 29 terms } =\frac{(203+399)}{2}\times29=8729$$

42. $\lim_{x\to0}\frac{\tan x-x}{x^2\tan x}$ is equal to **[2013]**

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Ans. (d)
Sol. $\lim_{x\to0}\frac{\tan x-x}{x^2\tan x}$

$$=\lim_{x\to0}\left[\frac{\sec^2x-1}{x^2\sec^2x+2x\tan x}\right]$$

Applying L' Hospital's rule,

$$=\lim_{x\to0}\left[\frac{1-\cos^2x}{x^2+2x\sin x\cos x}\right]$$

$$=\lim_{x\to0}\left[\frac{\sin^2x}{x^2+2x\sin x\cos x}\right]$$

$$=\lim_{x\to0}\left[\frac{\sin^2x/x^2}{1+\frac{2\sin x}{2}\cdot\cos x}\right]=\frac{1}{3}$$

43. Two fair dice are tossed. What is the probability that the total score is a prime number? **[2013]**

(a) $\frac{1}{6}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{7}{9}$

Ans. (b)
Sol. Sum is a prime number i.e. the sum of the dice outcomes can be $2,3,5,7,11$.
These sums are shown in following pairs
$(1,1),(1,2),(2,3),(3,4),(5,6)$
$(5,2),(4,1),(2,1),(4,3),(6,5)$
$(2,5),(1,4),(3,2),(1,6),(6,1)$

$\therefore$ Required probability $=\frac{15}{36}=\frac{5}{12}$ $[2$ dice $=6\times6=36]$

44. Find the equation of the circle which passes through $(-1,1)$ and $(2,1)$ and having centre on the line $x+2y+3=0$. **[2013]**

(a) $2x^2+2y^2-2x+7y-13=0$

(b) $x^2+y^2-2x+7y-13=0$

(c) $2x^2+2y^2+2x+7y-13=0$

(d) $x^2+y^2+2x+7y-13=0$

Ans. (a)

Sol. Since, centre lies on the line $x+2y+3=0$.

Let the centre be $(-2a-3,a)$, then its distance from the given two points $(-1,1)$ and $(2,1)$ is

$$(-2a-3+1)^2+(a-1)^2=(-2a-3-2)^2$$
$$+(a-1)^2(-2a-2)^2=(-2a-5)^2$$

$\Rightarrow a=-\frac{7}{4}$

Thus, the centre is $\left(\frac{1}{2},\frac{7}{4}\right)$.

Radius is $\sqrt{\frac{157}{16}}$.

So, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y+\frac{7}{4}\right)^2=\frac{157}{16}$$

$\Rightarrow 2x^2+2y^2-2x+7y-13=0$

45. If $\mathbf{a},\mathbf{b}$ and $\mathbf{c}$ are the position vectors of three vertices A,B and C of a triangle, respectively. Then, the area of this triangle is given by **[2013]**

(a) $\frac{1}{2}(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$ (b) $\frac{1}{2}|\mathbf{a}\times\mathbf{b}+\mathbf{c}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|$

(c) $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}$ (d) None of these

Ans. (b)

Sol. The two adjacent sides of the triangle are represented by $(\mathbf{b}-\mathbf{a})$ and $(\mathbf{c}-\mathbf{a})$, thus its area

$$=\frac{1}{2}|(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})|$$
$$=\frac{1}{2}|\mathbf{b}\times\mathbf{c}-\mathbf{b}\times\mathbf{a}-\mathbf{a}\times\mathbf{c}|=\frac{1}{2}|\mathbf{b}\times\mathbf{c}+\mathbf{a}\times\mathbf{b}+\mathbf{c}\times\mathbf{a}|$$

46. The sum of the focal distances of any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with eccentricity e, is given by **[2013]**

(a) $2ae$ (b) $2b$ (c) $2a$ (d) $2be$

Ans. (c)

Sol. ... Standard result regarding distance between two focal $PS+PS'=2a$

47. If $\sin x+\sin^2 x=1$, then $\cos^2 x+\cos^4 x$ is equal to **[2013]**

(a) 0 (b) 1 (c) -1 (d) 2

Ans. (b)

Sol. Given, $\sin x+\sin^2 x=1$

$\Rightarrow \sin x=1-\sin^2 x\Rightarrow \sin x=\cos^2 x$

$\therefore \cos^2 x=\sin x$

So, given equation $\cos^2 x+\cos^4 x$ can be written as $\cos^2 x+\cos^{2\times 2} x$ i.e. $\sin x+\sin^2 x$ where, $\sin x+\sin^2 x=1$

48. An experiment succeeds twice often as it fails. The probability that in the next six trials there will be atleast four successes, is **[2013]**

(a) $\frac{240}{729}$ (b) $\frac{496}{729}$ (c) $\frac{220}{729}$ (d) $\frac{233}{729}$

Ans. (b)

Sol. Suppose probability of success is $2p$ and failure is p.

Then,

$$2p+p=1\Rightarrow p=\frac{1}{3}$$

Hence, probability of getting success is $\frac{2}{3}$ and failure is $\frac{1}{3}$.

$\therefore$ Required probability $=P$ (atleast 4 success)

$$={}^6C_4\,(2/3)^4\left(\frac{1}{3}\right)^2+{}^6C_5(2/3)^5(1/3)^1+{}^6C_6(2/3)^6$$

$$=\frac{15\times16+6\times32+64}{3^6}=\frac{16(15+12+4)}{3^6}=\frac{496}{729}$$

49. Sum of 20 terms of the series $-1^2+2^2-3^2+4^2-\ldots$ is **[2013]**

(a) 180 (b) 200 (c) 210 (d) 220

Ans. (c)

Sol. Given, series $-1^2+2^2-3^2+4^2+\ldots$

$$=\{(-1)^2+(2)^2\}+\{(-3)^2+(4)^2\}+\ldots+\{(-19)^2+(20)^2\}$$
$$=3+7+11+\ldots+39$$

Required sum (for 10 terms)

$$=\left(\frac{3+39}{2}\right)\times10=210$$

50. 50. If $\tan\alpha=\frac{m}{m+1}$ and $\beta=\frac{1}{2m+1}$, then $a+b$ is equal to **[2013]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) π

Ans. (b)

Sol. Given, $\tan\alpha = \frac{m}{m+1}$ and $\beta = \frac{1}{2m+1}$

$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha - \tan\beta}$

$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)} = \frac{2m^2+2m+1}{2m^2+2m+1} = 1$

$\Rightarrow \tan(\alpha+\beta) = 1$

$\Rightarrow \alpha + \beta = \pi/4$

Analytical Ability and Logical Reasoning

51. A train takes 18 s to pass completely through a station 162 m long and 15 s through another station 120 m long, at the same speed. What is the length of the train in metre? **[2013]**

(a) 70 (b) 80 (c) 90 (d) 100

Ans. (c)

Sol. Let the length of the train be xm.

Then, $\frac{x+162}{18} = \frac{x+120}{15}$

$\Rightarrow \frac{x+162}{6} = \frac{x+120}{5}$

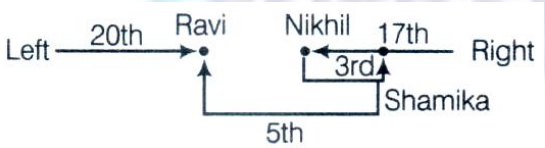
$\Rightarrow 6x + 720 = 5x + 810 \Rightarrow x = 90m$

52. In a row of children facing North, Shamika is third to the right of Nikhil, who is 17th from the right end of the row. Ravi is 5th to the left of Shamika and is 20th from the left end. Totally how many children are there in the row? **[2013]**

(a) 37 (b) 38 (c) 39 (d) None of these

Ans. (b)

Sol.



$\therefore$ Ravi's position from the left $= 20$ th

and Ravi's position from the right $= 17 + 2 = 19$ th

$\therefore$ Number of children in the row $= (20+19) - 1 = 38$

53. Given that

(i) Some apples are blackberries.

(ii) Some doughnuts are apples.

(iii) No coconut is a doughnut.

(iv) All blackberries are coconuts.

Which of the following statements is false? **[2013]**

(a) Some blackberries are doughnuts

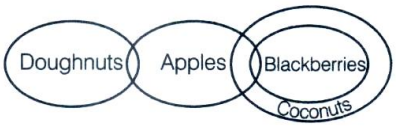
(b) Some coconuts are apples

(c) All coconuts are not apples

(d) All doughnuts are not coconuts

Ans. (a)

Sol. According to the statements,



Hence, some blackberries are doughnuts, is definitely, false.

Directions [Q. Nos. 54-56] In a family of 6 persons, there are two couples.

The lawyer is the head of the family and has two sons-Mukesh and Rakesh both teachers.

Mrs. Reena and her mother-in-law both are lawyers.

Mukesh's wife is a doctor and they have a son, Ajay.

54. Which of the following is definitely a couple? **[2013]**

(a) Lawyer-Teacher (b) Doctor-Lawyer

(c) Teacher-Teacher (d) None of the above

Ans. (a)

Sol. Rakesh (Teacher) and Reena (Lawyer) isdefinitely a couple.

Solutions [Q. Nos. 54 to 56]

Reena's mother-in-law $\rightarrow$ head of the family $\rightarrow$ Lawyer

Rakesh $\rightarrow$ Married $\rightarrow$ Teacher

Mukesh $\rightarrow$ Married $\rightarrow$ Teacher

Reena $\rightarrow$ Rakesh's wife $\rightarrow$ Lawyer

Mukesh's wife $\rightarrow$ Doctor

Ajay $\rightarrow$ Mukesh's son

Ajay's grandfather $\rightarrow$ Unknown profession

55. What is the profession of Rakesh's wife? **[2013]**

(a) Teacher (b) Doctor

(c) Lawyer (d) Cannot be determined

Ans. (c)

Sol. Rakesh's wife's profession is Lawyer.

56. What is/was Ajay's grandfather's occupation? **[2013]**

(a) Teacher (b) Lawyer

(c) Doctor (d) Cannot be determined

Ans. (d)

Sol. We cannot determine the Ajay's grandfather occupation.

57. Find the missing element is the series.

A, CD, GHI, ..., UVWXY **[2013]**

(a) LMNO (b) MNOP

(c) NOPQ (d) OPQR

Ans. (b)

Sol. The pattern of series is as follows:

58. In a code language, 'FRIEND' is coded as 'GTLISJ'. Which of the following is coded as 'HWDVI' in that language? **[2013]**

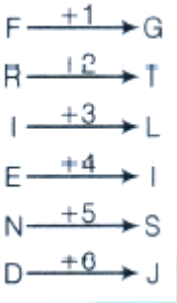
(a) HAPPY (b) GUARD

(c) BEADS (d) SPEED

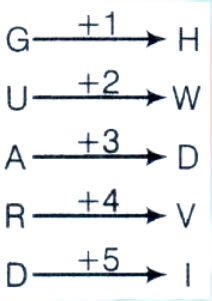
Ans. (b)

Sol.

As,



Similarly,



59. There are four brothers Alan, Bob, Carl and Dave. Dave is two years older than Bob, Bob is one year younger than Carl. Alan, who is 34, is two years younger than Carl. Who is the oldest? **[2013]**

(a) Alan (b) Bob (c) Carl (d) Dave

Ans. (d)

Sol. Given, Alan's age $=34$yr

Carl's age $=34+2=36$yr

Bob's age $=36-1=35$yr

Dave's age $=35+2=37$yr

Hence, Dave is the oldest among them

Directions [Q. Nos. 60-62]An employee has been assigned the task of allotting offices to six of the staff members. The offices are numbered 1-6. The offices are arranged in a row and they are separated from each other by six foot high dividers. Hence, voices sounds and cigarette smoke flow easily from one office to another.

Miss. Robert needs to use the telephone quite often throughout the day. Mr. Mike and Mr. Brown need adjacent offices as they need to consult each other often while working. Miss. Hardy is a senior employee and has to be allotted the office number 5 , having the biggest window. Mr. Donald requires silence in the offices next to his and Mr. Tim prefers to be as away as possible from Miss. Robert. Mr. Mike and Mr. Donald are all smokers. Miss. Hardy finds tobacco smoke allergic and consecutively the offices next to hers to be occupied by non-smokers. Unless specifically stated all the employees maintain an atmosphere of silence during office hours.

60. The ideal candidate to occupy the office farthest from Mr. Brown would be **[2013]**

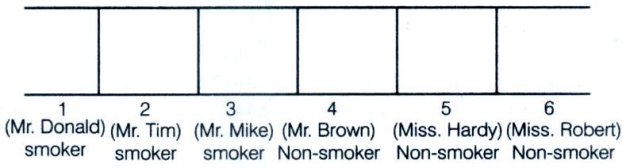
(a) Miss. Hardy (b) Mr. Mike

(c) Mr. Tim (d) Mr. Donald

Ans. (d)

Sol. Mr. Donald's office is farthest from Mr. Brown.

Solutions [Q. Nos. 60 to 62]



61. The three employees who are smokers, should be seated in the offices **[2013]**

(a) 1,2 and 4 (b) 2, 3 and 6

(c) 1, 2 and 6 (d) 1,2 and 3

Ans. (d)

Sol. The three smoker employers of the offices should be seated in 1, 2 and 3.

62. The ideal office for Mr. Mike would be **[2013]**

(a) 2 (b) 6 (c) 1 (d) 3

Ans. (d)

Sol. The ideal office for Mr. Mike would be 3.

63. A doctor said to his compounder ' I go to see the patients at their residence after every 3 h 30 min . I have already gone to the patient 1 h 20 min ago and next time I shall go at $1:40$pm.' At what time this information was given to the compounder by the doctor? **[2013]**

(a) 11:30 am (b) 11:20 am

(c) 10:10am (d) None of these

Ans. (a)

Sol. Doctor's previous time to see the patients $=1:40\text{pm}-3\text{ h}30\text{ m}=10:10\text{am}$

So, the information given to the compounder by the doctor

$=10:10\text{am}+1:20=11:30\text{am}$

64. Which pair of numbers comes next in the following series? **[2013]**

42 40 38 35 33 31 28

(a) 25 22 (b) 26 23 (c) 26 24 (d) 25 23

Ans. (c)

Sol. Series is increasing in three consecutive terms like 4240 38; 3533 31; 282624

Directions [Q. Nos. 65-66]

(i) All G's are H's

(ii) All G's are J's or K's

(iii) All J's and K's are G's

(iv) All L's are K's
(v) All N's are M's
(vi) No M's are G's

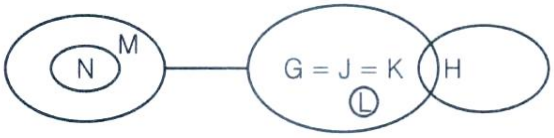
65. If no P's are K's, then which of the following must be true? **[2013]**

(a) All P's are J's
(b) If any P is a G, it is a J
(c) No P is an H
(d) If any P is an H, it is a G

Ans. (b)

Sol. If no P 's are K 's, then if any P is a G, it is a J, must be true.

Solutions [Q. Nos. 65 to 66]



66. Which of the following is inconsistent with one or more of the conditions? **[2013]**

(a) All H's are G's
(b) All H's that are not G's are M's
(c) Some H's are both M's and G's
(d) No M's are H's

Ans. (c)

Sol. Some H's are both M's and G's is inconsistent with one or more of the conditions.

67. Shyam is taller than Pradeep and Pradeep is as tall as Anurag. But Anand is shorter than Suresh, who is as tall as Anurag. If Pradeep is taller than Praveen, then who is the tallest of all? **[2013]**

(a) Pradeep (b) Praveen
(c) Suresh (d) Shyam

Ans. (d)

Sol. According to the question,
Shyam $>$ Pradeep $=$ Anurag $=$ Suresh $>$ (Anand, Praveen) Hence, Shyam is tallest among them.

68. When Rajeev was born his father was 32 yr older than his brother and his mother was 25 yr older than his sister. If Rajeev's brother is 6 yr older than Rajeev and his mother is 3 yr younger than his father, then how old was Rajeev's sister when he was born? **[2013]**

(a) 15 yr (b) 14 yr (c) 7 yr (d) 10 yr

Ans. (d)

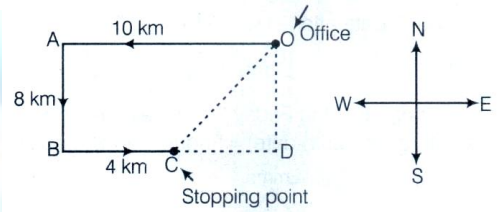
Sol. Suppose,
Rajeev's age $=0$ (Born)
Then, Rajeev's brother's age $=6\text{yr}$
Rajeev's father's age $=6+32=38\text{yr}$
Rajeev's mother's age $=38-3=35\text{yr}$
Rajeev's sister's age $=35-25=10\text{yr}$

69. Dhoni starts from his office at 8 am on a Sunday morning, travels 10 km towards West and then turns to his left and walks 8 km . Then, he again turns to his left and walks 4 km and then stops. What is the shortest distance to his office from the point, where he stopped? **[2013]**

(a) 18 km (b) 8 km
(c) 10 km (d) None of these

Ans. (c)

Sol. Dhoni's walking directions are as follows:



$\therefore \quad OD = AB = 8KM$

$\Rightarrow \quad CD = BD(AO) - BC$

$= (10 - 4\text{km})$

$= 6\text{km}$

$\therefore$ Required distance $(\text{OC}) = \sqrt{\text{OD}^2 + \text{CD}^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$

$= \sqrt{100} = 10 \text{ km}$

70. A treasure chest has less than 100 gold coins. The number of coins is **[2013]**

(i) one more than a multiple of 3
(ii) two more than a multiple of 4
(iii) three more than a multiple of 5 and
(iv) four more than a multiple of 6

How many coins are there in the chest?

(a) 58 (b) 88 (c) 98 (d) 38

Ans. (a)

Sol. By option, number 58 follows the statements rule.

$58 = \underline{3} \times 19 + \boxed{1}$

$58 = \underline{4} \times 14 + \boxed{2}$

$58 = \underline{5} \times 11 + \boxed{3}$

$58 = \underline{6} \times 9 + \boxed{4}$

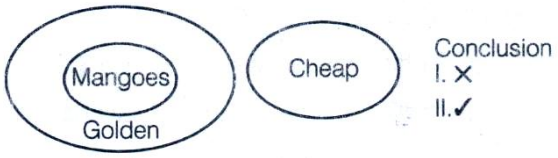
71. Read the statements and then decide which of the conclusions logically follow: **[2013]**

Statements
(i) All mangoes are golden in colour.
(ii) No golden coloured things are cheap.

Conclusions
(i) All mangoes are cheap.
(ii) Golden coloured mangoes are not cheap.

(a) Only conclusion I follows
(b) Only conclusion II follows

(c) Either I or II follows
(d) Neither I nor II follows
Ans. (b)
Sol. According to the statements, Venn diagram is as follows:



Hence, only conclusion II follows.

Directions [Q. Nos. 72-73]
A blacksmith has five iron articles A, B, C, D and E, each having a different weight.
A weighs twice as much as B.
B weighs four and half times as much as C.
C weighs half as much as D.
D weighs half as much as E.
E weighs less than A but more than C.

72. Which of the following article is heaviest in weight? **[2013]**
(a) A (b) B (c) C (d) D
Ans. (a)
Sol. A is the heaviest article in weight.

Solutions [Q. Nos. 72 to 73]
According to the question.
$A = 2B....(i)$
$B = 4.5C....(ii)$
$C = \frac{1}{2}D....(iii)$
$D = \frac{1}{2}E....(iv)$
$A > E > C....(v)$
Combining all the equations,
$A > B > E > D > C$

73. Which of the following represents the descending order of weights of the articles? **[2013]**
(a) A, B, E, D, C (b) B, D, E, A, C
(c) A, B, C, D, E (d) C, D, E, B, A
Ans. (a)
Sol. A, B, E, D, C

Directions [Q. Nos. 74-76]
There are three switches A, B and C which can be in ON/OFF position. Their settings change as per the following rules:
(i) If A is the only switch as ON, change B to ON.
(ii) If A and B are only switches as ON, change C to ON.
(iii) If all three switches are ON, change C to OFF.
(iv) For all other situations, all switches in ON are changed to OFF and all switches in OFF are changed to ON.

74. If switches A and B are ON and C is OFF, then their changed settings will be **[2013]**
(a) AON, BOFF, COFF
(b) A ON, B ON, C ON
(c) A ON, B OFF, C ON
(d) A OFF, B ON, C OFF
Ans. (b)
Sol. If switches A and B are ON and C is OFF, their changed settings will be AON, BON and CON.

Solutions [Q. Nos. 74 to 76]
A (only ON) $\rightarrow$ B(ON)
$A(ON) + B(ON) \rightarrow C(ON)....(i)$
$A(ON) + B(ON) + C(ON) \rightarrow C(OFF)....(ii)$
Except,
ON $\rightarrow$ OFF and OFF $\rightarrow$ ON

75. If only B is ON, then the changed setting will be **[2013]**
(a) AON, BON, CON (b) A ON, B ON, C OFF
(c) A ON, B OFF, C ON (d) A OFF, B OFF, C ON
Ans. (c)
Sol. If only B is ON, the changed setting will be $AON, BOFF$ and CON.

76. If only B is ON in the changed setting, then which of the following could have been the original setting? **[2013]**
(a) A ON, B ON, C ON (b) A ON, B OFF, C ON
(c) A OFF, B ON, C OFF (d) A OFF, B OFF, C ON
Ans. (b)
Sol. If only B is ON in the changed setting, the original setting. could have been $AON, BOFF$ and CON.

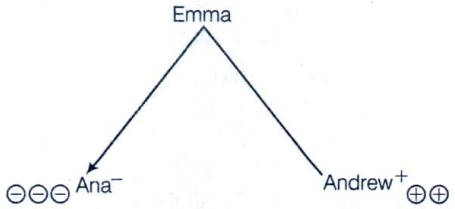
77. If the third day of a month falls on Friday, then what day will be on the fourth day after twenty first of the month? **[2013]**
(a) Monday (b) Tuesday
(c) Saturday (d) Thursday
Ans. (c)
Sol. $\therefore$ Fourth date after 21 st $= 21 + 4 = 25$ Then,

3	10	17	24
↑	↑	↑	↑
Ist Friday	2nd Friday	3rd Friday	4 th Friday

$\therefore$ Required day (25 th date) = Friday +1 = Saturday

78. Ana is a girl and has the same number of brothers and sisters. Andrew is a boy and has twice as many sisters as brothers. Ana and Andrew are the children of Emma. How many children does Emma have? **[2013]**
(a) 2 (b) 3 (c) 5 (d) 7

Ans. (d)
Sol.



$\therefore$ Number of children $= \Theta + \Theta + \Theta +$ Ana$^-$
$+$ Andrew $+ \Theta + \Theta$
$= 7$

Directions [Q. Nos. 79-81]
1. Anu is taller than Cini.
2. Eenu is shorter than Binu.
3. Anu is shorter than Dany.
Eenu is taller than Anu.

79. The best answer to 'who is the tallest ?' is **[2013]**
(a) Dany (b) Binu
(c) Dany or Binu (d) Both Dany and Binu
Ans. (c)
Sol. Dany or Binu is the tallest among them.
Solutions [Q. Nos. 79 to 81]
$Anu > Cini....(i)$
$Binu > Eenu....(ii)$
$Dany > Anu....(iii)$
$Eenu > Anu....(iv)$
Combining all the equations,
(Dany, Binu) > Eenu > Anu > Cini

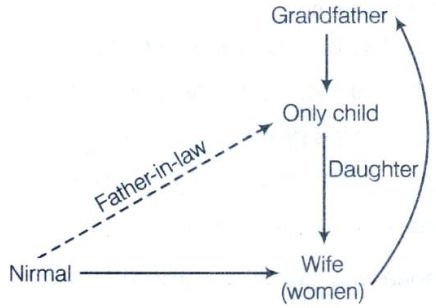
80. Who is the shortest? **[2013]**
(a) Cini (b) Anu or Cini
(c) Eenu (d) Insufficient data to conclude
Ans. (a)
Sol. Cini is the shortest among them.

81. Which of the following statements would help to logically order the persons according to their heights? **[2013]**
(a) Binu is 7 feet tall
(b) Dany and Binu do not have equal height
(c) Eenu is the tallest in the group
(d) Dany is the tallest in the group
Ans. (d)
Sol. Dany is the tallest in the group, which would help to logically order the persons according to their heights.

82. Karan and Arjun run a 100 m race, where Karan beats Arjun by 10 m . To do a favour of Arjun, Karan starts 10 m behind the starting line in a second 100 m race. They both run at their earlier speeds. Which of the following is true in connection with the second race? **[2013]**
(a) Karan and Arjun reach the finishing line simultaneously
(b) Arjun beats Karan by 1 m
(c) Arjun beats Karan by 11 m
(d) Karan beats Arjun by 1 m
Ans. (d)
Sol. In Ist 100 m race, distane covered by Karan $= 100$ m
and distance covered by Arjun $= 90$ m
In 2nd 100 m race, distance covered by Karan $= 110$ m
and distance covered by Arjun = ?
100 m of Karan $= 90$ m of Arjun
110 m of Karan $= \dfrac{90 \times 110}{100} = 99$ m of Arjun
$\therefore$ Karan beats Arjun by 1 m .

83. In a cricket season, India defeated Australia twice. West Indies defeated India twice. Australia defeated West Indies twice. India defeated New Zealand twice and West Indies defeated New Zealand twice. Which country has lost most number of times? **[2013]**
(a) India (b) Australia
(c) New Zealand (d) West Indies
Ans. (c)
Sol. India $(+2) \rightarrow$ Australia (-2)
West Indies $(+2) \rightarrow$ India (-2) $\begin{bmatrix} + \rightarrow \text{win} \\ - \rightarrow \text{loss} \end{bmatrix}$
Australia $(+2) \rightarrow$ West Indies (-2)
India $(+2) \rightarrow$ New Zealand (-2)
West Indies $(+2) \rightarrow$ New Zealand (-2)
Hence, New Zealand Country has lost 4 matches most number of times.

84. Pointing to a woman, Nirmal said, "She is the daughter of my wife's grandfather's only child". How is the woman related to Nirmal? **[2013]**
(a) Wife (b) Sister-in-law
(c) Sister (d) None of these
Ans. (a)
Sol.



So, women is related as wife of the Nirmal.

Directions [Q. Nos. 85-87]

There are five persons A, B, C, D and E standing on six steps numbered $1,2,3,4,5$ and 6 from the bottom. Atmost one person is standing on each step. The step number on which A is standing, is two less than that of C . Step number on which B is standing, is one more than that of D .

85. If A is standing on step 1 , which of the following is true? **[2013]**

(a) B is standing on step 2
(b) C is standing on step 4
(c) E is standing on step 3
(d) D is standing one step higher than C

Ans. (d)

Sol. According to condition IV, if A is standing on step 1, then D is standing one step higher than C.

Solutions [Q. Nos. 85 to 87]

Condition I	Condition II	Condition III	Condition IV
$6-C$	$6-$	$6\ldots B$	$6\ldots B$
$5-$	$5-C$	$5\ldots D$	$5\ldots B,D$
$4-A$	$4-$	$4-C$	$4\ldots D$
$3\ldots B$	$3-A$	$3-$	$3-C$
$2\ldots B,D$	$2\ldots B$	$2-A$	$2-$
$1-D$	$1\ldots D$	$1-$	$1-A$

86. If D is standing on step 1 , on which step A could be standing? **[2013]**

(a) 2 or 4 only (b) 3 or 5 only
(c) 3 or 4 only (d) 4 or 5 only

Ans. (c)

Sol. According to conditions I and II, if D is standing on step, 1 then A could be standing on either step 3 or 4 only.

87. If there are two steps in between the steps on which A and D are standing, A must be standing on which of the following steps? **[2013]**

(a) 3 (b) 4 (c) 5 (d) 6

Ans. (b)

Sol. According to conditions I, III and IV, if there are two steps in between the steps A and D, then A may be standing in 4 , 2 and 1 steps. So, in respect of option, we say that A must be standing on step 4.

88. From the information given below:
A * B means A and B are of the same age.
$A-B$ means B is younger than A.
$A+B$ means A is younger than B.
What does Sachin * Mohan - Ravi mean? **[2013]**

(a) Sachin is youngest (b) Ravi is youngest
(c) Sachin is oldest (d) Mohan is oldest

Ans. (b)

Sol. $A\times B=A=B$

$A-B\Rightarrow A>B$

$A+B\Rightarrow A<B$

Then, Sachin * Mohan - Ravi
$\Rightarrow$ Sachin $=$ Mohan $>$ Ravi
Hence, Ravi is youngest among them.

89. Jimmy saw the time while going to the tennis court. He saw the hour hand is 20° away from 4 . After he returned from tennis court, he noticed that the hour hand is 20° away from 4 . If he took ten minutes to go to tennis court and he walked at the same speed while going to the tennis court and while returning, how much time did he spent at the tennis court? **[2013]**

(a) 60 min (b) 80 min (c) 70 min (d) 50 min

Ans. (a)

Sol. Total movement of hour hand $=20^\circ+20^\circ=40^\circ$

So, hour hand covers $\frac{1^\circ}{2}$ in 1 min .

Also, hour hand covers 40° in 80 min .

Time duration to go tennis court and returning in $(10+10)$ min

$=20$ min

$\therefore$ Total spent time at the tennis court $=(80-20)$ min

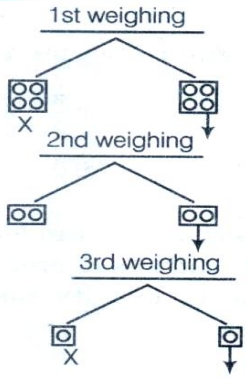
$=60$ min

90. There are 8 balls looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier ball is **[2013]**

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c)

Sol.



Hence, three times of weighing is required to identify the heavier ball.

General English

91. Out of the alternatives, choose the appropriate phrase to make the sentence meaningful. If you had joint accounts with _____ who died, then you will be responsible for the bills. **[2013]**
(a) everybody (b) anyone
(c) everyone (d) someone
Ans. (d)
Sol. 'Someone who died' is the correct phrase.

92. Choose the analogy that is closest in meaning to the pair: Diamond : Necklace **[2013]**
(a) Cars : Roads (b) Flowers : Bouquet
(c) Gold : Bangle (d) Books : Shop
Ans. (c)
Sol. Easy choice from among the given alternatives is Gold: Bangle.

93. Choose the suitable preposition for the blank to make a meaningful sentence. Suresh is angry ..... his servant. **[2013]**
(a) about (b) on (c) by (d) with
Ans. (d)
Sol. Angry with somebody is used.

94. Choose the correct alternative for the sentence below: The Earth is always revoling round the Sun. **[2013]**
(a) The Earth revolves round the Sun
(b) The Earth is revolving round the Sun
(c) The Earth revolving round the Sun
(d) None of the above
Ans. (a)
Sol. The Earth revolves round the Sun.

95. Choose the word that best expresses the meaning of the given idiom: 'A close shave.' **[2013]**
(a) A clean shave (b) A narrow escape
(c) A guarded secret (d) A sudden fall
Ans. (b)
Sol. A close shave means 'a narrow escape'.

96. Pick the part of the sentence that has an error: My elder brother is a MA whereas I am only a BA **[2013]**
(a) My elder brother (b) is a MA
(c) whereas I am (d) only a BA
Ans. (b)
Sol. MA begins with a vowel sound, so 'an' will be used.

97. Choose the suitable phrasal verb for the blank in the sentence below: **[2013]**
I _____ my hopes when untimely rain threatened my crops.
(a) gave in (b) gave out
(c) gave up (d) gave off
Ans. (c)
Sol. To gave up something is used.

98. Out of the given alternatives, choose the word that is opposite in meaning to the word
Affluent **[2013]**
(a) Reluctant (b) Poor
(c) Clear (d) Enthusiastic
Ans. (b)
Sol. Affluent means 'having a lot of money'.

99. Fill in the blank with appropriate form of noun: Don't blame yourself, it's not your. **[2013]**
(a) misunderstanding (b) error
(c) slip (d) fault
Ans. (d)
Sol. 'Fault' is the appropriate noun here.

100. Fill in the blank:
The instructor, alongwith the class _____ angry about the room change. **[2013]**
(a) are (b) have (c) has (d) is
Ans. (d)
Sol. Singular verb is required here.

101. Choose the suitable word for the blank to make it a meaningful statement.
What you say is _____ my comprehension? **[2013]**
(a) before (b) beside
(c) behind (d) beyond
Ans. (d)
Sol. 'beyond any comprehension' is the correct phrase.

102. Fill in the blank with a suitable preposition:
If you want to avoid traffic, you need to leave _____ 7:30 am. **[2013]**
(a) until (b) by (c) during (d) at
Ans. (b)
Sol. 'By 7:30 am' means not later than 7:30 am.

103. Choose the word the best expresses the meaning of the given idiom: "To smell a rat'. **[2013]**
(a) To suspect something bad
(b) To misunderstand
(c) To detect bad smell
(d) To forsake
Ans. (a)
Sol. To smell a rat means 'to suspect something bad'.

104. Out of the given alternatives, choose the word that best expresses the meaning of the word abridge.

[2013]

(a) Judge (b) Release
(c) Shorten (d) Dissolve

Ans. (c)

Sol. Abridge means to shorten a play/movie etc by leaving parts or sections out.

105. 'A dog's breakfast' means **[2013]**

(a) breakfast cooked for a dog
(b) breakfast cooked by a dog
(c) something that has been done very badly
(d) None of the above

Ans. (c)

Sol. A dog's breakfast means 'something that has been done very badly'.

106. Change the speech: **[2013]**

She says, " I like going to the sea side".

(a) She says she likes going to the sea side
(b) She says I like going to the sea side
(c) She says that she liked going to the sea side
(d) She says she like going to the sea side

Ans. (a)

Sol. She says that she likes going to the sea side.

107. Arrange the following to form a correct sentence.

P : will normally be granted
Q : candidates should note
R : that no request for
S : change of centre **[2013]**

(a) SRQP (b) PRQS (c) QSPR (d) QRSP

Ans. (d)

Sol. Candidates should note that no request for change of centre will normally be granted.

108. Rewrite the sentence after correcting the error.

She was one of the average student of the class. **[2013]**

(a) She was one of the average students of the class
(b) She is one of the average student of the class
(c) She was one among the average student of the class
(d) She is an average students of the class

Ans. (a)

Sol. She was one of the average students of the class.

109. Choose appropriate words to form a grammatically correct sentence.

The decoration of the new house, including the furniture and curtains ____ **[2013]**

(a) is more pleasing (b) are more pleasing
(c) is most pleasing (d) are pleasing

Ans. (c)

Sol. is most pleasing is correct.

110. Fill in the blank:

The President of the United States, accompanied by his advisors ____ enroute to Europe. **[2013]**

(a) were (b) are
(c) was (d) Both (a) and (c)

Ans. (c)

Sol. was is appropriate.

Computer Awareness

111. All digital circuits can be realised by using only **[2013]**

(a) Exclusive OR gates (b) Half adders
(c) Multiplexers (d) OR gate

Ans. (b)

Sol. All digital circuits can be realised by using only half adders. A half adder consists of an XOR gate and an AND gate.

112. The Boolean function $a+(a\cdot b)$ is equivalent to **[2013]**

(a) $a\cdot b$ (b) $a+b$ (c) $a\cdot\bar{b}$ (d) $\bar{a}+b$

Ans. (b)

Sol. The Boolean function $a+(a'\cdot b)$ is solved as

$$a+(a'\cdot b)$$
$$=(a'+a')\cdot(a+b) \quad [\text{using distributive law}]$$
$$=1\cdot(a+b) \quad [\because a+a'=1]$$
$$=a+b$$

113. Which of the following circuit is used as a memory device in computers? **[2013]**

(a) Flip-Flop (b) Rectifier
(c) Comparator (d) All of these

Ans. (a)

Sol. The flip-flop circuit is used as a memory device in computers.

114. Convert the hexadecimal number 4DF to its octal equivalent. **[2013]**

(a) 2333 (b) 2337 (c) 2773 (d) 2373

Ans. (b)

Sol. Conversion of hexadecimal $4DF$ to its octal as

$$\begin{matrix}4 & D & F\\ \downarrow & \downarrow & \downarrow\\ 0100 & 1101 & 1111\end{matrix}=(010011011111)_{16}$$

Now, $(010011011111)_{16}$ into octal number is

$$\frac{010}{2}\frac{011}{3}\frac{011}{3}\frac{111}{7}=(2337)_8$$

115. A tautology is a Boolean formula that is always true. Which of the following is a tautology? **[2013]**

(a) x (b) $\left(x+\bar{x}\right)y$

(c) $x+\bar{y}+\bar{x}$ (d) $\left(xy\right)+\bar{x}$

Ans. (c)

Sol. As we known a tautology is a Boolean formula that is always true. From the giving options, we solve all through the truth table.

Here, $x+\bar{y}+\bar{x}$ is satisfying the condition of tautology.

x	y	$\bar{x}$	$\bar{y}$	$x+\bar{y}+\bar{x}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

The output of $x+\bar{y}+\bar{x}$ is true.

116. Acronym of EEPROM is **[2013]**

(a) Extended Erasable Programmable Memory

(b) Electrically Erasable Read Only Memory

(c) Electrically Erasable Programmable Read Only Memory

(d) Extended Erasable Page-oriented Memory

Ans. (c)

Sol. EEPROM stands for Electrically Erasable Programmable Read Only Memory. It is a type of a non-volatile memory used in computers and other electronic devices.

117. For reproducing sound, a CD audio player uses a **[2013]**

(a) Quartz crystal (b) Titanium needle

(c) Barium ceramic (d) Laser beam

Ans. (d)

Sol. Compact Disc (CD) is a disc which contains digital data that is scanned by a laser beam for the reproduction of recorded sound.

118. When we open an internet site, we see WWW. What does WWW stand for ? **[2013]**

(a) World Wide Word (b) World Wide Web

(c) World Wide Webinar (d) Word Widing Works

Ans. (b)

Sol. WWW stands for World Wide Web.

119. The answer of the operation $(10111)_2 \times (1110)_2$ in hex equivalent is **[2013]**

(a) 150 (b) 14 C (c) 142 (d) 13 E

Ans. (c)

Sol. The multiplication of $(10111)_2 \times (1110)_2$ is

$(10111)_2 \times (1110)_2 = (000101000010)_2$

Now, the result is converted into hexadecimal as

$\underbrace{0001}_{1}\ \underbrace{0100}_{4}\ \underbrace{0010}_{2}$

So, the answer of the operation $(1011)_2 \times (1110)_2$ in hex equivalent is $(142)_{16}$.

120. The minimum number of bits to represent a character from ASCII code set is **[2013]**

(a) 2 (b) 8 (c) 5 (d) 7

Ans. (d)

Sol. The minimum number of bits to represent a character from ASCII code set is 7.