NIMCET-2014 SOLVED PAPER

Mathematics

1. A password consists of two alphabets from English followed by three digits chosen from 0 to 3 . Repetitions are allowed. The number of different passwords is **[2014]**

(a) ${}^{25}P_1, {}^{25}P_2, {}^{4}P_1, {}^{3}P_1, {}^{2}P_1$ (b) $({}^{26}P_1)^2, ({}^{4}P_1)^3$

(c) ${}^{26}P_1, {}^{26}P_2, {}^{4}P_1, {}^{4}P_2, {}^{4}P_3$ (d) $({}^{26}P_1 \cdot {}^{4}P_1)^2$

Ans. (b)

Sol. Total alphabet in English $= 26$

Two words can be selected from 26 alphabet in ${}^{26}P_1 \times {}^{26}P_1$ ways.

Now, next three digits from 0 to 3 i.e. $0,1,2,3$ are selected.

$$= {}^{4}P_1 \times {}^{4}P_1 \times {}^{4}P_1 \times {}^{4}P_1 = \left({}^{4}P_1\right)^3$$

$\therefore$ Password $$= {}^{26}P_1 \times {}^{26}P_1 \times \left({}^{4}P_1\right)^3$$

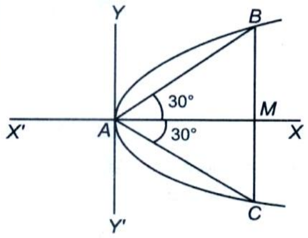
$$= \left({}^{26}P_1\right)^2 \times \left({}^{4}P_1\right)^3$$

2. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ such that one of the vertices of the triangle coincides with the vertex of the parabola. The length of the side of the triangle is **[2014]**

(a) $a\sqrt{3}$ (b) $2a\sqrt{3}$ (c) $4a\sqrt{3}$ (d) $8a\sqrt{3}$

Ans. (d)

Sol. Let length of side of ΔABC be I .



i.e. $AB = I$

Then, $\cos 30^{\circ} = \dfrac{AM}{AB}$

$$AM = x\cos 30^{\circ}$$

$$AM = I\frac{\sqrt{3}}{2}$$

and $\dfrac{BM}{AB} = \sin 30^{\circ}$

$$BM = \frac{I}{2}$$

$\therefore$ The coordinate of B are $\left(\dfrac{I\sqrt{3}}{2}, \dfrac{I}{2}\right)$

B lies on the parabola $y^2 = 4ax$

$$\therefore \frac{I^2}{4} = 4al\frac{\sqrt{3}}{2}$$

$$I = 8a\sqrt{3}$$

3. A chain of video stores sells three different brands of DVD players. Of its DVD player sales 50% are brand 1 , 30% are brand 2 and 20% are brand 3. Each manufacturer offers one year warranty on parts and labour. It is known that 25% of brand 1 DVD players require warranty repair work, whereas the corresponding percentage for brands 2 and 3 are 20% and 10% respectively. The probability that a randomly selected purchaser has a DVD player that will need repair while under warranty, is **[2014]**

(a) 0.795 (b) 0.205 (c) 0.1250 (d) 0.060

Ans. (b)

Sol. Let probability of sale of brand 1 is $P(A)$.

Similarly, for brand 2 is $P(B)$

and brand 3 is $P(C)$.

Given, $P(A) = \dfrac{50}{100} = \dfrac{1}{2}$

$$P(B) = \frac{30}{100} = \frac{3}{10} \text{....}(i)$$

$$P(C) = \frac{20}{100} = \frac{1}{5}$$

and also let the probability of required warranty work of brand (1) is $P(A)$.

Similarly, for brand 2 is $P(B)$

and for brand 3 is $P(C)$.

$$\therefore P(A) = \frac{25}{100} = \frac{1}{4}$$

$$P(B) = \frac{20}{100} = \frac{1}{5}$$

$$P(C) = \frac{10}{100} = \frac{1}{10}$$

Now, required probability

$$=\frac{1}{2}\times\frac{1}{4}+\frac{3}{10}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{10}$$
$$=\frac{1}{8}+\frac{3}{50}+\frac{1}{50}=\frac{50+24+8}{400}$$
$$=\frac{82}{400}$$
$$=0.205$$

4. The locus of intersection of two lines $\sqrt{3}x-y=4k\sqrt{3}$ and $k\left(\sqrt{3}x+y\right)=4\sqrt{3}$ for different values of k is a hyperbola. The eccentricity of the hyperbola is **[2014]**

(a) 1.5 (b) $\sqrt{3}$ (c) 2 (d) $\frac{\sqrt{3}}{2}$

Ans. (c)

Sol. Given, lines are

$$\sqrt{3}x-y=4k\sqrt{3}...(i)$$

And $k\left(\sqrt{3}x\right)+ky=4\sqrt{3}$

$$\Rightarrow\sqrt{3}x+y=\frac{4\sqrt{3}}{k}...(ii)$$

From Eqs. (i) and (ii),

$$3x^2-y^2=48$$
$$\frac{x^2}{16}-\frac{y^2}{48}=1$$

which is a hyperbola.

$$a=4,b=4\sqrt{3}$$
$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{16+48}{16}}=\sqrt{\frac{64}{16}}=2$$

5. Constant forces $\mathbf{P}=2\hat{\mathbf{i}}-5\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ and $\mathbf{Q}=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ act on a particle. The work done when the particle is displaced from $\mathbf{A}$ whose position vector is $4\hat{\mathbf{i}}-3\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ to $\mathbf{B}$ whose position vector is $6\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}$, is **[2014]**

(a) 10 units (b) -15 units
(c) -50 units (d) 25 units

Ans. (b)

Sol. We have,

$$\mathbf{P}=2\hat{\mathbf{i}}-5\hat{\mathbf{j}}+6\hat{\mathbf{k}}$$
$$\mathbf{Q}=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

Position vector of

$$\mathbf{A}=4\hat{\mathbf{i}}-3\hat{\mathbf{j}}-2\hat{\mathbf{k}}$$
$$\mathbf{B}=6\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}$$

and $\mathbf{d}=\mathbf{AB}=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$$\therefore\ \mathbf{F}=\mathbf{P}+\mathbf{Q}=\hat{\mathbf{i}}-3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$$

Work done $=\mathbf{F}\cdot\mathbf{d}=\left(\hat{\mathbf{i}}-3\hat{\mathbf{j}}+5\hat{\mathbf{k}}\right)\cdot\left(2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)$

$=2-12-5=-15$ units

6. The value of $\int\sqrt{x}e^{\sqrt{x}}dx$ is equal to **[2014]**

(a) $2\sqrt{x}-e^{\sqrt{x}}-4\sqrt{xe^{\sqrt{x}}}+C$
(b) $\left(2x-4\sqrt{x}+4\right)e^{\sqrt{x}}+C$
(c) $\left(2x+4\sqrt{x}+4\right)e^{\sqrt{x}}+C$
(d) $\left(1-4\sqrt{x}\right)e^{\sqrt{x}}+C$

Ans. (b)

Sol. We have,

$$1=\int\sqrt{x}e^{\sqrt{x}}dx...(i)$$

Put $\sqrt{x}=t$

$$\Rightarrow\quad\frac{1}{2\sqrt{x}}dx=dx$$
$$\Rightarrow\quad dx=2\sqrt{x}dt$$
$$I\quad=2\int\sqrt{x}e^t tdt=2\int t^2\cdot e^t dt$$
$$=2\left[t^2\cdot e^t-2\int te^t dt\right]$$
$$\therefore\quad=2[t^2e^t-2\{te^t-e^t]\}+C$$
$$=2e^t\left[t^2-2t+2\right]+C$$
$$=e^{\sqrt{x}}\left[2x-4\sqrt{x}+4\right]+C$$

7. For the vector $\mathbf{a}=-4\hat{\mathbf{i}}+2\hat{\mathbf{j}},\mathbf{b}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\mathbf{c}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}$, if $\mathbf{c}=m\mathbf{a}+n\mathbf{b}$, then the value of $m+n$ is **[2014]**

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

Ans. (c)

Sol. We have,

$$\mathbf{a}=-4\hat{\mathbf{i}}+2\hat{\mathbf{j}}...(i)$$
$$b=2\hat{\mathbf{i}}+\hat{\mathbf{j}}...(ii)$$
$$c=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}...(iii)$$
$$c=ma+nb...(iv)$$

From Eqs. (i), (ii), (iii) and (iv),

$$2\hat{\mathbf{i}}+3\hat{\mathbf{j}}=m\left(-4\hat{\mathbf{i}}+2\hat{\mathbf{j}}\right)+n\left(2\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$$

$\Rightarrow 2=-4m+2n$...(v)

$\Rightarrow 3=2m+n$...(vi)

$\Rightarrow 2=-4m+2(3-2m)$

$\Rightarrow 2=-4m+6-4m$

$\Rightarrow 8m=4$

$\Rightarrow m=\frac{1}{2}$

From Eq. (v),

$$2=-2+2n$$

$$\Rightarrow \quad n=2$$

$$m+n=\frac{1}{2}+2$$

$$\Rightarrow \quad m+n=\frac{5}{2}$$

8. The value of $\int_0^{\pi/4}\log(1+\tan x)\,dx$ is equal to **[2014]**

(a) $\frac{\pi}{4}\log 2$ (b) $\frac{\pi}{6}\log 2$ (c) $\frac{\pi}{8}\log 2$ (d) $\frac{\pi}{2}\log 2$

Ans. (c)

Sol. We have, $\int_0^{\pi/4}\log(1+\tan x)\,dx$

Let $I=\int_0^{\pi/4}\log(1+\tan x)\,dx$

$=\int_0^{\pi/4}\log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$

$=\int_0^{\pi/4}\log\left(1+\frac{\tan\pi/4-\tan x}{1+\tan\pi/4\tan x}\right)dx$

$=\int_0^{\pi/4}\log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$

$=\int_0^{\pi/4}\log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right)dx$

$=\int_0^{\pi/4}\log\left(\frac{2}{1+\tan x}\right)dx$

$=\int_0^{\pi/4}\log 2\,dx-\int_0^{\pi/4}\log(1+\tan x)\,dx$

$\Rightarrow 2I=\log 2[x]_0^{\pi/4}$

$\Rightarrow I=\log 2[\pi/4]$

$\therefore\ \mathrm{I}=\frac{\pi}{8}\log 2$

9. The number of ways in which 5 days can be chosen each of the 12 months of a non-leap year, is **[2014]**

(a) $\left({}^{30}C_5\right)^4\left({}^{31}C_5\right)^7\left({}^{28}C_5\right)$

(b) $\left({}^{30}C_5\right)^6\left({}^{28}C_5\right)^6$

(c) $\left({}^{30}C_5\right)^7\left({}^{31}C_5\right)^4\left({}^{28}C_5\right)$

(d) $\left({}^{30}C_5\right)^5\left({}^{31}C_5\right)^6\left({}^{28}C_5\right)$

Ans. (a)

Sol. In a leap year, 7 months have 31 days

4 months have 30 days

and 1 month (Feb) has 28 days

Total numbers of ways

$=\left({}^{31}C_5\right)^7\left({}^{30}C_5\right)^4\left({}^{28}C_5\right)^1$

10. If $[x]$ represents the greatest integer not exceeding x, then $\int_0^9[x]dx$ is **[2014]**

(a) 32 (b) 36 (c) 40 (d) 28

Ans. (b)

Sol. We have,

$I=\int_1^9[x]dx$

$=\int_1^2 1dx+\int_2^3 2dx+\int_3^4 3dx+\int_4^5 4dx$

$+\int_5^6 5dx+\int_6^7 6dx+\int_7^8 7dx+\int_8^9 8dx$

$=(2-1)+2(3-2)+3(4-3)+4(5-4)+5(6-5)$

$+6(7-6)+7(8-7)+8(9-8)$

$=1+2+\ldots+8$

$=\frac{8\times 9}{2}=36$

11. In a group of 200 students, the mean and standard deviation of scores were found to be 40 and 15, respectively. Later on it was found that two scores 43 and 35 were misread as 34 and 53 , respectively. The corrected mean of scores is **[2014]**

(a) 40.95 (b) 39.0 (c) 39.95 (d) 43

Ans. (c)

Sol. We have,

Total number of students, $n=200$

Combined mean, $\bar{x}=40$

Misread data, $(x_1,x_2)=34,53$

Correct data, $(x_3,x_4)=(43,35)$

$$\therefore\ \text{Correct mean} =\frac{(\bar{x})n-(34+53)+(43+35)}{200}$$

$$=\frac{200\times 40-87+78}{200}$$

$$=\frac{7991}{200}=39.95$$

12. If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & K & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has an inverse matrix, then the value of K is **[2014]**

(a) K is any real number (b) $K \neq -4$
(c) $K = -4$ (d) $K \neq 4$

Ans. (b)

Sol. If given matrix $A = \begin{bmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{bmatrix}$ is invertible, then $|A| \neq 0$.

$\therefore |A| = -1(5k+12) - 3(3+5) + 2(4-k)$
$= -5k - 12 - 24 + 8 - 2k$
$= -28 - 7k$
$\because |A| \neq 0$
$\therefore 7k \neq -28$
$\Rightarrow k \neq -4$

13. The mean deviation from the mean of the AP$a, a+d$, $a+2d, \ldots, a+2nd$ is **[2014]**

(a) $\frac{n}{n+1}d$ (b) $\frac{n(n+1)}{2n+1}d$
(c) $\frac{n+1}{2n+1}d$ (d) $\frac{n(n+1)}{2n+1}d$

Ans. (b)

Sol. Given, AP is
$a, a+d, a+2d, \ldots, a+2nd$

Mean of the AP,

$$\bar{x} = \frac{1}{2n+1}[a + a + d + \ldots + a + 2nd]$$
$$= \frac{1}{2n+1}\left[\frac{2n+1}{2}(a + a + 2nd)\right]$$
$$= a + nd$$

$\therefore$ Mean deviation from the mean

$$\frac{1}{2n+1}\sum_{r=0}^{2n}|(a+rd)-(a+nd)|$$
$$= \frac{1}{2n+1}\sum_{r=0}^{2n}|(r-n)d|$$
$$= \frac{1}{2n+1}2d(1+2+3+\ldots+n)$$
$$= \frac{1}{2n+1}\frac{2d[n(n+1)]}{2}$$
$$= \frac{n(n+1)d}{2n+1}$$

14. If (x_0, y_0) is the solution of the following equations: **[2014]**

$(2x)^{\ln 2} = (3y)^{\ln 3}$
$3^{\ln x} = 2^{\ln y}$

Then, x_0 is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 6

Ans. (c)

Sol. We have,

and $(2x)^{\log 2} = (3y)^{\log 3} \ldots (i)$
$3^{\log x} = 2^{\log y} \ldots (ii)$

If x_0, y_0 are the solution of these equations.

Now, $(2x_0)^{\log 2} = (3y_0)^{\log 3}$

Taking log on both sides,

$$\log 2 \log(2x_0) = \log 3 \log(3y_0)$$
$$\Rightarrow \quad \frac{\log 2}{\log 3}\log(2x_0) = \log(3y_0)$$
$$\frac{\log 2}{\log 3}(\log 2 + \log x_0) = \log 3 + \log y_0 \quad \ldots\text{(iii)}$$

Taking log on both sides of Eq. (ii)

$$\log x_0 \log 3 = \log y_0 \log 2$$
$$\Rightarrow \quad \Rightarrow \log y_0 = \frac{\log 3}{\log 2}\cdot \log x_0 \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv),

$$\frac{\log 2}{\log 3}(\log 2 + \log x_0) = \log 3 + \frac{\log 3}{\log 2}\cdot \log x_0$$
$$\Rightarrow \log x_0\left[\frac{\log 2}{\log 3} - \frac{\log 3}{\log 2}\right]1 = \log 3 - \log 2 \cdot \frac{\log 2}{\log 3}$$
$$\Rightarrow \log x_0\left[\frac{\log 2}{\log 3} - \frac{\log 3}{\log 2}\right] = \log 2\left[\frac{\log 3}{\log 2} - \frac{\log 2}{\log 3}\right]$$
$$\Rightarrow \log x_0 = -\log 2$$
$$\Rightarrow \log x_0 = -\log 2^{-1}$$
$$\text{or} \quad x_0 = \frac{1}{2}$$

15. The value of $\tan 1^{\circ} \tan 2^{\circ\circ} \tan 3^{\circ\circ} \ldots \tan 89^{\circ}$ is **[2014]**

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) 2

Ans. (c)

Sol. We have,

$\tan 1^{\circ} \tan 2^{\circ} \ldots \tan 89^{\circ}$
$= \tan(90^{\circ} - 89^{\circ})\tan 89^{\circ} \tan(90^{\circ} - 88^{\circ})\tan 88^{\circ} \ldots$
$= \cot 89^{\circ} \tan 89^{\circ\circ} - \tan 88^{\circ} \cot 88^{\circ} \ldots$
$= 1 \cdot 1$
$= 1$

16. If α and β are the roots of the equation $2x^2+2px+p^2=0$ where, p is a non-zero real number and α^4 and β^4 are the roots of $x^2-rx+s=0$, then the roots of $2x^2-4p^2x+4p-2r=0$ are **[2014]**

(a) real and unequal (b) equal and zero
(c) imaginary (d) equal and non-zero

Ans. (d)

Sol. We have, $2x^2+2px+p^2=0$

$$\alpha+\beta=-p$$

$$\alpha\beta=\frac{p^2}{2}\ldots(i)$$

and α^4 and β^4 are the roots of $x^2-rx+s=0$.

$$\alpha^4+\beta^4=r$$

$$\alpha^4\beta^4=s\ldots(i)$$

Now, $2x^2-4p^2x+4p-2r=0$

$$\because D=B^2-4AC$$

$$D=16p^4-4\left[2\left(p^4-2r\right)\right]$$

$$=16p^4-8p^4+16r$$

$$=8\left(p^4+2r\right)$$

$$\therefore =8\left[4\alpha^2\beta^2+2\left(\alpha^4+\beta^4\right)\right]$$

$$=16\left[(\alpha+\beta)^2-2\alpha\beta\right]^2$$

$$=16\left(p^2-p^2\right)$$

$$=0$$

Hence, the roots of the equation $2x^2-4p^2x+4p-2r=0$ are equal.

17. The number of ways to arrange the letters of the English alphabet, so that there are exactly 5 letters a and b, is **[2014]**

(a) ${}^{24}P_5$ (b) ${}^{24}P_5 20\,!$
(c) ${}^{24}P_5 20!2$ (d) ${}^{24}P_5 24!2$

Ans. (c)

Sol. Total number of English alphabet $=26$

Total number of arrangements $={}^{24}P_5 20!\cdot 2$

18. Suppose, the system of linear equations

$$-2x+y+z=l$$
$$x-2y+z=m$$
$$x+y-2z=n$$

is such that $l+m+n=0$. Then, the system has **[2014]**

(a) a non-zero unique solution
(b) trivial solution
(c) infinitely many solutions
(d) no solution

Ans. (c)

Sol. We have,

$$-2x+y+z=1\ldots(i)$$
$$x-2y+z=m\ldots(ii)$$
$$x+y-2z=n\ldots(iii)$$
$$1+m+n=0$$

Given,
Coefficient matrix will be

$$[A_k]=\begin{bmatrix}-2 & 1 & 1 & I\\ 1 & -2 & 1 & m\\ 1 & 1 & -2 & n\end{bmatrix}$$

Applying $R_1\to R_1+R_2+R_3$

$$=\begin{bmatrix}0 & 0 & 0 & I+m+n\\ 1 & -2 & 1 & m\\ 1 & 1 & -2 & n\end{bmatrix}$$

$$[A_k]=\begin{bmatrix}0 & 0 & 0 & 0\\ 1 & -2 & 1 & m\\ 1 & 1 & -2 & n\end{bmatrix}[\because I+m+n=0]$$

$\therefore p[A_k]=2$

i.e. $p[A_k]<$ number of variables.

Hence, it has infinitely many solutions.

19. $\mathbf{A}=4\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}},\mathbf{B}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$, then the unit vector $\hat{\mathbf{N}}$ perpendicular to vector $\mathbf{A}$ and $\mathbf{B}$ such that $\mathbf{A},\mathbf{B},\hat{\mathbf{N}}$ form a right handed system, is **[2014]**

(a) $\frac{1}{\sqrt{185}}\left[7\hat{\mathbf{i}}-6\hat{\mathbf{j}}-10\hat{\mathbf{k}}\right]$ (b) $\frac{1}{7}\left[6\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}\right]$

(c) $\frac{1}{\sqrt{21}}\left[2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}\right]$ (d) $\frac{1}{\sqrt{21}}\left[-2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+\hat{\mathbf{k}}\right]$

Ans. (a)

Sol. We have

$$A=4\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

$$B=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$$

$\mathbf{N}$ is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

i.e. $\hat{\mathbf{N}} = \lambda(\mathbf{A}\times\mathbf{B})$

$$\therefore \hat{\mathbf{N}} = \lambda\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 3 & 1 \\ 2 & -1 & 2 \end{vmatrix}$$

$$= \lambda\left[\hat{\mathbf{i}}(6+1)-\hat{\mathbf{j}}(8-2)+\hat{\mathbf{k}}(-4-6)\right]$$

$$\because = \lambda[7\hat{i}-6\hat{j}-10\hat{k}]$$

$$\therefore |\hat{\mathbf{N}}| = 1 \qquad [\text{ given}]$$

$$|\hat{\mathbf{N}}| = \lambda\sqrt{185} = 1$$

$$\Rightarrow \lambda = \frac{1}{\sqrt{185}} = 1$$

$$\therefore \hat{\mathbf{N}} = \frac{1}{\sqrt{185}}\left(7\hat{\mathbf{i}}-6\hat{\mathbf{j}}-10\hat{\mathbf{k}}\right)$$

20. The value of $\int\frac{(x+1)}{x(xe^x+1)}dx$ is equal to **[2014]**

(a) $\log\frac{1+xe^x}{xe^x}+C$ (b) $\log\left[xe^x(1+xe^x)\right]+C$

(c) $\log\left[\frac{1}{1+xe^x}\right]+C$ (d) $\log\left[\frac{xe^x}{1+xe^x}\right]+c$

Ans. (d)

Sol. Let $I = \int\frac{(x+1)}{x(xe^x+1)}dx$

Put $xe^x = t$

$$\Rightarrow (xe^x+e^x)dx = dt$$

$$\therefore \text{ I} = \int\frac{dt}{t(t+1)} = \int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$$

$$= \log t - \log(t+1) + C$$

$$= \log\frac{t}{t+1}$$

$$= \log\left(\frac{xe^x}{xe^x+1}\right)+C$$

21. The sum of two vectors $\mathbf{a}$ and $\mathbf{b}$ is a vector $\mathbf{c}$ such that $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=2$. Then, the magnitude of $\mathbf{a}-\mathbf{b}$ is equal to **[2014]**

(a) $2\sqrt{3}$ (b) 2 (c) $\sqrt{3}$ (d) 0

Ans. (a)

Sol. We have,

$\mathbf{c} = \mathbf{a}+\mathbf{b}$ and $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=2$

$$|\mathbf{c}|^2 = |\mathbf{a}|^2+|\mathbf{b}|^2+2|\mathbf{a}|\cdot|\mathbf{b}|\cos\theta$$

$$\Rightarrow 4 = 4+4+2\cdot2\cdot2\cos\theta$$

$$\Rightarrow -4 = 8\cos\theta$$

$$\Rightarrow \cos\theta = -\frac{1}{2}$$

$$\Rightarrow |\mathbf{a}-\mathbf{b}|^2 = |\mathbf{a}|^2+|\mathbf{b}|^2-2|\mathbf{a}||\mathbf{b}|\cos\theta$$

$$= 4+4-8\cos\theta = 8+8\times\frac{1}{2}$$

$$\Rightarrow |\mathbf{a}-\mathbf{b}|^2 = 12$$

$$\Rightarrow |\mathbf{a}-\mathbf{b}| = 2\sqrt{3}$$

22. If x and y are positive real numbers satisfying the system of equations **[2014]**

$x^2+y\sqrt{xy}=336, y^2+x\sqrt{xy}=112$, then $x+y$ is

(a) $\sqrt{448}$ (b) $\sqrt{224}$ (c) 20 (d) 40

Ans. (c)

Sol. We have,

$$x^2+y\sqrt{xy}=336$$

$$\Rightarrow \sqrt{x}\left(x\sqrt{x}+y\sqrt{y}\right)=336 \qquad \text{...(i)}$$

And $y^2+x\sqrt{xy}=112$

$$\Rightarrow \sqrt{y}\left(y\sqrt{y}+x\sqrt{x}\right)=112...(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\sqrt{x}}{\sqrt{y}}=3 \text{ or } x=9y$$

From Eq. (i), $81y^2+y\sqrt{9y^2}=336$

or $81y^2+3y^2 = 336$

$\Rightarrow$ $y^2 = 4,\ y=2$

$\Rightarrow$ $x=18$

$\therefore$ $x+y=18+2=20$

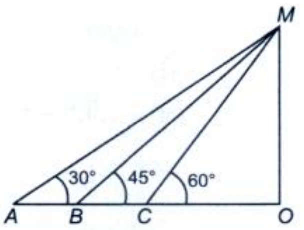
23. From three collinear points A, B and C on a level ground which can be on the same side of a tower, the angles of elevation of the top of the tower are $30^\circ, 45^\circ$ and 60°, respectively. If $BC = 60$ m, then AB is **[2014]**

(a) $15\sqrt{3}$ m (b) $30\sqrt{3}$ m

(c) $45\sqrt{3}$ m (d) $60\sqrt{3}$ m

Ans. (d)

Sol.



$BC = 60$ m [given]

Let $AB = x, OC = d, OM = h$

Now, in $\triangle AOM$,

$$\tan 30^\circ = \frac{OM}{AO}$$

$$\Rightarrow \qquad OA = \sqrt{3}OM \quad ...(i)$$

In $\triangle BOM$,

$$\tan 45^\circ = \frac{OM}{60+d}$$

$$\Rightarrow \qquad OM = 60 + d$$

$$\Rightarrow \qquad h = 60 + d \qquad ...(ii)$$

In $\triangle MOC$,

$$\tan 60^\circ = \frac{OM}{OC}$$

$$\Rightarrow \qquad \sqrt{3}\frac{h}{d}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{60+d}{d}$$

$$d = 30\left(\sqrt{3}+1\right) \quad ...(iii)$$

From Eq. (i)

$$x + 60 + d = \sqrt{3}h$$

$$\Rightarrow \qquad x + 60 + 30\left(\sqrt{3}+1\right) = \sqrt{3}\left[60 + 30\left(\sqrt{3}+1\right)\right]$$

$$\Rightarrow \qquad x = 60\sqrt{3} + 90 + 30\sqrt{3} - 60 - 30\sqrt{3} - 30$$

$$\Rightarrow \qquad x = 60\sqrt{3}$$

$$\therefore \qquad AB = 60\sqrt{3}$$

24. If $x = 1$ is the directrix of the parabola $y^3 = kx - 8$, then k is **[2014]**

(a) $\frac{1}{8}$ (b) 8 (c) 4 (d) $\frac{1}{4}$

Ans. (c)

Sol. We have,

$$y^2 = kx - 8$$

$$\Rightarrow \qquad y^2 = k\left(x - \frac{8}{k}\right)$$

which is a form of $y^2 = 4AX$

So, $X = x - 8/k, A = k/4$

$\therefore$ Equation of directrix, $X = -A$

$$\Rightarrow x - \frac{8}{k} = -\frac{k}{4}$$

$$\Rightarrow x = \frac{8}{k} - \frac{k}{4}$$

$$\Rightarrow 1 = \frac{8}{k} - \frac{k}{4}$$

$$\Rightarrow 4k = 32 - k^2$$

$$\Rightarrow k^2 + 4k - 32 = 0$$

$$\Rightarrow k^2 + 8k - 4k - 32 = 0$$

$$\Rightarrow (k+8)(k-4) = 0$$

$$\Rightarrow k = 4, -8$$

25. If $\sin x + a\cos x = b$, then $|a \cdot \sin x - \cos x|$ is **[2014]**

(a) $\sqrt{a^2 + b^2 + 1}$ (b) $\sqrt{a^2 - b^2 + 1}$

(c) $\sqrt{a^2 + b^2 - 1}$ (d) None of these

Ans. (b)

Sol. We have,

$\sin x + a\cos x = b$

On squaring both sides, we get

$$\sin^2 x + a^2\cos^2 x + 2a\sin x\cos x = b^2$$

$$\Rightarrow 1 - \cos^2 x + a^2 - a^2\sin^2 x + 2a\sin x\cos x = b^2$$

$$\Rightarrow a^2\sin^2 x + \cos^2 x - 2a\sin x\cos x = 1 + a^2 - b^2$$

$$\Rightarrow (a\sin x - \cos x)^2 = 1 + a^2 - b^2$$

$$\Rightarrow |a\sin x - \cos x| = \sqrt{1 + a^2 - b^2}$$

26. A condition that $x^3 + ax^2 + bx + c$ may have no extremum is **[2014]**

(a) $a^2 \geq 3b$ (b) $b^2 < 3a$

(c) $a^2 < 3b$ (d) $b^2 \geq 3a$

Ans. (c)

Sol. We have,

$$f(x) = x^3 + ax^2 + bx + c$$

$$\Rightarrow \qquad f'(x) = 3x^2 + 2ax + b$$

Condition for no extremum

$$b^2 - 4ac < 0$$

$$\Rightarrow \qquad 4a^2 - 4 \times 3b < 0$$

$$\Rightarrow \qquad a^2 - 3b < 0$$

$$\Rightarrow \qquad a^2 < 3b$$

27. If n and r are integers such that $1 \leq r \leq n$, then the value of $n\left({}^{n-1}C_{r-1}\right)$ is **[2014]**

(a) ${}^nC_r$ (b) $r\left({}^nC_r\right)$

(c) $n\left({}^nC_r\right)$ (d) $(n-1)\left({}^nC_r\right)$

Ans. (b)

Sol. We have,

$$n\,{}^{n-1}C_{r-1} = n\cdot\frac{(n-1)!}{(r-1)!(n-1-r+1)!}$$
$$=\frac{n!}{(r-1)!(n-r)!}$$
$$=\frac{r\cdot n!}{r\cdot(r-1)!(n-r)!}$$
$$=\frac{r\cdot n!}{r!(n-r)!}$$
$$=r\cdot\left({}^{n}C_{r}\right)$$

28. If the foci of the ellipse $b^2x^2+16y^2=16b^2$ and the hyperbola $81x^2-144y^2=\frac{81\times144}{25}$ coincide, then the value of b is **[2014]**

(a) 1 (b) $\sqrt{5}$ (c) $\sqrt{7}$ (d) 3

Ans. (c)

Sol. Since, foci coincide,

$$\therefore \quad 16-b^2=\frac{144}{25}+\frac{81}{25}$$
$$\Rightarrow \quad 16-b^2=\frac{225}{25}$$
$$\Rightarrow \quad b^2=16-\frac{225}{25}$$
$$=\frac{400-225}{25}$$
$$=\frac{175}{25}=7$$
$$\therefore \quad b=\sqrt{7}$$

29. There are 8 students appearing in an examination of which 3 have to appear in Mathematics paper and the remaining 5 in different subjects. Then, the number of ways they can be made to sit in a row, if the candidates in Mathematics cannot sit next to each other is **[2014]**

(a) 2400 (b) 16200 (c) 4200 (d) 14400

Ans. (d)

Sol. Total number of students $=8$

Those who appear in Mathematics $=3$ and other subjects $=5$

$\therefore$ Total number of ways $={}^{6}C_3\times5!\times3$

$=14400$

30. If x is so small that x^2 and higher powers of x can be neglected, then $\frac{(9+2x)^{1/2}(3+4x)}{(1-x)^{1/5}}$ is approximately equal to **[2014]**

(a) $9+\frac{74}{15}x$ (b) $9+\frac{74}{5}x$

(c) $3+\frac{74}{15}x$ (d) $3+\frac{74}{5}x$

Ans. (b)

Sol. We have,

$$\frac{(9+2x)^{1/2}(3+4x)}{(1-x)^{1/5}}=\frac{9\left(1+\frac{2}{9}x\right)^{1/2}\left(1+\frac{4}{3}x\right)}{(1-x)^{1/5}}$$
$$=\frac{9\left[1+\frac{2}{9}\cdot\frac{1}{2}x+\ldots\right]\left[1+\frac{4}{3}x\right]}{(1-x)^{1/5}}$$
$$=\frac{9\left[1+\frac{x}{9}+\frac{4}{3}x\right]}{(1-x)^{1/5}}$$
$$=\frac{9\left[1+\frac{13x}{9}\right]}{(1-x)^{1/5}}$$
$$=\frac{9(1-x)^{-13/9}}{(1-x)^{1/5}}$$
$$=9(1-x)^{-\frac{13}{9}-\frac{1}{5}}$$
$$=9(1-x)^{-74/45}$$
$$=9\left(1+\frac{74}{45}x\right)$$

[neglecting higher powers of x]

$$=9+\frac{74x}{5}$$

31. If the sets A and B are defined as $A=\{(x,y)|\ y=1/x, 0\neq x\in R\}$, $B=\{(x,y)|\ y=-x\in R\}$, then **[2014]**

(a) $A\cap B=\phi$ (b) $A\cap B=B$

(c) $A\cap B=A$ (d) None of these

Ans. (a)

Sol. We have,

$$A=\left\{(x,y)/\,y=\frac{1}{x},0\neq x\in R\right\}\ldots(i)$$

And $B=\{(x,y)/\,y=-x\in R\}\ldots(ii)$

It is clear that fromEqs. (i) and (ii),

$A\cap B=\phi$

32. If A,B and C is three angles of a ΔABC, whose area is Δ. Let a,b and c be the sides opposite to the angles A, B and C respectively. If $s=\frac{a+b+c}{2}=6$, then the product $\frac{1}{3}s^2(s-a)(s-b)(s-c)$ is equal to **[2014]**

(a) 2Δ (b) $2\Delta^2$ (c) $\sqrt{2}\Delta$ (d) Δ^2

Ans. (b)

Sol. We have, $s=\frac{a+b+c}{2}$

$2s=a+b+c...(i)$

and $\frac{1}{3}s^2(s-a)(s-b)(s-c)$

$=\frac{1}{3}s\left[s(s-a)(s-b)(s-c)\right]$

$=\frac{1}{3}s\left[\Delta^2\right]$

$\left[\because \Delta=\sqrt{s(s-a)(s-b)(s-c)}\right]$

$=\frac{1}{3}\times 6\times\Delta^2$

$=2\Delta^2$

33. A normal to the curve $x^2=4y$ passes through the point $(1,2)$. The distance of the origin from the normal is **[2014]**

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{3}{\sqrt{2}}$

Ans. (a)

Sol. We have, $x^2=4y$

On differentiating, $2x=4\frac{dy}{dx}$

or $\frac{dy}{dx}=x/2$

Slope of normal $=-2/x$

Let the common point of curve and normal be (h,k)

Then, $h^2=4k...(i)$

Equation of normal, $\frac{y-k}{x-h}=\frac{-2}{h}$

Normal passes through $(1,2)$, $\therefore$ $\frac{2-k}{1-h}=\frac{-2}{h}$

$2h-kh=-2+2h$

From Eq. (i), $2h-\frac{h^3}{4}=-2+2h$

or $\frac{h^3}{4}=2\Rightarrow h=2$

$k=1$, Equation of normal $x+y-3=0$

Now, distance form origin $(0,0)$

$=\left|\frac{0+0-4}{\sqrt{1+1}}\right|=\left|\frac{-2}{\sqrt{(2)}}\right|=\sqrt{2}$ units

34. Suppose r integers, $0<r<10$, are chosen from ($0,1,2$ ...,9) at random and with replacement. The probability that no two are equal, is **[2014]**

(a) $\frac{10!}{10!r!}$ (b) $\frac{10!}{10!(10-r)!}$

(c) $\frac{10!}{r!(10-r)!}$ (d) $\frac{10!}{10^r!(10-r)!}$

Ans. (d)

Sol. We have,

$0,1,2,\ldots,9$

Total number of digits = 10

Now, total number of ways when r integers are taken from these digits, is ${}^{10}C_r$.

$\therefore$ Required probability $=\frac{10!}{10^r!(10-r)!}$

35. If $x^2+2ax+10-3a>0$ for all $x\in IR$, then **[2014]**

(a) $-5<a<2$ (b) $a<-5$

(c) $a>5$ (d) $2<a<5$

Ans. (a)

Sol. $f(x)=x^2+2ax+10-3a>0$

$f(x)>0, \forall x\in R$

Then, $D<0$

$\Rightarrow b^2-4ac<0$

$\Rightarrow 4a^2-4(10-3a)<0$

$\Rightarrow 4a^2-40+12a<0$

$\Rightarrow a^2+3a-10<0$

$\Rightarrow a^2+5a-2a-10<0$

$\Rightarrow a(a+5)-2(a+5)<0$

$\Rightarrow (a+5)(a-2)<0$

$\Rightarrow -5<a<2$

36. A box contains 3 coins, one coin is fair, one coin is two headed and one coin is weighted, so that the probability of heads appearing is $\frac{1}{3}$. A coin is selected at random and tossed, then the probability that head appears, is **[2014]**

(a) $\frac{11}{18}$ (b) $\frac{7}{18}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

Ans. (a)

Sol. Probability of choosing fair coin $=\frac{1}{3}$

Probability of choosing two headed coin $= \frac{1}{3}$

Probability of choosing weighted coin $= \frac{1}{3}$

$\therefore$ Required probability

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{1}{6} + \frac{1}{3} + \frac{1}{9} = \frac{3+6+2}{18} = \frac{11}{18}$$

37. If a vector a makes an angle with the coordinate axes and has magnitude 3, then the angle between **a** and each of the three coordinate axes is **[2014]**

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

Ans. (a)

Sol. We have, $|\mathbf{a}| = 3$

Let α, β and γ be the angles between the given vector and coordinate axes respectively.

Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$\therefore \quad \cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \alpha = \beta = \gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

38. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$, where $[x]$ is the largest integer but not larger than x, then $\lim_{x\to 0} f(x)$ is **[2014]**

(a) -1 (b) 0 (c) 1 (d) Does not exist

Ans. (d)

Sol. If $f(x) = \begin{cases} \frac{\sin(-1)}{-1} = & \text{sin, when} & -1 \le x < 0 \\ 0 \Rightarrow & \text{when} & 0 \le x < 1 \end{cases}$

$$\Rightarrow \lim_{(x\to 0^-)} f(x) = \sin 1 \text{ and } \lim_{(x \to 0^+)} f(x) = 0$$

$$\therefore \text{ LHS} \neq \text{RHS}$$

So, limit does not exist.

39. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then $\cot(A-B)$ is equal to **[2014]**

(a) $\frac{1}{x} + \frac{1}{y}$ (b) $\frac{1}{x} - \frac{1}{y}$

(c) $-\frac{1}{x} + \frac{1}{y}$ (d) $-\frac{1}{x} - \frac{1}{y}$

Ans. (a)

Sol. We have

$$\tan A - \tan B = x ...(i)$$

and $$\cot B - \cot A = y ...(ii)$$

Then, $$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$= \frac{1 + \cot A \cot B}{y} ...(iii)$$

From Eq. (i),

$$\frac{1}{\cot A} - \frac{1}{\cot B} = x$$

$$\Rightarrow \frac{\cot B - \cot A}{\cot A \cot B} = x$$

$$\Rightarrow \cot A \cot B = \frac{y}{x}$$

From Eq. (iii),

$$\cot(A-B) = \frac{1 + \frac{y}{x}}{y} = \frac{1}{x} + \frac{1}{y}$$

40. If $a = \log_{12} 18, b = \log_{24} 54$, then $ab + 5(a-b)$ is **[2014]**

(a) 1 (b) 0 (c) 2 (d) $\frac{3}{2}$

Ans. (a)

Sol. We have,

$a = \log_{12} 18$

$\Rightarrow b = \log_{24} 54$

$\therefore ab + 5(a-b)$

$= (\log_{12} 18)(\log_{24} 54) + 5[\log_{12} 18 - \log_{24} 54]$

$$= \frac{\log_e 9 \times 2}{\log_e 4 \times 3} \cdot \frac{\log_e 9 \times 6}{\log_e 8 \times 3} + 5\left[\frac{\log_e 9 \times 2}{\log_e 4 \times 3} - \frac{\log_e 9 \times 6}{\log_e 8 \times 3}\right]$$

Solving this, we get

$$ab + 5(a-b) = 1$$

41. A student takes a quiz consisting of 5 multiple choice questions. Each question has 4 possible answers. If a student is guessing the answer at random and answer todifferentare independent, then the probability of atleast one correct answer is **[2014]**

(a) 0.237 (b) 0.00076 (c) 0.7623 (d) 1

Ans. (c)

Sol. Let probability of guessing be A.

$\therefore$ Probability guessing correct answer,

$$P(A)=\frac{1}{4}$$

Answer is not correct $=P\left(\bar{A}\right)=1-\frac{1}{4}$

$$P\left(\bar{A}\right)=\frac{3}{4}$$

$\therefore$ Required probability $=1-P$ (no answer is correct)

$$=1-{}^5C_0\left[\{P(A)\}^0\{P\left(\bar{A}\right)\}^5\right]$$

$$=1-{}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5$$

$$=1-\frac{243}{1024}=1-0.2370$$

$$=0.7623$$

42. The condition that the line $lx+my+n=0$ becomes a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is **[2014]**

(a) $a^2l+b^2m+n=0$ (b) $al^2+bm^2=n^2$

(c) $al+bm=n$ (d) $a^2l^2+b^2m^2=n^2$

Ans. (d)

Sol. We have equation of line,

$$Ix+my+nz=0$$

and

$$y=\frac{-I}{m}x+\left(\frac{-n}{m}\right)z \quad ...(i)$$

We know that, if the line $y=mx+c$ touches the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Then,

$$c^2=a^2m^2+b^2$$

$$\therefore \quad \frac{n^2}{m^2}=a^2\frac{l^2}{m^2}+b^2$$

$$\Rightarrow \quad n^2=a^2l^2+b^2m^2$$

43. The value of $\sin 20^\circ \sin 40^\circ \sin 80^\circ$ is **[2014]**

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{8}$ (d) $\frac{1}{8}$

Ans. (c)

Sol. We have,

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ$$

$$=\sin 20^\circ \sin\left(60^\circ-20^\circ\right)\sin\left(60^\circ+20^\circ\right)$$

$$=\sin 20^\circ\left(\sin^2 60^\circ-\sin^2 20^\circ\right)$$

$$=\sin 20^\circ\left[\frac{3}{4}-\sin^2 20^\circ\right]$$

$$=\frac{1}{4}\left[3\sin 20^\circ-\sin^3 20^\circ\right]$$

$$=\frac{1}{4}\sin 3\times 20^\circ=\frac{1}{4}\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{8}$$

44. Two non-negative numbers whose sum is 9 and the product of the one number and square of the other number is maximum, are **[2014]**

(a) 5 and 4 (b) 3 and 6

(c) 1 and 8 (d) 7 and 2

Ans. (b)

Sol. Let two numbers be x and y, where $x>0, y>0$.

Given,

$$x+y=9...(i)$$

$$z=x-y^2...(ii)$$

$$\Rightarrow \quad z=x(9-x)^2$$

$$=x\left(81+x^2-18x\right)=x^3-18x^2+81x$$

$$\therefore \frac{dz}{dx}=3x^2-36x+81$$

$$=3\left(x^2-12x+27\right)$$

$$\because \frac{dz}{dx}=0$$

$$\Rightarrow x^2-9x-3x+27$$

$$\Rightarrow\left(x-9\right)\left(x-3\right)=0$$

$$\Rightarrow \quad x=3, x=9$$

$$x=3 \quad \left[\because x=9\, not\ possible\right]$$

and $y=6$

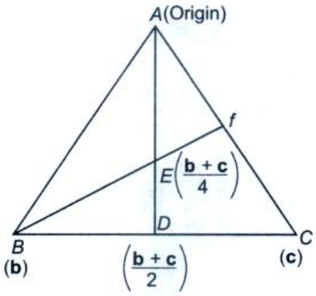
$\therefore$ So, numbers are 3 and 6 .

45. The median AD of ΔABC is bisected at E and BE is produced to meet the side AC at F. Then, $AF:FC$ is **[2014]**

(a) $2:1$ (b) $1:2$ (c) $3:1$ (d) $1:3$

Ans. (d)

Sol. Let the position vector of B and C are $\mathbf{b}$ and $\mathbf{c}$ respectively.



Equation of AC

i.e.

$$\mathbf{r}=\mathbf{b}+\lambda\left(\frac{\mathbf{b}+\mathbf{c}}{4}-\mathbf{b}\right)$$

and $\mathbf{r}=0+\mu C$

$$\Rightarrow \quad 1-\frac{3\lambda}{4}=0 \text{ and } \frac{\lambda}{4}=\mu$$

$$\Rightarrow \quad \lambda=\frac{4}{3} \text{ and } \mu=\frac{1}{3}$$

Therefore, the positive vector of $\mathbf{F}$ is

$$r=\frac{1}{3}c$$

$$AF=\frac{c}{3} \quad \Rightarrow AF=\frac{1}{3}AC$$

$$\Rightarrow AF:AC=1:3$$

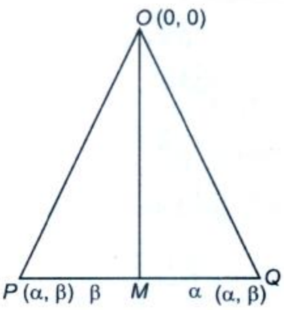
46. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that OPQ is an equilateral triangle, where O is the centre of the hyperbola, then which of the following is true? **[2014]**

(a) $b^2>\frac{-a^2}{\sqrt{3}}$ (b) $b^2>\frac{a^2}{3}$

(c) $b^2<\frac{a^2}{3}$ (d) $b^2<\frac{-a^2}{3}$

Ans. (b)

Sol. Let the vertex P be (α,β), so that Q is $(\alpha,-\beta)$ such that $\triangle OPQ$ is equilateral.



$$OP=OQ=PQ=2\beta$$

$$\sqrt{\alpha^2+\beta^2}=2\beta$$

$$\alpha^2=3\beta^2 \quad \text{...(i)}$$

or

Hence, point (α,β) lies on hyperbola.

$$\therefore \quad \frac{\alpha^2}{a^2}-\frac{\beta^2}{b^2}=1$$

$$\Rightarrow \quad \frac{3\beta^2}{a^2}-\frac{\beta^2}{b^2}=1 \qquad [from\, Eq.(i)]$$

$$\Rightarrow \quad \frac{3\beta^2}{a^2}-1=\frac{\beta^2}{b^2}=\pm1+\text{ ve}>0,$$

$$\Rightarrow \quad \frac{b^2}{a^2}>\frac{1}{3}\Rightarrow b^2>\frac{a^2}{3}$$

47. In ΔABC, if $a=2, b=4$ and $\angle C=60^\circ$, then A and B are respectively equal to **[2014]**

(a) $90^\circ, 30^\circ$ (b) $45^\circ, 75^\circ$

(c) $60^\circ, 60^\circ$ (d) $30^\circ, 90^\circ$

Ans. (a)

Sol. In $\triangle ABC$,

$$a=2, b=4, \angle C=60^\circ$$

$$\because \cos C=\frac{a^2+b^2-c^2}{2ab}$$

$$\because \cos 60^\circ=\frac{4+16-c^2}{2\times2\times4}$$

$$\Rightarrow \frac{1}{2}=\frac{20-c^2}{16}$$

$$\Rightarrow c^2=12$$

$$\Rightarrow c=2\sqrt{3}$$

Now, $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$

$$\Rightarrow \frac{\sin A}{a}=\frac{\sin C}{c}$$

$$\Rightarrow \frac{\sin A}{2}=\frac{\sin 60^\circ}{2\sqrt{3}}$$

$$\Rightarrow \sin A=\frac{1}{2}$$

$$\Rightarrow \angle A=30^\circ$$

Now $\angle A+\angle B+\angle C=180^\circ$

$$\Rightarrow 30^\circ+\angle B+60^\circ=180^\circ$$

$$\Rightarrow \angle B=90^\circ$$

$$\therefore \angle A=30^\circ$$

and $\angle B=90^\circ$

48. If $\int\frac{xe^x}{\sqrt{1+e^x}}dx=f(x)\sqrt{1+e^x}-2\log\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}+C$, then $f(x)$ is **[2014]**

(a) $2x-1$ (b) $2x-4$ (c) $x+4$ (d) $x-4$

Ans. (b)

Sol. Let $I = \int \frac{xe^x}{\sqrt{1+e^x}} dx$

Put $1+e^x = t^2$

$\Rightarrow e^x dx = 2t dt$

$x = \log(t^2-1)$

$\therefore \; I = 2\int \log\frac{(t^2-1)}{t} t\, dt = 2\int \log(t^2-1) dt$

$= 2\left[t\log(t^2-1) - 2\int \frac{t^2}{t^2-1} dt \right]$

$= 2\left[t\log(t^2-1) - 2t\int \left(1+\frac{1}{t^2-1}\right) dt \right]$

$= 2[t\log(t^2-1) - 2t - \log\left(\frac{t-1}{t+1}\right)\} + C$

$= 2x\sqrt{1+e^x} - 4\sqrt{1+e^x} - 2\log\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$

$= (2x-4)\sqrt{1+e^x} - 2\log\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$

Hence, $f(x) = 2x-4$

49. The average marks of boys in a class is 52 and that of girls is 42 . The average marks of boys and girls combined is 50 . The percentage of boys in the class is **[2014]**

(a) 80% (b) 60% (c) 40% (d) 20%

Ans. (a)

Sol. Given, combined average of class is 50 . Let number of boys in class be x and number of girls in class be y . By combined average formula,

$$50 = \frac{52\times x + 42\times y}{x+y}$$

$\Rightarrow \quad 50x + 50y = 52x + 42y$

$\Rightarrow \quad 8y = 2x$

$\Rightarrow \quad \frac{x}{y} = \frac{4}{1}$

$\Rightarrow \quad \frac{x}{x+y} = \frac{4}{5}$

Hence, ratio of boys to total number of students is $\frac{4}{5}$

and percentage $= \frac{4}{5}\times 100 = 80\%$.

50. How many even integers between 4000 and 7000 have four different digits? **[2014]**

(a) 672 (b) 840 (c) 504 (d) 728

Ans. (d)

Sol. Between 4000 to 5000 , choose unit digit in 4 ways $(0,2,6,8) = 4$

Choose tenth place of remaining 8 numbers $=8$ (because 2 even number out of 10 numbers are already taken)

Choose hundred place by remaining 7 numbers $=7$ ways

Total number ways between 4000 to 5000

$= 4\times 8\times 7 = 224$

Similarly, for 5000 to 6000 ,

Unit digit is chosen in 5 ways

$(0,2,6,8,4) = 5$

Tenth digit is chosen in 8 ways $=8$

Thousandth digit is chosen by remaining 7 numbers $=7$ ways

Total number ways $= 5\times 8\times 7 = 280$

Similarly, for 6000 to 7000

Total number ways $4\times 8\times 7 = 224$

Hence, total number ways for 4000 to 7000

$= 224 + 280 + 224 = 728$

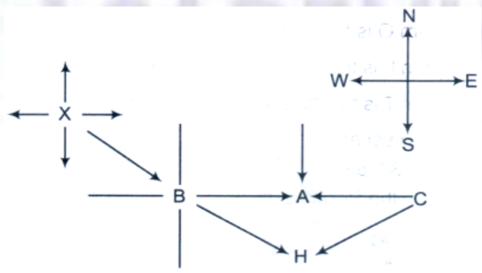
Analytical Ability and Logical Reasoning

51. A road network has parallel roads, which are equidistant from each other and running North-South or East-West only. The road junctions A, B, C, H and X are such that A is East of B and West of C. H is South-West of C and South-East of B. B is South-East of X. Which of the junctions are the farthest South. and the farthest East? **[2014]**

(a) H, B (b) H, C (c) C, H (d) B, H

Ans. (b)

Sol. On the basis of the given information regarding directions, we have the following figure.



It is clear from the figure that H is at the farthest South and C is at the farthest East.

So, option (b) is correct.

52. Four players A, B, C and D have to form into two pairs, however, no pair can play together more than seven times in a row A and B have played seven games in a row. C and D have three in a row. C does not to work with A . Who should play with B ? [2014]

(a) A (b) D (c) C (d) Cannot be determined

Ans. (c)

Sol. It is very clear from the given information that C does not work with A, therefore C should be the partner with B as they can form pairs i.e. AD and CB . So, according to the question,

Number of possibility for each player is

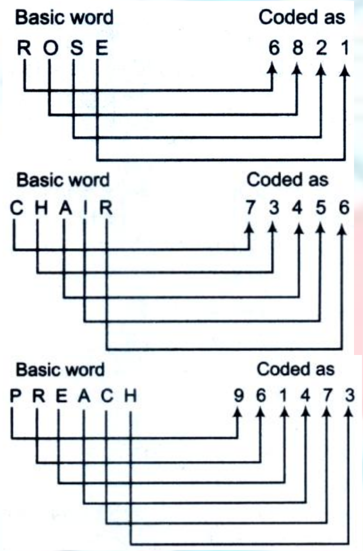
	A	B	C	D
A	×	×	×	√
B	×	×	√	×
C	×	×	×	√
D	√	×	×	×

53. If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, then the code for SEARCH is [2014]

(a) 216473 (b) 214673 (c) 214763 (d) 246173

Ans. (b)

Sol.



Therefore, the coding for word SEARCH will be 214673. Hence, option (b) is the correct choice.

Directions [Q. Nos. 54 to 56]

Cricket clubs in five towns A, B, C, D and E have one team each named P, Q, R, S and T , not necessarily in the same order.

The team in A has beaten R, P and S, Q has beaten the teams in E, C and A . Team R is in B and the team in C is not S.

54. Where is the team Q ? [2014]

(a) A (b) B (c) C (d) D

Ans. (d)

Sol. Team Q is from town D, according to the above table.

Solutions [Q. Nos. 54 to 56]

A table is drawn on the basis of the given information.

Town	A	B	C	D	E
Team	T	R	P	Q	S

55. Where is the team P ? [2014]

(a) A (b) B (c) C (d) D

Ans. (c)

Sol. Team P is from the town C, according to the above table.

56. Which team is in A? [2014]

(a) P (b) Q (c) S (d) T

Ans. (d)

Sol. Team T is from the town A, according to the above table.

57. Find the number that comes next in the series $120, 99, 80, 63, 48, \ldots.$ [2014]

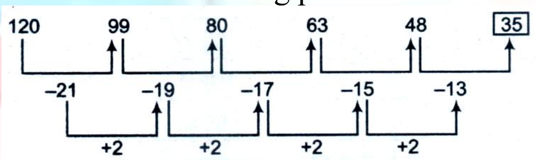
(a) 35 (b) 38 (c) 39 (d) 40

Ans. (a)

Sol. Given series

$120, 99, 80, 63, 48, \ldots$

We have the following pattern



Hence, the number that comes next in the series is 35 .

58. In a certain school, the number of students in each section was 24 . After admitting some students, three new sections have been started and now there are 16 sections with 21 students in each. What is the number of newly admitted students? [2014]

(a) 14 (b) 24 (c) 16 (d) 26

Ans. (b)

Sol. Total number of students when there were 24 students in each of 13 sections $= 13 \times 24 = 312$

Total number of students when three new section were added with 21 students in each of 16 sections $= 21 \times 16 = 336$

Number of newly admitted students $= 336 - 312 = 24$

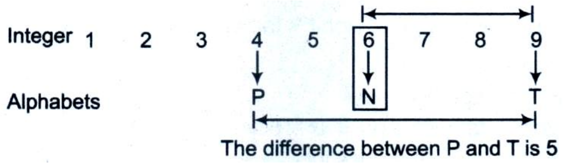
59. The nine alphabets L, M, N, O, P, Q, R, S and T are assigned to nine integers 1 to 9 not necessarily in the same order 4 is assigned to P. The difference between P and T is 5 . The difference between N and T is 3 .

What is the integer assigned to N ? [2014]

(a) 7 (b) 6 (c) 5 (d) 4

Ans. (b)

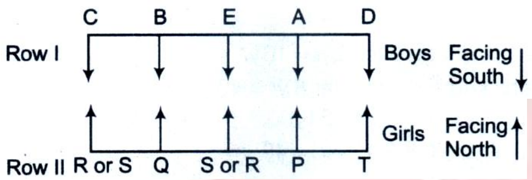
Sol. On the basis of the given information, the integer assigned to N is 6.
This is explained below
The difference between N and T is 3



Directions [Q. Nos. 60 to 63]
Five boys A, B, C, D, E and five girls P, Q, R, S, T are standing in two rows facing each other not necessarily in the order. E is not at any ends. C is to the immediate right of B and D is to the immediate left of A, who is facing P. There are as many girls between P and Q as between R and S. A is second to the left of B.S and R are not facing either B or D.

60. Which pair of boys are standing at the ends of the row? **[2014]**
(a) C and D (b) C and B
(c) D and B (d) None of these
Ans. (a)
Sol. C and D are standing at the ends of the row.

Solutions [Q. Nos. 60 to 63]
On the basis of the given information, we have the standing arrangement as shown below.



61. Which of the following is definitely true? **[2014]**
(a) C is third to the right of D (b) D is facing P
(c) C is facing S (d) None of these
Ans. (d)
Sol. None of the option is correct.

62. Who is standing to the immediate right of A ? **[2014]**
(a) E (b) C (c) D (d) B
Ans. (a)
Sol. E is standing to the immediate right of A.

63. Who is facing B ? **[2014]**
(a) R (b) S (c) Q (d) T
Ans. (c)
Sol. Q is facing B.

64. The sum of ages of a daughter and mother is 63 yr. Four years back mother's age was 4 times that of daughter's age at that time. What is present age of mother? **[2014]**
(a) 46 yr (b) 48 yr (c) 50 yr (d) 59 yr
Ans. (b)
Sol. Let the present age of daughter $= xyr$
and present age of mother $= y$yr
According to the question,
$x + y = 63 ...(i)$
4yr back,
Daughter 's age $= (x-4)$ yr
Mother 's age $= (y-4)$ yr
$\therefore 4(x-4) = y-4$
$4x - 16 = y - 4$
$4x - y = 12 ...(ii)$
From Eqs. (i) and (ii), we get
$x + y = 63$
$\Rightarrow \quad 4x - y = 12$
$\therefore \quad x = 15yr$
$\because \quad x + y = 63$
$\Rightarrow \quad 15 + y = 63$
$\Rightarrow \quad y = 48yr$
Hence, present age of mother $= 48$yr.

65. A watch gains 10 s in 5 min was set correct at 9.00 am. When the watch indicated 20 min past 7.00 pm in the same evening, the correct time is **[2014]**
(a) 7.00 pm (b) 7.40 pm
(c) 7.10 pm (d) 8.00 pm
Ans. (a)
Sol. Time gained in 5 min $= 10$ s
So, time gained in 60 min $= \frac{10}{5} \times 60$
$= 120$ s $= 2$ min
So, time gained in 1 h $= 2$ min
Time gained in 10 h $= 2 \times 10$ min $= 20$ min
Now,
Present time $= 7:20$pm
So, correct time is 7 pm.

66. Father is aged three times more than the age of his son Rohit. After 8 yr, he would be two and a half times of Rohit's age. After further 8 yr, how many times would he be of Rohit's age? **[2014]**
(a) 2 times (b) 3 times
(c) 2.5 times (d) 3.5 times
Ans. (a)
Sol. Let the present age of Rohit $= xyr$
and present age of Rohit 's father $= x + 3x = 4x$ yr
After 8 yr,
Rohit's age $= x + 8yr$
Rohit's father 's age $= 4x + 8$yr
According to the question,

$\Rightarrow \quad \frac{5}{2}(x+8)=4x+8$

$\Rightarrow \quad \frac{5x}{2}+20=4x+8$

$\therefore \quad x=8$

i.e. Rohit age after $8\text{yr}=16\text{yr}$

Rohit's father age after $8\text{yr}=40\text{yr}$

Now, after further 8 yr

Rohit age $=16+8=24\text{yr}$

Rohit 's father $=40+8=48\text{yr}$

67. What is the number that comes next in the series?
$1,2,3,6,11,20,37,68,$ **[2014]**

(a) 105 (b) 124 (c) 125 (d) 126

Ans. (c)

Sol. $1,2,3,6,11,20,37,68,..$

The above series is showing the following pattern. We have to the sum the three numbers

i.e.

$1+2+3 \quad =6$

$2+3+6 \quad =11$

$3+6+11 \quad =20$

$11+20+37 \quad =68$

$20+37+68 \quad =125$

So, the number that comes next in the series, is 125 .

Directions [Q. Nos. 68 to 69]

Six friends A, B, C, D, E and F are sitting round a hexagonal table. F , who is sitting exactly opposite A , is to be immediate right of B, D is between A and B and is exactly opposite to C .

68. Who are sitting next to A ? **[2014]**

(a) D and E (b) D and F

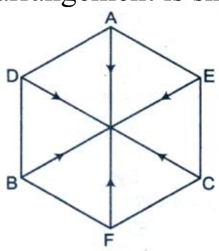
(c) C and E (d) B and D

Ans. (a)

Sol. D and E are sitting next to A.

Solutions [Q. Nos. 68 to 69]

On the basis of the given information, sitting arrangement is shown below :



69. Who is sitting opposite to B ? **[2014]**

(a) A (b) C (c) E (d) F

Ans. (c)

Sol. E is sitting opposite to B .

70. The arithmetic mean of 2^{10} and 2^{20} is **[2014]**

(a) 2^{15} (b) 2^5+2^{10}

(c) 2^9+2^{20} (d) 2^9+2^{19}

Ans. (d)

Sol. Arithmetic mean of 2^{10} and $2^{20}=\frac{2^{10}+2^{20}}{2}$

$=\frac{2(2^9+2^{19})}{2}=2^9+2^{19}$

71. There are five different boxes of different unknown weights each less than 100 kg . These boxes were weighted in pairs and the weights obtained are 110,112 , 113,114,115,116,117,118,120 and 121 kg . What is the weight in kg of the heaviest box? **[2014]**

(a) 60 (b) 62 (c) 64 (d) 61

Ans. (b)

Sol. Let the weights of the five boxes, in increasing order be A,B,C,D and E .

i.e. $\quad A<B<C<D<E$

Each of the boxes can be paired up with another box in a total of 4 ways.

If we sum up all the given weights, we get

$$4(A+B+C+D+E)=(110+112+113+114+115+116$$
$$+117+118+120+121=1156 \text{ kg}$$

$\Rightarrow A+B+C+D+E=289\text{kg}...(i)$

Now, 121 kg is the sum of the weight of the boxes D and E and 110 is the sum of the weights of the boxes A and B .

i.e. $\quad A+B=110$ kg

and $\quad D+E=121$ kg

$\therefore A+B+D+E=231\ kg...(ii)$

Subtracting Eq. (ii) from Eq. (i), we get

$C=289-231=58$ kg

Now, 120 kg is the sum of the weights of the boxes C and E i.e.

$C+E=120$ kg

$\Rightarrow \quad E=(120-58)\text{kg}=62$ kg

Directions [Q. Nos. 72 to 76]

All the roads of a city are either perpendicular or parallel to one another. The roads are all straight. Roads A, B, C, D and E are parallel to one another. Roads F, G, H, I, J, K, L and M are parallel to one another.

Road A is 1 km East of road B.

Road B is 1/2 km West of road C.

Road D is 1 km West of road E.

Road G is 1/2 km South of road H.

Road I is 1 km North of road J.

Road K is 1/2 km North of road L .

Road K is 1 km South of road M.

72. Which of the following is necessarily true? **[2014]**

(a) E and B intersect
(b) D is 2 km West of B
(c) D is atleast 2 km West of A
(d) M is 1.5 km North of L

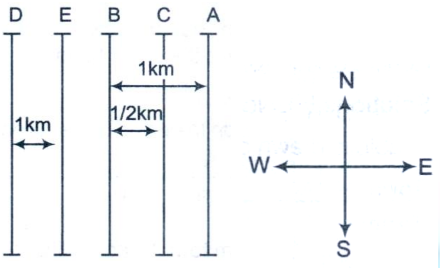
Ans. (d)

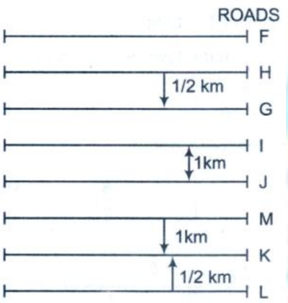
Sol. M is 1.5 km North of L.

Solutions [Q. Nos. 72 to 76]

On the basis of the given information in the question, we have the figure as shown below.

ROADS $\rightarrow$





73. If E is between B and C, then which of the following is false? **[2014]**

(a) D is 2 km West of A
(b) C is less than 1.5 km from D
(c) Distance from E to B added to distance of E to C is $1/2$ km
(d) E is less than 1 km from A

Ans. (a)

Sol. " D is 2 km West of A " is not true.

74. If road E is between B and C, then the distance between A and D is **[2014]**

(a) less than 1 km
(b) C is 1· km West of D
(c) between $1/2$ km and 2 km
(d) more than 2 km

Ans. (c)

Sol. The distance between A and D lies between $\frac{1}{2}$ and 2 km.

75. Which of the following possibilities would make two roads coincide? **[2014]**

(a) L is $1/2$ km North of I
(b) C is 1 km West of D
(c) I is $1/2$ km North of K
(d) E and B are $1/2$ km apart

Ans. (c)

Sol. If I is $1/2$ km North of (b), there is possibility of coinciding of the roads J and L.

76. If K is parallel to I, K is $1/2$ km South of J and 1 km North of G, then which of the following two roads would be $1/2$ km apart? **[2014]**

(a) I and K
(b) J and G
(c) I and G
(d) J and K

Ans. (d)

Sol. J and K would be $1/2$ km apart.

77. The students in three classes are in the ratio $2:3:5$. If 20 students are increased in each class, the ratio changes to $4:5:7$. The total number of students before the increase were **[2014]**

(a) 10 (b) 90 (c) 100 (d) None of these

Ans. (c)

Sol. Let the students in three classes be $2x, 3x$ and $5x$.

On increasing the students in each classes by 20, the students will be $2x+20, 3x+20$ and $5x+20$.

Now, according to the question,

$$2x+20:3x+20:5x+20::4:5:7$$

$$\because \frac{2x+20}{3x+20}=\frac{4}{5}$$

$$10x+100=12x+80$$

$$x=10$$

Total number of students before increase were

$$=2x+3x+5x=10x=10(10)=100$$

78. Ajith is three times older than Babita. Chetu is half the age of Das. Babita is older than Chetu. Which of the following additional information is needed to estimate the age of Ajith?

I. Chetu is 10 yr old.

II. Both Babita and Das are older than Chetu by the same number of years. **[2014]**

(a) Only I
(b) Only II
(c) I and II
(d) None of these

Ans. (c)

Sol. Let the ages of Ajith, Babita, Chetu, Das are A, B, C, D yr. Age of Ajith is three times that of Babita i.e. $A=3B$

Chetu is half the age of Das i.e. $C=\frac{D}{2}$

Babita is older than Chetu i.e. $B>C$.

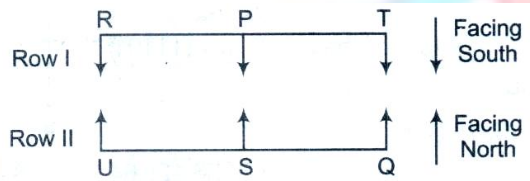
Statement I Chetu is 10 yr old. This statement is necessary to find the age of Ajith.

Statement II This statement is again necessary to find the age of Ajith.

Hence, both statements I and II are necessary to estimate the age of Ajith.

Directions [Q. Nos. 79 to 82]
Six friends P, Q, R, S, T and U are standing in two rows facing one another. P is the middle of one row. U is to the left to S and facing R, Q and T are not in the same row. Only one person is in between R and T.

79. Which of the following are in the same row? **[2014]**
(a) U, S and T (b) R, P and T
(c) U, Q and P (d) U, R, and Q
Ans. (b)
Sol. R, P and T are in the same row.
Solutions [Q. Nos. 79 to 82]
On the basis of the information given for the questions, we have the following standing arrangement of six friends i.e. P, Q, R, S, T, U.

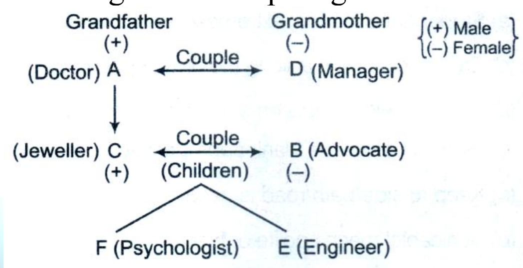


80. Who is to the left of S ? **[2014]**
(a) P (b) U (c) S (d) Q
Ans. (b)
Sol. U is standing to the left of S.

81. Who faces P ? **[2014]**
(a) Q (b) T (c) S (d) U
Ans. (c)
Sol. S is facing P.

82. Which of the following pairs are facing each other? **[2014]**
(a) RS (b) TU (c) PU (d) TQ
Ans. (d)
Sol. T is standing opposite to Q, i.e. TQ are facing each other.

Directions [Q. Nos. 83 to 87]
Six members of a family A, B, C, D, E and F are Psychologist, Manager, Advocate, Jeweller, Doctor and Engineer but not necessarily in same order.
Doctor is the grandfather of F , who is Psychologist.
Manager D is married to A.
Jeweller C is married to Advocate.
B is the mother of F and E.
There are two married couples in the family.
83. Which is the profession of A? **[2014]**
(a) Manager (b) Engineer
(c) Can't be determined (d) None of these
Ans. (d)
Sol. Profession of A is doctor. But it is not given in any of the options.
Solutions [Q. Nos. 83 to 87]
On the basis of the given information, we have the following relationship diagram.



84. What is the profession of **E**? **[2014]**
(a) Manager (b) Engineer
(c) Doctor (d) None of these
Ans. (b)
Sol. Profession of E is engineer.

85. How is A related to E? **[2014]**
(a) Grandmother (b) Wife
(c) Grandfather (d) None of these
Ans. (c)
Sol. A is grandfather of E.

86. How many male members are there in the family? **[2014]**
(a) Two (b) Three (c) Four (d) Can't be determined
Ans. (d)
Sol. This cannot be determined.

87. Who are the two couples in the family? **[2014]**
(a) AD and CB (b) AB and CD
(c) AC and BD (d) None of these
Ans. (a)
Sol. AD and CB are the two couples in the family.

Directions [Q. Nos. 88 to 90]
At a small company, parking spaces are reserved for the top executives: CEO, President, Vice-President, Secretary and Treasurer with the spaces lined up in that order. The parking lot guard can tell at a glance, if the cars are parked correctly by looking at the colour of the cars. The cars are yellow, green, purple, red and blue and the executive names are Alice, Bert, Cheryl, David and Enid.
The car in the first space is red.
A blue car is parked between the red car and the green car.
The car in the last space is purple.
The secretary drives a yellow car.
Alice's is parked next to David's.
Enid drives a green car.

Bert's car is parked between Cheryl's and Enid's.
David's car is parked in the last space.

88. Who is the secretary? **[2014]**
(a) Enid (b) David (c) Cheryl (d) Alice
Ans. (d)
Sol. Alice is the secretary.

Solutions [Q. Nos. 88 to 90]
Based on the given information, following table is drawn.

Execu-tive	CEO	Presi-dent	Vice -Presi-dent	Secre-;tary	Treas-urer
Execu-tive names	Che-ryl	Bert	Enid	Alice	David
Cars	Red	Blue	Green	Yellow	Purple
Order	1	2	3	4	5

89. Who is the CEO? **[2014]**
(a) Alice (b) Bert (c) Cheryl (d) David
Ans. (c)
Sol. Cheryl is the CEO.

90. What colour is the Vice-President car? **[2014]**
(a) Green (b) Yellow (c) Blue (d) Purple
Ans. (a)
Sol. Vice-President's car is of green colour.

General English

91. Fill in the blank with a correct word.
The kitten was soaked to the skin from the ____ .
[2014]
(a) craven (b) storm (c) abyss (d) wind
Ans. (b)
Sol. In the context of the sentence 'storm' fits the blank appropriately.

92. Fill in the blank with the correct word. The ship was attacked by ____ near a deserted Island. **[2014]**
(a) burglars (b) gangsters (c) pirates (d) thieves
Ans. (c)
Sol. 'Pirate' means a person who attacks ships at sea in order to steal from them.

93. From the given alternatives, chosen the one which best express the given sentence in indirect/direct speech. The boy said, 'Who dare call you a thief ?'
[2014]
(a) The boy enquired who dared call him a thief
(b) The boy asked who called him a thief
(c) The boy told that who dared call him a thief
(d) The boy wondered who dared call him a thief
Ans. (a)
Sol. Other three options are clearly in appropriate.

94. Choose the one which can be substituted for the sentence. 'The study of ancient societies'. **[2014]**
(a) Anthropology (b) Archeology
(c) History (d) Etnology
Ans. (a)
Sol. Anthropology means 'the study of ancient societies'.

Directions [Q. Nos. 95 to 96]
Population explosion, malnutrition and ill health are the problems that modern scientists examine for solutions. The agriculture scientists are required to concentrate not only on large production, but also more on improved varieties and protein-rich foods to ward off the ills of malnutrition. The medical scientists responsibilities is not limited to the manufacture of drugs to cure diseases, they must invent medicines to prot zumanity from epidemics. UoJess important is the area of war and weapons. The large scale devastation in Japan by the atom bomb is a stigma on the lair name of scientist. The modem scientist must make a point not to help in the proliferation of atomic weapons. They should rather
devote their energies to the peaceful uses of atomic energy for the emancipation of humanity from hunger and diseases. They must realise that the benefit of their researches and inventions should reach the hands of all, the rich and poor alike.

95. Modern scientists must make point not to help.
[2014]
(a) In the peaceful use of atomic energy
(b) In the prevention of malnutrition
(c) In the proliferation of atomic weapons
(d) In the removal of ill health
Ans. (c)
Sol. Clearly stated in the passage.

96. What does the expression 'malnutrition' used in the passage mean? **[2014]**
(a) Excessive nourishment
(b) Prevention of epidemics
(c) Proliferation of diseases
(d) Lack of proteins
Ans. (d)
Sol. Easy choice from among the given options.

97. Change the voice,
Why did your brother write such a letter? **[2014]**
(a) Why was such a letter written by your brother?
(b) Why did your brother write such a letter?
(c) Why was such a letter wrote by your brother?
(d) Why does your brother write such a letter?

Ans. (a)
Sol. Why was such a letter written by your brother?

98. The first and the last parts of a sentence are numbered as 1 and 6 . The rest of the sentence is split into four parts named P, Q, R, S. These four parts are not given in their proper order. Read the sentence and find out which of the four combinations in correct.
1. Let's never
P. that food
Q. virtually impossible
R. forget
S. is seductive and
6. to resist **[2014]**
(a) SRPQ (b) PSRQ (c) QSRP (d) RPSQ
Ans. (d)
Sol. The correct sequence is RPSQ .

99. Arrange the given words to form a meaningful sentence **[2014]**
(a) dejected (b) students (c) lot (d) of
(e) a (f) were
(a) dbfeac (b) abfecd (c) ecdbfa (d) afebcd
Ans. (c)
Sol. A lot of students were dejected.

100. Fill in the blank with appropriate question tag.
She lives in Chennai now, ____ **[2014]**
(a) lives she ? (b) doesn't she?
(c) does she ? (d) she does?
Ans. (b)
Sol. The appropriate question tag is 'doesn't she?'

101. Pick out the correct word that best expresses the meaning of 'prudent'. **[2014]**
(a) Skillful (b) Efficient
(c) Wise (d) Profitable
Ans. (c)
Sol. 'Prudent' means 'sensible and careful when you make judgements'.

102. Choose the correct article for the sentence below.
"Many ____ flower is born to blush unseen." **[2014]**
(a) an (b) the (c) a (d) No article
Ans. (c)
Sol. Many a is used.

103. The synonym of 'stupendous' is **[2014]**
(a) astounding (b) horrible
(c) appealing (d) comforting
Ans. (a)
Sol. Stupendous means 'extremely large or impressive'.

104. Select the pair with same relationship AFTER : BEFORE **[2014]**
(a) FIRST : SECOND
(b) CONTEMPORARY : HISTORIC
(c) PRESENT : PAST
(d) SUCCESSOR : PREDECESSOR
Ans. (d)
Sol. Easy choice from among the given options.

105. Choose the one which can be substituted for the phrase "A person who insists on something"?**[2014]**
(a) Disciplinarian (b) Stickler
(c) Instantaneous (d) Boaster
Ans. (b)
Sol. A person who insists on something is called stickler.

106. Choose the correct form of verb for the sentence below. I propose that the meeting ____ put off till sunday next. **[2014]**
(a) will be (b) is to be
(c) should be (d) be
Ans. (c)
Sol. 'Should be' fits the blank appropriately.

107. Fill in the blank with correct preposition.
The policeman told me to keep ____ the left. **[2014]**
(a) for (b) of (c) to (d) by
Ans. (c)
Sol. Keep to side/path/road is used.

108. Choose the most suitable synonym for the word "Amicable". **[2014]**
(a) Just (b) Pleasant
(c) Peaceful (d) Complete
Ans. (b)
Sol. 'Amicable' means polite or friendly.

109. Choose the most suitable antonym for the word Rude. **[2014]**
(a) Sweet (b) Polite (c) Decent (d) Gentle
Ans. (b)
Sol. 'Rude' means 'impolite'.

110. Choose the word that matches suitably with the word underlined in the given sentence.
"Developing indigenous technology is important to lead the nation to self-sufficiency." **[2014]**
(a) Intelligent (b) Native
(c) Capitalistic (d) Wise
Ans. (b)
Sol. 'Native' is the correct match.

Computer Awareness

111. The decimal equivalent of the hexadecimal operation $\text{A10}+\text{B21}$ is **[2014]**
(a) 5425 (b) 5246 (c) 2849 (d) 5344
Ans. (a)
Sol. The hexadecimal operation $\text{A10}+\text{B21}$

$$A10 = 101000010000$$
$$B21 = 101100100001$$

Now, we add these two binary numbers

$$\begin{array}{r} 101000010000 \\ +101100100001 \\ \hline 1010100110001 \\ \hline \end{array}$$

Decimal number of 1010100110001 is 5425.

112. What is the 2's complement of 00110101 1001 1100 ? **[2014]**
(a) 1100 1010 1100 1011
(b) 1100 1010 0110 0011
(c) 1100 1010 0110 0100
(d) 1100 1010 1111 1111
Ans. (c)
Sol. The given number is 0011010110011100.
Now, change this number into one's complement as,

$$\begin{array}{rl} & 0011010110011100 \\ \text{1's} & \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ & 1100101001100011 \end{array}$$

Now add 1 to the one's complement to obtain two's complement representation of the given number as

$$\begin{array}{rr} & 1100101001100011 \\ & +1 \\ \hline \text{2's} & 1100101001100100 \end{array}$$

113. Multiplication of 111_2 by 101_2 is **[2014]**
(a) 110011_2 (b) 100011_2
(c) 111100_2 (d) 000101_2
Ans. (b)
Sol. Multiplication of two binary numbers, which are given as

$111_2 \times 101_2$

$$\begin{array}{r} 111 \\ 101 \\ \hline 111 \\ 000\times \\ 111\times\times \\ \hline 100011 \\ \hline \end{array}$$

So, the output is $(100011)_2$.

114. What is the 8 bit 2 's complement representation of the negative integer-93? **[2014]**
(a) 1010011 (b) 10100010
(c) OXA2 (d) None of these
Ans. (d)
Sol. The given number is -93 .
The equivalent binary representation of 93 in byte is 01011001.
Now change it into 1's complement.

$$\begin{array}{cccccccc} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \end{array}$$

Therefore, the one's complement representation of -93 is 10100110
Now found 2's complement.

$$\begin{array}{r} 10100110 \\ +1 \\ \hline 10100111 \\ \hline \end{array}$$

This value does not match with other options. So, the answer is none of these.

115. Consider the values $A = 2.0\times10^{30}, B = -2.0\times10^{30}$, $C = 1.0\times10^{30}$. Assume, that the floating point numbers are represented with 32 bits. What are the values of X and Y, when the following sequence of operations are executed on a computer? **[2014]**

$X = A + B$
$Y = A + C$
$X = X + C$
$Y = Y + B$

(a) $X = 1.0, Y = 1.0$ (b) $X = 1.0, Y = 0.0$
(c) $X = 0.0, Y = 1.0$ (d) $X = 0.0, Y = 0.0$
Ans. (a)
Sol. (a)

116. The Boolean expression $X\cdot(X+Y)$ is same as **[2014]**
(a) $X\cdot(1+Y)$ (b) X
(c) $X\cdot 1$ (d) All of these
Ans. (b)
Sol. The Boolean expression $X\cdot(X+Y)$

$X\cdot(X+Y)$
$=(X\cdot X)+(X\cdot Y)$ [using distributive law]
$= X+(X\cdot Y)$ $[\because X\cdot X = X]$
$= X$ [using absorption law]

So, the expression $X\cdot(X+Y)$ is equal to X.

117. How many bytes are there in a nibble? **[2014]**
(a) one-fourth (b) half (c) 2 (d) 4
Ans. (b)
Sol. A nibble is equal to half byte

As we know, 1 byte = 2 nibble

118. The number of bit strings of length 8 , that start with the bit 0 or end with the bits 11 is **[2014]**
(a) 132 (b) 180 (c) 256 (d) 160
Ans. (a)
Sol. (a)

119. The result of multiplication of the numbers $(10101)_2$ and $(11101)_2$ in hexadecimal form is
[2014]
(a) 609 (b) 216 (c) 261 (d) 906
Ans. (c)
Sol. The result of multiplication of the numbers $(10101)_2$ and $(11101)_2$.
$(10101)_2 = (21)_{10}$
$(11101)_2 = (29)_{10}$

120. The binary equivalent of $(531.53125)_{10}$ is
[2014]
(a) $(1001010011.100001)_2$ (b) $(1000010011.10011)_2$
(c) $(1010010011.11001)_2$ (d) $(1000010011.10001)_2$
Ans. (b)
Sol. $(531.53125)_{H_0}$

512 256 128 64 32 16 8 4 2 1
531 binary number is 1 0 0 0 0 1 0 0 1 1
.53125 binary number is $= .10011$
$(531.53125)_{10} = (1000010011.10011)$