NIMCET-2017 SOLVED PAPER

General English

Directions (Q. Nos. 1 to 5) Read the following passage and answer the questions.

Anthoropologists have pieced together the little they know about the history of left handedness and right handedness from indirect evidence. Through early men and women did not leave written records, they did leave tools, bones, and pictures. Stone Age hand axes and hatchets were made from stones that were carefully chipped away to form sharp cutting edges. In some the pattern of chipping shows that these tools and weapons were made by right handed people designed to fit comfortable into a right hand. Other Stone Age implements were made by or for left handers prehistoric pictures painted on the walls of caves provide further clues to the handedness of ancient people.

A right hander finds it easier to draw faces of people and animals facing toward the left whereas, a left hander finds it easier to draw faces facing toward the right. Both kinds of faces have been found in ancient painting.

On the whole the evidence seems to indicate the prehistoric people were either ambidextrous or about equally likely to be left or right handed. But in the Bronze Age, the picture changed. the tools and weapons found from that period are mostly made for right handed use. The predominance of right handedness among humans today had apparently already been established.

1. What is the indirect evidence through which the preferred handedness of the Stone Age people could be understood? **[2017]**

(a) Petrified forms of vegetation
(b) Patterns of stone chipping
(c) Fossilised waste material
(d) Fossilised footprints

Ans. (b)

Sol. 'Patterns of stone chipping' is correct. As it is clearly described in lines 5 and 6 .

2. According to the passage, a person who is right handed is more likely to draw people and animals that are facing **[2017]**

(a) upward (b) downward
(c) towards the right (d) towards the left

Ans. (d)

Sol. 'Toward the left' is correct.
As it is mentioned in lines 9 and 10 of the passage.

3. What does the world 'the picture' refer to which of the following? **[2017]**

(a) Faces of animals and people
(b) People's view from inside a cave
(c) People's tendency to work with either hand
(d) The kinds of paint used on cave walls

Ans. (c)

Sol. 'People's tendency to work with either hand' is correct.
As it is mentioned in the passage :
That in the Bronze Age, the picture-changed i.e., right handedness became prominent.

4. The author implies that which of the following developments occurred around the time of the Bronze Age. **[2017]**

(a) The establishment of written records
(b) A change in the styles of cave painting
(c) An increase in human skill in the handling of tools
(d) The prevalence of right-handedness

Ans. (d)

Sol. 'The prevalence of right handedness' is correct. As it is mentioned in the last three lines of the passage.

5. What is the main idea conveyed through the passage? **[2017]**

(a) The purpose of ancient implements
(b) The significance of prehistoric cave paintings
(c) The development of right handedness and left handedness
(d) The pattern of chipping ancient tools

Ans. (c)

Sol. 'The development of right handedness and left handedness' is correct. As the passage, has mainly focussed on pattern of writing styles using hands.

6. Which of the following refers to the idiom 'under the Sun'? **[2017]**

(a) Anything and everything
(b) A large number of things
(c) A few things
(d) Something

Ans. (a)

Sol. 'Anything and everything' is correct.

As the idiom 'under the Sun' means all that is on and around the Earth. e.g. Google seems to know each and everything under the Sun.

7. Choose a phrasal verb to replace the explanation in brackets: When we arrive at the station, we (descend from) _____ the train. **[2017]**

(a) Get down (b) Stand down
(c) Get off (d) Stand out

Ans. (a)
Sol. 'Get down' is correct.
As 'get down' means to descend from.
e.g. She got down the Metro and hired an auto rickshaw for home.

8. Choose the suitable word from the following and fill in the blank:
The medal was awarded for the student's _____ conduct and courage. **[2017]**

(a) non-receptive (b) exemplary
(c) unreliable (d) disputable

Ans. (b)
Sol. 'Exemplary' is correct.
As 'exemplary' means 'so good that it might serve as a model for others, so exemplary conduct and courage, makes a meaningful phrase.

9. Which of the following is a correctly spelt word? **[2017]**

(a) Hiderence (b) Hindrence
(c) Hindarrence (d) Hindrance

Ans. (d)
Sol. 'Hindrance' is correct.
As 'Hindrance' means barrier, obstacle or deterrent while the rest with the given spellings are meaningless.

10. Which one of the following statements is grammatically correct? **[2017]**

(a) The Earth revolves round the Sun
(b) I have not seen him since four years
(c) She met an one-eyed man
(d) One of the books borrowed by the Students are famous

Ans. (a)
Sol. 'The Earth revolves round the Sun' is correct.
As this sentence has no grammatical error while in sentence (b) 'since' will be replaced by 'for'. In sentence (c) 'an' will be replaced by 'a' and in sentence (d) 'are' will be replaced by 'is' to make them grammatically correct.

11. Choose the set words from among the alternatives given. Which when inserted in the sentence best suit the meaning of the sentence? **[2017]**
The _____ of evidence was on the side of plaintif since all but one witness testify his story was _____

(a) paucity, accurate
(b) prosperity, far fetched
(c) prepondenerance, correct
(d) accuracy, insufficient

Ans. (c)
Sol. 'Preponderance, correct' is correct.
As 'preponderance' means 'prevalence, dominance or advantage' which supports evidence duly and suitably 'Correct' means 'right'. So, the given set of words forms a meaningfully correct sentence i.e.
The preponderance of evidence was on the side of planning since all but one witness testified his story was correct.

12. Choose the one which is nearest in meaning to the word 'TURN UP'? **[2017]**

(a) Show up (b) Come up
(c) Land up (d) Crop up

Ans. (a)
Sol. 'Show up' is correct.
As both 'turn up' and 'show up' mean 'appear, attend, come in or arrive'.

13. The phrase 'ready to believe' means **[2017]**

(a) Credulous (b) Creditable
(c) Credible (d) Ineredible

Ans. (a)
Sol. 'Credulous' is correct.
As 'Credulous' means 'gullible, overtrusting, unsuspicious or spontaneous'.
All of these are similar to 'ready to believe' in meaning.

14. Choose the appropriate word from among the choice to fill in the blank in sentence; "If you drink to much, it will _____ your judgement." **[2017]**

(a) impair (b) impede
(c) impose (d) impel

Ans. (a)
Sol. 'Impair' is correct.
As 'impair' means 'damage, destroy, affect, devalue or harm. So, it makes the sentence meaningfully correct i.e..
If you drink too much, it will impair your judgement.

15. Choose the set of words for each blank that best fits the meaning of the following sentence as a whole:
_____ green and black tea are obtained from the same plant, there are quite a few significant difference _____ them. **[2017]**

(a) Since, among (b) However, in
(c) Though, between (d) Because, across

Ans. (c)
Sol. 'Though, between' is correct.

As 'Though symbolises contrast despite similarity and 'between' is used to connect to things. So, 'Though green and black tea .... differences between them' makes a meaningful sentence.

16. Choose the correct alternative which can be substituted for the given word/sentence.
A person who travels to a sacred place as an act of religious devoation. **[2017]**
(a) Hermit (b) Pilgrim
(c) Saint (d) Medicant
Ans. (b)
Sol. 'Pilgrim' is correct.
As 'Pilgrim' means 'a religious traveller'.

17. Pick out the most effective word from the given words to fill in the blank to make the sentence meaningfully complete:
Some people ____ themselves into believing that they are indispensable to the organisation they work for. **[2017]**
(a) keep (b) fool
(c) delude (d) denigrate
Ans. (c)
Sol. 'Delude' is correct.
As 'delude' means 'deceive, fool, beguile one's ownself to live in unreal world or world of one's own fancies.

18. Fill in the blanks with appropriate phrase to make the sentence meaningfully complete.
_____ bad weather, the trip will be postponed to next week. **[2017]**
(a) In case (b) In case of
(c) In case to (d) In case from
Ans. (b)
Sol. 'In case of' is correct.
As 'In case of' means 'in the event of.
So, 'In case of bad weather, the trip will be postponed to next week' is a meaningful sentence.

19. In the following sentence, choose the most suitable one word for the expression: "A book containing summarised information on all branches of knowledge". **[2017]**
(a) Dictionary (b) Anthology
(c) Encyclopedia (d) Directory
Ans. (c)
Sol. 'Encyclopedia' is correct.
As 'Encyclopedia' means 'a book of facts'.
So, it matches the given information in the question.

20. Pick out the most effective word from the given words to fill in the blanks to make the sentence meaningful completely:
The man was about to move his bike into the compound of his apartment when a passes by _____ down the motor cycle. **[2017]**
(a) forced (b) fell
(c) turned (d) knocked
Ans. (d)
Sol. 'Knocked' is correct.
As 'knock' means 'to strike' hit or push. So, "The man was about to move his bike into the compound of his apartment when a passes by knocked down the motorcycle.' is correct.

Computer Awareness

21. Which one of the following boolean algebraic rule is correct? **[2017]**
(a) $A \cdot A' = 1$ (b) $A + AB = A + B$
(c) $A + A'B = A + B$ (d) $A(A+B) = B$
Ans. (c)
Sol. $A \cdot A' = 0$
So, option (a) is wrong
$A + AB$
$A(1+B) = A$
So, option (b) is wrong.
$A + A'B$
$(A+A')(A+B)$
$1(A+B) = A+B$
So, option (c) is right.
$A(A+B)$
$A \cdot A + B = A + B$
So, option (d) is wrong.

22. The representation of a floating point binary number +1001.11 in 8-bit fraction and 6 -bit exponent format is **[2017]**
(a) Fraction : 01001110 exponent: 000100
(b) Fraction : 00001001 exponent : 000011
(c) Fraction : 10010000 exponent : 110000
(d) Fraction : 00100100 exponent : 011000
Ans. (a)
Sol. The binary number +1001.11 is represented with an 8-bit fraction and 6 -bit exponent as follows :

Fraction	Exponent
01001110	000100

The fraction has a 0 in the left most position to denote positive. The binary point of the fraction follows the sign bit but is not shown in the register. The exponent has the equivalent binary number +4 . The floating point number is equivalent to

$$m \times 2^e = +(.1001110)_2 \times 2^{+4}$$

23. Which term is redundant in the expression $AB + A'C + BC$? **[2017]**

(a) BC (b) $A'C$ (c) AB (d) None of these

Ans. (d)

Sol. $AB + A'C + BC = AB + A'C + BC(A + A')$

[by Complementary law]

$= AB + A'C + ABC + A'BC$

$= AB + ABC + A'C + A'BC$

[by Associative law]

$= AB(1+C) + A'C(1+B)$[by Absorption law]

$= AB + A'C$

24. Let the memory access time is 10 milliseconds and cache access time is 10 microseconds. Assume the cache hit ratio 15%. The effective memory access time is **[2017]**

(a) 2 milliseconds (b) 1.5 milliseconds

(c) 1.85 microseconds (d) 1.85 milliseconds

Ans. (*)

Sol. Hit Ratio $= 15\%$

Cache memory time $= 10$ microseconds

Memory access time = 10 milliseconds

Effective Memory Access Time (EAT) = Hit Ratio (Cache memory time + Memory access time) + (sign hit) * (Cache memory time $+2^*$ Memory access time)

$\text{EAT} = 0.15 \quad (0.01+10) + (1-0.15)(0.01+2\times10)$

$= (0.15)(10.01) + (0.85)(20.01)$

$= 1.5015 + 17.0085 = 18.51$ milliseconds

None option is correct.

25. Which of the following is the representation of decimal number (-147) in 2's complement notation on a 12 bit machine? **[2017]**

(a) 111101101100 (b) 110001001101

(c) 111101101101 (d) 000001101101

Ans. (c)

Sol. $(147)_{10}$ convert into $(?)_2$

2	147	
2	73	1
2	36	1
2	18	0
2	9	0
2	4	1
2	2	0
	1	0

$(147)_{10} = (10010011)_2$

First bit (the left most) indicates the sign,

$1 =$ negative, $0 =$ positive

Positive binary computer representation on 12 -bit if needed, add extra 0_s in front of the base 2 number :

So, $(147)_{10} = 000010010011$

For 1's complement, replace all the bits on 0 with 1 s and all the bits set on 1 with 0 s .

1 's complement of

$(-147) = 111101101100$

For 2's complement, add 1 to the number of 1's complement.

2's complement of (-147)

$$\begin{array}{r} 111101101100 \\ +1 \\ \hline 111101101101 \end{array}$$

26. The first instruction of bootstrap loader program of an operating system is stored in **[2017]**

(a) RAM (b) Hard disk

(c) BIOS (d) None of these

Ans. (a)

Sol. RAM

27. Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is **[2017]**

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c)

Sol. Changing (123) base 5 into base 10 .

$= 1\times5^2 + 2\times5^1 + 3\times5^0$

$= 1\times25 + 2\times5 + 3\times1^0$

$= 25 + 10 + 3 = 38 \quad$ ...(i)

Changing $x8$ base y in decimal

$= x^*y^1 + 8^*y^0$

$= xy + 8 \quad$ (ii)

From Eqs. (i) and (ii), we get

$xy + 8 = 38$

$\Rightarrow \quad xy = 30$

So, possible combinations $= (1,30),(2,15),(3,10)$

but we have 8 present in $x8$, so base $y > 8$

Therefore, total soltuion $= 3$

28. The smallest integer that can be represented by an 8- bit number in 2 's complement form is **[2017]**

(a) -256 (b) -128 (c) -127 (d) -255

Ans. (b)

Sol. The range of numbers that can be represented by n -bits in 2's complement form is $(-2)^{n-1}$ to $(2^{n-1})-1$

Hence, the smallest number is $(-2)^{n-1}$

$= (-2)^{8-1} = (-2)^7 = -128$

29. Which of the following in a functionally complete set of gates? **[2017]**

I. NAND II. NOR

(a) I but not II (b) II but not I

(c) Neither I nor II (d) Both I and II

Ans. (d)

Sol. A functionally complete set of Boolean function consists of a set of Boolean functions from which you can construct all Boolean functions. {AND, NOT}, {OR, NOT}, {NAND}, {NOR} are four functionally complete sets.

30. The total number of binary function that can be defined using n boolean variables is **[2017]**

(a) 2^{n-1} (b) 2^n (c) 2^{n+1} (d) None of these

Ans. (d)

Sol. For n-Boolean variables, there are 2^n possible boolean inputs. Each input can generate either True or False as the output. Hence, total number of different boolean function $=2^{2^n}$.

Analytical Ability and Logical Reasoning

31. Two persons S and M have made the following statements among themselves.

- S says that I am certainly not over 40 years.
- M says that I am 38 years and you are at least 5 year old than me.
- S says you are at least 39 years.

If all the above statements are wrong, what are the ages of M and S ? **[2017]**

(a) 36 and 40 (b) 36 and 41

(c) 37 and 40 (d) Cannot be determined

Ans. (d)

Sol. If all the given statement are false, then the information will be as follows

(i) $\mathrm{S}>40$ years

(ii) S is $1,2,3,4$ or 5 years older than M and $\mathrm{M}\neq 38$ years

(iii) $\mathrm{M}\le 39$ years

According to the information and from options, Ms age can be of 36 or 37 or 39 years.

And S's age can be of 40 or 41 or 42 or 43 or 44 years.

Hence, exact age of M and S cannot be determined.

32. What is the largest number of positive integers to be picked up randomly so that the sum of difference of any two of the chosen numbers is divisible by 10 ? **[2017]**

(a) 2 (b) 5 (c) 7 (d) 10

Ans. (b)

Sol. Let largest number is P and any two choosen numbers x and y.

According to the question $P+x-y$ is divisible by 10 .

So,

P is largest then $\begin{array}{l} P+x-y\ge 10....(i) \\ x-y\le P...(ii) \end{array}$

These inequalities can be written in equation form

$$P+x-y=10....(iii)$$

and $x-y=P.....(iv)$

On subtracting eq. (iv) by (iii), we get

$$P+x-y-x+y=10-P$$

$$\Rightarrow P=10-P$$

$$\Rightarrow 2P=10$$

$$\Rightarrow P=\frac{10}{2}=5$$

33. Five children were administered psychological tests to know their intellectual levels. In the report psychologists pointed that child A is less intelligent than child B . The child C is less intelligent than child D. The child B is less intelligent than child C and child A is less intelligent than child E . Which child is most intelligent? **[2017]**

(a) D only (b) E only

(c) D or E (d) Neither D nor E

Ans. (c)

Sol. According to the given information,

$$\mathrm{A<B, C<D,\ B<C \text{ and } A<E}$$

$$\therefore \quad \mathrm{A<B<C<D \text{ and } A<E}$$

$\because$ E's position cann't be determined.

Hence, most intelligent child is either D or E.

34. From a group of 7 men and 6 women, a committee of 5 persons with more males than females is to be formed. In how many ways can this be done? **[2017]**

(a) 564 (b) 645 (c) 735 (d) 756

Ans. (d)

Sol. There are three number of ways by which we can form the committee.

(i) 3 Males and 2 Females

(ii) 4 Males and 1 Female

(iii) 5 Males + 0 Female

Hence, total number of ways

$$={}^7C_3\,{}^6C_2+{}^7C_4\,{}^6C_1+{}^7C_5\,{}^6C_0$$

$$=\frac{7\times6\times5}{3\times2\times1}\times\frac{6\times5}{2\times1}+\frac{7\times6\times5}{3\times2\times1}\times\frac{6}{1}+\frac{7\times6}{2\times1}\times1$$

$$=525+210+21=756$$

Hence, this can be done by 756 ways.

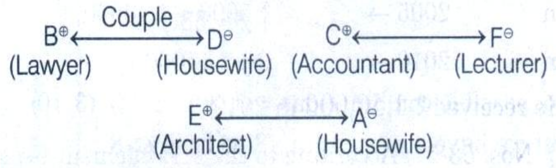
35. A, B, C, D, E and F are 6 friends from a club. There are two houses wives, one lecturer, one architect, one accountant and one lawyer in the group. There are three married couples. The lawyer B is married to D, who is a houses wife. No lady is either an architect or an accountant. C , the accountant is married to F , who is lecturer. If E is not a house wife, what is the profession of E ? **[2017]**

(a) Lawyer (b) Architect

(c) Lecturer (d) Accountant

Ans. (b)

Sol. According to the information, arrangement is as follows



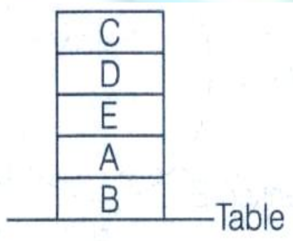
E is an architect.

36. There are five books A, B, C, D and E placed on a table. If A is placed below E, C is placed above D, B is placed below A and D is placed above E , then which of the following books touches the surface of the table? **[2017]**

(a) C (b) B (c) A (d) E

Ans. (b)

Sol. According to the information, the arrangement is as follows



Hence, book B touches the surface of the table.

37. The following series is obtained by considering representations of decimal 99 in different number system. The next two numbers in the sequence are

99, 90, 83, 78 _____ , _____ **[2017]**

(a) 71,69 (b) 69,57

(c) 67,59 (d) 69,63

Ans. (a)

Sol. Decimal 99 is represented in different number system in the series.

99	90	83	78	$\boxed{71}$	$\boxed{69}$
Base = 10	Base = 11	Base = 12	Base = 13	Base = 14	Base =15
$=9\times10+9$	$=9\times11+0$	$=8\times12+3$	$=7\times13+8$	$=7\times14+1$	$=6\times15+9$
$=(99)_{10}$	$=(99)_{10}$	$=(99)_{10}$	$=(99)_{10}$	$=(99)_{10}$	$=(99)_{10}$

Hence, $(71, 69)$ is the correct answer.

Directions (Q. Nos. 38 to 40) Questions are based on the following

- In a family of six person A, B, C, D, E and F , there are two married couples.
- D is the grandmother of A and mother of B
- C is wife of B and mother of F

F is granddaughter of E

38. Who is C to A ? **[2017]**

(a) Daughter (b) Mother

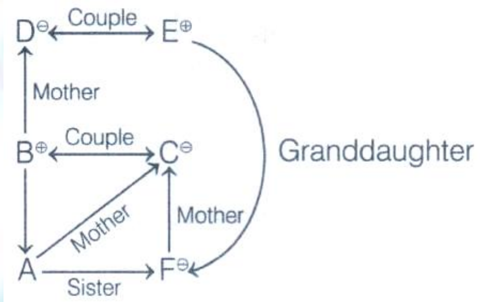
(c) Father (d) Cannot be determined

Ans. (b)

Sol. C is the mother of A .

(Q. Nos. 38-40)

According to the information, arrangement is as follows



39. Which of the following is true? **[2017]**

(a) A is brother of F (b) A is sister of F

(c) B has two daughters (d) None of these

Ans. (d)

Sol. As gender of A is not known, therefore none of the option is correct.

40. Who among the following is one of the couples? **[2017]**

(a) CD (b) DE (c) EB (d) None of these

Ans. (b)

Sol. DE and BC are the couples in the family.

41. The missing number in the following series 336,210 , 120, 60, ____ 6 is **[2017]**

(a) 24 (b) 30 (c) 34 (d) 40

Ans. (a)

Sol. The pattern of the series is as follows

336	210	120	60	$\boxed{24}$	6
↓	↓	↓	↓	↓	↓
$=7^3-7$	$=6^3-6$	$=5^3-5$	$=4^3-4$	$=3^3-3$	$=2^3-2$

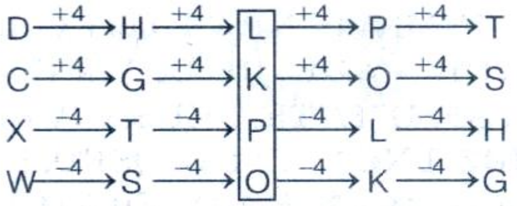
42. If the day after the day after tomorrow is three days before Friday, then today is **[2017]**

(a) Tuesday (b) Thursday

(c) Saturday (d) Monday

Ans. (c)

Sol. Three day before Friday is Tuesday.
$\therefore$ The day after the day after tomorrow = Tuesday
$\therefore$ The day after the day tomorrow = Monday
$\therefore$ The day on tomorrow = Sunday
$\therefore$ Today = Saturday

43. Find the missing term of the following series
DCXW, HGTS, ____ POLK, TSHG **[2017]**
(a) KLOP (b) LKOP
(c) KLPO (d) LKPO
Ans. (d)
Sol.


$\therefore$ The missing term = LKPO

44. Four passengers in a train find that they form an interesting group. Two of them are lawyers and the other two are doctors. Two of them speak Bengali and the other two speak Hindi and no two of the same profession speak the same language. They also found that two of them are Christians and two are Muslims and no two of the same religion speak the same languages. The Hindi speaking doctor is a Christian. Then, which of the following statement logically follows? **[2017]**
(a) The Bengali speaking lawyer is a Muslim
(b) The Christian lawyers speaks Bengali
(c) The Bengali speaking doctor is a Christian
(d) The Bengali speaking doctor is a Muslim
Ans. (b)
Sol. According to the information, arrangement is as follows

Profession	Language	Religion
Lawyer	Bengali	Christian/Muslim
Lawyer	Hindi	Muslim
Doctor	Bengali	Muslim/Christian
Doctor	Hindi	Christian
Profession	Language	Religion

$\because$ Lawyer is either Christian or Muslim, he will definitely speaks Bengali.
Hence, option (b) is the correct answer.

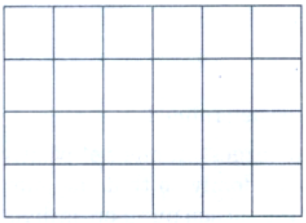
Directions (Q. Nos. 45 to 47) Questions are based on the following
In an amusement park seven friends-Feroz, Gautam, Harish, Javed, Kumar, Laxman and Mohan are deciding who will ride the roller coaster. There is time for only one ride before the park closes.
- If Feroz rides Gautam must ride.
- If Gautam and Harish both ride, Javed cannot ride.
- If Harish and Javed both ride, Laxman cannot ride.
- If Javed rides, either Kumar or Mohan must ride.
- Kumar and Laxman cannot both ride, but one of them must ride.
- Kumar and Mohan cannot both ride.

45. Which of the following is an acceptable combination of riders if only three people ride? **[2017]**
(a) Harish, Javed and Laxman
(b) Harish, Javed and Kumar
(c) Feroz, Gautam and Javed
(d) Gautam, Kumar and Laxman
Ans. (b)
Sol. From all the conditions given in the questions,
Option (a) is not possible as Harish, Javed and Laxman cannot ride together given in condition (iii).
Option (c) is not possible as in condition (v), in Kumar or Laxman one person is compulsory. No one is given in option (c).
Option (d) is not possible as Kumar and Laxman cannot ride together.
Hence, option (b) is correct which fulfills all the conditions.

46. If Javed and Mohan both ride, which of the following is true? **[2017]**
(a) Gautam cannot ride (b) Harish must ride
(c) Feroz cannot ride (d) Laxman must ride
Ans. (d)
Sol. If Javed rides then either Kumar or Mohan will definitely ride by condition (iv).
But, Javed and Mohan is already given. Thus, Kumar will not ride.
But in condition (v), one person from Kumar or Laxman will definitely ride. Hence, Laxman must ride.

47. If Feroz and Harish both ride, what is the greatest number of people who can ride? **[2017]**
(a) 5 (b) 7 (c) 4 (d) 6
Ans. (a)
Sol. If Feroz and Harish both ride, then Gautam must ride. By condition (ii), Javed will not ride.
By condition (vi), either Kumar or Mohan will ride and by condition (v), either Kumar or Laxman is compulsory.
If Mohan rides, then Laxman will ride.
And, if Kumar rides then Laxman cannot ride.
Hence, Feroz, Harish, Gautam, Mohan and Laxman can ride.

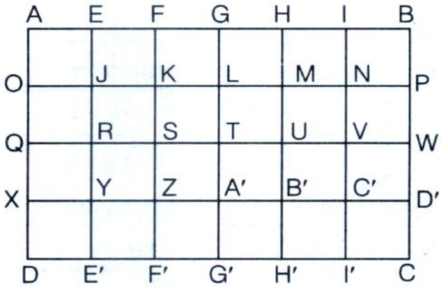
48. The number of squares in the following 4×6 grid is **[2017]**



(a) 36 (b) 44 (c) 51 (d) 54

Ans. (c)

Sol.

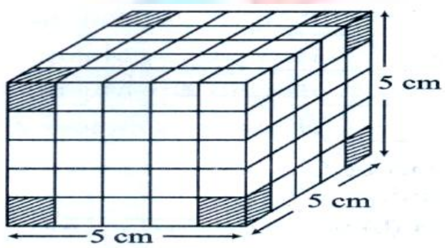


Total number of squares
= AEJO, EFKJ, FGLK, GHML, HINM, IBPN, JRQO, JKSR, KLTS, LMUT, MNVU, NPWV, QRYX, RSZY, STAZ, TUB'A', UVC'B', VWD'C', DEYX, YZF'E', ZAGF', AB'HG', B'C T'H', CD'CI', AFSQ, EGTR, FHUS, GIVT, HBWU, OKZX, JLAY, KMBZ, LNC'A', MPD'B', QSF'D, RTGE', SUHF', TVIG', UWCH', AGAX, EHBY, FICZ, GBD'A', OLGD, JMHE', KNI'F', LPCG', AHHD, EIIE', FBCF'.

49. A cube is made up of 125 , one cm , smaller cubes placed on a table. How many smaller cubes are visible only on three sides? **[2017]**
(a) 4 (b) 8 (c) 12 (d) 16

Ans. (a)

Sol. Four smaller cubes at the corners of the larger cube are visible only on three sides when cube placed on a table.



50. Three thieves rob a bakery of bread, one after the other. Each thief takes half of what is present and half bread. If 3 breads remain at the end, what is the number of breads that were present initially? **[2017]**
(a) 24 (b) 31 (c) 37 (d) 41

Ans. (b)

Sol. Let number of breads available be ' x '.
According to question,
Number of breads robbed by 1st thief,

$$=\frac{x}{2}+\frac{1}{2}=\frac{x+1}{2}$$

Now, number of breads left after 1st robbery,

$$=x-\frac{x+1}{2}=\frac{x-1}{2}$$

Again, number of breads robbed by 2 nd thief,

$$=\frac{1}{2}\left(\frac{x-1}{2}\right)+\frac{1}{2}$$

$$=\frac{x-1}{4}+\frac{1}{2}=\frac{x+1}{4}$$

So, number of bread left after 2 nd robbery,

$$=\left(\frac{x-1}{2}\right)-\left(\frac{x+1}{4}\right)$$

$$=\frac{2x-2-x-1}{4}=\frac{x-3}{4}$$

Now, number of breads robbed by 3rd thief,

$$=\frac{1}{2}\left(\frac{x-3}{4}\right)+\frac{1}{2}$$

$$=\frac{x-3}{8}+\frac{1}{2}=\frac{x+1}{8}$$

Hence, number of breads left after 3rd robbery,

$$=\left(\frac{x-3}{4}\right)-\left(\frac{x+1}{8}\right)$$

$$=\frac{x-7}{8}$$

Now,

$$\frac{x-7}{8}=3$$

$$x=31$$

$\Rightarrow$

Therefore, total number of breads present initially is 31

51. A caterpillar crawls up a pole of 75 inches high standing from the ground. Each day it crawls up 5 inches and each night it slides down 4 inches. When will it reach the top of the pole? **[2017]**
(a) At the end of 70 days (b) At the end of 71 days
(c) At the end of 72 days (d) At the end of 73 days

Ans. (b)

Sol. Caterpillar crawls upward (5-4), 1 inch in a day. Upto 70th day, caterpillar crawls upward 70 inches. On 71st day, caterpillar crawls upward 75 cm at the end of the day.

52. A man's investments doubles in every 5 years. If the invested ₹ 5,000 in each of the years 1990, 1995, 2000 and 2005 , then what was the total amount received by him in 2010? **[2017]**
(a) ₹ 140000 (b) ₹ 30000
(c) ₹ 70000 (d) ₹ 150000

Ans. (d)

Sol.

	Investment	Amount he get
In	1990 $\rightarrow$	₹ 5000

In	1995 $\rightarrow$	₹ 5000 + ₹ 10000
In	2000 $\rightarrow$	₹ 5000 + ₹ 30000
In	2005 $\rightarrow$	₹ 5000 + ₹ 70000
In	2010 $\rightarrow$	0 + ₹ 150000

He received ₹ 1,50,000 in 2010.

Directions (Q. Nos. 53 to 57) Questions are based on the following information

A, B, C, D, E, F, G and H are sitting around a circular table facing the centre. Each one of them has a different profession viz, doctor, engineer, architect, teacher, clerk, shopkeeper, banker and businessman.

- A sits third to right of teacher.
- D sits second of left of G.
- G is not an immediate neighbor of teacher.
- Only one person sit between B, the shopkeeper and the teacher.
- The one who is an architect sits third to right of the shopkeeper.
- H sits between architect and engineer.
- E is not an immediate neighbour of H.
- Engineer sits third to the right of clerk.
- Only one person sits between the businessman and F
- E is neither a businessman nor a doctor.

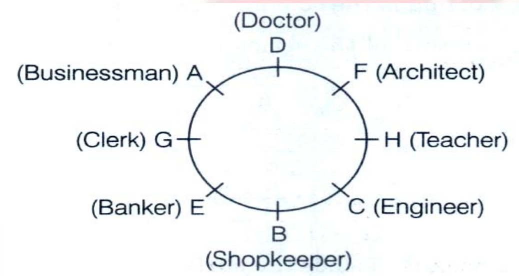
53. Who amongst the following is the clerk? **[2017]**

(a) C (b) D (c) E (d) G

Ans. (d)

Sol. G is the clerk.

(Q. Nos. 53-57) According to the arrangement is as follows



54. Which of the following is true with respect to the given sitting arrangement? **[2017]**

(a) **E** is an immediate neighbour of the engineer.

(b) E is an architect.

(c) The clerk is an immediate neighbour of the banker.

(d) The teacher sits between H and the engineer.

Ans. (c)

Sol. The clerk is an immediate neighbour of the banker, is true.

55. What is the profession of H ? **[2017]**

(a) Architect (b) Shopkeeper

(c) Banker (d) Teacher

Ans. (d)

Sol. The profession of H is teacher.

56. Who sits exactly between the architect and businessman? **[2017]**

(a) C and H (b) Clerk

(c) Banker and Shopkeeper (d) Doctor

Ans. (d)

Sol. Doctor sits between the architect and businessman.

57. Who sits immediately right of the businessman? **[2017]**

(a) Teacher (b) Doctor (c) Clerk (d) Banker

Ans. (c)

Sol. Clerk sits immediately right of the businessman.

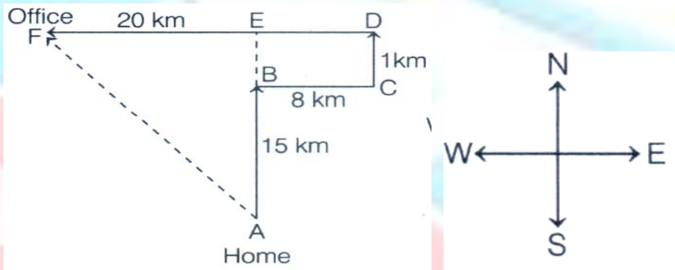
58. Raghav left his home for office in car. He drove 15 km straight towards North and then turned Eastwards and covered 8 km . He then turned to left and covered 1 km . He again turned left and drove for 20 km and reached office. How far and in which direction is his office from the home? **[2017]**

(a) 20 km North-West (b) 15 km North-West

(c) 30 km North-West (d) 25 km North

Ans. (a)

Sol.



$$AF = \sqrt{(FE)^2 + (AE)^2}$$

$$AF = \sqrt{(20-8)^2 + (15+1)^2}$$

$$\Rightarrow \quad = \sqrt{144 + 256}$$

$$= \sqrt{400}$$

$$= 20 \text{ km}$$

Raghav's office is 20 km away from his home and in North-West direction.

59. John is 20 years older than Steve. In 10 years, Steve's age will be half that of John. What is Steve's age now? **[2017]**

(a) 2 (b) 8 (c) 10 (d) 20

Ans. (c)

Sol. Let the age of Steve be 'S' and John be 'J'.

According to the question,

$$J = S + 20 \quad ....(i)$$

And, $$S + 10 = \frac{J + 10}{2}$$

$$\Rightarrow \quad 2S + 20 = J + 10$$

$$\Rightarrow \quad 2S + 10 = S + 20$$

$$\Rightarrow \quad S = 10$$

Present age of Steve is 10 years.

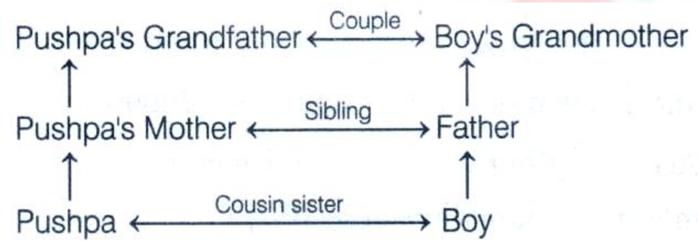
60. Pointing to a boy, Aruna said to Pushpa, "The mother of his father is the wife of your maternal grand father". How is Pushpa related to the boy? **[2017]**

(a) Sister (b) Niece

(c) Cousin sister (d) Wife

Ans. (c)

Sol.



Hence, Pushpa is the cousin sister of boy in the photograph.

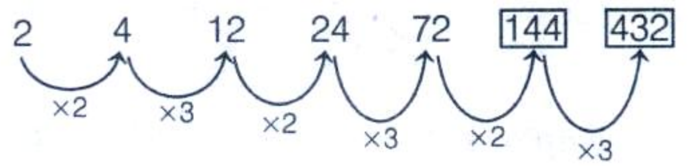
61. Which of the following pairs of numbers follow the number in the series $2, 4, 12, 24, 72, ...?$ **[2017]**

(a) 144,432 (b) 288,332

(c) 332,288 (d) 432,144

Ans. (a)

Sol. The pattern of the series is as follows



62. P, Q, R, S, T and U are sitting in two rows, three in each row facing each other.

- R is second to the left of P.
- Q and T are facing each other.
- S and P are diagonally opposite to each other.
- Q is not a neighbour of R.

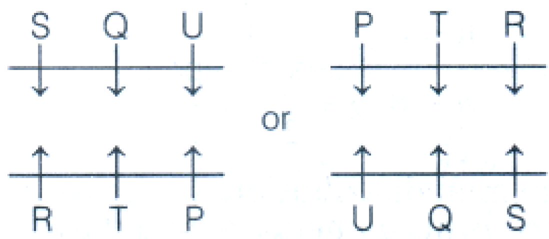
Which of the following are sitting in a row? **[2017]**

(a) P, Q, R (b) P, U, S

(c) U, T, S (d) P, T, R

Ans. (d)

Sol. The arrangement is as follows:



Hence, P, T, R are sitting in a row.

Directions (Q. Nos. 63 to 66) Questions are based on the following information

There are six teachers A, B, C, D, E and F ina school. Each teacher has to teach two subjects, one compulsory and the other optional.

D's optional is History, while three other have it as compulsory subject. E and F have Physics as one of their subjects. F's compulsory subject is Mathematics, which is an optional subject of both C and E .

History and English are A's subjects but in term of compulsory and optional subjects, they are reverse of D's. Chemistry is an optional subject of one of the teachers. There is only one female teacher, who has English as her compulsory subject

63. What is C's compulsory subject? **[2017]**

(a) Physics (b) Chemistry

(c) English (d) History

Ans. (d)

Sol. History is D's compulsory subject.

(Q. Nos. 63-66) According to the information, arrangement is as follows:

	Compulsory Subject	Optional Subject
A	History	English
B	History	Chemistry
C	History	Mathematics
D	English	History
E	Physics	Mathematics
F	Mathematics	Physics

64. Who among the following has Chemistry as a subject? **[2017]**

(a) A (b) B (c) C (d) D

Ans. (b)

Sol. B has Chemistry as a subject.

65. Which of the following groups of teachers has History as the compulsory subjects? **[2017]**

(a) B, C and D (b) C and D

(c) A, B and C (d) A, C and D

Ans. (c)

Sol. Group 'A, B and C' has history as the compulsory subject.

66. Disregarding which is compulsory or optional subject who has the same two subject combination as that of F ? **[2017]**

(a) B (b) E (c) D (d) A

Ans. (b)

Sol. E has the same two subject combination as that of F .

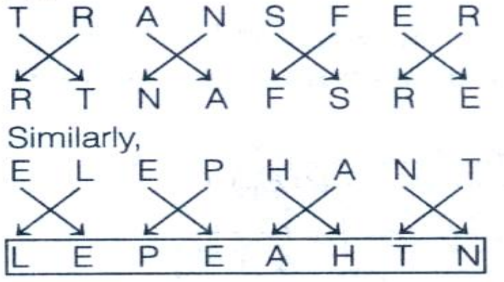
67. If TRANSFER is coded as RTNAFSRE, the ELEPHANT would be coded as **[2017]**

(a) LEPEHATN (b) LEPEAHTN

(c) LEEPAHTN (d) LEPEAHNT

Ans. (b)

Sol. As,



68. Which two of the following numbers comes in the next in the following sequence.

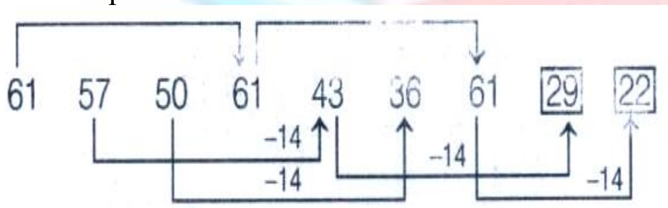
$61, 57, 50, 61, 43, 36, 61, \ldots\ldots\ldots$ **[2017]**

(a) 29,61 (b) 29,20

(c) 29,22 (d) 31,61

Ans. (c)

Sol. The pattern of the series is as follows



69. How many minimum number of colours will be required to paint all the sides of a cube without the adjacent sides having the same colours? **[2017]**

(a) 3 (b) 4 (c) 5 (d) 6

Ans. (a)

Sol. Three different colours are required to point all the sides of a cube so that no two adjacent sides having the same colours. Opposite faces have same colour.

70. In the following sequence, which pair of numbers fill in the blanks? **[2017]**

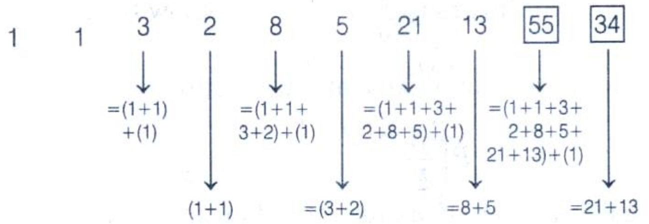
$1, 1, 3, 2, 8, 5, 21, 13$

(a) 54,33 (b) 34,55

(c) 55,34 (d) 33,54

Ans. (c)

Sol. The pattern of the series is as follows



Mathematics

71. A and B are independent witness in a case. The chance that A speaks truth is x and B speaks truth is y. If A and B agree on certain statement, the probability that the statement is true, is **[2017]**

(a) $\dfrac{xy}{xy+(1-x)(1-y)}$ (b) $\dfrac{xy}{(1-x)(1-y)}$

(c) $\dfrac{(1-x)(1-y)}{xy+(1-y)(1-x)}$ (d) $\dfrac{x+y}{xy+(1-x)(1-y)}$

Ans. (a)

Sol. $P(A$ speaks truth$) = x$

$P(B$ speaks truth $) = y$

Since, both A and B agree on certain statement

$\therefore$ Total probability

$$= P(A)\cdot P(B) + P(A')P(B')$$

$$= xy + (1-x)(1-y)$$

If statement is true then it means both A and B speaks truth.

$\therefore$ Required probability

$$= \frac{xy}{xy+(1-x)(1-y)}$$

72. The harmonic mean of two numbers is 4 . Their arithmetic mean A and the geometric mean G satisfy the relation $2A + G^2 = 27$, then the two numbers are **[2017]**

(a) 4 and 2 (b) 6 and 3

(c) 5 and 7 (d) 4 and 1

Ans. (b)

Sol. Let the two number be a and b.

According to the question, we have

and $\dfrac{2ab}{a+b} = 4$

$$2A + G^2 = 27 \left[\because A = \frac{a+b}{2}, G = \sqrt{ab}\right]$$

$$\therefore\ ab = 2(a+b) \qquad ....(i)$$

and

$$a+b+ab = 27 \qquad ...(ii)$$

$$\Rightarrow\ a+b+2(a+b) = 27$$

$$\Rightarrow 3(a+b) = 27$$

$$\Rightarrow a+b=9 \quad ....(iii)$$

From Eqs. (i) and (iii), we have

$$ab=18 \quad ...(iv)$$

Putting, $b=\frac{18}{a}$ from Eqs. (iv) in Eqs. (iii), we get

$$a+\frac{18}{a}=9$$

$$\Rightarrow \quad a^2-9a+18=0$$

$$\Rightarrow \quad (a-3)(a-6)=0$$

$$\Rightarrow \quad a=3,6$$

$$\therefore \quad b=6,3$$

$\therefore$ Required numbers are 3 and 6 .

73. In an entrance test there are multiple choice questions, with four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If the student gets the correct answer to a question, then the probability that he was guessing is **[2017]**

(a) $\frac{37}{40}$ (b) $\frac{1}{37}$ (c) $\frac{36}{37}$ (d) $\frac{1}{9}$

Ans. (b)

Sol. Let E_1 be the event that student knows the answer and E_2 be the event that student guesses the answer.

Also, A be the event that he answered correctly.

Now, according to the question

$$P(E_1)=0.90, P(E_2)=0.10$$

$$P\left(\frac{A}{E_1}\right)=1, P\left(\frac{A}{E_2}\right)=0.25$$

Required probability,

$$P\left(\frac{E_2}{A}\right)=\frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$=\frac{0.10\times0.25}{0.90\times1+0.10\times0.25}$$

$$=\frac{25\times10}{90\times100+10\times25}=\frac{25}{900+25}=\frac{1}{36+1}=\frac{1}{37}$$

74. A man is known to speak the truth 2 out of 3 times. He threw a dice cube with 1 to 6 on its faces and reports that it is 1 . Then the probability that it is actually 1 is **[2017]**

(a) $\frac{1}{2}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) $\frac{5}{6}$

Ans. (c)

Sol. Let E_1 and E_2 be the events that man speaks truth and lie, respectively. Also, let A be the event that 1 appear on dice.

Now, according to the question, we know that

$$P(E_1)=\frac{2}{3}, P(E_2)=\frac{1}{3}$$

$$P\left(\frac{A}{E_1}\right)=\frac{1}{6}, P\left(\frac{A}{E_2}\right)=\frac{5}{6}$$

$\therefore$ Required probability,

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{2}{3}\times\frac{1}{6}}{\frac{2}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{5}{6}}=\frac{2}{2+5}=\frac{2}{7}$$

75. Let A and B be two events such that $P(\overline{A\cup B})=\frac{1}{6}, P(A\cap B)=\frac{1}{4}$ and $P\left(\bar{A}\right)=\frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then the events A and B are **[2017]**

(a) independent but not equally likely
(b) mutually exclusively and independent
(c) equally likely and mutually exclusive
(d) equally likely but not independent

Ans. (a)

Sol. We have,

$$P\left(\overline{A\cup B}\right)=\frac{1}{6}$$

$$P(A\cap B)=\frac{1}{4}$$

$$P\left(\bar{A}\right)=\frac{1}{4}$$

Now, $P\left(\bar{A}\right)=\frac{1}{4}$

$$\Rightarrow 1-P(A)=\frac{1}{4}$$

$$\Rightarrow P(A)=\frac{3}{4}$$

Again, $P\left(\overline{A\cup B}\right)=\frac{1}{6}$

$$\Rightarrow 1-P(A\cup B)=\frac{1}{6}$$

$$\Rightarrow P(A\cup B)=1-\frac{1}{6}=\frac{5}{6}$$

Now, we know that

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow \quad \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$$

$$\Rightarrow \quad P(B)=\frac{5}{6}-\frac{1}{2}=\frac{1}{3}$$

Again, $P(A)\cdot P(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}=P(A\cap B)$

and $P(A)\neq P(B)$

$\therefore A$ and B are independent but not equally likely.

76. The mean and variance of a random variable X having binomial distribution are 4 and 2 , respectively. Then $P(X=1)$ is **[2017]**

(a) $\frac{1}{32}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

Ans. (a)

Sol. According to the questions, we have

Mean $=np=4$ and variance $=npq=2$

$$\therefore \quad 4q=2$$

$$\Rightarrow \quad q=\frac{1}{2}$$

Now, $p=1-q=1-\frac{1}{2}=\frac{1}{2}$

and $np=4$

$$\Rightarrow \quad n\times\frac{1}{2}=4$$

$$\Rightarrow \quad n=8$$

$$\therefore \quad n=8, p=\frac{1}{2}, q=\frac{1}{2}$$

Now, by binomial distribution, we know that

$$P(X=r)={}^{n}C_{r}p^{r}q^{n-r}$$

$$\therefore P(X=1)={}^{8}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{8-1}=8\times\left(\frac{1}{2}\right)^{8}=\frac{1}{32}$$

77. If $\bar{x}$ is the mean of distribution of X, then usual notation $\sum_{i=1}^{n} f_i\left(x_i-\bar{x}\right)$ is **[2017]**

(a) Mean deviation about mean

(b) Standard deviation

(c) 1

(d) 0

Ans. (d)

Sol. By property of arithmetic mean, the algebraic sum of all the deviation taken about the arithmetic mean is always zero.

$$\therefore \sum_{i=1}^{n} f_i\left(x_i-\bar{x}\right)=0$$

78. If E_1 and E_2 are two events associated with a random experiment such that $P(E_2)=0.35, P(E_1$ or $E_2)=0.85$ and $P(E_1$ and $E_2)=0.15$, then $P(E_1)$ is **[2017]**

(a) 0.25 (b) 0.35 (c) 0.65 (d) 0.75

Ans. (c)

Sol. We have,

$$P(E_2)=0.35$$

$$P(E_1 \text{ or } E_2)=0.85$$

$$P(E_1 \text{ and } E_2)=0.15$$

We know that,

$$P(E_1 \text{ or } E_2)=P(E_1)+P(E_2)-P(E_1 \text{ and } E_2)$$

$$\Rightarrow 0.85=P(E_1)+0.35-0.15$$

$$\Rightarrow P(E_1)=0.85-0.35+0.15=0.65$$

79. Find a matrix X such that $2A+B+X=0$, where $A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$ **[2017]**

(a) $\begin{bmatrix}1 & 2\\ 7 & 13\end{bmatrix}$ (b) $\begin{bmatrix}-1 & -2\\ -7 & -13\end{bmatrix}$

(c) $\begin{bmatrix}13 & 2\\ 7 & 1\end{bmatrix}$ (d) $\begin{bmatrix}-13 & -2\\ -7 & -1\end{bmatrix}$

Ans. (b)

Sol. We have,

$$A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$$

Now,

$$2A+B+X=0$$

$$X=-(2A+B)$$

$$\Rightarrow \quad =-\left(2\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}\right)$$

$$=-\left(\begin{bmatrix}-2 & 4\\ 6 & 8\end{bmatrix}+\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}\right)$$

$$=-\begin{bmatrix}1 & 2\\ 7 & 13\end{bmatrix}=\begin{bmatrix}-1 & -2\\ -7 & -13\end{bmatrix}$$

80. If in a triangle ABC, the altitudes from the vertices A, B, C on opposite sides are in HP, then $\sin A, \sin B$, $\sin C$ are in **[2017]**

(a) HP (b) Arithmetico-Geometric progression

(c) AP (d) GP

Ans. (c)

Sol. Let, the altitude from A, B, C are h_1, h_2, h_3 respectively.

We know that,

Area of triangle, $\Delta = \frac{1}{2}ah_1 = \frac{1}{2}bh_2 = \frac{1}{2}bh_3$

$\therefore\ a = \frac{2\Delta}{h_1}, b = \frac{2\Delta}{h_2}, c = \frac{2\Delta}{h_3}$

Since, h_1, h_2, h_3 are in HP

$\Rightarrow\ \frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{h_3}$ are in AP

$\Rightarrow\ \frac{2\Delta}{h_1}, \frac{2\Delta}{h_2}, \frac{2\Delta}{h_3}$ are in AP

$\Rightarrow a, b, c$ are in AP

$\Rightarrow k\sin A, k\sin B, k\sin C$ are in AP

$\Rightarrow \sin A, \sin B, \sin C$ are in AP .

81. α, β are the roots of an equation $x^2 - 2x\cos\theta + 1 = 0$, then the equation having α^n and β^n is **[2017]**

(a) $x^2 - (2\cos n\theta)x + 1 = 0$

(b) $2x^2 - (2\cos n\theta)x - 1 = 0$

(c) $x^2 + (2\cos n\theta)x + 1 = 0$

(d) $x^2 + (2\cos n\theta)x - 1 = 0$

Ans. (a)

Sol. We have,

$$x^2 - 2x\cos\theta + 1 = 0$$

$$\Rightarrow\quad x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \frac{2\cos\theta \pm 2i\sin\theta}{2}$$

$$= \cos\theta \pm i\sin\theta$$

$\therefore\quad \alpha = \cos\theta + i\sin\theta$

and $\quad \beta = \cos\theta - i\sin\theta$

$\therefore\quad \alpha^n = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

and $\quad \beta^n = (\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$

[using De-Moivre's theorem]

Now, $\alpha^n + \beta^n = 2\cos n\theta$ and $\alpha^n \cdot \beta^n = 1$

$\therefore$ Equation having α^n and β^n as roots is

$$x^2 - (\alpha^n + \beta^n)x + \alpha^n\beta^n = 0$$

$$\Rightarrow\quad x^2 - 2\cos n\theta x + 1 = 0$$

82. The equation $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$ has **[2017]**

(a) all three real roots

(b) one real and two imaginary roots

(c) three real roots, namely $x = a, y = b, z = c$

(d) None of the above

Ans. (b)

Sol. Let

$$f(x) = (x-a)^3 + (x-b)^3 + (x-c)^3$$

$$\therefore\quad f'(x) = 3(x-a)^2 + 3(x-b)^2 + 3(x-c)^2$$

$$= 3\left[(x-a)^2 + (x-b)^2 + (x-c)^2\right]$$

$$\therefore\quad f'(x) > 0 \forall x \in R$$

$\therefore f'(x) = 0$ has no real roots.

Hence, $f(x) = 0$ has two imaginary and one real root.

83. Three positive numbers whose sum is 21 are in arithmetic progression. If $2, 2, 14$ are added to them respectively, then resulting numbers are in geometric progression. Then which of the following is not among the three numbers? **[2017]**

(a) 25 (b) 13 (c) 1 (d) 7

Ans. (b)

Sol. Let the three numbers in AP are $a-d, a, a+d$.

It is given that

$$a - d + a + a + d = 21$$

$$\Rightarrow\quad 3a = 21$$

$$\Rightarrow\quad a = 7$$

$\therefore$ Number are $7-d, 7, 7+d$.

Again, it is given that

$7-d+2, 7+2, 7+d+14$ are in G.P.

$\Rightarrow\quad 9-d, 9, 21+d$ are in G.P.

$$\Rightarrow\quad 9^2 = (9-d)(21+d)$$

$$\Rightarrow\quad 81 = 189 + 9d - 21d - d^2$$

$$\Rightarrow\quad d^2 + 12d - 108 = 0$$

$$\Rightarrow\quad (d-6)(d+18) = 0$$

$$\Rightarrow\quad d = 6, -18$$

Putting $d = 6$ in $7-d, 7, 7+d$, we get $1, 7, 13$.

$\therefore 25$ is not among the three numbers.

84. If $\sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1}n$, then **[2017]**

(a) $n = \frac{(a-b)}{(1+ab)}$ (b) $n = \frac{ab}{(a-b)}$

(c) $n = \frac{(a+b)}{(1-ab)}$ (d) $n = \frac{(1-ab)}{(1+ab)}$

Ans. (c)

Sol. We have,

$\Rightarrow \sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}n$

$\Rightarrow 2\tan^{-1}a+2\tan^{-1}b=2\tan^{-1}n$

$\Rightarrow \left[\because 2\tan^{-1}x=\sin^{-1}\frac{2x}{1+x^2}\right]$

$\Rightarrow \tan^{-1}a+\tan^{-1}b=\tan^{-1}n$

$\Rightarrow \tan^{-1}\left(\frac{a+b}{1-ab}\right)=\tan^{-1}n$

$\Rightarrow \frac{a+b}{1-ab}=n$

85. The value of A that satisfies the equation $a\sin A+b\cos A-c$ is equal to **[2017]**

(a) $\tan^{-1}\left(\frac{a}{b}\right)\pm\cos^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)$

(b) $\tan^{-1}\left(\frac{c}{b}\right)\pm\sin^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right)$

(c) $\tan^{-1}\left(\frac{a}{b}\right)\pm\sin^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)$

(d) None of the above

Ans. (d)

Sol. Given,

Let $a\sin A+b\cos A-c=0...(i)$

$a=r\cos\alpha$

$b=r\sin\alpha$

$\Rightarrow r=\sqrt{a^2+b^2}$

$\tan\alpha=\frac{b}{a}$

$\Rightarrow \alpha=\tan^{-1}\left(\frac{b}{a}\right)$

Substituting these values in Eq. (i), we get

$r\cos\alpha\sin A+r\sin\alpha\cos A=c$

$\Rightarrow r\sin(A+\alpha)=c$

$\Rightarrow \sin(A+\alpha)=\frac{c}{\sqrt{a^2+b^2}}$

$\Rightarrow A+\alpha=\sin^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)$

$\Rightarrow A=\sin^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)-\tan^{-1}\left(\frac{b}{a}\right)$

86. If $\tan x=\frac{-3}{4}$ and $\frac{3\pi}{2}<x<2\pi$, then the value of $\sin 2x$ is **[2017]**

(a) $7/25$ (b) $-7/25$ (c) $24/25$ (d) $-24/25$

Ans. (d)

Sol. Given that,

$\tan x=-\frac{3}{4}$ and $\frac{3\pi}{2}<x<2\pi$

Since, x lies in the fourth quadrant, then

$\sin x=-\frac{3}{5}$ and $\cos x=\frac{4}{5}$

As we know that,

$\sin 2x=2\sin x\cos x$

$=2\times\left(-\frac{3}{5}\right)\times\frac{4}{5}=-\frac{24}{25}$

87. Find the principal value of $\cot^{-1}\left(-\sqrt{3}\right)$. **[2017]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) $\frac{5\pi}{6}$

Ans. (d)

Sol. Since, $\cot^{-1}(-x)=\pi-\cot^{-1}(x)$

$\therefore \cot^{-1}\left(-\sqrt{3}\right)=\pi-\cot^{-1}\left(\sqrt{3}\right)$

$=\pi-\cot^{-1}\left(\cot\frac{\pi}{6}\right)=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$

88. If $\cos\theta=\frac{4}{5}$ and $\cos\phi=\frac{12}{13}$, with θ and ϕ both in the fourth quadrant, the value of $\cos(\theta+\phi)$ is **[2017]**

(a) $-\frac{16}{65}$ (b) $\frac{-33}{65}$ (c) $\frac{33}{65}$ (d) $\frac{16}{65}$

Ans. (c)

Sol. Given, $\cos\theta=\frac{4}{5},\cos\phi=\frac{12}{13}$

It is given that θ and ϕ both lies in fourth quadrant.

$\therefore \sin\theta=-\frac{3}{5}$ and $\sin\phi=-\frac{5}{13}$

Now, $\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$

$=\frac{4}{5}\times\frac{12}{13}-\left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right)=\frac{48}{65}-\frac{15}{65}=\frac{33}{65}$

89. The value of $\sin 36^\circ$ is **[2017]**

(a) $\frac{\sqrt{10+2\sqrt{5}}}{4}$ (b) $\frac{\sqrt{10-2\sqrt{5}}}{4}$

(c) $\frac{\left(\sqrt{5}+1\right)}{4}$ (d) $\frac{\left(\sqrt{5}-1\right)}{4}$

Ans. (b)

Sol. Since, $\sin 18^\circ=\frac{\sqrt{5}-1}{4}$

As we know that,

$$\cos 36^\circ = 1-2\sin^2 18^\circ$$

$$= 1-2\left(\frac{\sqrt{5}-1}{4}\right)^2 = 1-2\left(\frac{5+1-2\sqrt{5}}{16}\right)$$

$$= 1-\left(\frac{6-2\sqrt{5}}{8}\right) = \frac{8-6+2\sqrt{5}}{8}$$

$$= \frac{2+2\sqrt{5}}{8} = \frac{\sqrt{5}+1}{4}$$

$$\sin^2 36^\circ + \cos^2 36^\circ = 1$$

$$\sin^2 36^\circ = 1-\cos^2 36^\circ$$

$$= 1-\left(\frac{\sqrt{5}+1}{4}\right)^2 = 1-\left(\frac{5+1+2\sqrt{5}}{16}\right)$$

$$= \frac{16-6-2\sqrt{5}}{16} = \frac{10-2\sqrt{5}}{16}$$

$$\Rightarrow \sin 36^\circ = \sqrt{\frac{10-2\sqrt{5}}{16}} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

90. Express $(\cos 5x - \cos 7x)$ as a product of sines or cosines or sines and cosines. **[2017]**

(a) $2\cos 4x \cos x$ (b) $2\sin 4x \sin x$
(c) $2\sin 6x \sin x$ (d) $2\cos 6x \cos x$

Ans. (c)

Sol. Given that,

$\cos 5x - \cos 7x$

We know that,

$$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$$

$$\cos 5x - \cos 7x = 2\sin\left(\frac{5x+7x}{2}\right)\sin\left(\frac{7x-5x}{2}\right)$$

$$= 2\sin\left(\frac{12x}{2}\right)\sin\left(\frac{2x}{2}\right)$$

$$= 2\sin 6x \sin x$$

91. If non-zero numbers a,b,c and in A.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point, then the point is **[2017]**

(a) $(1,-2)$ (b) $\left(1,-\frac{1}{2}\right)$ (c) $(-1,2)$ (d) $(-1,-2)$

Ans. (*)

Sol. Given, numbers a,b,c are in AP

$\therefore 2b = a+c ...(i)$

Now, the given straight line,

$$\Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}=-\frac{1}{c}$$

$$\Rightarrow \frac{x}{\left(-\frac{a}{c}\right)}+\frac{y}{\left(-\frac{b}{c}\right)}=1..(ii)$$

By using condition (i), we cannot determine that line (ii) passes through from which point. So we can say that given condition for question is wrong for concluding a definite result.

92. If the lines $x+(a-1)y+1=0$ and $2x+a^2y-1=0$ are perpendicular, then the condition satisfied by a is **[2017]**

(a) $|a|=2$ (b) $0<a<1$
(c) $-1<a<0$ (d) $a=-1$

Ans. (d)

Sol. Given equations of line are

$$x+(a-1)y+1=0$$

and $2x+a^2y-1=0$

Since, line (i) and line (ii) are perpendicular.

$\therefore$ Slope of line $1x$

Slope of line $2=-1$

$\therefore$ Slope of line $x+(a-1)y+1=0$

$$m_1 = \frac{-1}{a-1}$$

and slope of line $2x+a^2y-1=0$

$$m_2 = \frac{-2}{a^2}$$

$$\because \quad m_1 m_2 = -1$$

$$\therefore \quad \frac{1}{(a-1)}\cdot\frac{2}{a^2} = -1$$

$$\Rightarrow 2 = -a^2(a-1)$$

$$\Rightarrow 2 = -a^3 + a^2$$

$$\Rightarrow a^3 + a^2 - 2 = 0...(iii)$$

On solving Eq. (iii), we get

$a=-1$

93. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is circumradius of the triangle ABC, then $2(r+R)$ equals **[2017]**

(a) $a+c$ (b) $a+b+c$ (c) $a+b$ (d) $b+c$

Ans. (c)

Sol. Given,

$$\angle C = \frac{\pi}{2}$$

$\because \quad R = \frac{c}{2\sin C} = \frac{c}{2\sin 90^\circ} = \frac{c}{2}$

and $\quad r = (s-c)\tan\frac{C}{2} = (s-c)\tan 45^\circ$

$\therefore \quad r = (s-c)$

$$2(r+R) = 2\left(\frac{c}{2} + s - c\right)$$
$$= 2s - c$$
$$= a + b + c - c$$
$$= a + b$$

94. If $x^2 + 3xy + 2y^2 - x - 4y - 6 = 0$ represents a pair of straight lines, their point of intersection is **[2017]**

(a) $(0,0)$ (b) $(8,5)$ (c) $(8,-5)$ (d) $(-2,5)$

Ans. (c)

Sol. As we know that the pair of straight line $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect each other at $\left(\frac{hf - bg}{ab - h^2}, \frac{hg - af}{ab - h^2}\right)$

Therefore, point of intersection of pair of straight line $x^2 + 3xy + 2y^2 - x - 4y - 6 = 0$ is

$$= \left(\frac{\frac{3}{2}\times(-2) - (2)\left(-\frac{1}{2}\right)}{(1\times 2) - \left(\frac{3}{2}\right)^2}, \frac{\frac{3}{2}\times\left(-\frac{1}{2}\right) - 1\times(-2)}{1\times 2 - \left(\frac{3}{2}\right)^2}\right)$$

$$\frac{-3+1}{2-\frac{9}{4}}, \frac{-\frac{3}{4}+2}{2-\frac{9}{4}}$$

$$= \left(\frac{-2}{-\frac{1}{4}}, \frac{\frac{5}{4}}{-\frac{1}{4}}\right) = (8, -5)$$

95. The equation of the tangent line to the curve $y = 2x\sin x$ at the point $\left(\frac{\pi}{2}, \pi\right)$ is **[2017]**

(a) $y = 2x + 2\pi$ (b) $y = 2x$

(c) $y = -2x + 2\pi$ (d) $y = -2x$

Ans. (b)

Sol. Given equation of curve is

$$y = 2x\sin x$$

Slope at point $\left(\frac{\pi}{2}, \pi\right)$

$$\left(\frac{dy}{dx}\right)_{\left(\frac{\pi}{2},\pi\right)} = 2\sin x + 2x\cos x$$
$$= 2\sin\frac{\pi}{2} + 2\times 0$$
$$= 2 + 0 = 2$$

$\therefore$ Equation of tangent at point $\left(\frac{\pi}{2}, \pi\right)$ is

$$y - \pi = \left(\frac{dy}{dx}\right)_{\left(\frac{\pi}{2},\pi\right)}\left(x - \frac{\pi}{2}\right)$$

$\Rightarrow \quad y - \pi = 2\left(x - \frac{\pi}{2}\right)$

$\Rightarrow \quad y - \pi = 2x - \pi$

$\Rightarrow \quad y = 2x$

96. If the graph of $y = (x-2)^2 - 3$ is shifted by 5 units up along Y -axis and 2 units to the right along the X -axis, then the equation of the resultant graph is **[2017]**

(a) $y = x^2 + 2$ (b) $y = (x-2)^2 + 5$

(c) $y = (x+2)^2 + 2$ (d) $y = (x-4)^2 + 2$

Ans. (d)

Sol. We have,

$$y = (x-2)^2 - 3 ...(i)$$

Since, y is shifted up along Y -axis by 5 units and x is shifted along right by 2 units

$\therefore$ Put $y = y - 5$ and $x = x - 2$ in Eq. (i), we get

$$y - 5 = (x - 2 - 2)^2 - 3$$

$\Rightarrow \quad y - 5 = (x-4)^2 - 3$

$\Rightarrow \quad y = (x-4)^2 + 2$

97. The direction cosines of the vector $a = \left(-2\hat{i} + \hat{j} - 5\hat{k}\right)$ are **[2017]**

(a) $-2, 1, -5$ (b) $\frac{1}{3}, \frac{-1}{6}, \frac{-5}{6}$

(c) $\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}$ (d) $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

Ans. (d)

Sol. We have,

$$\vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$$

$\therefore \quad |\vec{a}| = \sqrt{(-2)^2 + (1)^2 + (-5)^2}$

$$= \sqrt{4 + 1 + 25} = \sqrt{30}$$

$\therefore$ Unit vector, $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

$$=\frac{-2\hat{i}+\hat{j}-5\hat{k}}{\sqrt{30}}=\frac{-2}{\sqrt{30}}\hat{i}+\frac{1}{\sqrt{30}}\hat{j}-\frac{5}{\sqrt{30}}\hat{k}$$

$\therefore$ DC's of $\vec{a}=\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}$

98. The equation of the hyperbola with centre at the origin, length of the transverse axis is 6 and one focus $(0,4)$ is **[2017]**

(a) $\frac{y^2}{9}+\frac{x^2}{7}=1$ (b) $\frac{y^2}{9}-\frac{x^2}{7}=1$

(c) $\frac{y^2}{7}+\frac{x^2}{9}=1$ (d) $\frac{y^2}{7}-\frac{x^2}{9}=1$

Ans. (b)

Sol. We have,

Focus $=(0,4)$ and length of transverse axis $=6$

$\therefore \quad be=4 \text{ and} 2b=6$

$\Rightarrow \quad be=4$ and $b=3$

$\Rightarrow \quad e=\frac{4}{3}$ and $b=3$

Also, $e^2=1+\frac{a^2}{b^2}$

$\Rightarrow \quad \frac{16}{9}=1+\frac{a^2}{9}$

$\Rightarrow \quad a^2=7$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$$

$$\Rightarrow \frac{x^2}{7}-\frac{y^2}{9}=-1$$

$$\Rightarrow \frac{y^2}{9}-\frac{x^2}{7}=1$$

99. If $\vec{a},\vec{b},\vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7$, $|\vec{b}|=5, |\vec{c}|=3$, then the angle between the vectors $\vec{b}$ and $\vec{c}$ is **[2017]**

(a) 60° (b) 30° (c) 45° (d) 90°

Ans. (a)

Sol. We have,

$|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$

Now, $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow |\vec{b}+\vec{c}|=|\vec{a}|$

$\Rightarrow |\vec{b}+\vec{c}|^2=|\vec{a}|^2$

$\Rightarrow (\vec{b}+\vec{c})\cdot(\vec{b}+\vec{c})=|\vec{a}|^2$

$\Rightarrow |\vec{b}|^2+|\vec{c}|^2+2\vec{b}\cdot\vec{c}=|\vec{a}|^2$

$\Rightarrow 25+9+2|\vec{b}||\vec{c}|\cos\theta=49$

[where θ is angle between $\vec{b}$ and $\vec{c}$]

$\Rightarrow 2\times5\times3\cos\theta=15$

$\Rightarrow \cos\theta=\frac{1}{2}$

$\Rightarrow \theta=60^\circ$

100. If $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}, \hat{i}+\hat{j}+c\hat{k}\ (a\neq b\neq c\neq 1)$ are coplanar, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is **[2017]**

(a) -1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1

Ans. (d)

Sol. If $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}, \hat{i}+\hat{j}+c\hat{k}\ (a\neq b\neq c\neq 1)$ are coplanar, then

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$$

$$=\begin{vmatrix} a & 1-a & 0 \\ 1 & b-1 & 1-b \\ 1 & 0 & c-1 \end{vmatrix}=0 \begin{pmatrix} C_2\to C_2-C_1 \\ C_3\to C_3-C_2 \end{pmatrix}$$

$$=(1-a)(1-b)(1-c)\begin{vmatrix} \frac{a}{1-a} & 1 & 0 \\ \frac{1}{1-b} & -1 & 1 \\ \frac{1}{1-c} & 0 & -1 \end{vmatrix}=0$$

$$=\frac{a}{1-a}(1-0)-\frac{1}{1-b}(-1-0)+\frac{1}{1-c}(1-0)=0$$

$$\Rightarrow \frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$

$$\Rightarrow \frac{a}{1-a}+1+\frac{1}{1-b}+\frac{1}{1-c}=1$$

$$\Rightarrow \frac{a+1-a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

101. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vector having magnitudes 1,1 and 2 , respectively. If $\vec{a}\times(\vec{a}\times\vec{c})-\vec{b}=0$, then the acute angle between $\vec{a}$ and $\vec{c}$ is **[2017]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) None of these

Ans. (b)

Sol. We have,

$|\vec{a}|=1, |\vec{b}|=1, |\vec{c}|=2$

Now, $\vec{a}\times(\vec{a}\times\vec{c})-\vec{b}=0$

$\Rightarrow \vec{a}\times(\vec{a}\times\vec{c})=\vec{b}$

$\Rightarrow |\vec{a}\times(\vec{a}\times\vec{c})|=|\vec{b}|$

$\Rightarrow |\vec{a}||\vec{a}\times\vec{c}|\sin 90^\circ=|\vec{b}| \quad [\because \vec{a}\perp(\vec{a}\times\vec{c})]$

$\Rightarrow 1\times|\vec{a}\times\vec{c}|\times 1=1$

$\Rightarrow |\vec{a}\times\vec{c}|=1$

$\Rightarrow |\vec{a}||\vec{c}|\sin\theta=1$

[where θ is angle between $\vec{a}$ and $\vec{c}$]

$\Rightarrow 1\times 2\times\sin\theta=1$

$\Rightarrow \sin\theta=\frac{1}{2}$

$\Rightarrow \theta=\frac{\pi}{6}$

102. Let $\vec{a},\vec{b},\vec{c}$ be vector such that $|\vec{a}|=2, |\vec{b}|=3, |\vec{c}|=5$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. The value of $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{c}\vec{a}$ is **[2017]**

(a) 38 (b) -38 (c) 19 (d) -19

Ans. (d)

Sol. We have,

$|\vec{a}|=2, |\vec{b}|=3, |\vec{c}|=5$ and $\vec{a}+\vec{b}+\vec{c}=0$

Now,

$\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})=\vec{a}\cdot 0$

$\Rightarrow |\vec{a}|^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=-4....(i)$

Similarly, $\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{c}=-9...(ii)$

and $\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}=-25...(iii)$

On adding Eqs. (i), (ii) and (iii), we get

$2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-38$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-19$

103. If $\vec{a}=(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{b}=(3\hat{i}-\hat{j}+2\hat{k})$ then the angle between $(a+b)$ and $(a-b)$ is **[2017]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

Ans. (c)

Sol. We have,

$\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$

$\vec{a}+\vec{b}=(\hat{i}+2\hat{j}-3\hat{k})+(3\hat{i}-\hat{j}+2\hat{k})$

$\therefore \quad =4\hat{i}+\hat{j}-\hat{k}$

$\vec{a}-\vec{b}=(\hat{i}+2\hat{j}-3\hat{k})-(3\hat{i}-\hat{j}+2\hat{k})$

$=-2\hat{i}+3\hat{j}-5\hat{k}$

Let θ be the angle between $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$.

$\therefore \quad \cos\theta=\frac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}$

$=\frac{(4\hat{i}+\hat{j}-\hat{k})\cdot(-2\hat{i}+3\hat{j}-5\hat{k})}{|4\hat{i}+\hat{j}-\hat{k}||-2\hat{i}+3\hat{j}-5\hat{k}|}$

$=\frac{-8+3+5}{\sqrt{4^2+1^2+(-1)^2}\sqrt{(-2)^2+3^2+(-5)^2}}=0$

$\therefore \theta \quad =90^\circ$ or $\frac{\pi}{2}$

104. The number of elements in the power set $P(S)$ of the set $S=\{2,(1,4)\}$ is **[2017]**

(a) 2 (b) 4 (c) 8 (d) 10

Ans. (b)

Sol. We have,

$S \quad =\{2,(1,4)\}$

$\therefore \quad n(s) \quad =2$

$\therefore$ Number of elements in power set $P(s)$

$=n(P(s))=2^{n(s)}=2^2=4$

105. If $(1-x+x^2)^n=a_0+a_1x+a_2x^2+.....+a_{2n}x^{2n}$ then $a_0+a_2+a_4+......+a_{2n}$ is **[2017]**

(a) $\frac{3^n+1}{2}$ (b) $\frac{3^n-1}{2}$ (c) $\frac{1-3^n}{2}$ (d) $3^n+\frac{1}{2}$

Ans. (a)

Sol. We have,

$(1-x+x^2)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}......(i)$

Put $x=1$ in Eq. (i), we get

$(1-1+1)^n=a_0+a_1+a_2+\ldots+a_{2n}$

$\Rightarrow \quad a_0+a_1+a_2\ldots a_{2n}=1 \quad$ (ii)

Again, put $x=-1$ in Eq. (i), we get

$(1-(-1)+(-1)^2)^n=a_0-a_1+a_2-\ldots+a_{2n}$

$\Rightarrow \quad a_0-a_1+a_2-......+a_{2n}=3^n \quad$ (iii)

On adding Eq. (ii) and Eq. (iii), we get

$$2(a_0+a_2+\ldots\ldots a_{2n})=3^n+1$$

$$\Rightarrow \quad a_0+a_2+\ldots\ldots a_{2n}=\frac{3^n+1}{2}$$

106. M distinct animals of a circus have to be placed in m cages, one is each cage. There are n small cages and p small animal ($n<p<m$). The large animals are so large that they do not fit in small cage. However, small animals can be put in any cage. The number of putting the animals into cage is **[2017]**

(a) $\left\{{}^{(m-n)}P_p\right\}\left\{{}^{(m-p)}P_{(m-p)}\right\}$ (b) ${}^{(m-n)}C_p$

(c) $\left\{{}^{(m-n)}C_p\right\}\left\{{}^{(m-p)}C_{(m-p)}\right\}$ (d) ${}^{(m-n)}P_p$

Ans. (a)

Sol. According to the questions,
Required number of ways $=$ Number of ways of putting p small animals in n cages $\times$ Number of ways of putting remaining animals in the cages

$$={}^{m-n}P_p\times{}^{m-p}P_{m-p}$$

107. Let A and B two sets containing four and two elements, respectively. The number of subsets of the $A\times B$, each having at least three elements is **[2017]**

(a) 270 (b) 239 (c) 219 (d) 256

Ans. (c)

Sol. Given,

$$n(A)=4 \text{ and } n(B)=2$$

$$\therefore \quad n(A\times B)=n(A)\cdot n(B)=4\times 2=8$$

$\therefore$ Number of subsets of $A\times B=2^{n(A\times B)}$

$$=2^8=256$$

$\therefore$ The number of subsets of the set $A\times B$, having atleast three elements is

$$=256-\left({}^8C_0+{}^8C_1+{}^8C_2\right)$$

$$=256-(1+8+28)=256-37=219$$

108. The slope of the function

$$f(x)=\begin{cases} x^2\sin\left(\dfrac{1}{x}\right), & x\neq 0\\ 0 & x=0\end{cases}$$ **[2017]**

(a) 1 (b) 0 (c) -1 (d) None of these

Ans. (b)

Sol. We have,

$$f(x)=\begin{cases} x^2\sin\left(\dfrac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$$

Now, (LHL at $x=0$)

$$=\lim_{x\to 0^-}x^2\sin\left(\frac{1}{x}\right)=\lim_{h\to 0}(0-h)^2\sin\left(\frac{1}{0-h}\right)$$

$$=\lim_{h\to 0}h^2\sin\left(\frac{-1}{h}\right)=0$$

(RHL at $x=0$)

$$=\lim_{x\to 0^+}x^2\sin\left(\frac{1}{x}\right)=\lim_{h\to 0}(0+h)^2\sin\left(\frac{1}{0+h}\right)$$

$$=\lim_{h\to 0}h^2\sin\left(\frac{1}{h}\right)=0$$

and

$$f(0)=0$$

$\therefore$ LHL $=$ RHL $=f(0)$

$\therefore f(x)$ is continuous at $x=0$.

Again,

(LHD at $x=0$) $\lim\limits_{x\to 0^-}\dfrac{f(x)-f(0)}{x-0}$

$$=\lim_{x\to 0^-}\frac{x^2\sin\left(\frac{1}{x}\right)-0}{x}$$

$$=\lim_{x\to 0^-}x\sin\left(\frac{1}{x}\right)$$

$$=\lim_{h\to 0}(0-h)\sin\left(\frac{1}{0-h}\right)$$

$$=\lim_{h\to 0}h\sin\left(\frac{1}{h}\right)=0$$

and (RHD at $x=0$) $\lim\limits_{x\to 0^+}\dfrac{f(x)-f(0)}{x-0}$

$$=\lim_{x\to 0^+}\frac{x^2\sin\left(\frac{1}{x}\right)-0}{x}=\lim_{x\to 0^+}x\sin\left(\frac{1}{x}\right)$$

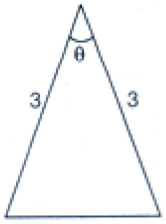
$$=\lim_{h\to 0}(0+h)\sin\left(\frac{1}{0+h}\right)$$

$$=\lim_{h\to 0}h\sin\frac{1}{h}=0$$

$\therefore$ LHD $=$ RHD $=0=f'(x)$

$\therefore$ Slope of $f(x)=f'(x)=0$.

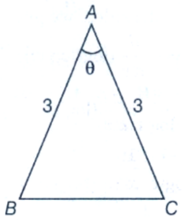
109. What is the largest area of an isosceles triangle with two edges of length 3 ? **[2017]**



(a) 3 (b) $\frac{3}{2}$ (c) 9 (d) $\frac{9}{2}$

Ans. (d)

Sol.



Area of $\triangle ABC = \frac{1}{2}(AB)(AC)\sin A$

$$= \frac{1}{2} \times 3 \times 3 \sin\theta = \frac{9}{2}\sin\theta$$

Area will be maximum when, $\theta = \frac{\pi}{2}$

$\therefore$ Maximum area of $\triangle ABC = \frac{9}{2}\sin\frac{\pi}{2} = \frac{9}{2} \times 1 = \frac{9}{2}$

110. The value of $\int_9^\pi x^3 \sin dx$ is **[2017]**

(a) $\pi^3 - 6\pi$ (b) $-\pi^3 - 6\pi$

(c) $-\pi^3 + 6\pi$ (d) $\pi^3 + 6\pi$

Ans. (a)

Sol. Let, $I = \int \underset{I}{x^3} \underset{II}{\sin x} dx$

$$= x^3[-\cos x] - \int(-\cos x) \cdot 3x^2 dx$$

$$-x^3\cos x + 3\int_I x^2 \cos_{II}^{x} dx$$

$$= -x^3 \cdot \cos x + 3\left[x^2(\sin x) - \int \sin x \cdot (2x) dx\right]$$

$$= -x^3\cos x + 3x^2\sin x - 6\int \underset{I}{x} \underset{II}{\sin x} dx$$

$$= -x^3\cos x + 3x^2\sin x - 6\left[x(-\cos x) - \int(-\cos x) \cdot 1 dx\right]$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x$$

$$\therefore \int_0^\pi x^3 \sin x dx = \left[-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x\right]_0^\pi$$

$$= \left(-\pi^3\cos\pi + 3\pi^2\sin\pi + 6\pi\cos\pi - 6\sin\pi\right)$$

$$= (0 + 0 + 0 - 6\sin 0)$$

$$= \pi^3 + 0 - 6\pi + 0 + 0 = \pi^3 - 6\pi$$

111. Let $f(a)$ be a polynomial of degree four, having extreme value at $x = 1$ and $x = 2$. If **[2017]**

$\lim_{x\to 0}\left[1 + \frac{f(x)}{x^2}\right] = 3$, then $f(2)$ is

(a) 0 (b) 4 (c) -8 (d) -4

Ans. (a)

Sol. We have,

$$\lim_{x\to 0}\left[1 + \frac{f(x)}{x^2}\right] = 3$$

$\therefore \quad x^2 + f(x) = ax^4 + bx^3 + 3x^2$

$\Rightarrow \quad f(x) = ax^4 + bx^3 + 2x^2$

$\Rightarrow \quad f'(x) = 4ax^3 + 3bx^2 + 4x$

Also, $f'(x) = 0$ at $x = 1$ and 2

$\therefore \quad f'(1) = 0 \Rightarrow 4a + 3b + 4 = 0 \ldots(i)$

and $\quad f'(2) = 0$

$\Rightarrow \quad 32a + 12b + 8 = 0 \ldots\ldots(ii)$

On solving Eqs. (i) and (ii), we get

$$a = \frac{1}{2} \text{ and } b = -2$$

$$\therefore f(x) = \frac{1}{2}x^4 - 2x^3 + 3x^2$$

$$\therefore \quad f(2) = \frac{1}{2}(2)^4 - 2(2)^3 + 3(2)^2 = \{8 - 16 + 8\} = 0$$

112. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin(x/2) + \cos(x/2)$ is **[2017]**

(a) 4 (b) $3 + \sqrt{2}$ (c) 9 (d) $4 + \sqrt{2}$

Ans. (d)

Sol. Let $f(x) = 4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$

$$= 3 + \sin^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$$

$$= 3 + \sin^2 x + \sqrt{1 + \sin x}$$

$$f'(x) = 2\sin x\cos x + \frac{1}{2\sqrt{1 + \sin x}} \cdot \cos x$$

$$= \cos x\left[2\sin x + \frac{1}{2\sqrt{1 + \sin x}}\right]$$

Now, $f'(x) = 0$

$$\Rightarrow \quad \cos x\left[2\sin x + \frac{1}{2\sqrt{1 + \sin x}}\right] = 0$$

$$\Rightarrow \quad \cos x = 0$$

$$\Rightarrow \quad x = \frac{\pi}{2}$$

Again, $f'\left(\frac{\pi}{2}\right) < 0$

$\therefore f(x)$ attains maximum value at $x = \frac{\pi}{2}$

$\therefore f_{\max} = f\left(\frac{\pi}{2}\right) =$

$= 3 + \sin^2\frac{\pi}{2} + \sqrt{1+\sin\frac{\pi}{2}}$

$= 3 + 1 + \sqrt{2} = 4 + \sqrt{2}$

113. The solution of $(e^x+1)ydy = (y+1)e^x dx$ is **[2017]**

(a) $e^x = c(e^x+1)(y+1)$ (b) $e^y = e^x + y + 1$

(c) $y = (e^x+1)(y+1)$ (d) None of these

Ans. (d)

Sol. We have,

$(e^x+1)ydy = (y+1)e^x dx$

$\Rightarrow \frac{y}{y+1}dy = \frac{e^x}{e^x+1}dx$

On integrating both the sides, we get

$\int \frac{y}{y+1}dy = \int \frac{e^x}{e^x+1}dx$

$\Rightarrow \int\left(1-\frac{1}{y+1}\right)dy = \int \frac{e^x}{e^x+1}dx$

$\Rightarrow y - \log(y+1) = \log(e^x+1) + \log c$

$\Rightarrow y = \log(y+1) + \log(e^x+1) + \log c$

$\Rightarrow y = \log\left[c(y+1)(e^x+1)\right]$

$\Rightarrow e^y = c(y+1)(e^x+1)$

114. Evaluate $\int_0^1 x(1-x)^n dx$ **[2017]**

(a) $\frac{-1}{(n+1)(n+2)}$ (b) $\frac{1}{(n+1)(n+2)}$

(c) $(n+1)(n+2)$ (d) $(n-1)(n-2)$

Ans. (a)

Sol. Let $I = \int_0^1 x(1-x)^n dx$

$= \int_0^1 (1-x)x^n dx \left[\int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$= \int_0^1 \left(x^n - x^{n+1}\right)dx$

$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$

$= \frac{1}{n+1} - \frac{1}{n+2}$

$= \frac{1}{(n+1)(n+2)}$

115. The critical point and nature for the function $f(x,y) = x^2 - 2x + 2y^2 + 4y - 2$ is **[2017]**

(a) $(1,1)$ maximum (b) $(1,-1)$ maximum

(c) $(1,1)$ minimum (d) $(1,-1)$ minimum

Ans. (d)

Sol. Given,

$f(x,y) = x^2 - 2x + 2y^2 + 4y - 2$.

For critical points

$\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

$\therefore 2x - 2 = 0$ and $4y + 4 = 0$

$\Rightarrow x = 1$ and $y = -1$

Hence, critical point of the given function is $(1,-1)$.

The given function is maximum at $(1,-1)$, if

$rt - s^2 > 0$ and $r < 0$

And minimum at $(1,-1)$, if $rt - s^2 > 0$ and $r > 0$

For maximum and minimum

and

$r = \frac{\partial^2 f}{\partial x^2} = 2$

$t = \frac{\partial^2 f}{\partial y^2} = 4$

$s = \frac{\partial^2 f}{\partial x \partial y} = 0$

Clearly, by putting the value of r, t and s, we get $r > 0$ and $rt - s^2 > 0$.

Hence, the given function is minimum at $(1,-1)$.

116. If $y = \cos^2 x^2$, find $\frac{dy}{dx}$ **[2017]**

(a) $4x^2 \sin x^2 \cos x^2$ (b) $-4x^2 \cos x^2 \sin x^2$

(c) $2x \sin x^2 \cos x^2$ (d) $-2x \cos x^2 \sin x^2$

Ans. (*)

Sol. Given $y = \left(\cos x^2\right)^2$

Using chain rule,

$\frac{dy}{dx} = 2\left(\cos x^2\right)^{2-1}\frac{dy}{dx}\left(\cos x^2\right)$

$= 2\cos x^2\left(-\sin x^2\right)\frac{d}{dx}\left(x^2\right)$

$= -2\cos x^2 \sin x^2 \times 2x$

$= -4x \cos x^2 \sin x^2$

No option match.

117. The derivative of $\left(x^3 + e^x + 3^x + \cot x\right)$ with respect to x is **[2017]**

(a) $3x^2+e^x+3^x(\log 3)-\text{cosec}^2x$

(b) $3x^2+e^x+3^x(\log 3)+\text{cosec}^2x$

(c) $3x^2+e^x+3^x(\log 3)-\sec^2x$

(d) $3x^2+e^x+3^x(\log 3)+\sec^2x$

Ans. (a)

Sol. Let given function is

$$y=x^3+e^x+3^x+\cot x$$

Now,

$$\frac{dy}{dx}=3x^2+e^x+3^x\log 3-\text{cosec}^2x$$

118. The solution of the differential equation $\frac{dy}{dx}=e^{x+y}+x^2e^y$ is **[2017]**

(a) $e^{x-y}+\frac{x^3}{3}=C$ (b) $e^x+e^{-y}+\frac{x^3}{3}=C$

(c) $e^x-e^{-y}=\frac{x^3}{3}+C$ (d) None of these

Ans. (b)

Sol. Given differential equation is

$$\frac{dy}{dx}=e^{x+y}+x^2e^y$$

$$=e^y\left(x^2+e^x\right)$$

$$\Rightarrow \frac{dy}{e^y}=\left(x^2+e^x\right)dx$$

[using variable separable form]

Integrating both sides, we get

$$\int e^{-y}dy=\int\left(x^2+e^x\right)dx$$

$$\Rightarrow -e^{-y}=\frac{x^3}{3}+e^x+C$$

$$\Rightarrow e^x+e^{-y}+\frac{x^3}{3}=C \text{ [where } C \text{ is constant]}$$

119. Differentiate $[-\log(\log x), x>1]$ with respect to x. **[2017]**

(a) $\frac{-1}{(x\log x)}$ (b) $\frac{1}{\log x}$ (c) $\frac{1}{x}$ (d) $x\log x$

Ans. (a)

Sol. Let $y=-\log(\log x), x>1$

On differentiating both sides, we get

$$\frac{dy}{dx}=-\frac{d}{dx}(\log(\log x))$$

$$=-\frac{1}{\log x}\frac{d}{dx}(\log x)$$

$$=\frac{-1}{\log x}\cdot\frac{1}{x}$$

$$=-\frac{1}{x\log x}$$

120. Evaluate $\lim_{x\to 0}\frac{x\tan x}{(1-\cos x)}$ **[2017]**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) -2 (d) 2

Ans. (d)

Sol. Given, $\lim_{x\to 0}\frac{x\tan x}{(1-\cos x)}$

It is clear that function is of the form $\frac{0}{0}$ therefore by using L' Hopital rule.

$$=\lim_{x\to 0}\frac{x\sec^2x+\tan x}{\sin x}$$

Again apply L' Hopital rule,

$$=\lim_{x\to 0}\frac{x\left(2\sec^2x\tan x+\sec^2x+\sec^2x\right)}{\cos x}$$

Putting $x=0$, we get

$$\lim_{x\to 0}f(x)=2$$