NIMCET-2015 SOLVED PAPER

Mathematics

1. A professor has 24 text books on computer science and is concerned about their coverage of the topics (P) compilers, (Q) data structures and (R) operating systems. The following data gives the number of books that contain material on these topics :

$n(P)=8, n(Q)=13$,

$n(R)=13, n(P\cap Q)=3, n(P\cap Q\cap R)=2$, where

$\mathrm{n}(\mathrm{x})$ is the cardinality of the set x. Then, the number of text books that have no material no compiler is **[2015]**

(a) 4 (b) 8 (c) 12 (d) 16

Ans.(*)

Sol. None option is correct.

2. The value of $\tan\left(\frac{7\pi}{8}\right)$ is **[2015]**

(a) $1-\sqrt{2}$ (b) $1+\sqrt{2}$

(c) $\sqrt{2}+\sqrt{3}$ (d) $\sqrt{2}-\sqrt{3}$

Ans.(a)

Sol. We have, $\tan=\frac{7\pi}{8}$

$$=\tan\left(\pi-\frac{\pi}{8}\right)=-\tan\frac{\pi}{8}=-\left(\frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}}\right)$$

$$\Rightarrow \quad -\tan\frac{\pi}{8}=-\left(\frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)=1-\sqrt{2}$$

3. If $\mathbf{a}$ and $\mathbf{b}$ are vectors such that $|\mathbf{a}|=13, |\mathbf{b}|=5$ and $\mathbf{a}\cdot\mathbf{b}=60$, then the value of $|\mathbf{a}\times\mathbf{b}|$ is **[2015]**

(a) 625 (b) 225 (c) 45 (d) 25

Ans.(d)

Sol. We have, $|\mathbf{a}|=13, |\mathbf{b}|=5, \mathbf{a}\cdot\mathbf{b}=60$

$$|\mathbf{a}\times\mathbf{b}| = \sqrt{|\mathbf{a}|^2\cdot|\mathbf{b}|^2-(\mathbf{a}\cdot\mathbf{b})^2}$$

$$|\mathbf{a}\times\mathbf{b}| = \sqrt{13^2\times5^2-(60)^2}$$

$$=\sqrt{169\times25-3600}$$

$$=\sqrt{4425-3600}=\sqrt{625}=25$$

4. Two towers face each other separated by a distance of 25 m . As seen from the top of the first tower, the angle of depression of the second tower's base is 60° and that of the top is 30°. The height (in meters) of the second tower is **[2015]**

(a) $\frac{50}{\sqrt{3}}$ (b) $\frac{25}{\sqrt{3}}$ (c) 50 (d) $25\sqrt{3}$

Ans.(a)

Sol. Let the height of first tower is PQ.

$\therefore \quad PQ=AD$

In given figure

In $\triangle APB, \tan60^\circ=\frac{AB}{PA}$

$\Rightarrow \sqrt{3}PA=AB \quad ...(i)$

In $\triangle BQD$,

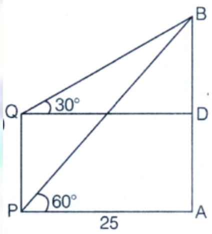
$\tan30^\circ=\frac{BD}{QD}$

$\frac{1}{\sqrt{3}}=\frac{BD}{QD}$

$QD=\sqrt{3}BD$

$\Rightarrow \; PA=\sqrt{3}BD$

$\Rightarrow \; PA=\sqrt{3}[AB-AD] \quad ...(ii)[\because QD=PA]$



From Eq. (i) and (ii), we get

$$PA=\sqrt{3}\left(\sqrt{3}PA-AD\right)$$

$$PA=3PA-\sqrt{3}AD$$

$$\sqrt{3}AD=2PA \quad [\because PA=25]$$

$$AD=\frac{2\times25}{\sqrt{3}}$$

$$AD=\frac{50}{\sqrt{3}}$$

Hence, height of first tower is $\frac{50}{\sqrt{3}}$ m.

5. If $\mathbf{a}=4\hat{\mathbf{i}}+6\hat{\mathbf{j}}$ and $\mathbf{b}=3\hat{\mathbf{i}}+4\hat{\mathbf{k}}$, then the vector form of the component of $\mathbf{a}$ along $\mathbf{b}$ is **[2015]**

(a) $\frac{18}{10\sqrt{13}}\left(3\hat{\mathbf{i}}+4\hat{\mathbf{k}}\right)$ (b) $\frac{18}{5}\left(3\hat{\mathbf{i}}+4\hat{\mathbf{k}}\right)$

(c) $\frac{18}{\sqrt{13}}\left(3\hat{\mathbf{i}}+4\hat{\mathbf{k}}\right)$ (d) $\left(3\hat{\mathbf{i}}+4\hat{\mathbf{k}}\right)$

Ans.(d)

Sol. Since, $\mathbf{a}=4\hat{\mathbf{i}}+6\hat{\mathbf{j}}$ and $\mathbf{b}=3\hat{\mathbf{i}}+4\hat{\mathbf{k}}$

Vector component of $\mathbf{a}$ along $\mathbf{b}$ is $\frac{(\mathbf{a}\cdot\mathbf{b})\mathbf{b}}{\mathbf{b}}$

$$=\frac{\left(4\hat{\mathbf{i}}+6\hat{\mathbf{j}}\right)\cdot\left(3\hat{\mathbf{i}}+4\hat{\mathbf{k}}\right)}{\left(\sqrt{3^2+4^2}\right)^2}\left(3\hat{\mathbf{i}}+4\hat{\mathbf{k}}\right)$$

$$=\frac{36}{25}\left(3\hat{\mathbf{i}}+4\hat{\mathbf{k}}\right)$$

6. The value of

$\sin^{-1}\frac{1}{\sqrt{2}}+\sin^{-1}\frac{\sqrt{2}-\sqrt{1}}{\sqrt{6}}+\sin^{-1}\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}}+\ldots$ to infinity is equal to **[2015]**

(a) π (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Ans.(c)

Sol. We have,

$$\sin^{-1}\frac{1}{\sqrt{2}}+\sin^{-1}\frac{\sqrt{2}-\sqrt{1}}{\sqrt{6}}+\sin^{-1}\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}}+\ldots\infty$$

$$\Rightarrow\sum_{n=1}^{\infty}\sin^{-1}\left(\frac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x(x+1)}}\right)$$

$$=\sum_{n=1}^{\infty}\tan^{-1}\sqrt{x}-\tan^{-1}\sqrt{x-1}$$

$$\Rightarrow\sum_{n=1}^{\infty}\tan^{-1}\sqrt{x}-\tan^{-1}\sqrt{x-1}=\tan^{-1}\infty=\frac{\pi}{2}$$

7. If two circle $x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g'x+2f'y=0$ touch each other, then which of the following is true? **[2015]**

(a) $gf=gf'$ (b) $g'f=gf'$

(c) $gg'=ff'$ (d) None of these

Ans.(c)

Sol. Given, equation of circles are

$x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g'x+2f'y=0$

Two circle touch each other

$\therefore$ Distance between centre of circle $=$ Sum of radius of two circles

i.e. $c_1c_2=r_1+r_2$ [$\because c_1$ and c_2 are centre of circle]

$$\Rightarrow\sqrt{(g-g')^2}+\sqrt{(f-f')^2}=\sqrt{(g^2+f^2)}+\sqrt{(g^{12}+f^{12})}$$

Squaring on both sides,

$$g^2+g'^2-2gg'+f^2+f^{12}-2f'=g^2+f^2+g'^2+f^{12}$$
$$+2\sqrt{(g^2+f^2)(g'^2+f'^2)}$$
$$\Rightarrow -(gg'+ff')=\sqrt{(g^2+f^2)(g'^2+f'^2)}$$

Again squaring, we get

$$g^2g'^2+f^2f'^2+2gg'ff'=g^2g^{12}+g^2f^{12}+g^{12}f^2+f^2f^{12}$$
$$\Rightarrow g^2g'^2+f^2f'^2-2gg'ff'+f'=0$$
$$\Rightarrow (gg'-ff')^2=0\Rightarrow gg'=ff'$$

8. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to **[2015]**

(a) $\frac{\pi}{2}$ (b) 1 (c) -1 (d) $-\frac{\pi}{2}$

Ans. Aanswer is not available in this paper.

Sol. Solution is not available in this paper.

9. In a right-angled triangle, the hypotenuse is four times the perpendicular drawn to it from the opposite vertex. The value of one of the acute angles is **[2015]**

(a) 45° (b) 30° (c) 15° (d) None of these

Ans.(c)

Sol. Let ABC is a right angle triangle and $\angle B=90^\circ$.

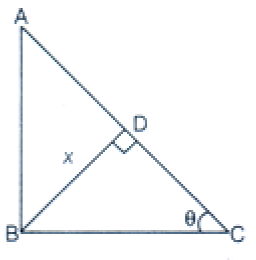
We have, $AC=4AD$.

Let $BD=x$

$\therefore AC=4x$

$\angle BDC=90^\circ$

In □BDC,



$$\sin\theta=\frac{BD}{BC}$$

$$\Rightarrow BC=\frac{x}{\sin\theta}\quad\ldots(i)$$

In □ABC,

$$\cos\theta = \frac{BC}{AC}$$

$$\Rightarrow \quad \cos\theta = \frac{BC}{4x}$$

$$\Rightarrow \quad BC = 4x\cos\theta ...(ii)$$

From Eqs. (i) and (ii) we get

$$4x\cos\theta = \frac{x}{\sin\theta}$$

$$2\sin\theta\cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \sin 2\theta = \frac{1}{2}$$

$$\Rightarrow \quad 2\theta = 30^\circ \Rightarrow \theta = 15^\circ$$

10. A a targeting B, B and C are targeting A. Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$ respectively. If A is hit, then the probability that B hit the target and does not, is **[2015]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Ans. Aanswer is not available in this paper.
Sol. Solution is not available in this paper.

11. If the angles of a triangle are in the ratio $2:3:7$, then the ratio of the sides opposite to these angles is **[2015]**

(a) $\sqrt{2}:2:\sqrt{3}+1$ (b) $2:\sqrt{2}:\sqrt{3}+1$

(c) $2:\sqrt{2}:\frac{\sqrt{2}}{\sqrt{3}-1}$ (d) $\frac{1}{\sqrt{2}}:2:\frac{\sqrt{3}+1}{2}$

Ans.(a)
Sol. Angles of a triangle are in the ratio $2:3:7$

$$\therefore \quad 2x+3x+7x = 180^\circ$$

$$\Rightarrow \quad x = 15^\circ$$

Since, angles of triangle are $30^\circ, 45^\circ, 105^\circ$

$$\therefore \quad \frac{a}{\sin 30^\circ} = \frac{b}{\sin 45^\circ} = \frac{c}{\sin 105^\circ}$$

$$\Rightarrow \quad \frac{a}{1/2} = \frac{b}{1/\sqrt{2}} = \frac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$\therefore \quad a:b:c = \sqrt{2}:2:\sqrt{3}+1$$

12. Suppose that, A and B are two events with probabilities $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$. Then, which of the following is true? **[2015]**

(a) $\frac{1}{3} \le P(A\cap B) \le \frac{1}{2}$ (b) $\frac{1}{4} \le P(A\cap B) \le \frac{1}{3}$

(c) $\frac{1}{6} \le P(A\cap B) \le \frac{1}{3}$ (d) $\frac{1}{4} \le P(A\cap B) \le \frac{1}{2}$

Ans.(b)

Sol. We have, $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$

Now, $P(A) > P(B)$

$\therefore P(A\cap B) \le P(B)$

$\Rightarrow P(A\cap B) \le \frac{1}{3}$ 13.

13. The number of one-to-one function from $\{1,2,3\}$ to $\{1,2,3,4,5\}$ is **[2015]**

(a) 125 (b) 243 (c) 10 (d) 60

Ans. (d)

Sol. Let $A = \{1,2,3\}$ and $B = \{1,2,3,4,5\}$

$\therefore$ Number of one-to-one function is

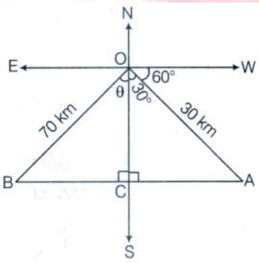
$$^5P_3 = \frac{5!}{2!} = 60$$

14. A harbour lies in a direction 60° South of West from a fort and at a distance 30 km from it, a ship sets out from the harbour at noon and sails due East at 10 km an hour. The time at which the ship will be 70 km from the fort is **[2015]**

(a) 7 PM (b) 8 PM (c) 5 PM (d) 10 PM

Ans.(b)

Sol.



In $\triangle OAC$,

$$\sin 30^\circ = \frac{AC}{OA}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AC}{30}$$

$$\Rightarrow \quad AC = 15 \text{ km}$$

Also, $\cos 30^\circ = \frac{OC}{OA}$

$$= \frac{\sqrt{3}}{2} = \frac{OC}{30}$$

$$\Rightarrow OC = 15\sqrt{3}$$

In $\triangle OBC$,

$$\cos\theta = \frac{OC}{OB} = \frac{15\sqrt{3}}{70}$$

Also, In $\triangle OBC$,

$$\sin\theta = \frac{BC}{OB}$$

$$\Rightarrow \quad \sin\theta = \frac{BC}{70}$$

$$\Rightarrow \quad 1-\cos^2\theta = \frac{BC^2}{4900}$$

$$\Rightarrow \quad 1-\frac{(15\sqrt{3})^2}{4900} = \frac{BC^2}{4900}$$

$$\Rightarrow \quad BC^2 = 4225$$

$$\Rightarrow \quad BC = 65$$

$\therefore$ Total distance travelled $= AC + BC = 15 + 65 = 80$

The time at which ship will cover 70 km from the fort

$= \frac{80}{10} = 8h$ [$\because$ Speed of ship $= 10$ km/h]

$\therefore$ The ship will reach at 8 pm .

15. If x, y and z are three consecutive positive integers, then $\log(1+xz)$ is **[2015]**

(a) $\log y$ (b) $\log\frac{y}{2}$ (c) $\log(2y)$ (d) $2\log(y)$

Ans.(d)

Sol. Since, x, y and z are three consecutive positive numbers.

$\therefore x = y-1$ and $z = y+1$

Now, $\log(1+xz) = \log[1+(y-1)+(y+1)]$

$\log \quad = (1+y^2-1)$

$= \log y^2 = 2\log y$

16. If a, b, and c are in geometric progression, then $\log_{ax}x, \log_{bx}x$ and $\log_{cx}x$ are in **[2015]**

(a) arithmetic progression
(b) geometric progression
(c) harmonic progression
(d) arithmetico-geometric progression

Ans.(a)

Sol. Since, a, b and c are in GP.

$\therefore ax, bx$ and cx will also in GP.

and $\log ax, \log bx$ and $\log cx$ are in AP.

[$\because a, b, c$ are in AP, then $\log a$ and $\log b$ and $\log c$ are in AP]

Hence, $\log_{ax}x, \log_{bx}x$ and $\log_{cx}x$, are in AP.

17. If $\mathbf{a}$ and $\mathbf{b}$ are vector in space, given by $a = \frac{\hat{\mathbf{i}}-2\hat{\mathbf{j}}}{\sqrt{5}}$ and $\mathbf{b} = \frac{2\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{\sqrt{14}}$, then the value of $(2\mathbf{a}+\mathbf{b})\cdot[(\mathbf{a}\times\mathbf{b})\times(\mathbf{a}-2\mathbf{b})]$ is **[2015]**

(a) 3 (b) 4 (c) 5 (d) 6

Ans.(c)

Sol. We have,

$$\vec{a} = \frac{\hat{i}-2\hat{j}}{\sqrt{5}} \text{ and } b = \frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$$

$$(2a+b)\cdot[(a\times b)\times(a-2b)]$$

$$\Rightarrow (2a+b)\cdot[a\cdot(a-2b)b\cdot(a-2b)a]$$

$$[\because (a\times b)\times c = (a-c)b-(b-c)a)]$$

$$\Rightarrow (2a+b)\cdot[(a^2-2a\cdot b)b-(b\cdot a-2b^2)a]$$

$$\Rightarrow [2a+b]\cdot[(1-0)b-(1-2)a]\,[\because a^2 = 1 = b^2, a\cdot b = 0]$$

$$[2a+b]\cdot[b+2a]$$

$$\Rightarrow 4a^2+4a\cdot b+b^2$$

$$\Rightarrow \quad 4+0+1 = 5$$

18. The value of the sum **[2015]**

$$\frac{1}{2\sqrt{1}+1\sqrt{2}}+\frac{1}{3\sqrt{2}+2\sqrt{3}}+\frac{1}{4\sqrt{3}+3\sqrt{4}}+\ldots+\frac{1}{25\sqrt{24}+24\sqrt{25}}$$

is

(a) $\frac{9}{10}$ (b) $\frac{4}{5}$ (c) $\frac{14}{15}$ (d) $\frac{7}{15}$

Ans.(b)

Sol. We have,

$$\frac{1}{2\sqrt{1}+1\sqrt{2}}+\frac{1}{3\sqrt{2}+2\sqrt{3}}+\frac{1}{4\sqrt{3}+3\sqrt{4}}\ldots+\frac{1}{25\sqrt{24}+24\sqrt{25}}$$

Now,

$$T_n = \frac{1}{(n+1)\sqrt{n}+n\sqrt{n+1}}$$

$$\Rightarrow T_n = \frac{1}{\sqrt{n}\sqrt{n+1}(\sqrt{n+1}+\sqrt{n})}$$

$$\Rightarrow T_n = \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n}\sqrt{n+1}} = \frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}$$

$$S = \frac{1}{\sqrt{1}}-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\cdots\frac{1}{\sqrt{24}}-\frac{1}{\sqrt{25}}$$

$$= 1-\frac{1}{\sqrt{25}} = 1-\frac{1}{5} = \frac{4}{5}$$

19. If $\mathbf{a}=\hat{\mathbf{i}}-\hat{\mathbf{k}}, \mathbf{b}=x\hat{\mathbf{i}}+\hat{\mathbf{j}}+(1-x)\hat{\mathbf{k}}$ and $\mathbf{c}=y\hat{\mathbf{i}}+x\hat{\mathbf{j}}+(1+x-y)\hat{\mathbf{k}}$, then $[\mathbf{abc}]$ depends on **[2015]**

(a) Neither x nor y (b) Only x
(c) Only y (d) Both x and y

Ans. (a)

Sol. We have, $\mathbf{a}=\hat{\mathbf{i}}-\hat{\mathbf{k}}, \mathbf{b}=x\hat{\mathbf{i}}+\hat{\mathbf{j}}+(1-x)\hat{\mathbf{k}}$

and $\mathbf{c}=y\hat{\mathbf{i}}+\hat{\mathbf{j}}+(1+x-y)\hat{\mathbf{F}}$

Now,

$$[\mathbf{abc}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$$c_1 \to c_1 + c_2 \quad = \begin{vmatrix} 0 & 0 & -1 \\ 1 & 1 & 1-x \\ 1+x & x & 1+x-y \end{vmatrix}$$

Expand along R_1,

$-1[x-1+x]=1$

$\therefore$ Neither depends on x nor on y.

20. If $42\left({}^nP_2\right)={}^nP_4$, then the value of n is **[2015]**

(a) 2 (b) 4 (c) 9 (d) 42

Ans. (c)

Sol. We have, $42\,{}^nP_2={}^nP_4$

$\Rightarrow 42\dfrac{n!}{(n-2)!}=\dfrac{n!}{(n-4)!}$

$\Rightarrow \dfrac{(n-2)!}{(n-4)!}=42$

$\Rightarrow \dfrac{(n-2)(n-3)(n-4)!}{(n-4)!}=42$

$\Rightarrow n^2-5n+6=42$

$n^2-5n-36$

$(n-9)(n+4)=0$

$n=9$

21. Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors. Which of the following vectors are not perpendicular to each other? **[2015]**

(a) $(\mathbf{a}\times\mathbf{b})$ and $\mathbf{a}$ (b) $(\mathbf{a}+\mathbf{b})$ and $\mathbf{a}\times\mathbf{b}$
(c) $(\mathbf{a}+\mathbf{b})$ and $\mathbf{a}-\mathbf{b}$ (d) $\mathbf{a}-\mathbf{b}$ and $\mathbf{a}\times\mathbf{b}$

Ans. (a)

Sol. $(a\times b)\cdot a=[a,b,a]=0$

(b) $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}\times\mathbf{b})=[\mathbf{aab}]+[\mathbf{bab}]=0+0=0$

(c) $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})=|\mathbf{a}|^2-|\mathbf{b}|^2\neq 0$

(d) $(\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}\times\mathbf{b})=[\mathbf{aab}]-[\mathbf{bab}]=0-0=0$

Hence, option (c) is correct.

22. If $A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a,b and c are real positive numbers such that $abc=1$ and $A^TA=1$, then the equation that holds true among the following is **[2015]**

(a) $a+b+c=1$ (b) $a^2+b^2+c^2=1$
(c) $ab+bc+ca=0$ (d) $a^3+b^3+c^3=4$

Ans. (b)

Sol. We have,

$$A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

It is given that, $abc=1$ and $A^TA=I$

$$A^{\bullet}A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$$\begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ac+ab+bc \\ ab+bc+ca & b^2+c^2+a^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & c^2+a^2+b^2 \end{bmatrix}$$

Now, $A^TA=I$

$$\begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore a^2+b^2+c^2=1$

but $ab+bc+ca\neq 0$[$\because a,b,c$ are real positive numbers]

Hence, $a^2+b^2+c^2=1$

23. The equation of the tangent at any point of curve $x=a\cos 2t, y=2\sqrt{2}a\sin t$, with m as its slope is **[2015]**

(a) $y=mx+a\left(m-\dfrac{1}{m}\right)$ (b) $y=mx-a\left(m+\dfrac{1}{m}\right)$

(c) $y=mx+a\left(a+\dfrac{1}{a}\right)$ (d) $y=amx+a\left(m-\dfrac{1}{m}\right)$

Ans. (b)

Sol. We have, $x=a\cos 2t$ and $y=2\sqrt{2}a\sin t$

$\dfrac{dx}{dt}=-2a\sin 2t$ and $\dfrac{dy}{dt}=2\sqrt{2}a\cos t$

$$\therefore \frac{dy}{dx} = \frac{2\sqrt{2}a\cos t}{-2a\sin 2t}$$

$$= \frac{-\sqrt{2}\cos t}{\sin^2 t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{2}\cos t}{2\sin t\cos t} = \frac{-1}{\sqrt{2}\sin t}$$

It is given that, m is the slope of tangent.

$$\therefore \quad m = -\frac{1}{\sqrt{2}\sin t}$$

$$\Rightarrow \quad \sqrt{2}\sin t = \frac{-1}{m}$$

Squaring both sides,

$$\Rightarrow 2\sin^2 t = \frac{-1}{m^2}$$

$$\Rightarrow 1-\cos 2t = \frac{1}{m^2}$$

$$\Rightarrow 1-\frac{1}{m^2} = \cos 2t$$

$$\Rightarrow \frac{m^2-1}{m^2} = \cos 2t$$

Equation of tangent of curve is

$$y - 2\sqrt{2}a\sin t = m\left(x - a\cos 2t\right)$$

$$y - 2a\left(\frac{-1}{m}\right) = m\left(x - a\left(\frac{m^2-1}{m^2}\right)\right)$$

$$\Rightarrow y = mx - ma\left(\frac{m^2-1}{m^2}\right) - \frac{2a}{m}$$

$$\Rightarrow y = mx - a\left(\frac{m^2-1+2}{m}\right)$$

$$\Rightarrow y = mx - a\left(m + \frac{1}{m}\right)$$

24. The locus of the mid-point of all chords of the parabola $y^2 = 4x$, which are drawn through its vertex is **[2015]**

(a) $y^2 = 8x$ (b) $y^2 = 2x$

(c) $x^2 + 4y^2 = 16$ (d) $x^2 = 2y$

Ans. (b)

Sol. Let $\left(h_1, k\right)$ be the mid-point of chord of parabola $y^2 = 4x$.

$\therefore$ Equation of chord is $yk - 2\left(x+h\right) = k^2 - 4h$

[$\because$ Equation of chord of parabola when mid-point $\left(x, y\right)$ is $T = S_1$]

$$\Rightarrow yk - 2x - 2h = k^2 - 4h....(i)$$

Chordare drawn through vertex

$$\therefore \quad h = \frac{x}{2} \text{ and } k = \frac{y}{2}$$

$$\Rightarrow \quad 2h = x \text{ and} 2k = y$$

Putting the values of x and y in Eq. (i), we get

$$2k^2 - 4h - 2h = k^2 - 4hk^2 = 2h$$

$\therefore$ Locus of the mid-point of the parabola is $y^2 = 2x$.

25. The value of $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a-x}-2\sqrt{x}}$ is **[2015]**

(a) $\frac{2}{3}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{3\sqrt{3}}{2}$ (d) $\frac{2}{3\sqrt{3}}$

Ans. (d)

Sol. We have, $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

$$\Rightarrow \lim_{x\to a}\frac{\left(\sqrt{a+2x}-\sqrt{3x}\right)\left(\sqrt{a+2x}+3\sqrt{x}\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left(\sqrt{3a+x}-2\sqrt{x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)\left(\sqrt{3a+x}\right)+2\sqrt{x}\right)}$$

$$\Rightarrow \lim_{x\to a}\frac{\left(a+2x-3x\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left(3a+x-4x\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$$

$$\Rightarrow \lim_{x\to a}\frac{\left(a-x\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{3\left(a-x\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$$

$$\Rightarrow \frac{1}{3}\times\frac{4}{2\sqrt{3}} = \frac{2}{3\sqrt{3}}$$

26. The value of $\int_{-\pi/3}^{\pi/3}\frac{x\sin x}{\cos^2 x}dx$ is **[2015]**

(a) $\frac{1}{3}\left(4\pi+1\right)$ (b) $\frac{4\pi}{3} - 2\log\tan\frac{5\pi}{12}$

(c) $\frac{4\pi}{3} + \log\tan\frac{5\pi}{12}$ (d) $\frac{4\pi}{3} - \log\tan\frac{5\pi}{12}$

Ans. (b)

Sol. We have, $\int_{-\pi/3}^{\pi/3}\frac{x\sin x}{\cos^2 x}dx$

Let $I = \int_{-\pi/3}^{\pi/3}\frac{x\sin x}{\cos^2 x}dx$

$$\Rightarrow I = 2\int_0^{\pi/3}\frac{x\sin x}{\cos^2 x}dx\left[\because \frac{x\sin x}{\cos^2 x}\text{ is an even function}\right]$$

$$\Rightarrow I = 2\int_0^{\pi/3}x\sec x\tan x dx$$

$$\Rightarrow I = 2\left[x\int\sec x\tan x dx - \int\frac{dx}{dx}\int\sec x\tan x dx\cdot dx\right]_0^{\pi/3}$$

$$\Rightarrow I = 2\left[\left[x\sec x\right]_0^{\pi/3} - \left[\log\left(\sec x + \tan x\right)\right]_0^{\pi/3}\right]$$

$$\Rightarrow I = 2\left[\frac{\pi}{3}\sec\frac{\pi}{3} - \log\left(\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right)\right]$$

$$\Rightarrow I=\frac{4\pi}{3}-2\log\left[\frac{1+\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}\right]$$

$$\Rightarrow I=\frac{4\pi}{3}-2\log\left[\frac{\left(\cos\frac{\pi}{6}+\sin\frac{\pi}{6}\right)^2}{\cos^2\frac{\pi}{6}-\sin^2\frac{\pi}{6}}\right]$$

$$\Rightarrow I=\frac{4\pi}{3}-2\log\left[\frac{\cos\frac{\pi}{6}+\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}-\sin\frac{\pi}{6}}\right]$$

$$\Rightarrow I=\frac{4\pi}{3}-2\log\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$\Rightarrow I=\frac{4\pi}{3}-2\log\tan\frac{5\pi}{12}$$

27. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then value of b^2 is **[2015]**

(a) 1 (b) 5 (c) 7 (d) 9

Ans. (c)

Sol. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$

and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide

$\therefore$ Foci of the ellipse and hyperbola lie on X-axis.

Foci of the ellipse $=\sqrt{16-b^2}$

and foci of the hyperbola $=\sqrt{\frac{144}{25}+\frac{81}{25}}$

Foci of the ellipse and hyperbola coincide

$\therefore \sqrt{16-b^2}=\sqrt{\frac{144}{25}+\frac{81}{25}}$

Squaring both sides,

$\Rightarrow \quad 16-b^2=\frac{225}{25}$

$\Rightarrow \quad b^2=16-9=7$

28. If $A+B+C=\pi$, then the value of

$\begin{vmatrix}\sin(A+B+C) & \sin B & \cos C\\ -\sin B & 0 & \tan A\\ \cos(A+B) & -\tan A & 0\end{vmatrix}$ is **[2015]**

(a) 0 (b) 1 (c) $2\sin A\sin B$ (d) 2

Ans. (a)

Sol. We have,

Let $A=\begin{vmatrix}\sin(A+B+C) & \sin B & \cos C\\ -\sin B & 0 & \tan A\\ \cos(A+B) & -\tan A & 0\end{vmatrix}$

$\Rightarrow A=\begin{vmatrix}\sin\pi & \sin B & \cos C\\ -\sin B & 0 & \tan A\\ \cos(\pi-c) & \tan A & 0\end{vmatrix}\ [\because A+B+C=\pi]$

$\Rightarrow A=\begin{vmatrix}0 & \sin B & \cos C\\ -\sin B & 0 & \tan A\\ -\cos C & \tan A & 0\end{vmatrix}$

$A=0$ [$\because A$ is skew-symmetrix C]

$|A|=0$ Matrix]

29. If the mean deviation of the number $1, 1+d, 1+2d, \ldots\ldots 1+100d$ from their mean is 225, then the value of d is **[2015]**

(a) 20.0 (b) 10.1 (c) 20.2 (d) 10.0

Ans. (d)

Sol. Mean $=\frac{1+1+d+1+2d+\ldots+1+100d}{101}$

Mean $=\frac{101+d(1+2+3+\ldots+100)}{101}$

$$\Rightarrow \text{Mean}=\frac{101+d\left(\frac{100\times101}{2}\right)}{101}=\frac{101(1+50d)}{101}=50d+1$$

$$\Rightarrow \text{MD}(\bar{x})=\frac{\Sigma\left|x_1-\bar{x}\right|}{x}$$

$$\Rightarrow \text{MD}(\bar{x})=\frac{|1-50d-1|+|1+d-50d-1|+\ldots |1+100d-50d-1|}{101}$$

$$\Rightarrow 225=\frac{(50d+49d+\ldots d)+(d+2d+3d\ldots50d)}{101}$$

$$\Rightarrow 225=\frac{2d(1+2+3+\ldots+50)}{101}$$

$$\Rightarrow 225=\frac{2d(50\times51)}{2\times101}\Rightarrow d=\frac{225\times101}{50\times51}=8.91$$

None option is correct.

30. If $P=\sin^{20}\theta+\cos^{48}\theta$, then the inequality that holds for all values of θ is **[2015]**

(a) $P\geq1$ (b) $0<P\leq1$

(c) $1<P<3$ (d) $0\leq P\leq1$

Ans. (a)

Sol. We have, equation of line is $x+y=1\ldots(i)$

A line perpendicular to the given line is $-x+y=\lambda$

This line is passing through $(2,4)$.

$\therefore \qquad -2+4=\lambda$

$\Rightarrow \qquad \lambda=2$

Hence, equation of line is $-x+y=2....(ii)$

Solving Eqs. (i) and (ii), we get

$x=\frac{-1}{2}$ and $y=\frac{3}{2}$

$\therefore$ foot of perpendicular is $\left(\frac{-1}{2},\frac{3}{2}\right)$.

31. The foot of the perpendicular from the point $(2,4)$ upon $x+y=1$ is **[2015]**

(a) $\left(\frac{1}{2},\frac{3}{2}\right)$ (b) $\left(-\frac{1}{2},\frac{3}{2}\right)$

(c) $\left(\frac{4}{3},\frac{1}{2}\right)$ (d) $\left(\frac{4}{3},-\frac{1}{2}\right)$

Ans. Aanswer is not available in this paper.
Sol. Solution is not available in this paper.

32. The value of k for which the equation ($k-2$) $x^2+8x+k+4=0$ has both real, distinct and negative and negative roots is **[2015]**

(a) 0 (b) 2 (c) 3 (d) -4

Ans. (c)

Sol. We have $(k-2)x^2+8x+k+4=0$ has both real, distinct and negative and negative roots.

There are three cases

(i) $D>0$

(ii) Sum of roots <0 i.e. $\frac{-b}{a}<0$

(iii) Product of roots >0 i.e. $\frac{c}{a}>0$

Case I

$D>0$

$64-4(k-2)(k+4)>0$

$16-(k^2+2k-8)>0$

$\Rightarrow k^2+2k-24<0$

$(k+6)(k-4)<0$

$-6<k<4 \qquad ....(i)$

Case ll Sum of roots <0

$\therefore \qquad \frac{-8}{k-2}<0$

$\Rightarrow \qquad \frac{8}{k-2}>0$

$\Rightarrow \quad k>2....(ii)$

Case III Product of roots >0

$\therefore \quad \frac{k+4}{k-2}>0$

$\therefore \quad k\in(-\infty,-4)\cup(2,\infty)....(iii)$

From Eqs. (i), (ii) and (iii) We get
$k=3$ is satisfied
Hence, value of $k=3$

33. If $(2,1),(-1,-2),(3,3)$ are the mid-points of the sides BC, CA, and AB of a triangle ABC, then equation of the line BC is **[2015]**

(a) $5x+4y+6=0$ (b) $5x-4y-6=0$

(c) $5x+4y-6=0$ (d) $5x-4y+6=0$

Ans. (b)

Sol. Let D,E and F are the mid-points of the sides BC, CA and AB of ΔABC.

BC is parallel to side EF and passes through $(2,1)$.

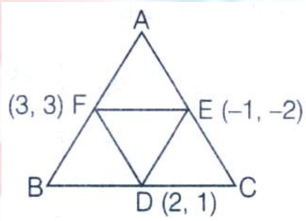
$\therefore$ Equation of line BC is

$$y-1=\frac{3+2}{3+1}(x-2)$$

$\Rightarrow \quad y-1=\frac{5}{4}(x-2)$

$\Rightarrow \quad 4y-4=5x-10$

$\Rightarrow \quad 5x-4y-6=0$



34. If a fair dice is rolled successively, then the probability that 1 appears in an even numbered throw is **[2015]**

(a) $\frac{5}{36}$ (b) $\frac{3}{11}$ (c) $\frac{1}{6}$ (d) $\frac{5}{11}$

Ans. (d)

Sol. Let A be the event 1 appear on the die

$\therefore \ \mathrm{P(A)}=\frac{1}{6}, \mathrm{P}\left(\bar{\mathrm{A}}\right)=1-\frac{1}{6}=\frac{5}{6}$

Now, 1 appears in an even numbered throw.
Required probability

$$=P\left(\bar{A}\right)\cdot P(A)+P\left(\bar{A}\right)\cdot P\left(\bar{A}\right)\cdot P\left(\bar{A}\right)\cdot P\left(\bar{A}\right)+\ldots$$

$$=\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\ldots$$

$$=\frac{5}{6}\times\frac{1}{6}\left[1+\frac{25}{36}+\left(\frac{25}{36}\right)^2\cdots\right]=\frac{5}{36}\left(\frac{1}{1-\frac{25}{36}}\right)=\frac{5}{11}$$

35. Let $\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{c}=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ be three vectors. A vector $\mathbf{v}$ in the plane of $\mathbf{a}$ and $\mathbf{b}$ whose projection on $\frac{\mathrm{c}}{|\mathrm{c}|}$ is $\frac{1}{\sqrt{3}}$, is **[2015]**

(a) $3\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ (b) $\hat{\mathbf{i}}-3\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

(c) $5\hat{\mathbf{i}}-2\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ (d) $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

Ans. (a)

Sol. We have,

$$\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

$$\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}} \text{ and}$$

$$\mathbf{c}=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

A vector $\mathbf{v}$ in plane of $\mathbf{a}$ and $\mathbf{b}$

$$\therefore \mathbf{v} = \lambda\mathbf{a}+\mu\mathbf{b}$$

$$\Rightarrow \mathbf{v} = \lambda\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)+\mu\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$$

$$=(\lambda+\mu)\hat{\mathbf{i}}+(\lambda-\mu)\hat{\mathbf{j}}+(\lambda+\mu)\hat{\mathbf{k}}$$

Projection of v and $\frac{c}{|c|}$ is $\frac{1}{\sqrt{3}}$

$$\therefore \frac{(\lambda+\mu)\hat{\mathbf{i}}+(\lambda-\mu)\hat{\mathbf{j}}+(\lambda+\mu)\hat{\mathbf{k}}\cdot\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)/\sqrt{3}}{\left|\frac{c}{|c|}\right|}=\frac{1}{\sqrt{3}}$$

$$\frac{(\lambda+\mu)-(\lambda-\mu)-(\lambda+\mu)}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \lambda-\mu=-1$$

$\therefore$ Option (a) is correct.

36. The number of bit strings of length 10 that contain either five consecutive 0 's or five consecutive 1's is **[2015]**

(a) 64 (b) 112 (c) 220 (d) 222

Ans. Aanswer is not available in this paper.

Sol. Solution is not available in this paper.

37. If $0<x<\pi$ and $\cos x+\sin x=\frac{1}{2}$, then the value of $\tan x$ **[2015]**

(a) $\frac{4-\sqrt{7}}{3}$ (b) $\frac{4+\sqrt{7}}{3}$

(c) $\frac{1+\sqrt{7}}{4}$ (d) $\frac{1-\sqrt{7}}{4}$

Ans. (b)

Sol. We have, $0<x<\pi$ and

$$\cos x+\sin x=\frac{1}{2}$$

$$\Rightarrow (\cos x+\sin x)^2=\frac{1}{4}$$

$$\Rightarrow 1+\sin 2x=\frac{1}{4}$$

$$\Rightarrow \sin 2x=\frac{-3}{4}$$

$$\Rightarrow \frac{2\tan x}{1+\tan^2 x}=\frac{-3}{4}$$

$$\Rightarrow 3\tan^2 x+8\tan x+3=0$$

$$\Rightarrow \tan x=\frac{-8\pm\sqrt{64-36}}{6}$$

$$\Rightarrow \tan x=\frac{-8\pm2\sqrt{7}}{6}=\frac{-4\pm\sqrt{7}}{3}$$

$$\text{or } \tan x=\frac{-\left(4+\sqrt{7}\right)}{3}$$

$$\frac{-4+\sqrt{7}}{3}$$

38. If a, b and c are the position vectors of the vertices A,B,C of a triangle ABC, then the area of the triangle ABC is **[2015]**

(a) $\frac{1}{2}|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|$ (b) $|\mathbf{a}\times\mathbf{b}|$

(c) $\frac{1}{2}|\mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{c}-\mathbf{c}\times\mathbf{a}|$ (d) $|\mathbf{a}\times(\mathbf{b}\times\mathbf{c})|$

Ans. (a)

Sol. We have a,b and c are the position vectors of the vertices of A,B and C of a $\triangle ABC$.

$$\text{Area of triangle}ABC =\frac{1}{2}|AB\times AC|$$

$$=\frac{1}{2}|(b-a)\times(c-a)|$$

$$=\frac{1}{2}|(b\times c)+(a\times b)+(c\times a)+a\times a|$$

$$=\frac{1}{2}|(a\times b)+b\times c+c\times a|$$

$$[\because \mathbf{a}\times\mathbf{a}=0]$$

39. If $\int e^x\left(f(x)-f'(x)\right)dx=\phi(x)$, then the value of $\int e^x f(x)dx$ is **[2015]**

(a) $\phi(x)+e^x f(x)$ (b) $\phi(x)-e^x f(x)$

(c) $\frac{1}{2}\left[\phi(x)+e^x f(x)\right]$ (d) $\frac{1}{2}\left[\phi(x)+e^x f'(x)\right]$

Ans. (c)

Sol. We have,

$$\int e^x\left[f(x)-f'(x)\right]dx=\phi x$$

$$\Rightarrow \int e^x\left[f(x)dx-\int e^x f'(x)dx=\phi(x)\right.$$

$$\Rightarrow \int e^x f(x)dx-\left[e^x\int f'(x)dx-\int\frac{de^x}{dx}\int f(x)dx.dx\right]=\phi x$$

$$\Rightarrow \int e^x f(x)dx-\left[e^x f(x)-\int e^x f'(x)dx\right]=\phi x$$

$$\Rightarrow \int e^x f(x)dx-e^x f(x)+\int e^x f(x)dx=\phi x$$

$$\Rightarrow 2\int e^x f(x)dx=\phi(x)+e^x f(x)$$

$$\Rightarrow \int e^x f(x)dx=\frac{1}{2}\left[\phi(x)+e^x f(x)\right]$$

40. If $3x+4y+K=0$ is a tangent to the hyperbola $9x^2-16y^2=144$, then the value of K is

(a) 0 (b) 1 (c) -1 (d) -3

Ans. (a)

Sol. We have, $3x+4y+k=0$ is tangent of the hyperbola

$9x^2-16y^2=144$

Equation of hyperbola is $9x^2-16y^2=144$

$$\Rightarrow \frac{x^2}{16}-\frac{y^2}{9}=1$$

Equation of tangent is $3x+4y+k=0$

We know that, $y=mx+c$ touches the hyperbola.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{ if } c^2=a^2m^2-b^2$$

$$\left(\frac{-k}{4}\right)^2=16\left(\frac{-3}{4}\right)^2-9$$

$$\frac{k^2}{16}=9-9$$

$$k=0$$

41. a,b, and c are positive integers such that $a^2+b^2-2bc=100$ and $2ab-c^2=100$. Then, the value of $\frac{a+b}{c}$ is **[2015]**

(a) 10 (b) 100 (c) 2 (d) 20

Ans. Aanswer is not available in this paper.

Sol. Solution is not available in this paper.

42. If $(-4,5)$ is one vertex and $7x-y+8=0$ is one diagonal of a square, then the equation of the other diagonal is **[2015]**

(a) $x+7y=21$ (b) $x+7y=31$

(c) $x+7y=31$ (d) $x+7y=35$

Ans. (b)

Sol. We have, $(-4,5)$ is one vertex of the square and equation of diagonal is $7x-y+8$. Diagonals of square are perpendicular to each other.

$\therefore$ Equation of other diagonal which is perpendicular to

$7x-y+8=0$ is

$x+7y=\lambda$

It passes through $(-4,5)$

$\therefore -4+35=\lambda \Rightarrow \lambda=31$

Hence, equation ofdiagonal is, $x+7y=31$.

43. Out of $2n+1$ ticket, which are consecutively numbered, three are drawn at random. Then, the probability that, the numbers on them are in arithmetic progression is **[2015]**

(a) $\frac{n^2}{4n^2-1}$ (b) $\frac{n}{4n^2-1}$

(c) $\frac{3n^2}{4n^2-1}$ (d) $\frac{3n}{4n^2-1}$

Ans. (d)

Sol. We have,

Total number of ticket $=(2n+1)$

Total number of outcomes of 3 numbers are drawn from

$(2n+1)$ tickets is ${}^{2n+1}C_3$

$$=\frac{(2n+1)}{3!(2n-2)!}=\frac{(2n+1)(2n)(2n-1)}{3!}$$

$$=\frac{n\left(4n^2-1\right)}{3}$$

Favourable outcomes are if three numbers are in AP

$(2n-1+2n-3+2n-5....+5+3+1)=n^2$

$$\therefore \text{ Required probability }=\frac{n^2}{\frac{n\left(4n^2-1\right)}{3}}=\frac{3n}{4n^2-1}$$

44. A circle touches the X-axis and also touches another circle with centre at $(0,3)$ and radius 2 . Then, the locus of the centre of the first circle is **[2015]**

(a) a parabola (b) a hyperbola

(c) a circle (d) an ellipse

Ans. (a)

Sol. Let the centre of circle is (h,k) and radius of circle is k.

Centre of another circle $(0,3)$ and radius is 2.

Both circles touch each other.

$\therefore \quad \sqrt{(h-0)^2+(k-3)^2}=k+2$

$\Rightarrow \quad \sqrt{h^2+k^2-6k+9}=k+2$

$\Rightarrow \quad h^2+k^2-6k+9=k^2+4k+4$

$\Rightarrow \quad h^2=10k-5$

$\Rightarrow x^2=10y-5$ which represent equation of parabola.

$\therefore$ Locus of centre of circle is parabola.

45. Let $\bar{P}$ and $\bar{Q}$ denote the complements of two sets P and Q. Then, the set $(P-Q)\cup(Q-P)\cup(P\cap Q)$ is **[2015]**

(a) $P\cup Q$ (b) $\bar{P}\cup\bar{Q}$ (c) $P\cap Q$ (d) $\bar{P}\cap\bar{Q}$

Ans. (a)

Sol. We have,

$(P-Q)\cup(Q-P)\cup(P\cap Q)$

$\Rightarrow \quad (P\cap Q')\cup(Q\cap P')\cup(P\cap Q)$

$\Rightarrow \quad (P\cap Q')\cup[(Q\cap P')\cup(Q\cap P)]$

$\Rightarrow \quad (P\cap Q')\cup[Q\cap(P\cup P')]$

$\Rightarrow \quad (P\cap Q')\cup(Q\cap\cup)$

$\Rightarrow \quad (P\cap Q')\cup Q$

$\Rightarrow \quad (P\cup Q)\cap(Q\cup Q')$

$\Rightarrow \quad (P\cup Q)\cap\cup=(P\cup Q)$

46. With the usual notation, $\frac{d^2x}{dy^2}$ is **[2015]**

(a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-2}$

(c) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

Ans. (d)

Sol. Since, $\frac{dx}{dy}=\frac{1}{\frac{dy}{dx}}=\left(\frac{dy}{dx}\right)^{-1}$

$\Rightarrow \quad \frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}\frac{dy}{dx}$

$\Rightarrow \quad \frac{d^2x}{dy^2}=-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}\left(\frac{dy}{dx}\right)$

$\Rightarrow \quad \frac{d^2x}{dy^2}=-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

47. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre at $(0,3)$ is **[2015]**

(a) 4 units (b) 3 units

(c) $\sqrt{12}$ units (d) $\frac{7}{2}$ units

Ans. (a)

Sol. We have,

Equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$

Here, $a^2=16$

and $b^2=9$

We know that, foci of ellipse of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $(\pm ae,0)$

$ae=\sqrt{a^2-b^2}$

$=\sqrt{16-9}$

$=\sqrt{7}$

$\therefore$ Foci of ellipse $(\pm\sqrt{7},0)$.

It is given that, centre of circle is $(0,3)$ and passes through $\pm\sqrt{7},0$

$r=\sqrt{(0+\sqrt{7})^2+(3-0)^2}$

$\therefore \quad =\sqrt{7+9}$

$=4$

Hence, radius of circle $=4$ units

48. A function $F:(0,\pi)\to R$ defined by $f(x)=2\sin x+\cos 2x$ has **[2015]**

(a) a local minimum but no local maximum brs

(b) a local maximum but no local minimum rl

(c) Both local minimum and local maximum?

(d) Neither a local minimum nor a local maximum

Ans. (c)

Sol. We have,

$f(x)=2\sin x+\cos 2x$

$f'(x)=2\cos x-2\sin 2x$

$f''(x)=-2\sin x-4\cos 2x$

For maxima or minima,

$f'(x)=0$

$\Rightarrow 2\cos x-2\sin 2x=0$

$\Rightarrow \cos x-2\sin x\cos x=0$

$\Rightarrow \cos x(1-2\sin x)=0$

$\cos x=0$ or $1-2\sin x=0$

$x=\frac{\pi}{2}$ or $x=\frac{\pi}{6},\frac{5\pi}{6}$

Now,

$$f''\left(\frac{\pi}{2}\right) = -2\sin\frac{\pi}{2} - 4\cos\left(\frac{2\pi}{2}\right)$$
$$= -2+4 = 2 > 0$$
$$f''\left(\frac{\pi}{6}\right) = -2\sin\frac{\pi}{6} - 4\cos\left(\frac{2\pi}{6}\right)$$
$$= -1-2 = -3 < 0$$
$$f''\left(\frac{5\pi}{6}\right) = -2\sin\frac{5\pi}{6} - 4\cos\left(\frac{10\pi}{6}\right)$$
$$= -1-2 = -3 < 0$$

Hence, $f(x)$ is minimum at $\frac{\pi}{2}$ and maximum at $\frac{\pi}{6}$.

$\therefore f(x)$ has both local minimum and local maximum.

49. A matrix M_r is defined as

$M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix} r \in N$, then the value of

$\det(M_1)+\det(M_2)+\ldots+\det(M_{2015})$ is **[2015]**

(a) 2014^2 (b) 2013^{29715}
(c) 2015 (d) 2015^2

Ans. (d)

Sol. We have,

$$M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}, r \in N$$
$$|M_r| = r^2-(r-1)^2$$
$$= r^2-r^2+2r-1 = 2r-1$$

Now,

$$|M_1|+|M_2|+|M_3|\ldots|M_{2015}|$$
$$\Rightarrow 1+3+5+7+\ldots+3029$$
$$= (2015)^2$$
$$\left[\because \sum_{n=1}^{n}(2n-1) = n^2\right]$$

50. If $\mathbf{AC} = 2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{BD} = -\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$, then the area of the quadrilateral $ABCD$ is **[2015]**

(a) $\frac{5}{2}\sqrt{3}$ (b) $5\sqrt{3}$ (c) $\frac{15}{2}\sqrt{3}$ (d) $10\sqrt{3}$

Ans. (b)

Sol. We have,

$$\mathbf{AC} = 2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$$
$$\mathbf{BD} = -\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$$

Area of quadrilateral $ABCD = |\mathbf{AC}\times\mathbf{BD}|$

$$\mathbf{AC}\times\mathbf{BD} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & 1 \\ -1 & 3 & 2 \end{vmatrix}$$
$$= (2-3)\hat{\mathbf{i}}-(4+1)\hat{\mathbf{j}}+(6+1)\hat{\mathbf{k}}$$
$$= \hat{\mathbf{i}}-5\hat{\mathbf{j}}+7\hat{\mathbf{k}}$$
$$|\mathrm{AC}\times\mathrm{BD}| = \sqrt{1+25+49}$$
$$= \sqrt{75} = 5\sqrt{3}$$

Analytical Ability and Logical Reasoning

Directions [Q. Nos. 51-54] A circular field with inner radius of 10 m and outer radius of 20 m is divided into 5 successive stages for ploughing. The ploughing at each stage, with starting points P1, P2, P3, P4 and P5 was alloted to one of the five farmers F1, F2, F3, F4 and F5 not necessarily in that order.
F5 was alloted the stage starting a point P4.
The stage from P5 to P3 was not the first stage.
F4 was allotted the work of the fourth stage.
Finishing point of stage 3 was P1 and the work was not allotted to F1.
F3 was allotted the work of stage ending at P5.

51. Which of the following is the finish point for farmer F2? **[2015]**

(a) P1 (b) P2 (c) P 3 (d) P4

Ans. (a)

Sol. P_1 is the finishing point of F_2.

Solutions [Q. Nos. 51 to 54]

The given information can be represented as:

Stage	Farmer	Starting point of the stage	Ending point of the stage
First	F_3	P_2	P_5
Second	F_1	P_5	P_3
Third	F_2	P_3	P_1
Fourth	F_4	P_1	P_4
Fifth	F_5	P_4	P_2

52. Which stage was ploughed by F5? **[2015]**

(a) 2 (b) 3 (c) 4 (d) 5

Ans. (d)

Sol. 5th stage is ploughed by F_5.

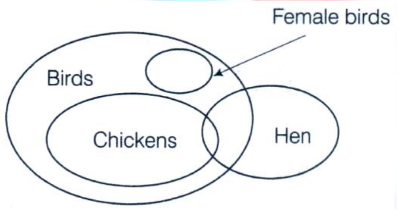
53. What are the starting and ending points of the field ploughed by F4? **[2015]**
(a) P1 and P2 (b) P1 and P4
(c) P4 and P2 (d) P2 and P4
Ans. (b)
Sol. P_1 and P_4

54. What is the starting point for stage 3 ? **[2015]**
(a) P2 (b) P 3
(c) P4 (d) Cannot be determined
Ans. (b)
Sol. P_3 is the starting point of stage 3 .

55. If Tuesday falls on the fourth of a month, then which day will fall three days after 24 th of the same month? **[2015]**
(a) Monday (b) Tuesday
(c) Thursday (d) Friday
Ans. (c)
Sol. The 4th day is Tuesday, then 11th, 12th and 25th also Tuesdays.
Then, 3 days after 24th is 27th, which is Thursday.

56. If the statements "All chickens are birds", "Some chickens are hens" and "Female birds lay eggs", are all facts, then which of the following must also be a fact? **[2015]**

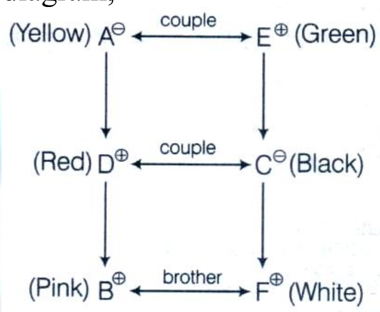
I. All birds lay eggs
II. Some hens are birds
III. Some chickens are not hens
(a) I and II (b) II and III
(c) I and III (d) Neither I nor II nor III
Ans. (b)
Sol.



So, only conclusion II and III follow.

Directions [Q. Nos. 57-60]
There is a family of six members A, B, C, D, E and F .
There are two married couples in the family and the family members represent three generations.
Each member has a distinct choice of a colour amongst Green, Yellow, Black, Red, White and Pink.
No lady member likes either Green or White.
C , who likes Black colour, is the daughter-in-law of E .
B is the brother of F and son of D and likes Pink.
A is the grandmother of F and F does not like Red.
Wife of the husband having a choice for Green colour likes Yellow.

57. Which of the following is the colour preference of ? **[2015]**
(a) Red (b) Yellow
(c) Either Red or Yellow (d) Cannot be determined
Ans. (b)
Sol. A likes yellow colour.
Solutions [Q. Nos. 57 to 60]
From the given information, we can draw a relation diagram,



58. Which of the following could be the colour combination of one of the couples? **[2015]**
(a) Yellow-Red (b) Green-Black
(c) Red-Yellow (d) Yellow-Green
Ans. (d)
Sol. Yellow-Hypen green is the colour combination of AE .

59. Which of the following is one of the married couples? **[2015]**
(a) C (b) AC
(c) AD (d) Cannot be determined
Ans. (a)
Sol. CD is the married couple.

60. Which of the following is true about ? **[2015]**
(a) Brother of B (b) Sister of B
(c) Daughter of C (d) Cannot be determined
Ans. (a)
Sol. F is the brother of B .

61. If the English word "EXAMINATION" is coded as 56149512965, then the word "GOVERNMENT" is coded as **[2015]**
(a) 7645954552 (b) 7654694562
(c) 7645955423 (d) 7654964526
Ans. (a)
Sol. As ,

E	X	A	M	I	N	A	T	I	O	P
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
5	6	1	4	9	5	1	2	9	6	5

The pattern in the above word is as follows.

$$E=5$$
$$X=24=2+4=6$$
$$A=1$$
$$M=13=1+3=4$$
$$I=9$$
$$N=14=1+4=5$$
$$A=1$$
$$T=20=2+0=2$$
$$I=9$$
$$O=15=1+5=6$$
$$N=14=1+4=5$$

$\therefore$ The sum of the placing value of the words will be the code of the words. Similarly,

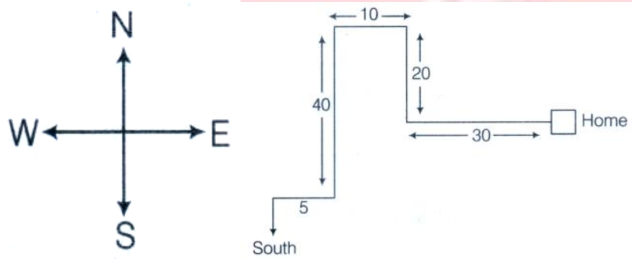
G	O	V	E	R	N	M	E	N	T
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
7	6	4	5	9	5	4	5	5	2

62. Gopal starts from his house towards West. After walking a distance 30 m , he turned towards right and walked 20 m . He turned left and after moving a distance of 10 m , turned to his left again and walked 40 m . Then, he turned left and walked 5 m . Finally, he turns to his left. In which direction is the walking now? **[2015]**

(a) North (b) South (c) East (d) South East

Ans. (b)

Sol. According to the information given in the question, we can draw a direction diagram.



From the above diagram, it is clear that Gopal is walking in South direction.

63. Read the conclusion and then decide which of the given conclusions logically follows from the two given statements (i) and (ii) disregarding commonly known facts. **[2015]**

Statements

(i) No women teacher can play.

(ii) Some women teachers are athletes.

Conclusions

I. Male athletes can play.

II. Some athletes can play.

(a) Only conclusion I follows

(b) Only conclusion II follows

(c) Either I or II follows

(d) Neither I nor II follows

Ans. (d)

Sol.

So, no conclusion follows.

64. Which of the following come next in the series $8,6,9,23,87,\ldots\ldots$? **[2015]**

(a) 28 (b) 226 (c) 324 (d) 429

Ans. (d)

Sol. $8,6,9,23,87$?

The pattern is as follows

$$8\times1-2=6$$
$$6\times2-3=9$$
$$9\times3-4=23$$
$$23\times4-5=87$$
$$87\times5-6=429$$

So, the next term will be 429 .

65. In an examination there are 100 questions divided into 3 parts A, B and C and each part should contain atleast one question. Each question in parts A, B and C carry 1, 2 and 3 marks respectively. Part A is for atleast 60% of the total marks and part B should contain 23 questions. How many questions must be set in part C ?

[2015]

(a) 1 (b) 2

(c) 3 (d) Cannot be determined

Ans. (a)

Sol. Suppose, the number of questions in group A,B and C be x,y, and z , respectively.

Then, $x+y+z=100$ (given)

Total marks would be $x+2y+3z$

Given, $y=23$

Total marks from section $B=46$

Different possible values for z are $1,2,3\ldots..$

$\therefore$ Corresponding values for x are $76,75,74\ldots$

(Since, total questions are 100)

when $z=1,x=76,y=23$

Total marks from groups A,B and C are $76,46,3$ respectively.

Percentage marks from $A=\dfrac{76}{76+46+3}$

$=\dfrac{75}{125}>60\%$

when $z=2,x=75,y=23$

Percentage marks from $A=\dfrac{75}{75+46+6}$

$=\frac{75}{127}<60\%$

For all other values, when z increases, x decreases and contribution of marks from A keeps decreasing.

$\therefore$ There is only 1 possible value for questions from group C .

66. If $\div$ means addition, - means division, $\times$ means subtraction and + means multiplication, then the value of $\frac{(36\times4)-8\times4}{4+8\times2+16\div1}$ is **[2015]**

(a) 0 (b) 8 (c) 12 (d) 16

Ans. (a)

Sol. Given,

$\div \rightarrow +$

$- \rightarrow \div$

$\times \rightarrow -$

$+ \rightarrow x$

Then,

$$\frac{(36\times4)-8\times4}{4+8\times2+16\div1}$$

$$=\frac{(36-4)\div8-4}{4\times8-2\times16+1}$$

$$=\frac{(32)\div8-4}{4\times8-2\times16+1}$$

$$=\frac{4-4}{4\times8-2\times16+1}=0$$

67. Which letter is the word CYBERNETICS occupies the same position as it does in the English alphabet? **[2015]**

(a) C (b) E (c) I (d) T

Ans. (c)

Sol. In the word CYBERNETICS,

I occupies the same position as it does in the English alphabet. The place value of ' I ' is 9.

68. The remainder when 2^{31} is divided by 5 is **[2015]**

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c)

Sol. Here, 2^{31} is divided by 5

$$2^{31}=2^{30}\times2^{1}$$

$$=\left(2^{2}\right)^{15}\times2$$

$$=(4)^{15}\times2$$

$$=(-1+5)\times2=4\times2=8\div5=3$$

Directions [Q. Nos. 69-70] The letters of English alphabet from A to M were written, leaving space for one letter between every two letters and then the remaining letters were inserted beginning with N and ending the series with Z after M .

69. Which letter would be 3rd to the right of the 7th letter from the left? **[2015]**

(a) C (b) O (c) R (d) S

Ans. (c)

Sol. 7th letter from the left is D and 3rd letter to the right of D would be ' R '.

Solutions [Q. Nos. 69 to 70]

AN BO CPD QE RF SGT HU IV JW KXLY MZ

70. Which letter would be exactly in the middle of eighteenth letter from the beginning and fifteenth from the end? **[2015]**

(a) G (b) H (c) J (d) L

Ans. (b)

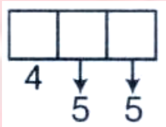
Sol. The eighteenth letter from the beginning is V and fifteenth letter from the last is S . So, H will be exactly in between S and V .

71. How many 3-digit numbers divisible by 5 , can be formed using the digits $2,3,5,6,7$ and 9 , without repetition of digits? **[2015]**

(a) 216 (b) 20 (c) 120 (d) 24

Ans. (b)

Sol. Here



is fixed.

So, total 3 digits number divisible by 5

$=5\times4\times1=20$

72. Using only $2,5,10,25$ and 50 paise coins, what is the smallest number of coins required to pay exactly 78 paise, 69 paise and ₹1.01 to three different persons? **[2015]**

(a) 19 (b) 20 (c) 17 (d) 18

Ans. (a)

Sol. Here,

We have, to find LCM of 78 paise, 69 paise and 101 paise.

And,

$78=1\times50+2\times10+2\times4=5$ coins

$69=1\times50+1\times10+1\times5+2\times2=5$

$101=1\times101=1\times50+4\times10+1\times5+3\times2=9$

So, LCM $=2\times3\times13\times23\times101=181194$

73. Which of the following two patterns will fit in the blanks of the series ZA_5, Y_4 B, XC_6, W_3D , ____ ? **[2015]**

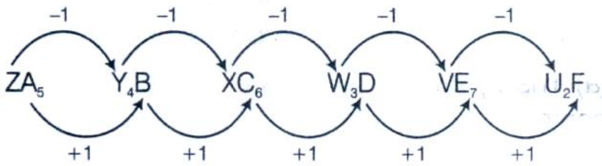
(a) VE_7 and U_2E (b) V_2E and U_7 F

(c) VE_7 and U_2F (d) VF_7 and U_2E

Ans. (c)

Sol. $ZA_5, Y_4B, XC_6, W_3D --$

The pattern is as follows:



So, VE_7 and U_2 F will be fit in the blanks.

74. Which of the following numbers comes next in the two-digit decimal number sequence $61, 52, 63, 94, \ldots$? **[2015]**

(a) 65 (b) 64 (c) 56 (d) 46

Ans. (d)

Sol. $61, 52, 63, 94$?

The number formed by reversing the digit of each number of the sequence is square of consecutive natural numbers starting from 4 . See how,

$61 \ldots .16 = 4^2$

$52 \ldots .25 = 5^2$

$63 \ldots 36 = 6^2$

$94 \ldots .49 = 7^2$

Therefore, the required number $= 8^2 = 64 = 46$.

75. Three ladies X, Y and Z marry three man A, B and $C. X$ is married to A, Y is not married to an engineer. Z is not married to a doctor, C is not a doctor and A is a lawyer. Then, which of the following statements is correct? **[2015]**

(a) X is married to a doctor

(b) Y is married to C , who is a doctor

(c) Z is married to B , who is an engineer

(d) None of the above

Ans. (d)

Sol. From the given information

Wife	Husband
X	A (Lawyer)
Y	B (Doctor)
Z	C (Engineer)

From the above table, no statement is correct.

76. There are five books A, B, C, D and E placed on a table. If A is placed below E, C is placed D, B is placed below A and D is placed between A and E, then which of the following books can be on the top? **[2015]**

(a) D or E (b) C or E

(c) A or E (d) None of these

Ans. (d)

Sol. From the statements, A is placed below E

E
A

C is placed below D

D
C

B is placed below A

A
B

D is placed between A and E, then

E
D
A
B
C

So, E will be on the top.

77. Among five children A, B, C, D and E, B is taller than E but shorter than D . A is shorter than C but taller than D . If all the children stand in a line according to their heights, then who would be the fourth if counted from the tallest one? **[2015]**

(a) D (b) C (c) B (d) A

Ans. (c)

Sol. Given that

B is taller than E but shorter than D.

$D > B > E$

A is shorter than C but taller than D.

$D < A < C$

Then the order is

$$\underset{1}{C} > \underset{2}{A} > \underset{3}{D} > \underset{4}{B} > \underset{5}{E}$$

So, B is fourth.

Directions [Q. Nos. 78-81]

In a family of six person A, B, C, D, E and F there are two married couples.

D is grandmother of A and mother B .

C is wife of B and mother F .

F is the grand daughter of E .

78. What is C to A ? **[2015]**

(a) Daughter (b) Grandmother

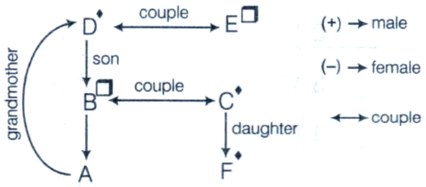
(c) Mother (d) Cannot be determined

Ans. (c)

Sol. ' C ' is the mother of A

Solutions [Q. Nos. 78 to 81]

From the given information in the question, we can draw a relation diagram,



79. How many male members are there in the family? **[2015]**

(a) Two (b) Three
(c) Four (d) Cannot be determined

Ans. (d)

Sol. It cannot be determined because the gender of A is not defined.

80. Who among the following is one of the couples? **[2015]**

(a) CD (b) DE
(c) EB (d) Cannot be determined

Ans. (d)

Sol. DF is the couple.

81. Which of the following is true? **[2015]**

(a) A is brother of F (b) A is sister of F
(c) B has two daughters (d) None of these

Ans. (d)

Sol. No statement is true.

Directions [Q. Nos. 82-85] A, B, C, D, E, F and G are seven girls having different amount of money from among ₹ $10, 20, 40, 60, 80, 120$ and 200 with them.
They had 3 chocolates, 2 toffees and 2 lollipops together, each other having one of these seven items.
B and F do not have chocolates and they have ₹ 200 and ₹ 80 , respectively.
C has ₹ 60 with her and G has an amount which neither ₹ 40 nor ₹ 120 .
A has ₹10 and does not have a toffee.
The girl having 40 with her is the only one other than A to have the same type of item.
E and girl having ₹ 20 with her have the same find of item.

82. How much amount does G have with her? **[2015]**

(a) ₹ 20 (b) ₹ 10 (c) ₹ 60 (d) None of these

Ans. (a)

Sol. From the table, it is clear that G has ₹ 20.

Solutions [Q. Nos. 82 to 85]

The given information in the question can be summarised as :

Girl	Money	Item
A	10	Lollipops
B	200	Toffees
C	60	Chocolates
D	40	Lollipops
E	120	Chocolates
F	80	Toffees
G	20	Chocolates

83. Which of the following girl have chocolates with them? **[2015]**

(a) F, C, G (b) C, G, E
(c) C, G, D (d) G, D, E

Ans. (b)

Sol. From the table, it is clear that C, G and E has chocolates with them.

84. Which of the following combination is definitely correct? **[2015]**

(a) C-chocolate ₹ 60 (b) G-toffee- ₹ 20
(c) C-chocolate-₹ 40 (d) None of these

Ans. (a)

Sol. Combination in option (a) is correct.

85. Which girls has ₹ 40 with her? **[2015]**

(a) E (b) A (c) D (d) None of these

Ans. (c)

Sol. From the table, it is clear that D has ₹ 40 with her.

86. P, Q, R, S, T, U and V are sitting in a row facing North. In order todetermine, who is sitting exactly in the middle of the row, which of the following information is needed? **[2015]**

I. T and U are sitting at extreme ends of the row.
II. S is third to the right of T
III. Q is four places to the left of R and P is two places of the left of V

(a) I and II only are sufficient
(b) I and III only are sufficient
(c) I and either II or III are sufficient
(d) I, II and III

Ans. (a)

Sol. From the statement I

I - T and U are sitting at extreme ends of the row.

Case I

T .... U

Case II

U I

From statement II

S is third to the right of T.

Then, from case I

$T \ldots . \underline{S} \ldots . U$

So, S is sitting exactly in the middle.

So, only statement I and II are sufficient.

87. How many times do the hour and the minute hands of a clock overlap in 24 h ? **[2015]**

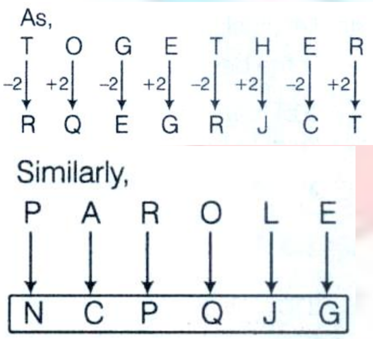
(a) 24 (b) 22 (c) 26 (d) 20

Ans. (b)
Sol. The hands of a clock coincide 11 times in every 12 h (Since, between 11 and 1, they coincide only once i.e. at 12 O' clock).

AM	PM
12:00	12:00
1:05	1:05
2:11	2:11
3:16	3:16
4:22	4:22
5:27	5:27
6:33	6:33
7:38	7:38
8:44	8:44
9:49	9:49
10:55	10:55

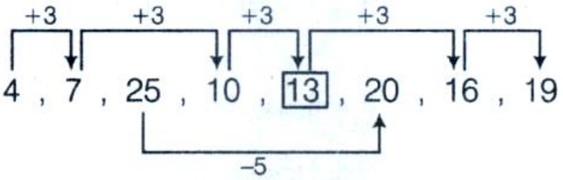
The hands overlap about 65 min , not every 60 min . So, the hand coincide 22 times in a day.

88. In a certain code, TOGETHER is coded as RQEGRJCT. In the same code, PAROLE will be written as **[2015]**
(a) NCPQJG (b) NCQPJG
(c) RCPQJK (d) RCTQNG
Ans. (a)
Sol.



89. A drawer contains 10 black and 10 brown socks which are all mixed up. What is the smallest number of socks to be taken from the drawer to decided without seeing them, to be sure that there is atleast one pair of socks of the same colour? **[2015]**
(a) 11 (b) 10
(c) 3 (d) Cannot be determined
Ans. (c)
Sol. Even if first 2 are different colours, 3rd one has to be black or brown. So, the smallest number of socks will be 3 .

90. Find the missing number in the series :
4,7,25,10,…., 20,16,19 **[2015]**
(a) 13 (b) 15 (c) 20 (d) 28
Ans. (a)
Sol. 4,7,25,10 _ 20,16,19
The pattern is as follows:



Two series alternate here, with every third number following a different patterns. In the main series, 3 is added to each number to arrive at the next. In the alternating series, 5 is subtracted from each number to arrive at the next.

General English

91. Identify the type of error in the sentence: "The cost of this project will be much lesser than 5% more than that predicted earlier". **[2015]**
(a) syntactical error (b) punctuation error
(c) grammatical error (d) conflicting words
Ans. (a)
Sol. The syntax of the sentence is wrong as it contains two contrast comparisons.

92. Insert appropriate prepositions in the blanks to complete the sentence. "This property has been ____ the possession ____ the royal family ____ generations." **[2015]**
(a) with, of, of (b) in, of, for
(c) in, with, by (d) of, by, since
Ans. (b)
Sol. To show possession the preposition of position 'in' and that of possessive case 'of' are correct in the first two blanks; whereas to show the 'time period' of action 'for' is used.

93. Choose the right word to fill in the blank in the sentence. "The mermaid legend ____ have originated with a group of mammals collectively known to science as Srinians". **[2015]**
(a) should (b) may (c) need (d) can
Ans. (b)
Sol. 'may' denotes 'possibility' here.

94. Identify appropriate word to fill the blanks in the sentence. "The feeling of guilt left a ____ impression in the life?" **[2015]**
(a) perennial (b) parennial
(c) perennial (d) perinial
Ans. (a)
Sol. 'perennial' means 'that which lasts for a long time'. Hence, it is the best option.

95. Which of the following sentences is grammatically incorrect? **[2015]**
(a) He is smiling (b) He smiles
(c) He always smiles (d) He is always smiling
Ans. (d)
Sol. 'Continous tense' is used to denote the action which is progressive at the time of speaking. Hence, the use of 'always' is not appropriate with it.

96. Find the most suitable phrasal verb to be filled in the following sentence.
"Left too long in the sun, the leaves had all" **[2015]**
(a) Shrugged off (b) Shared out
(c) Shrivelled up (d) Skived off
Ans. (c)
Sol. 'Shrivel up' means to become dry and wrinkled as a result of exposure to heat, cold, etc. or becoming old.

97. Fill in the blank from among the choices in the sentence.
A 'Couch potato' is a person who ____ **[2015]**
(a) spends a lot of time watching television
(b) spends money on potatoes
(c) likes potatoes
(d) is lazy, but intelligent
Ans. (a)
Sol. 'A couch potato' means a person who spends a lot of time in sitting and watching television.

98. Which of the following sentence is grammatically incorrect? **[2015]**
(a) She never travelled abroad for fear of becoming ill through eating foreign food
(b) She avoids foreign travel as she fears she will become ill through eating foreign food
(c) She never travelled abroad due to her fear of becoming ill through eating foreign food
(d) She never travelled abroad in fear for becoming ill with eating foreign food
Ans. (d)
Sol. Here, the use of prepositions 'in' and 'for' are not correct.

99. Match the most suitable pharsal Verb from Group L to each word in Group M. **[2015]**

	Group L		Group M
1.	Call out	P.	A Footballer
2.	Stands in for	Q.	A Criminal
3.	Send down	R.	A Colleague
4.	Send off	S.	A Doctor

(a) $3-R, 2-S, 1-P, 4-Q$ (b) 1-S, 2-R, 3-Q, 4-P
(c) $1-P, 2-Q, 3-R, 4-S$ (d) 2-P, 3-S, 4-R, 1-Q
Ans. (b)
Sol. Explanation :
1. 'call out' means to summon to deal with an emergency or a special work at an unusual time. Hence, 'to call out a doctor' is correct.
2. 'Stands in for' means 'acts as a substitute of'. Hence, 'Stands in for a colleague' i.e. 'proxy a colleague' is correct.
3. 'Send down' means to dispatch somebody to some place or a lower level.
Hence, 'Send down a criminal' seems appropriate.
4. 'Send off a footballer' is also appropriate.
Hence, option ' b ' is the best answer.

100. Choose the one which best expresses the following sentence in passive/active voice.
"You can play with these kittens quite safely" **[2015]**
(a) These kittens can be played with quite safely
(b) These kittens can play with you quite safely
(c) These kittens can be played with you quite safely
(d) These kittens can played with quite safely
Ans. (b)
Sol. The passive voice of 'can/may/might+first form of verb' is done as 'can/may/might+be+third form of the verb'.
Hence, the passive voice of the given sentence will be "These kittens can be played with safely (by you)."

101. Which of the following terms refers to the original inhabitants of a place? **[2015]**
(a) Originals (b) Aborigines
(c) Abominables (d) Cannibals
Ans. (b)
Sol. 'Aborigines' means the individuals belonging to or originating from the indigenous or the oldest known population or a place.
Hence, ' b ' is the correct option.

102. Replace the underlined word with one of the choice given without changing the meaning of the sentence. "The news of our success was met with exuberant cries" **[2015]**
(a) Excited (b) Pathetic
(c) Exclusive (d) Poignant
Ans. (a)
Sol. 'Exuberant' means 'full of unrestrained energy or joy'.
'Excited' and 'Poignant' have the similar meanings. But 'Poignant' is usually meant for deep emotions of pity or sorrow.
Hence, ' a ' is the best option.

103. Select the word that is furthest is meaning to the word AFFLUENCE. **[2015]**
(a) Stagnation (b) Misery
(c) Neglect (d) Poverty
Ans. (b)
Sol. 'Affluence' means 'richness'.
Its opposite words are 'poverty' and 'misery'. But the latter is the extreme form of the former.
Hence, ' b ' is the best option.

Directions [Q. Nos. 104-106]
The proud warrior class of the samurai (meaning "those who serve") grew from a band of mercenaries hired by feudal landowners in the 11th century to win them the
control of Honshu, Japan's main island. These mercenaries lived by the cult of the sword, worshipping athletic prowess and martial skills. They developed a fierce loyalty to their masters and a fearlessness that made them formidable adversaries. They fought in elaborate armour, wielding their most prized possession, a double-edged sabre with which they could cut a man in half.
Later the spartan principles of Zen Buddhism, with its love of nature softened their fighting zeal. It became fashionable for them to live sparce and frugal lives during the Kamakura era (1192-1333), when the ruling warrior family Minamato moved their seat to power of the eastern city of Kamakura.
104. Who are usually referred to as mercenaries? **[2015]**
(a) Soldiers with martial skills
(b) Proud warriors
(c) Soldiers who fight for money
(d) Loyal warriors
Ans. (a)
Sol. It is clear from the sentence 'These mercenaries lived by the cult of the sword, worshipping athletic prowess and martial skills'.

105. Which of the following best describes the warriors? **[2015]**
(a) Proud, greedy (b) Fearless, worshipful
(c) Loyal, fearless (d) Possessive, soft
Ans. (c)
Sol. It is clear from the passage that the soldiers were loyal and fearless.

106. In the kamakura period it became fashionable for these warriors to live **[2015]**
(a) Zealour lives (b) Austere lives
(c) Powerful lives (d) Natural lives
Ans. (b)
Sol. It is clear from the sentence, 'It became fashionable for them to live sparce and frugal lives.
____

107. Rearrange the parts of a sentence referred to by P, Q, R and S to form a complete and meaningful sentence "I enclose ____ ". **[2015]**
P : and the postage Q : a postal order
R : the price of books S : which will cover
(a) $RPSQ$ (b) Q S P R
(c) Q S R P (d) Q P S R
Ans. (c)
Sol. The correct sentence will be "I enclose a postal order which will cover the price of books and the postage."

108. Which of the following is the antonym of the word 'Exigency'? **[2015]**
(a) Penchant (b) Emergency
(c) Earnestness (d) Indifference
Ans. (b)
Sol. The correct option is (b).

109. Which of the following prepositions fills up the blanks in the sentence? **[2015]**
"Quinine is an effective antidote ____ Malaria".
(a) to (b) against (c) for (d) None of these
Ans. (a)
Sol. The preposition 'to' is always used with the word 'antidote'.

110. In the sentence, "The defence labs have showcased many new innovations this year ____
there is an error of **[2015]**
(a) redundancy (b) word order
(c) collocation (d) omission
Ans. (d)
Sol. The correct spelling is 'innovations'.

Computer Awareness

111. When the value 37 H is divided by 17 H , the remainder is **[2015]**
(a) COH (b) 03 H (c) 07 H (d) 09 H
Ans. (d)
Sol. 37 H means $(00110111)_2$
17 H means $(00010111)_2$
Now, we convert $37H, 17H$ binary number into decimal number.
$(00110111)_2$ decimal number $= (55)_{10}$

$(00010111)_2$ decimal number $=(23)_{10}$

$(55)_{10} \div (23)_{10} = 23\overline{)55}(2 = (9)_{10}$

46 or

$\overline{9}$ $(1001)_2$

$(1001)_2$ Hexadecimal $=09$, remainder $=09$H On division of 37 H by 17 H the remainder is 09 H .

112. The number of Boolean function possible with n binary variables is equal to **[2015]**

(a) 2^{2^n} (b) 2^n (c) $2^{2^{n+1}}$ (d) 2^{n+1}

Ans. (b)

Sol. If there are p possibilities for choice 1 and q possibilities for choice 2 , then there are a total of $p^* q$ different way of doing both. It is trivial that this can be extended to n choices.

So, there would be 2^n boolean functions.

113. Consider 4 -bit gray code representation of numbers. Let $h_3h_2h_1h_0$ be the gray code representation of a number n and $g_3g_2g_1g_0$ be the gray code representation of the number $(n+1)$ modulo 15 . Which one of the following functions is correct? **[2015]**

(a) $g_0(h_3h_2h_1h_0) = \Sigma(1,2,3,6,10,13,14,15)$

(b) $g_1(h_3h_2h_1h_0) = \Sigma(4,9,10,11,12,13,14,15)$

(c) $g_2(h_3h_2h_1h_0) = \Sigma(2,4,5,6,7,12,13,15)$

(d) $g_3(h_3h_2h_1h_0) = \Sigma(0,1,6,7,10,11,12,13)$

Ans. (b)

Sol.

Binary	h	$h_3h_2h_1h_0$	$(n+1)\text{mod}16$	$g_3g_2g_1g_0$
0000	0	0000	1	0001
0001	1	0001	2	0011
0010	2	0011	3	0010
0011	3	0010	4	0110
0100	4	0110	5	0111
0101	5	0111	6	0101
0110	6	0101	7	0100
0111	7	0100	8	1100
1000	8	1100	9	1101
1001	9	1101	10	1111
1010	10	1111	11	1110
1011	11	1110	12	1010
1100	12	1010	13	1011
1101	13	1011	14	1001
1110	14	1001	15	1000
1111	15	1000	0	0000

This gives the solution option (b).

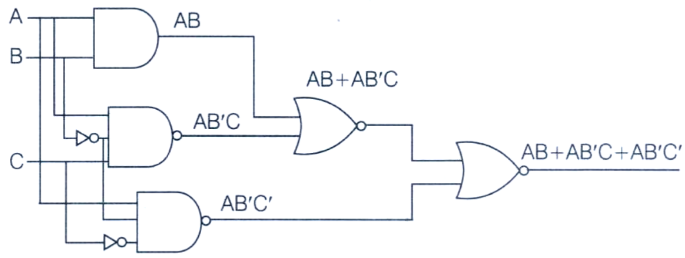
$g_1 \cdot (h_3h_2h_1h_0) = \Sigma(4,9,10,11,12,13,14,15)$

114. The minimum number NAND gates required to realize $AB + AB'C + AB'C'$ is **[2015]**

(a) 3 (b) 2 (c) 1 (d) 0

Ans. (b)

Sol.



As, we can see in Logic Gates,

There are two NAND required.

115. Which optical phenomenon is utilized in the operation of the latest write-once optical storage medium called digital paper? **[2015]**

(a) Polarisation (b) Interference

(c) Internal reflection (d) Diffraction

Ans. (d)

Sol. The diffraction beam can be machine read by a reading device which produces a reading light beam directed at the grating. The reading device has a plurality of detectors positioned to be illuminated by the diffracted light beam. Information can be stored in the optical memory by locally changing the optional properties of the grating.

116. P is a 16 -bit signed integer. The 2 's complement representation of P is $(\text{F87 B})_{18}$. The 2's complement representation of 8 P is **[2015]**

(a) $(\text{C3D8})_{36}$ (b) $(187B)_{16}$

(c) $(187B)_{16}$ (d) $(987B)_{16}$

Ans. (a)

Sol.

$P = (\text{F87B})_{16} = 1111100001111011$

2's complement $=$ 1111100001111011

$+1$

$\overline{0000011110000101}$

$(0000\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 1)_2$

$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$

1024 512 256 128 64 32 16 8 4 2 1

Value in decimal is $(1925)_{10}$, Hence, $P=-1925$

So, now doing $8^* P = (-15400)_{10}$

Since, a negative number we have to find the 2's complement of -15400 .

Binary of $15400 = 0011110000101000$

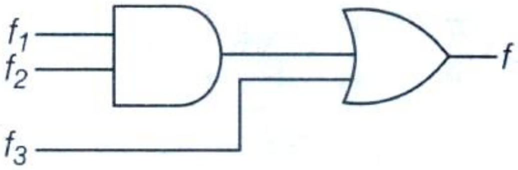
2's complement

= 1100 0011 1101 1000

C 3 D 8

$= (C3D8)_{16}$

117. Given, f_1, f_3 and f in canonical sum of products form (in decimal) from the circuit **[2015]**



$f_1 = \Sigma m(4,5,6,7,8), f_3 = \Sigma m(1,6,15)$

and $f = \Sigma m(1,6,8,15)$, then f_2 is

(a) $\Sigma(4,6)$ (b) $\Sigma(4,8)$ (c) $\Sigma(6,8)$ (d) $\Sigma(4,6,8)$

Ans. (c)

Sol.

From the diagram

$f \quad = f_1 f_2 + f_3$ (put value)

$= (4,5,6,7,8) f_2 + (1,6,15) = (1,6,8,15)$

Now, as you know that the number $0,1,2,\ldots 14,15$ represent the minterms like $\Sigma_m(6,8)$.

(i) If you AND$(4,6)$ with $(4,5,6,7,8)$ it will result in $(4,6)$ but we don't need 4 , so option (a) is wrong.

(ii) If you AND (4,8) with $(4,5,6,7,8)$ it will result in (4,8) but we don't need 4 , so option (b) is wrong.

(iii) If you AND$(4,6,8)$ with $(4,5,6,7,8)$ it will result in $(4,6,8)$ but we don't need 4 here too, so option (d) is wrong.

(iv) If you AND (6, 8) with (4, 5, 6, 7, 8) it will result in (6, 8) now when OR (6,8) with (1,6,15), you get (1,6,8,15) which is f , so option (c) is correct

118. Which of the following is equivalent to the expression $\left(\overline{\overline{X+Y}+Z}\right)$? **[2015]**

(a) $\left(\bar{X}+\bar{Y}\right)Z$ (b) $(X+Y)\bar{Z}$

(c) $\left(\bar{X}+\bar{Y}\right)\bar{Z}$ (d) $(X+Y)Z$

Ans. (d)

Sol. Given, expression $\left(\overline{\overline{x+y}+\bar{z}}\right)$

As, we know De-morgan's law says

So, let

$\overline{a+b} = \bar{a}\cdot\bar{b}$

So, let

$x+y=a$

$z=b$

$\Rightarrow \left(\overline{\overline{x+y}+\bar{z}}\right) = \overline{\overline{x+y}}\cdot\bar{\bar{z}}$

$\downarrow\downarrow \quad \downarrow \quad \downarrow$

$\overline{a+b} \quad \bar{a}\cdot\bar{b}$

$\Rightarrow \overline{\overline{x+y}}\cdot\bar{\bar{z}} = (x+y)\cdot z$

Hence, $\overline{\overline{x+y}+\bar{z}} = (x+y)\cdot z$

119. $\{p \to q \vee r, q \to s, r \to s\}$ is logically equivalent to **[2015]**

(a) $q \to r$ (b) $r \to q$

(c) $p \to s$ (d) $s \to p$

Ans. (c)

Sol. Since, $q \to s$ and $r \to s$, thus $(q \vee r) \to s, p \to (q \vee r)$ is given, then by rule of 'Syllogism'.

$p \to s$ is deducted.

120. The minimum number of MOS transistors required to make a dynamic RAM cell is **[2015]**

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

Sol. Dynamic random access memories (DRAMs) are semiconductor integrated circuits (ICs) that operate like a bank of capacitors. DRAM consist atleast 1 MOS transistor necessarily.