

Mathematics

1. The point of intersection of circle [2018]

$x^2 + y^2 + 10x - 12y + 51 = 0$ and the line $3y + x = 3$ is

- (a) $(-6, 3)$ (b) $(3, -6)$
(c) $(6, -3)$ (d) $(-3, 6)$

Ans. (a)

Sol. We have,

$$x^2 + y^2 + 10x - 12y + 51 = 0 \text{ and } 3y + x = 3$$

To find the intersection point of circle, put $x = 3 - 3y$ in equation of circle, we have

$$(3 - 3y)^2 + y^2 + 10(3 - 3y) - 12y + 51 = 0$$

$$\Rightarrow 9 + 9y^2 - 18y + y^2 + 30 - 30y - 12y + 51 = 0$$

$$\Rightarrow 10y^2 - 60y + 90 = 0$$

$$\Rightarrow y^2 - 6y + 9 = 0$$

$$\Rightarrow (y - 3)^2 = 0$$

$$\Rightarrow y = 3$$

$$\therefore x = 3 - 3y = 3 - 3 \times 3 = -6$$

$\therefore$ Intersection point is $(-6, 3)$,

2. The number of solutions of the equation

$\sin x + \sin 5x = \sin 3x$ lying in the interval $[0, \pi]$ is [2018]

- (a) 4 (b) 6 (c) 5 (d) 2

Ans. (b)

Sol. We have,

$$\sin x + \sin 5x = \sin 3x$$

$$\Rightarrow 2 \sin \left(\frac{x + 5x}{2} \right) \cos \left(\frac{x - 5x}{2} \right) = \sin 3x$$

$$[\because \cos(-x) = \cos x]$$

$$2 \sin 3x \cos 2x = \sin 3x$$

$$\Rightarrow 2 \sin 3x \cos 2x - \sin 3x = 0$$

$$\Rightarrow \sin 3x (2 \cos 2x - 1) = 0$$

$$\Rightarrow \sin 3x = 0 \text{ or } \cos 2x = \frac{1}{2}$$

$$\Rightarrow 3x = 0, \pi, 2\pi, 3\pi \text{ or } 2x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\Rightarrow x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$\therefore$ Number of solution is 6.

3. In an acute angled $\triangle ABC$ the least value of $\sec A + \sec B + \sec C$ is [2018]

- (a) 6 (b) 8 (c) 3 (d) 2

Ans. (a)

Sol. Generally, maximum or minimum values of triangles occurs when all the angles of triangle are equal.

$$\therefore A = B = C = \frac{\pi}{3}$$

$\therefore$ Minimum value of $\sec A + \sec B + \sec C$

$$= \sec \frac{\pi}{3} + \sec \frac{\pi}{3} + \sec \frac{\pi}{3} = 2 + 2 + 2 = 6$$

4. Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and

$Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then [2018]

- (a) $P \subset Q$ and $Q - P \neq \emptyset$ (b) $P \not\subset Q$
(c) $Q \not\subset P$ (d) $P = Q$

Ans. (d)

Sol. We have,

$$P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\} \text{ and}$$

$$Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$$

Now,

$$\sin \theta - \cos \theta = \sqrt{2} \cos \theta$$

$$\Rightarrow (\sin \theta - \cos \theta)^2 = 2 \cos^2 \theta$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = 2 \cos^2 \theta$$

$$\Rightarrow \sin^2 \theta = \cos^2 \theta + 2 \sin \theta \cos \theta$$

$$\Rightarrow 2 \sin^2 \theta = \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$$

$$\Rightarrow 2 \sin^2 \theta = (\sin \theta + \cos \theta)^2$$

$$\Rightarrow \sqrt{2} \sin \theta = \sin \theta + \cos \theta$$

$$\therefore P = Q \text{ [taking positive square root]}$$

5. If $\frac{\tan x}{2} = \frac{\tan y}{3} = \frac{\tan z}{5}$ and $x + y + z = \pi$, then the value of $\tan^2 x + \tan^2 y + \tan^2 z$ is [2018]

- (a) $\frac{38}{3}$ (b) 38 (c) 114 (d) None of these

Ans. (a)

Sol. Given, $x + y + z = \pi$

$$\tan(x+y+z) = \tan\pi$$

$$\therefore \tan x + \tan y + \tan z = \tan x \tan y \tan z \quad \dots(i)$$

$$\text{Let } \frac{\tan x}{2} = \frac{\tan y}{3} = \frac{\tan z}{5} = K$$

$$\Rightarrow \tan x = 2K, \tan y = 3K \text{ and } \tan z = 5K$$

Put these values in Eq. (i), we get

$$\Rightarrow 2K + 3K + 5K = 2K \times 3K \times 5K$$

$$\Rightarrow 10K = 30K^3$$

$$\Rightarrow K^2 = \frac{1}{3}$$

$$\text{So, } \tan^2 x + \tan^2 y + \tan^2 z$$

$$= (2K)^2 + (3K)^2 + (5K)^2$$

$$= 4K^2 + 9K^2 + 25K^2$$

$$= K^2 (38)$$

$$= \frac{38}{3} \left(\because K^2 = \frac{1}{3} \right)$$

6. The circles whose equations are $x^2 + y^2 + c^2 = 2ax$ and $x^2 + y^2 + c^2 - 2by = 0$ will touch one another externally if **[2018]**

$$(a) \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a^2}$$

$$(b) \frac{1}{c^2} + \frac{1}{a^2} = \frac{1}{b^2}$$

$$(c) \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$$

$$(d) \text{None of these}$$

Ans. (c)

Sol. Given equation of circle is

$$x^2 + y^2 - 2ax + c^2 = 0 \quad \dots(i)$$

On comparing to the standard equation of circle with

$$x^2 + y^2 + 2gx + 2fy + C_1 = 0$$

we get, $g = -a, f = 0, C_1 = c^2$

Here, centre $C_1 = (-g, -f) = (a, 0)$

and radius $(r_1) = \sqrt{g^2 + f^2 - C_1} = \sqrt{a^2 - c^2}$

Equation of another circle is

$$x^2 + y^2 - 2by + c^2 = 0 \quad \dots(ii)$$

Here, centre $C_2 = (0, b)$

and radius $(r_2) = \sqrt{b^2 - c^2}$

As we know that, two circles touch each other externally, if

$$C_1 C_2 = r_1 + r_2$$

$$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{a^2 - c^2} + \sqrt{b^2 - c^2}$$

Squaring both sides, we get

$$\Rightarrow a^2 + b^2 = a^2 - c^2 + b^2 - c^2 + 2\sqrt{a^2 - c^2} \sqrt{b^2 - c^2}$$

$$\Rightarrow 2c^2 = 2\sqrt{a^2 - c^2} \sqrt{b^2 - c^2}$$

$$\Rightarrow c^2 = \sqrt{a^2 - c^2} \sqrt{b^2 - c^2}$$

Again squaring both sides, we get

$$\Rightarrow c^4 = (a^2 - c^2)(b^2 - c^2)$$

$$\Rightarrow c^4 = a^2 b^2 - a^2 c^2 - b^2 c^2 + c^4$$

$$\Rightarrow a^2 c^2 + b^2 c^2 = a^2 b^2 \quad \dots(iii)$$

Divide Eq. (iii) by $a^2 b^2 c^2$, we get

$$\Rightarrow \frac{1}{b^2} + \frac{1}{a^2} = \frac{1}{c^2} \Rightarrow \frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

7. The locus of the orthocentre of the triangle formed by the lines $(1+p)x - py + p(1+p) = 0$, **[2018]**

$(1+q)(x-q) + q(1+q) = 0$ and $y = 0$ where $p \neq q$ is

(a) a hyperbola

(b) a parabola

(c) an ellipse

(d) a straight line

Ans. (*)

Sol. Given lines

$$(1+p)x - py + p(1+p) = 0 \quad \dots(i)$$

$$(1+q)(x-q) + q(1+q) = 0 \quad \dots(ii)$$

$$y = 0 \quad \dots(iii)$$

and

From Eq. (i)

$$py = (1+p)x + p(1+p)$$

$$\Rightarrow py = (1+p)(x+p) \quad \dots(iv)$$

From Eq. (ii)

$$x - q + q = 0 \quad [\text{divide by } (1+q)]$$

$$\Rightarrow x = 0 \quad \dots(v)$$

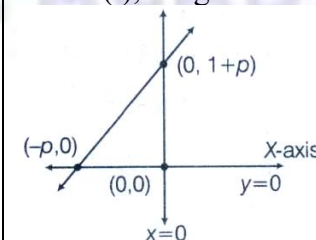
From Eqs. (iv) and (v), we get

$$py = (1+p)(0+p)$$

$$\Rightarrow y = (1+p)$$

So, intersecting point of lines (i) and (ii) is $(0, 1+p)$

Now, intersecting point of lines (i) and (iii), put $y = 0$ in line (i), we get



$$\Rightarrow (1+p)x - 0 + p(1+p) = 0$$

$$\Rightarrow x = -p$$

So, intersecting point $(-p, 0)$.

And intersecting point of lines (ii) and (iii) is $(0, 0)$.

So, orthocentre of the triangle formed by given line is

(0,0).

None option is correct.

Note Under the consideration of the given condition in question such as in line (ii) only one variable x is used doing solving the question. We get conclusion about the locus of the orthocentre is not match with option. So, we can say question has some conditional error.

8. Equation of a common tangent with positive slope to the circle $x^2 + y^2 - 8x = 0$ as well as to the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{4} = 1 \text{ is} \quad [2018]$$

- (a) $2x - \sqrt{5}y - 20 = 0$ (b) $2x - \sqrt{5}y + 4 = 0$
(c) $3x - 4y + 8 = 0$ (d) $4x - 3y + 4 = 0$

Ans. (b)

Sol. Let the equation of line $y = mx + c$

If Eq. (i) is tangent to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$

$$\therefore c = \pm \sqrt{9m^2 - 4} \quad [\because a = 3, b = 2]$$

So, equation of line (i) is

$$y = mx \pm \sqrt{9m^2 - 4} \quad \dots(ii)$$

It is also tangent to the circle $x^2 + y^2 - 8x = 0$ here, centre $c = (4, 0)$ and radius $(r) = 4$.

$\therefore$ Perpendicular distance from centre to the tangent = Radius

$$\therefore \frac{|4m \pm \sqrt{9m^2 - 4}|}{\sqrt{1 + m^2}} = 4$$

$$(4m \pm \sqrt{9m^2 - 4})^2 = 16(1 + m^2) \quad \dots(iii)$$

[squaring both sides]

By solving Eq. (iii), we get

$$m = \frac{2}{\sqrt{5}}$$

Put the value of $m = \frac{2}{\sqrt{5}}$ in Eq.(i)

$$\begin{aligned} \therefore y &= \frac{2}{\sqrt{5}}x \pm \sqrt{9 \times \frac{4}{5} - 4} \\ &= \frac{2}{\sqrt{5}}x \pm \sqrt{\frac{16}{5}} = \frac{2x}{\sqrt{5}} \pm \frac{4}{\sqrt{5}} \\ \Rightarrow \sqrt{5}y &= 2x \pm 4 \\ \Rightarrow 2x \pm 4 - \sqrt{5}y &= 0 \\ \Rightarrow 2x - \sqrt{5}y + 4 &= 0 \end{aligned}$$

9. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is [2018]

(a) $2/3$ sq. unit

(b) 1 sq. unit

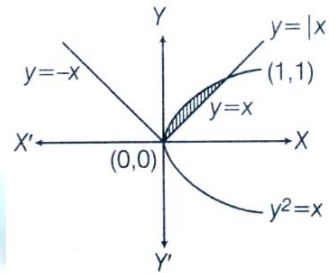
(c) $1/6$ sq. unit

(d) $1/3$ sq. unit

Ans. (c)

Sol. We have,

$$y^2 = x \text{ and } y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



The intersecting point of given curve is given by

$$\begin{aligned} x^2 &= x \\ \Rightarrow x^2 - x &= 0 \\ \Rightarrow x(x-1) &= 0 \\ \Rightarrow x &= 0, 1 \\ \Rightarrow y &= 0, 1 \end{aligned}$$

$\therefore$ Point of intersection are $(0,0)$ and $(1,1)$.

$$\begin{aligned} \therefore \text{Required area} &= \int_0^1 (\sqrt{x} - x) dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2} \right]_0^1 \\ &= \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ sq. unit} \end{aligned}$$

10. Equation of the line perpendicular to $x - 2y = 1$ and passing through $(1,1)$ is [2018]

- (a) $x + 2y = 3$ (b) $x + y = 2$
(c) $y = 2x + 3$ (d) $y = -2x + 3$

Ans. (d)

Sol. Given equation of line is $x - 2y = 1$

$$\therefore \text{Slope, } m = \frac{-1}{(-2)} = \frac{1}{2}$$

$$\left[\because \text{for a line } Ax + By + C = 0, \text{ Slope} = \frac{-A}{B} \right]$$

Since, required line is perpendicular to the given line,

$$\therefore \text{Slope of required line} = \frac{-1}{m} = -2$$

$\therefore$ Equation of line passing through $(1,1)$ and having slope -2 is given by

$$y - 1 = -2(x - 1)$$

[$\because$ equation of line is $y - y_1 = m(x - x_1)$]

$$\begin{aligned} \Rightarrow y - 1 &= -2x + 2 \\ \Rightarrow 2x + y &= 3 \\ \Rightarrow y &= -2x + 3 \end{aligned}$$

11. If $A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + \dots + x^{16}$,

then $f(A) =$ [2018]

- (a) 0 (b) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 5 \\ 1 & 1 \end{bmatrix}$

Ans. (b)

Sol. Given, $A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$

$$\text{and } f(x) = 1 + x + x^2 + \dots + x^{16}$$

$$\therefore f(A) = 1 + A + A^2 + \dots + A^{16} \text{ [put } x = A \text{]}$$

Put the value of $m = \frac{2}{\sqrt{5}}$ in Eq. (i)

$$\text{Now, } A^2 = A \cdot A$$

$$= \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = A^3 = \dots = A^{16} = 0$$

$$\text{Hence, } f(A) = I + A$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} \quad [\because |I| = 1]$$

$$= \begin{bmatrix} 1+0 & 0+5 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$$

12. 9 balls are to be placed in 9 boxes and 5 of the balls cannot fit into 3 small boxes. The number of ways of arranging one ball in each of the boxes is [2018]

- (a) 18720 (b) 18270 (c) 17280 (d) 12780

Ans. (c)

Sol.



Given that 5 balls cannot fit in small boxes.

So, we have only 4 balls to arrange in 3 small boxes.

Which can be arranged in ${}^4C_3 \times 3!$

Now, we have (6) balls left, which can be arranged in 6! ways.

Hence, the total number of ways

$${}^4C_3 \times 3! \times 6! = 4 \times 6 \times 720 = 17280$$

13. Which of the following functions is inverse of itself? [2018]

- (a) $f(x) = \frac{1-x}{1+x}$ (b) $f(x) = 3^{\log x}$
(c) $f(x) = 3^{x(x+1)}$ (d) None of these

Ans. (a)

Sol. As we know that any function will be inverse of itself if $f(f(x)) = x$.

By taking option (a) $f(x) = \frac{1-x}{1+x}$

$$= \frac{1 - \left(\frac{1-x}{1+x} \right)}{1 + \left(\frac{1-x}{1+x} \right)} = \frac{\frac{1+x-1+x}{1+x}}{\frac{1+x+1-x}{1+x}} = \frac{2x}{2}$$

$$\Rightarrow f(f(x)) = x$$

14. A student council has 10 members. From this one President, one Vice-President, one Secretary, one Joint-Secretary and two Executive Committee members have to be elected. In how many ways this can be done? [2018]

- (a) 151200 (b) 75600 (c) 37800 (d) 18900

Ans. (b)

Sol. Given that,

Student council has total numbers = 10

To select one President, one Vice-President, one Secretary, one Joint-Secretary and two Executive Committee members from 10 members.

Firstly, we have to select 6 members from 10 members.

Which can be done in total number of ways $= {}^{10}C_6$

But there are two Executive (repetition) members.

$\therefore$ That number of ways will be $= \frac{6!}{2!}$

Hence, total number of ways in which we can appoint

$$\text{three members} = {}^{10}C_6 \times \frac{6!}{2!}$$

$$= \frac{10!}{4!6!} \times \frac{6!}{2!} \left[\because {}^nC_r = \frac{n!}{(n-r)!r!} \right]$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{2} = 10 \times 9 \times 8 \times 7 \times 5 \times 3 = 75600$$

15. In a survey where 100 students reported which subjects they like, 32 students in total liked Mathematics, 38 students liked Business and 30 students liked Literature. Moreover 7 students liked both Mathematics and Literature, 10 students liked both Mathematics and Business, 8 students liked both Business and Literature, 5 students liked all three subjects. Then the number of people who liked exactly one subject is [2018]

- (a) 60 (b) 65 (c) 70 (d) 78

Ans. (b)

Sol. Let M, B and L be the set of students studying Mathematics, Business and Literature, respectively.

$$\therefore n(M) = 32, n(B) = 38, n(L) = 30$$

$$n(M \cap L) = 7, n(M \cap B) = 10, n(B \cap L) = 8 \text{ and } n(M \cap B \cap L) = 5$$

Hence, total number of students studying exactly one

$$\begin{aligned} \text{subject} &= n(M) + n(B) + n(L) - 2\{n(M \cap B) + n(B \cap L) \\ &\quad + n(M \cap L)\} + 3n(M \cap B \cap L) \\ &= 32 + 38 + 30 - 2(7 + 10 + 8) + 3(5) \\ &= 100 - 50 + 15 = 65 \end{aligned}$$

16. The number of natural numbers which are smaller than 2×10^8 and which contain only the digits 1 and 2 is **[2018]**

- (a) 786 (b) 666 (c) 766 (d) 1066

Ans. (c)

Sol. According to question, following cases arises nine digit number (less than 2×10^8) having only digits 1 and 2 = 2^8

Eight digit number (less than 2×10^8) having only digits 1 and 2 = 2^8

Similarly, Seven digit numbers = 2^7

Six digit numbers = 2^6 , Five digit numbers = 2^5 ,

Four digit numbers = 2^4 , Three digit numbers = 2^3 ,

Two digit numbers = 2^2 and One digit numbers = 2

So, number of all such natural numbers

$$\begin{aligned} &= 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^8 \\ &= 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 256 = 766 \end{aligned}$$

17. If $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the value of a

and b are

[2018]

- (a) $a \in \mathbb{R}, b = 2$ (b) $a = 1, b \in \mathbb{R}$
(c) $a \in \mathbb{R}, b \in \mathbb{R}$ (d) $a = 1, b = 2$

Ans. (b)

Sol. Given that, $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$

LHS is the form of 1^∞ , therefore

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow \infty} f(x)$$

$$\text{Thus, } \lim_{x \rightarrow \infty} \left(\frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(2a + \frac{2b}{x}\right) = e^2$$

$$\Rightarrow e^{(2a+0)} = e^2$$

On comparing exponent power both sides, we get
 $a = 1, b \in \mathbb{R}$

18. If $f(x) = \sin^5 x + \sin^3 x$ and $g(x) = \cos^5 x + \sin^3 x$, then the value of

$$\int_0^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) + g(-x)] dx \text{ is } \quad \textbf{[2018]}$$

- (a) 0 (b) > 1 (c) 0 and 1 (d) less than 0

Ans. (a)

Sol. Given,

$$f(x) = \sin^5 x + \sin^3 x$$

$$\begin{aligned} \text{and } f(-x) &= \sin^5(-x) + \sin^3(-x) \\ &= -\sin^5 x - \sin^3 x \\ &= -(\sin^5 x + \sin^3 x) \end{aligned}$$

$$\Rightarrow f(-x) = -f(x)$$

$$\Rightarrow f(-x) + f(x) = f(x) - f(x) = 0$$

$$\begin{aligned} \therefore \int_0^{\pi/2} [f(x) + f(-x)][g(x) + g(-x)] dx \\ = \int_0^{\pi/2} 0 \cdot [g(x) + g(-x)] dx = 0 \end{aligned}$$

19. $\frac{d^2 x}{dy^2}$ equals **[2018]**

- (a) $\left(\frac{d^2 y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2 y}{dx^2}\right)^{-1} \left(\frac{dy}{dx}\right)^{-3}$
(c) $\left(\frac{d^2 y}{dx^2}\right) \left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2 y}{dx^2}\right) \left(\frac{dy}{dx}\right)^{-3}$

Ans. (d)

Sol. Given,

$$\frac{d^2 x}{dy^2}$$

$$\therefore \frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$$

Differentiate both sides w.r.t y , we get

$$\Rightarrow \frac{d}{dy} \left(\frac{dx}{dy}\right) = \frac{d}{dy} \left\{ \left(\frac{dy}{dx}\right)^{-1} \right\}$$

$$\Rightarrow \frac{d^2 x}{dy^2} = \frac{d}{dx} \left\{ \left(\frac{dy}{dx}\right)^{-1} \right\} \frac{dx}{dy}$$

$$\Rightarrow \frac{d^2 x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-2} \frac{d}{dx} \left(\frac{dy}{dx}\right) \left(\frac{dy}{dx}\right)^{-1}$$

$$\Rightarrow \frac{d^2 x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-3} \left(\frac{d^2 y}{dx^2}\right)$$

20. Differential coefficient of $\log_{10} x$ w.r.t $\log_x 10$ is **[2018]**

- (a) $-\frac{(\log x)^2}{(\log 10)^2}$ (b) $\frac{(\log_{10} x)^2}{(\log 10)^2}$
(c) $\frac{(\log_x 10)^2}{(\log 10)^2}$ (d) $-\frac{(\log 10)^2}{(\log x)^2}$

Ans. (a)

Sol. Let $u = \log_{10} x$ and $v = \log_x 10$

$$\therefore u = \frac{\log_e x}{\log_e 10}, v = \frac{\log_e 10}{\log_e x}$$

$$\frac{du}{dx} = \frac{1}{\log_e 10} \cdot \frac{1}{x} \quad [\text{differentiating w.r.t. } x]$$

$$\text{and } \frac{dv}{dx} = \log_e 10 \cdot \frac{1}{(\log_e x)^2} \cdot \frac{1}{x}$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\frac{1}{\log_e 10} \cdot \frac{1}{x}}{\log_e 10 \cdot \frac{1}{(\log_e x)^2} \cdot \frac{1}{x}}$$

$$= -\frac{1}{\log_e 10} \cdot \frac{1}{x} \cdot \frac{x}{\log_e 10} (\log_e x)^2$$

$$= -\frac{(\log_e x)^2}{(\log_e 10)^2}$$

21. $f(x) = x + |x|$ is continuous for [2018]

- (a) $x \in (-\infty, \infty)$ (b) $x \in (-\infty, \infty) - \{0\}$
(c) Only $x > 0$ (d) No value of x

Ans. (a)

Sol. Given, $f(x) = x + |x|$

$$= \begin{cases} x+x, x \geq 0 \\ x-x, x < 0 \end{cases} = \begin{cases} 2x, x \geq 0 \\ 0, x < 0 \end{cases}$$

Here, we see that $f(x)$ is a polynomial functions, so it is continuous for every value of x except 0.

Now, we have to check the continuity only at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0^-} f(0-h) = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0^+} f(0+h) = \lim_{h \rightarrow 0^+} 2(0+h) = 0$$

$$\text{and } f(0) = 0$$

$$\therefore \text{LHL} = \text{RHL} = f(0)$$

So, $f(x)$ is continuous at $x = 0$ also.

Hence, $f(x)$ is continuous at $x \in (-\infty, \infty)$.

22. If a, b and c are unit vectors, then

$|a-b|^2 + |b-c|^2 + |c-a|^2$ does not exceed [2018]

- (a) 4 (b) 9 (c) 8 (d) 6

Ans. (b)

Sol. Given, $|a| = |b| = |c| = 1$

Since,

$$|a+b+c| = 0$$

$$0 \leq |a|^2 + |b|^2 + |c|^2 + 2(a \cdot b + b \cdot c + c \cdot a)$$

$$1+1+1+2(a \cdot b + b \cdot c + c \cdot a) > 0$$

$$(a \cdot b + b \cdot c + c \cdot a) > -\frac{3}{2}$$

$$|a-b|^2 + |b-c|^2 + |c-a|^2$$

$$= 2(|a|^2 + |b|^2 + |c|^2) - a \cdot b - b \cdot c - c \cdot a$$

$$\leq 2(1+1+1)+3$$

$$|a-b|^2 + |b-c|^2 + |c-a|^2 \leq 9$$

23. The vector $\vec{a} = \alpha \hat{i} + 2\hat{j} + \beta \hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then, which one of the following gives possible values of α and β ? [2018]

- (a) $\alpha = 2, \beta = 2$ (b) $\alpha = 1, \beta = 2$
(c) $\alpha = 2, \beta = 1$ (d) $\alpha = 1, \beta = 1$

Ans. (d)

Sol. We have, $\vec{a} = \alpha \hat{i} + 2\hat{j} + \beta \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$

Now, unit vector along angle bisector of $\vec{b}$ and $\vec{c}$ is

$$\vec{d} = \frac{1}{2} \left(\frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|} \right) = \frac{1}{2} \left(\frac{\hat{i} + \hat{j}}{\sqrt{1^2 + 1^2}} + \frac{\hat{j} + \hat{k}}{\sqrt{1^2 + 1^2}} \right)$$

$$= \frac{1}{2\sqrt{2}} (\hat{i} + 2\hat{j} + \hat{k})$$

Since, $\vec{a}$ is along $\vec{d}$,

$$\therefore \vec{a} = \lambda \vec{d}$$

$$\Rightarrow \alpha \hat{i} + 2\hat{j} + \beta \hat{k} = \frac{\lambda}{2\sqrt{2}} (\hat{i} + 2\hat{j} + \hat{k})$$

On comparing the coefficient of i, j and k , we get

$$\lambda = 2\sqrt{2}$$

$$\therefore \alpha = 1 \text{ and } \beta = 1$$

24. Forces $4\hat{i} - 3\hat{j} + 7\hat{k}$ and $-2\hat{i} + 2\hat{j} - 8\hat{k}$ are acting on a particle and displace it from the point $(5, 7, 1)$ to

$(2, 5, -6)$, then the work done by the force is [2018]

- (a) 25 (b) 9 (c) 15 (d) 7

Ans. (*)

Sol. Let

$$\vec{F}_1 = 4\hat{i} - 3\hat{j} + 7\hat{k}, \vec{F}_2 = -2\hat{i} + 2\hat{j} - 8\hat{k}$$

$\therefore$ Resulting force

$$\vec{F}_1 + \vec{F}_2 = (4\hat{i} - 3\hat{j} + 7\hat{k}) + (-2\hat{i} + 2\hat{j} - 8\hat{k})$$

$$= 2\hat{i} - \hat{j} - \hat{k}$$

Since, point is displace from $A(5, 7, 1)$ to $B(2, 5, -6)$

$$\therefore \vec{AB} = (2-5)\hat{i} + (5-7)\hat{j} + (-6-1)\hat{k}$$

$$= -3\hat{i} - 2\hat{j} - 7\hat{k}$$

$$\begin{aligned}\therefore \text{Work done} &= \vec{F} \cdot \vec{AB} \\ &= (2\hat{i} - \hat{j} - \hat{k}) \cdot (-3\hat{i} - 2\hat{j} - 7\hat{k}) \\ &= -6 + 2 + 7 = 3 \text{ units}\end{aligned}$$

No option match.

25. A bird is flying in a straight line with velocity vector $10\hat{i} + 6\hat{j} + \hat{k}$, measured in km/h. If starting points is $(1, 2, 3)$, how much time does it take to reach a point in space that is 13 m high from ground? [2018]

- (a) 600 seconds (b) 360 seconds
(c) 36 seconds (d) 60 seconds

Ans. (c)

Sol. Here, velocity vector of bird is

$$\vec{v} = 10\hat{i} + 6\hat{j} + \hat{k} \text{ or } |\vec{v}_x| = 10 \text{ km/h}$$

$$|\vec{v}_y| = 6 \text{ km/h}$$

$$|\vec{v}_z| = 1 \text{ km/h}$$

Initial position vector of the bird

$$\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$$

When bird reach a point in space that is 13 m high from the ground then final position vector of the bird will be

$$\vec{OB} = \hat{i} + 2\hat{j} + 13\hat{k}$$

$$\therefore \vec{AB} = \vec{OB} - \vec{OA} = 10\hat{k}$$

$$\text{or } |\vec{AB}| = 10 \text{ m}$$

Time taken by bird to cover a distance of 10 m along the vertical is

$$t = \frac{|\vec{AB}|}{|\vec{v}_z|} = \frac{10}{1 \times \frac{5}{18}} = \frac{10 \times 18}{5} = 36 \text{ seconds}$$

26. If A and B are two events and $P(A \cup B) = 5/6$,

$P(A \cap B) = 1/3$, $P(B) = 1/2$, then A and B are two

events which are [2018]

- (a) Dependent (b) Independent
(c) Mutually exclusive (d) Equally likely

Ans. (b)

Sol. Given that, $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and

$$P(B) = \frac{1}{2}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3}$$

$$\Rightarrow P(A) = \frac{5}{6} + \frac{1}{3} - \frac{1}{2}$$

$$= \frac{5+2-3}{6}$$

$$\Rightarrow P(A) = \frac{4}{6} = \frac{2}{3}$$

$$\therefore P(A) \cdot P(B) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3} = P(A \cap B)$$

$\therefore A$ and B are independent events.

27. If $a_1, a_2, a_3, \dots, a_n$ are positive real numbers whose product is a fixed number C , then the minimum value of $a_1 + a_2 + \dots + 2a_n$ is [2018]

- (a) $n(2C)^{1/n}$ (b) $(n+1)C^{1/n}$
(c) $2nC^{1/n}$ (d) $(n+1)(2C)^{1/n}$

Ans. (a)

Sol. We know that,

$$A \cdot M \geq G \cdot M$$

$$\therefore \frac{a_1 + a_2 + \dots + a_{n-1} + 2a_n}{n} \geq (a_1 a_2 \dots 2a_n)^{1/n}$$

$$\Rightarrow \frac{a_1 + a_2 + \dots + a_{n-1} + 2a_n}{n} \geq (2C)^{1/n} [\because a_1 a_2 a_3 \dots a_n = C]$$

$$\Rightarrow a_1 + a_2 + \dots + a_{n-1} + 2a_n \geq n(2C)^{1/n}$$

28. If a, b, c are the roots of the equation

$x^3 - 3x^2 + 3x + 7 = 0$, then the value of

$$\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} \text{ is [2018]}$$

- (a) 9 (b) 27 (c) 81 (d) 0

Ans. (d)

Sol. Given equation,

$$\left. \begin{aligned} x^3 - 3x^2 + 3x + 7 &= 0 \\ a + b + c &= 3 \\ ab + bc + ca &= 3 \\ abc &= -7 \end{aligned} \right\} \dots (i)$$

$$\text{Now, } \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & c & b \\ -b & a & c \\ -c & b & a \end{vmatrix} \\
 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \\
 = [a(bc - a^2) + b(ac - b^2) + c(ab - c^2)] \\
 = [3abc - a^3 - b^3 - c^3]^2 = 0 \quad [\text{using (i)}]$$

29. The coefficient of x^n in the expansion of $(1 - 2x + 3x^2 - 4x^3 + \dots \text{ to } \infty)^{-n}$ is **[2018]**

- (a) $\frac{(2n)!}{n!(n-1)!}$ (b) $\frac{(2n)!}{[(n-1)!]^2}$
 (c) $\frac{(2n)!}{(n!)^2}$ (d) None of these

Ans. (c)

Sol. $\therefore (1 - 2x + 3x^2 - 4x^3 + \dots \text{ to } \infty) = (1+x)^{-2}$
 [by Binomial expansion]

$$\therefore (1 - 2x + 3x^2 - 4x^3 + \dots \text{ to } \infty)^{-n} = (1+x)^{-2n}$$

Now, r^{th} term in the expansion of $(1+x)^{-2n}$
 $= {}^{2n}C_r (1)^{2n-r} (x)^r$

For x^n , put $r = n$

$\therefore$ Coefficient of x^n is ${}^{2n}C_n$

$$\text{and } {}^{2n}C_n = \frac{2n!}{(2n-n)!n!} = \frac{(2n)!}{(n!)^2}$$

30. Let α, β be the roots of the equation

$x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation

$x^2 - qx + r = 0$. Then, the value of r is **[2018]**

- (a) $\frac{2}{9}(p-q)(2q-p)$ (b) $\frac{2}{9}(q-p)(2q-p)$
 (c) $\frac{2}{9}(q-2p)(2q-p)$ (d) $\frac{2}{9}(2p-q)(2q-p)$

Ans. (d)

Sol. Given that, α, β are the roots of equation

$$x^2 - px + r = 0$$

$$\therefore \alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = p \quad \dots (i)$$

$$\text{and } \alpha\beta = \frac{\text{constant}}{\text{Coefficient of } x^2} = r$$

Also, $\frac{\alpha}{2}$ and 2β are roots of equation $x^2 - qx + r = 0$.

$$\therefore \frac{\alpha}{2} + 2\beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = q$$

$$\Rightarrow \alpha + 4\beta = 2q \quad \dots (ii)$$

Now, subtract Eq.(i) from Eq. (ii), we get

$$3\beta = 2q - p$$

$$\Rightarrow \beta = \frac{2q - p}{3}$$

Put the values of β in Eq. (i),

$$\Rightarrow \alpha = p - \frac{2q - p}{3} = \frac{3p - 2q + p}{3} = \frac{4p - 2q}{3}$$

$$= \frac{2}{3}(2p - q)$$

$$\therefore r = \alpha\beta$$

$$\Rightarrow r = \frac{2}{3}(2p - q) \cdot \frac{(2q - p)}{3}$$

$$= \frac{2}{9}(2p - q)(2q - p)$$

31. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is **[2018]**

- (a) 6/17 (b) 3/17 (c) 4/17 (d) 5/17

Ans. (a)

Sol. We have,

$$\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$$

$$= \cot\left[\tan^{-1}\frac{1}{\sqrt{\left(\frac{5}{3}\right)^2 - 1}} + \tan^{-1}\frac{2}{3}\right]$$

$$\left[\because \operatorname{cosec}^{-1}x = \tan^{-1}\frac{1}{\sqrt{x^2 - 1}}\right]$$

$$= \cot\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right]$$

$$= \cot\left[\tan^{-1}\left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \times \frac{2}{3}}\right)\right]$$

$$= \cot\left(\tan^{-1}\frac{17}{6}\right)$$

$$\cot\left(\cot^{-1}\frac{6}{17}\right) = \frac{6}{17} \left[\because \tan^{-1}x = \cot^{-1}\frac{1}{x}\right]$$

32. If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of

$\tan(\theta + \alpha) + 2\tan\alpha$ is **[2018]**

- (a) 3 (b) 2 (c) -1 (d) 0

Ans. (d)

Sol. We have,

$$\sin \theta = 3 \sin (\theta + 2\alpha)$$

$$\Rightarrow \frac{\sin (\theta + 2\alpha)}{\sin \theta} = \frac{1}{3}$$

Apply componendo and dividendo,

$$\Rightarrow \frac{\sin (\theta + 2\alpha) + \sin \theta}{\sin (\theta + 2\alpha) - \sin \theta} = \frac{1 + 3}{1 - 3}$$

$$\Rightarrow \frac{2 \sin \left(\frac{\theta + 2\alpha + \theta}{2} \right) \cos \left(\frac{\theta + 2\alpha - \theta}{2} \right)}{2 \cos \left(\frac{\theta + 2\alpha + \theta}{2} \right) \sin \left(\frac{\theta + 2\alpha - \theta}{2} \right)} = \frac{4}{-2}$$

$$\Rightarrow \frac{2 \sin (\theta + \alpha) \cos \alpha}{2 \cos (\theta + \alpha) \sin \alpha} = -2$$

$$\Rightarrow \frac{\tan (\theta + \alpha)}{\tan \alpha} = -2$$

$$\Rightarrow \tan (\theta + \alpha) = -2 \tan \alpha$$

$$\Rightarrow \tan (\theta + \alpha) + 2 \tan \alpha = 0$$

33. In a $\triangle ABC$, $\angle A = 90^\circ$ and D is mid-point of AC .

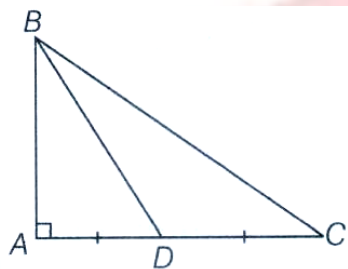
The value of $BC^2 - BD^2$ is equal to

[2018]

- (a) AD^2 (b) $2AD^2$ (c) $3AD^2$ (d) $4AD^2$

Ans. (c)

Sol. Given, D is mid-point of $\triangle ABC$ right angled at A .



In $\triangle ABC$,

$$BC^2 = AB^2 + AC^2 \quad \dots(i)$$

and in $\triangle ABD$

$$BD^2 = AB^2 + AD^2 \quad \dots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$\begin{aligned} BC^2 - BD^2 &= AB^2 + AC^2 - AB^2 - AD^2 \\ &= AC^2 - AD^2 \\ &= (2AD)^2 - AD^2 \quad [\because AC = 2AD] \\ &= (2AD)^2 - AD^2 \\ &= 4AD^2 - AD^2 = 3AD^2 \end{aligned}$$

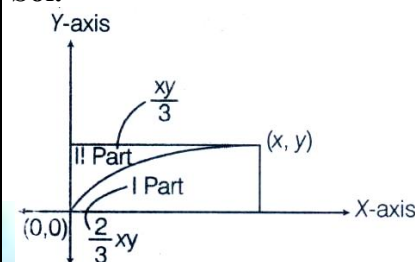
34. Through any point (x, y) of a curve which passes through the origin, lines are drawn parallel to the coordinate axes. The curve given that divides the rectangle formed by the two lines and the axes into two

areas, one of the which is twice the other, represents a family of [2018]

- (a) circles (b) parabolas
(c) hyperbolas (d) straight lines

Ans.(b)

Sol.



Area of rectangle $= xy$

Area of first part $= 2 \times$ Area of II part

$$\therefore \int_0^x y dx = \frac{2xy}{3}$$

$$\Rightarrow y = \frac{2}{3} \left[y + x \frac{dy}{dx} \right]$$

$$\Rightarrow \frac{y}{3} = \frac{2x}{3} \frac{dy}{dx}$$

$$\Rightarrow y = 2x \frac{dy}{dx}$$

$$\Rightarrow \frac{dx}{x} = \frac{2dy}{y}$$

$$\Rightarrow \log x + \log c = 2 \log y$$

$$\Rightarrow \log cx = \log y^2$$

$$\Rightarrow y^2 = cx$$

$\Rightarrow$ Which is a parabola.

35. A line through $(4, 2)$ meets the coordinate axes at P and Q . Then the locus of the circumcentre of $\triangle OPQ$ is [2018]

(a) $\frac{1}{x} + \frac{1}{y} = 2$ (b) $\frac{2}{x} + \frac{1}{y} = 1$

(c) $\frac{1}{x} + \frac{2}{y} = 1$ (d) $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$

Ans.(b)

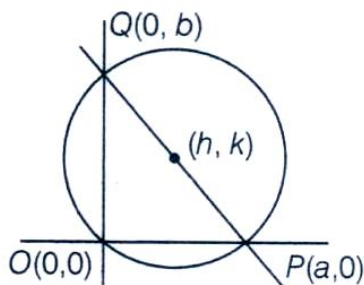
Sol. Let the equation of line having intercept a, b , on coordinates axes is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots(i)$$

$\therefore$ This intercept line meet the coordinate axes at $P(a, 0)$ and $Q(0, b)$, respectively.

Since, Eq. (i) passes through $(4, 2)$

$$\therefore \frac{4}{a} + \frac{2}{b} = 1 \quad \dots(ii)$$



Since, (h, k) is circumcentre of $\triangle OPQ$. Therefore (h, k) is the mid-point of PQ .

$$\therefore h = \frac{0+a}{2} = \frac{a}{2} \text{ and } k = \frac{0+b}{2} = \frac{b}{2} \quad \dots(iii)$$

$$\Rightarrow a = 2h \text{ and } b = 2k$$

$\therefore$ From Eq. (ii) and Eq. (iii), we get

$$\frac{4}{2h} + \frac{2}{2k} = 1$$

$$\Rightarrow \frac{2}{h} + \frac{1}{k} = 1$$

$$\therefore \text{Locus of the circumcentre is } \frac{2}{x} + \frac{1}{y} = 1$$

$$36. 6 + \log_{\frac{1}{4}} \frac{1}{\sqrt{2}} \left[\sqrt{1 - \frac{1}{\sqrt{2}}} \sqrt{1 - \frac{1}{\sqrt{2}}} \sqrt{1 - \frac{1}{\sqrt{2}}} \dots \right] =$$

[2018]

$$(a) 6 \quad (b) \frac{13}{2} \quad (c) 4 \quad (d) \frac{25}{4}$$

Ans.(b)

Sol. Given that,

$$6 + \log_{\frac{1}{4}} \frac{1}{\sqrt{2}} \left[\sqrt{1 - \frac{1}{\sqrt{2}}} \sqrt{1 - \frac{1}{\sqrt{2}}} \sqrt{1 - \frac{1}{\sqrt{2}}} \dots \right] \quad \dots(i)$$

$$\text{Let } y = \frac{1}{\sqrt{2}} \sqrt{1 - \frac{1}{\sqrt{2}}} \sqrt{1 - \frac{1}{\sqrt{2}}} \dots (y > 0)$$

Squaring both sides, we get

$$\Rightarrow 2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$$

$$\Rightarrow y = \frac{-1 \pm \sqrt{1+8}}{4}$$

$$\therefore = \frac{-1+3}{4}, \frac{-1-3}{4} = \frac{1}{2}, -1$$

$$y > 0 \Rightarrow y = \frac{1}{2}$$

So, putting the value of y in Eq. (i),

$$6 + \log_{\frac{1}{4}} \frac{1}{2} = 6 + \log_{\left(\frac{1}{2}\right)^2} \frac{1}{2}$$

$$= 6 + \frac{1}{2} = \frac{12+1}{2} = \frac{13}{2}$$

37. The function $f(x) = \log(x + \sqrt{x^2 + 1})$ is [2018]

- (a) an even function
- (b) an odd function
- (c) a periodic function
- (d) Neither an even nor an odd function

Ans.(b)

Sol. Given that,

$$f(x) = \log(x + \sqrt{x^2 + 1})$$

$$\therefore f(-x) = \log(-x + \sqrt{x^2 + 1})$$

$$= \log \left(\frac{\sqrt{x^2 + 1} - x}{1} \times \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1} + x} \right)$$

[by rationalisation]

$$= \log \left(\frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + x} \right)$$

$$\log(x + \sqrt{x^2 + 1})^{-1}$$

$$= -\log(x + \sqrt{x^2 + 1})$$

$$\therefore f(-x) = -f(x)$$

Hence, $f(x)$ is an odd function.

38. Two persons A and B agree to meet on 20th April 2018 between 6 P.M. and 7 P.M. with the understanding that they will wait no longer than 20 minutes for the other. What is the probability that they meet? [2018]

$$(a) 5/9 \quad (b) 7/9 \quad (c) 2/9 \quad (d) 4/9$$

Ans.(a)

Sol. There are total minutes between 6 P.M to 7 P.M = 60 min.

Probability of meeting longer than 20 minutes of 2 persons

$$= \left(\frac{40}{60} \right)^2 = \frac{4}{9}$$

So, the probability of meeting no longer than 20

$$\text{minutes will be } = 1 - \frac{4}{9} = \frac{5}{9}$$

39. Three numbers a, b and c are chosen at random (without replacement) from among the numbers 1, 2, 3, 99. The probability that $a^3 + b^3 + c^3 - 3abc$ is divisible by 3 is [2018]

$$(a) \frac{3 \cdot {}^{33}C_3 + ({}^{33}C_1)^3}{{}^{99}C_3} \quad (b) \frac{3 \cdot {}^{33}C_3 - ({}^{33}C_1)^3}{{}^{99}C_3}$$

$$(c) \frac{2 \cdot {}^{33}C_3 + ({}^{33}C_1)^3}{{}^{99}C_3} \quad (d) \frac{2 \cdot {}^{33}C_3 - ({}^{33}C_1)^3}{{}^{99}C_3}$$

Ans.(a)

Sol. Total number of ways of choosing 3 numbers from the given set of numbers $1, 2, \dots, 99 = {}^{99}C_3$

Let us divide the given numbers into 3 groups G_1, G_2, G_3 as follows.

$G_1 : 3, 6, 9, \dots, 99$ [33 elements]

$G_2 : 1, 4, 7, \dots, 97$ [33 elements]

$G_3 : 2, 5, 8, \dots, 98$ [33 elements]

Given that $a^3 + b^3 + c^3 - 3abc$ is to be divisible by 3 .
If $a^3 + b^3 + c^3 - 3abc$ is divisible by 3 , then which is possible in the following cases.

- (i) All the numbers belong to the first group.
- (ii) All the numbers belong to the second group
- (iii) All the numbers belong to the third group
- (iv) One number belong to the first group, one belong to second group and one belong to the third group.

So, favourable number of cases are $3 \cdot {}^{33}C_3 + ({}^{33}C_1)^3$

$$\therefore \text{Required probability} = \frac{3 \cdot {}^{33}C_3 + ({}^{33}C_1)^3}{{}^{99}C_3}$$

40. A and B play a game where each is asked to select a number from 1 to 25 . If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is [2018]

- (a) $\frac{1}{25}$ (b) $\frac{24}{25}$ (c) $\frac{2}{25}$ (d) None of these

Ans.(b)

Sol. There are A and B have to select as number from set of numbers $\{1, 2, \dots, 25\}$.

According to given condition, they will win prize if they choose the same numbers, i.e., then probability of choosing same numbers is $\frac{1}{25}$.

Hence, the probability that they will not win a prize in a single trial $= 1 - \frac{1}{25} = \frac{25-1}{25} = \frac{24}{25}$

41. The quadratic equation whose roots are $\sin^2 18^\circ$ and $\cos^2 36^\circ$ is [2018]

- (a) $16x^2 - 12x + 1 = 0$ (b) $16x^2 + 12x + 1 = 0$
- (c) $16x^2 - 12x - 1 = 0$ (d) $16x^2 + 10x + 1 = 0$

Ans. (a)

Sol. Given that roots are $\sin^2 18^\circ$ and $\cos^2 36^\circ$.

$$\therefore \text{Sum of roots} = \sin^2 18^\circ + \cos^2 36^\circ$$

$$= \left(\frac{\sqrt{5}-1}{4} \right)^2 + \left(\frac{\sqrt{5}+1}{4} \right)^2$$

$$= \frac{5+1-2\sqrt{5}+5+1+2\sqrt{5}}{16} = \frac{12}{16} = \frac{3}{4}$$

Product of roots $= (\sin^2 18^\circ) \times (\cos^2 36^\circ)$

$$= \left(\frac{\sqrt{5}-1}{4} \right)^2 \times \left(\frac{\sqrt{5}+1}{4} \right)^2 = \frac{1}{16}$$

Hence, required equation will be

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$$

$$\Rightarrow x^2 - \frac{3}{4}x + \frac{1}{16} = 0$$

$$\Rightarrow 16x^2 - 12x + 1 = 0$$

42. Sum to infinity of a geometric progression is twice the sum of the first two terms. Then, what are the possible values of the common ratio? [2018]

- (a) $\pm \frac{1}{\sqrt{2}}$ (b) $\pm \frac{1}{2}$ (c) $\pm \frac{1}{\sqrt{3}}$ (d) $\pm \frac{1}{3}$

Ans.(a)

Sol. According to question,

$$\text{G.M} = 2(a + ar)$$

$$\Rightarrow \frac{a}{1-r} = 2(a + ar)$$

$$\Rightarrow \frac{1}{1-r} = 2(1+r)$$

$$\Rightarrow (1+r)(1-r) = \frac{1}{2}$$

$$\Rightarrow 1-r^2 = \frac{1}{2}$$

$$\Rightarrow r^2 = \frac{1}{2}$$

$$\Rightarrow r = \pm \frac{1}{\sqrt{2}}$$

43. Suppose that m and n are fixed numbers such that the m^{th} term a_m is equal to n and the n^{th} term a_n is equal to $m(m \neq n)$, then the $(m+n)^{\text{th}}$ term is [2018]

- (a) $(m+n)/mn$ (b) $mn/(m+n)$
- (c) $(m+n)/n$ (d) $(m+n)/m$

Ans.(b)

Sol. Given that, m^{th} term of H.P

$$\frac{1}{n} = \frac{1}{a} + (m-1)d \quad \dots(i)$$

and n^{th} term $\frac{1}{m} = \frac{1}{a} + (n-1)d \quad \dots(ii)$

Subtract Eq. (ii) from Eq. (i), we get

$$\Rightarrow \frac{1}{n} - \frac{1}{m} = \frac{1}{a} + (m-1)d - \frac{1}{a} - (n-1)d$$

$$\Rightarrow \frac{m-n}{mn} = d[m-1-n+1]$$

$$\Rightarrow d = \frac{1}{mn}$$

Put the value of d in Eq. (i)

$$\frac{1}{n} = \frac{1}{a} + \frac{m-1}{mn}$$

$$\frac{1}{a} = \frac{1}{n} - \frac{(m-1)}{mn} = \frac{m-m+1}{mn} = \frac{1}{mn}$$

$$a = mn$$

$$T_{m+n} = \frac{1}{\frac{1}{mn} + (m+n-1)\frac{1}{mn}}$$

$$= \frac{1}{\frac{1+m+n-1}{mn}} = \frac{mn}{m+n}$$

44. If A is an invertible skew symmetric matrix, then A^{-1} is a [2018]

- (a) symmetric matrix (b) skew-symmetric matrix
(c) zero matrix (d) identity matrix

Ans. (b)

Sol. We have, A is skew-symmetric matrix.

$$\therefore A^T = -A$$

$$\Rightarrow (A^T)^{-1} = (-A)^{-1}$$

$$\Rightarrow (A^{-1})^T = -A^{-1}$$

$\therefore A^{-1}$ is also skew-symmetric matrix.

45. If the mean of the squares of first n natural numbers be 11, then n is equal to [2018]

- (a) $\frac{-13}{2}$ (b) 11 (c) 5 (d) 4

Ans. (c)

Sol. Given that,

Mean [First n natural number] = 11

$$\Rightarrow \frac{n(n+1)(2n+1)}{6n} = 11$$

$$\Rightarrow (n+1)(2n+1) = 66$$

$$\Rightarrow 2n^2 + 3n + 1 = 66$$

$$\Rightarrow 2n^2 + 3n - 65 = 0$$

$$\Rightarrow n = \frac{-3 \pm \sqrt{9+520}}{4}$$

$$\Rightarrow = \frac{-3 \pm \sqrt{529}}{4}$$

$$\text{As, } = \frac{-3 \pm 23}{4} = \frac{20}{4} = 5$$

$$\therefore n = \frac{-3-23}{4} = \frac{-26}{4} [\because n \text{ cannot be negative}]$$

$$\therefore n = 5$$

46. The set of points, where $f(x) = \frac{x}{1+|x|}$ is

differentiable is

[2018]

- (a) $(-\infty, -1) \cup (-1, -\infty)$ (b) $(-\infty, \infty)$
(c) $(0, \infty)$ (d) $(-\infty, 0) \cup (0, \infty)$

Ans. (b)

Sol. We have,

$$f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x > 0 \end{cases}$$

Given function f is differentiable at all points except possible at $x=0$.

$$\therefore f'(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0^-} \frac{\frac{x}{1-x}}{1-x} = \lim_{x \rightarrow 0^-} \frac{1}{x}$$

$$Rf'(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$\therefore \lim_{x \rightarrow 0^+} \frac{\left(\frac{x}{1+x}\right)}{x} = \lim_{x \rightarrow 0^+} \frac{1}{1+x} = 1$$

$$Lf'(0) = Rf'(0) = 1,$$

we get

f is differentiable at $x=0$.

Thus, f is differentiable for all $x \in (-\infty, \infty)$.

47. $\int_0^\pi xf(\sin x) dx$ is equal to [2018]

- (a) $\pi \int_0^\pi f(\sin x) dx$ (b) $\frac{\pi}{2} \int_0^\pi f(\sin x) dx$
 (c) $\pi \int_0^{\pi/2} f(\cos x) dx$ (d) $\pi \int_0^\pi f(\cos x) dx$

Ans.(c)

Sol. Given, $I = \int_0^\pi xf(\sin x) dx$

$$= \int_0^\pi (\pi - x) f(\sin(\pi - x)) dx$$

$$= \int_0^\pi (\pi - x) f(\sin x) dx$$

$$= \int_0^\pi \pi f(\sin x) dx - \int_0^\pi xf(\sin x) dx$$

$$= \pi \int_0^\pi f(\sin x) dx - I$$

$$\Rightarrow 2I = \pi \int_0^{\pi/2} [f(\sin x) + f(\sin(\pi - x))] dx$$

$$= 2\pi \int_0^{\pi/2} f(\sin x) dx$$

$$\Rightarrow I = \pi \int_0^{\pi/2} f(\sin(\frac{\pi}{2} - x)) dx$$

$$\Rightarrow I = \pi \int_0^{\pi/2} f(\cos x) dx$$

48. Let $f: R \rightarrow R$ be defined by

$$f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ |x-2|, & \text{if } x \geq 0 \end{cases} \text{ Find } \int_{-2}^3 f(x) dx. [2018]$$

- (a) 0.5 (b) 2.5 (c) 4.5 (d) 6.5

Ans.(c)

Sol. Given, $f(x) = \begin{cases} x+2; x < 0 \\ -(x-2); 0 \leq x < 2 \\ x-2; x \geq 2 \end{cases}$

$$f(x) = \begin{cases} x+2; x < 0 \\ 2-x; 0 \leq x < 2 \\ x-2; x \geq 2 \end{cases}$$

$$\int_{-2}^0 f(x) dx + \int_0^2 f(x) dx + \int_2^3 f(x) dx$$

$$= \int_{-2}^0 (x+2) dx + \int_0^2 (2-x) dx + \int_2^3 (x-2) dx$$

$$= \left[\frac{x^2}{2} + 2x \right]_{-2}^0 + \left[2x - \frac{x^2}{2} \right]_0^2 + \left[\frac{x^2}{2} - 2x \right]_2^3$$

$$= \left[0 - \left(\frac{4}{2} - 4 \right) \right] + \left[\left(4 - \frac{4}{2} \right) - 0 \right] + \left[\left(\frac{9}{2} - 6 \right) - \left(\frac{4}{2} - 4 \right) \right]$$

$$= 2 + 2 - \frac{3}{2} + 2 = 6 - \frac{3}{2} = \frac{9}{2} = 4.5$$

49. Slope of two lines $6x^2 - 2xy - 2y^2 = 0$ differ by

[2018]

- (a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{7}{5}$ (d) -1

Ans. (b)

Sol. Let two slopes of lines may be m_1 and m_2 .

$$\therefore m_1 + m_2 = \frac{-2h}{b} = \frac{2}{(-2)} = -1$$

and $m_1 m_2 = \frac{a}{b} = \frac{6}{(-2)} = -3$

Now, $(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 m_2$

$$(m_1 - m_2)^2 = (-1)^2 - 4(-3)$$

$$\therefore (m_1 - m_2)^2 = 1 + 12 = 13$$

$$m_1 - m_2 = \sqrt{13} = 3.60 = \frac{7}{2} \text{ (approx.)}$$

50. If the radius of a circle changes at the rate of $\frac{-2}{\pi}$ m/s at what rate does the circle's area change

when the radius is 10 m ?

[2018]

- (a) 40 m²/s (b) 30 m²/s
 (c) -30 m²/s (d) -40 m²/s

Ans.(d)

Sol. Given that, $\frac{dr}{dt} = \frac{-2}{\pi}$ m/s

We know that,

$$\text{Area of circle} = \pi r^2$$

$$\therefore A = \pi r^2$$

Differentiate w.r.t to t, we get

$$\frac{dA}{dt} = \pi(2r) \cdot \frac{dr}{dt}$$

$$= 2\pi r \left(\frac{-2}{\pi} \right) \text{ m}^2/\text{s} = -\frac{4\pi}{\pi} (10) \text{ m}^2/\text{s} \text{ [given, } r=10 \text{]} = -40 \text{ m}^2/\text{s}$$

Analytical Ability and Logical Reasoning

51. Decide which of the assumptions is implicit in the statement and choose your answer accordingly.

Statement "Buy pure and natural honey of company X." - An advertisement in a newspaper.

Assumptions I. Artificial honey can be prepared.
 II. People do not mind paying more for pure and natural honey.

III. No other company supplies pure honey. [2018]

- (a) Only I is implicit (b) I and II are implicit
 (c) I and III are implicit (d) All are implicit

Ans.(a)

Sol. Artificial honey can be prepared as the word natural is mentioned in the advertisement. No

comparison regarding the price of natural honey is given.

Also, no information regarding other companies is given. Hence, only assumption I is implicit.

52. Choose the conclusion which logically follows from the given statement(s).

Statement All scientists working in America are talented. Some Indian scientists are working in America.

Conclusions

1. None of Indian scientists is talented.
2. Some talented Indian scientists have migrated to America.
3. All talented scientists are Indians.
4. Some Indian scientists are talented.

The conclusion(s) correctly drawn is/are **[2018]**

- (a) 1 only (b) 2 only (c) 2 and 3 (d) 2 and 4

Ans.(d)

Sol. From the given statement it is clear the some talented Indian Scientist have migrated to America and also some Indian Scientists are talented.

Hence, conclusion (2) and (4) are true.

53. One New York publisher has estimated that 50,000 to 60,000 people in the United States want an anthology that includes the complete works of William Shakespeare. And what accounts for this renewed interest in Shakespeare? As scholars point out, his psychological insights into both male and female characters are amazing even today.

This paragraph best supports the statement that **[2018]**

- (a) Shakespeare's characters are more interesting than fictional characters today
(b) People even today are interested in Shakespeare's work because of the characters
(c) Academic scholars are putting together an anthology of Shakespeare's work
(d) New Yorkers have a renewed interested in the work of Shakespeare

Ans.(b)

Sol. Clearly, the paragraph best supports the statement (b).

54. A runs $1\frac{2}{3}$ times as fast as B. If A gives B start of 80 m, how far must the winning post be so that A and B might reach it at the same time? **[2018]**

- (a) 200 m (b) 400 m (c) 300 m (d) 160 m

Ans.(a)

Sol. A runs $1\frac{2}{3}$ times as fast as B.

Hence, ratio of speed of A and B = 5 : 3 or we can say that A moves 5 m in the same time in which B moves 3 m.

Hence, A gains 2 m over B by running 5 m.

$$\therefore \text{Required distance} = \frac{5}{2} \times 80 = 200 \text{ m}$$

55. Two men and three boys can do piece of work in ten days; while three men and two boys can do the same work in eight days. In how many days can two men and one boy do the work? **[2018]**

- (a) 12.5 (b) 9 (c) 9.5 (d) 8.5

Ans.(a)

Sol. Let the efficiency of one man be 'M' and of one boy be 'B'.

According to the question,

$$\begin{aligned} (2M + 3B)10 &= (3M + 2B)8 \\ \Rightarrow 20M + 30B &= 24M + 16B \\ \Rightarrow 14B &= 4M \\ \text{or } 2M &= 7B \end{aligned}$$

or We can say 10 boys can complete the work in 10 days. Now, time taken to complete the work by 2 men and 1 boy = time taken to complete the work by 8 boys.

Now, 10 boys can do work in 10 days

$$\therefore 8 \text{ boys can do it in } \frac{10 \times 10}{8} = 12.5 \text{ days}$$

56. How many numbers between 1 and 1000 are divisible by all numbers 2, 3, 4, 5 and 6? **[2018]**

- (a) 16 (b) 32 (c) 17 (d) 33

Ans.(a)

Sol. LCM of 2, 3, 4, 5 and 6 = 60

Hence, Number between 1 and 1000 which are divisible by 60 = 16

$$\text{as } 1000 = 60 \times 16 + 40$$

57. There are 8436 steel balls, each with a radius of 1 cm, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth and so on. The number of horizontal layers in the pile is **[2018]**

- (a) 34 (b) 38 (c) 36 (d) 32

Ans.(c)

Sol. Number of balls in 1st layer = 1

Number of balls in 2nd layer = 3 = 1 + 2

Number of balls in 3rd layer = 6 = 1 + 2 + 3

Number of balls in 4th layer = 10 = 1 + 2 + 3 + 4

$$\text{Now, the } n\text{th term} = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$\text{Sum of } n \text{ terms of series} = \frac{1}{2} [\Sigma n^2 + \Sigma n]$$

$$= \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right] = \frac{n(n+1)(n+2)}{6}$$

Now,

$$\frac{n(n+1)(n+2)}{6} = 8436$$

$$n(n+1)(n+2) = 50616$$

Or The above condition is satisfied only for $n = 36$

58. A body travels from A to B in 10 s with a speed of 50 km/h and returns with a speed of 100 km/h in 5 s. The average speed and the average velocity for the whole journey is [2018]

- (a) $17.5 \text{ ms}^{-1}, 0 \text{ km/h}$ (b) $16.5 \text{ ms}^{-1}, 0 \text{ km/h}$
(c) $15.5 \text{ ms}^{-1}, 0 \text{ km/h}$ (d) $18.5 \text{ ms}^{-1}, 0 \text{ km/h}$

Ans.(d)

Sol. Speed while travelling from A to B = 50 km/h

Speed while travelling from B to A = 100 km/h

$\therefore$ Required average speed

$$= \frac{2 \times 50 \times 100}{50 + 100} \times \frac{1000}{3600} = 18.5 \text{ m/s}$$

Now, average velocity is 0 km/h as displacement is zero.

59. Find the odd one out. [2018]

- (a) DEHG (b) RSVU (c) JK NM (d) LMQP

Ans.(d)

Sol. Here,

D $\xrightarrow{+1}$ E $\xrightarrow{+3}$ H $\xrightarrow{-1}$ G
R $\xrightarrow{+1}$ S $\xrightarrow{+3}$ V $\xrightarrow{-1}$ U
J $\xrightarrow{+1}$ K $\xrightarrow{+3}$ N $\xrightarrow{-1}$ M
L $\xrightarrow{+1}$ M $\xrightarrow{+4}$ Q $\xrightarrow{-1}$ P

Hence, L M Q P is the odd one.

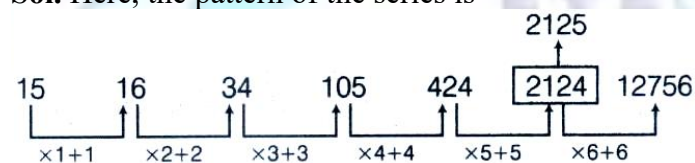
60. Find out the wrong number in the following series. [2018]

15, 16, 34, 105, 424, 2124, 12756

- (a) 16 (b) 34 (c) 424 (d) 2124

Ans.(d)

Sol. Here, the pattern of the series is



Hence, 2124 is the wrong number.

61. My mother is twice as old as my brother. I am five years younger to my brother but three years older to my sister. If my sister is twelve years of age, how old is my mother? [2018]

- (a) 24 (b) 30 (c) 40 (d) 50

Ans.(c)

Sol. Age of man's sister = 12yr

$\therefore$ Age of man = $12 + 3 = 15$ yr

$\therefore$ Age of man's brother = $15 + 5 = 20$ yr

Hence, age of man's mother = $20 \times 2 = 40$ yr

62. Find the missing term in the series.

3, 20, 63, 144, 275 _____ .

[2018]

- (a) 354 (b) 468 (c) 548 (d) 554

Ans.(b)

Sol. The pattern of the series is,

$$(1)^2 = 1 \times 3 = 3$$

$\downarrow +2$

$$(2)^2 = 4 \times 5 = 20$$

$\downarrow +2$

$$(3)^2 = 9 \times 7 = 63$$

$\downarrow +2$

$$(4)^2 = 16 \times 9 = 144$$

$\downarrow +2$

$$(5)^2 = 25 \times 11 = 275$$

$\downarrow +2$

$$(6)^2 = 36 \times 13 = 468$$

Directions (Q. Nos. 63 to 65) Questions are based on the following passage:

Nine individuals Z, Y, X, W, V, U, T, S and R are the only candidates who can serve on three committees A, B and C and each candidate should serve on exactly one of the committees.

Committee A should consist of exactly one member more than Committee B. It is possible that there are no members of Committee C. Among Z, Y and X none can serve on Committee A. Among W, V and U none can serve on Committee B. Among T, S and R none can serve on Committee C.

63. In case T and Z are the individuals serving on Committee B, how many of the nine individuals should serve on Committee C? [2018]

- (a) 3 (b) 4 (c) 5 (d) 6

Ans. (b)

Sol. Two members (T and Z) are serving committee B $\therefore$ Three members would serve committee A. and hence, 4 members would serve on committee C.

64. In case, T, S and X are the only individuals serving on committee B, the membership of Committee C should be [2018]

- (a) Z and Y (b) Z and W
(c) Y and V (d) X and V

Ans. (a)

Sol. T, S and X serve committee B.

$\therefore$ 4 members would serve committee A.

Now, Z, Y and X cannot serve committee A .
Hence, Z and Y serve on committee C .

65. In case R is the only individual serving on Committee B , which among the following should serve on Committee A ? [2018]

- (a) W and S (b) V and T
(c) U and S (d) T and S

Ans. (d)

Sol. R is the only individual serving committee B .
 $\therefore$ 2 members would serve committee A and the rest would serve committee C .

Now, T, S and R cannot serve on committee C .
Hence, T and S serve on committee A .

66. In a city, 40% of the adults are illiterate while 85% of the children are literate. If the ratio of the adults to that of the children is 2 : 3, then what per cent of the population is literate? [2018]

- (a) 20% (b) 25% (c) 50% (d) 75%

Ans. (d)

Sol. Let the total population be 100 .

$$\therefore \text{Number of adults} = \frac{2}{5} \times 100 = 40$$

and number of children = 60

Now, 60% of adults are literate.

$$\therefore \text{Number of literate adults} = \frac{60}{100} \times 40 = 24$$

$$\text{and number of literate children} = \frac{85}{100} \times 60 = 51$$

$$\therefore \text{Total number of literates} = 24 + 51 = 75$$

$$\therefore \text{Required percentage} = \frac{75}{100} \times 100 = 75\%$$

67. There are 50 students admitted to a nursery class. Some students can speak only English and some can speak only Hindi. 10 students can speak both English and Hindi. If the number of students who can speak English is 21, then how many students can speak Hindi, how many can speak only Hindi, and how many can speak only English? [2018]

- (a) 21, 11 and 29 respectively
(b) 28, 18 and 22 respectively
(c) 37, 27 and 13 respectively
(d) 39, 29 and 11 respectively

Ans. (d)

Sol. Total number of students = 50

Number of students who speak English = 21 and
number of students who speak both English and Hindi = 10

$$\therefore \text{Number of students who speak only English} = 21 - 10 = 11$$

$$\text{and number of students who speak Hindi} = 50 - 11 = 39$$

$$\text{and number of students who speak only Hindi} = 39 - 10 = 29$$

68. The last digit of the number obtained by multiplying the numbers $81 \times 82 \times 83 \times 84 \times 85 \times 86 \times 87 \times 88 \times 89$ will be [2018]
(a) 0 (b) 9 (c) 7 (d) 2

Ans. (a)

Sol. The last digit can be obtained by multiplying the last digits of the given numbers.

$$\text{Here, } 82 \times 85, \text{ i.e. } 2 \times 5 = 10$$

Hence, the last digit would be 0 .

69. It has been reported in recent years that a very large number of seats in the engineering colleges in the country remain vacant at the end of the admission session.

Which of the following may be the probable cause of the above effect? [2018]

- (a) There has been a considerable decrease in hiring of engineering graduates due to economic slowdown in the recent years
(b) Students have always preferred to complete graduation in three years time instead of four years for engineering
(c) The government has recently decided to provide post qualification professional training to all engineering graduates at its own cost
(d) None of the above

Ans. (a)

Sol. As there is a considerable decrease in hiring of engineering graduates, a large number of engineering seats remains vacant.

70. At what time, in minutes, between 30' clock and 40' clock, both the needles will coincide each other? [2018]

- (a) $5\frac{1}{11}$ (b) $12\frac{4}{11}$ (c) $13\frac{4}{11}$ (d) $16\frac{4}{11}$

Ans. (d)

Sol. At 3 O' clock both the hands of the clock are 15 min space apart.

To coincide, the minute hand must gain 15 min over the hour hand.

Now, 55 min are gained in 60 min .

$$\therefore 15 \text{ min would be gained in } \frac{60}{55} \times 15 = 16\frac{4}{11}$$

Hence, the needles will coincide after $16\frac{4}{11}$ min .

71. If $\log_3 2, \log_3 (2^x - 5), \log_3 \left(2^x - \frac{7}{2}\right)$ are in

arithmetic progression, then the value of x is equal to

[2018]

(a) 5 (b) 4 (c) 3 (d) 2

Ans.(c)

Sol. As, $\log_3 2, \log_3 (2^x - 5), \log_3 \left(2^x - \frac{7}{2}\right)$ are in arithmetic progression.

$$\therefore \log_3 (2^x - 5) - \log_3 2 = \log_3 \left(2^x - \frac{7}{2}\right) - \log_3 (2^x - 5)$$

$$= \log_3 \left(\frac{2^x - 5}{2}\right) = \log_3 \left(\frac{2^x - \frac{7}{2}}{2^x - 5}\right)$$

$$\left[\because \log \left(\frac{a}{b}\right) = \log a - \log b \right]$$

$$\Rightarrow (2^x - 5)(2^x - 5) = \left(2^x - \frac{7}{2}\right) \times 2$$

$$\Rightarrow (2^x - 5)^2 = 2^{x+1} - 7$$

$$\Rightarrow 2^{2x} + 25 - 2 \times 2^x \times 5 = 2^{x+1} - 7$$

$$\Rightarrow 2^{x+1} + 2^{x+1} \times 5 - 2^{2x} = 32$$

$$\Rightarrow 2^{x+1} (6) - 2^{2x} = 32$$

$$\Rightarrow 2^x (12 - 2^x) = 32$$

The above equation is satisfied for $x = 3$

72. Let n be the number of different 5 digit numbers, divisible by 4 that can be formed with the digits 1, 2, 3, 4, 5 and 6, with no digit being repeated. What is the value of n ?

[2018]

(a) 144 (b) 168 (c) 192 (d) 222

Ans.(c)

Sol. For a number to be divisible by 4 the last two digits must be divisible by 4.

$\therefore$ The possible cases are

----12 ----16

----24 ----32

----36 ----52

----56 ----64

$$\therefore \text{Value of } n = 4 \times 3 \times 2 \times 1 \times 8 = 192$$

73. Let S be the set of integers X such that

i. $100 \leq x \leq 200$ ii. x is odd andiii. x is divisible by 3 but not by 7How many elements does S contain?

[2018]

(a) 16 (b) 12 (c) 11 (d) 13

Ans.(d)

Sol. First number divisible by 3 between 100 and 200 = 102

Last number divisible by 3 between 100 and 200 = 198

Total numbers divisible by 3 between 100 and

200 = 33

Out of which 105, 126, 147, 168 and 189 are divisible by 7 and from remaining numbers

102, 108, 114, 120, 132, 138,

144, 150, 156, 162, 174, 180, 186, 192 and 198 are even numbers.

$$\therefore x = 33 - 20 = 13$$

74. Two pipes A and B can fill a cistern in 37.5 min and 45 min, respectively. Both pipes are opened. The cistern will be filled in just half an hour, if the pipe B is turned off after

[2018]

(a) 5 min- (b) 9 min (c) 10 min (d) 15 min

Ans.(b)

Sol. Pipe A can fill a cistern in 37.5 min

$$\therefore \text{Part of cistern filled by pipe A in 1 min} = \frac{1}{37.5}$$

$$\text{Similarly, part of cistern filled by pipe B in 1 min} = \frac{1}{45}$$

Now, the cistern will be filled in half an hour if pipe B is turned off after some time.

Let pipe B is turned off after x min.

$$\therefore \frac{30}{37.5} + \frac{x}{45} = 1$$

$$0.8 + \frac{x}{45} = 1$$

$$x = 45 \times 0.2 = 9 \text{ min}$$

75. Three persons A, B and C wear shirts of Black, Blue and Orange colours (not necessarily in the order) and pants of Green, Yellow and Orange (not necessarily in that order). No person wore shirt and pants of the same colour. Further, it is given that,

1. A did not wear shirt of Black colour.

2. B did not wear shirt of Blue colour.

3. C did not wear shirt of Orange colour.

4. A did not wear the pants of Green colour.

5. B wore pants of Orange colour.

What were the colours of the pants and shirt worn by C respectively?

[2018]

(a) Orange and Black

(b) Green and Blue

(c) Yellow and Blue

(d) Yellow and Black

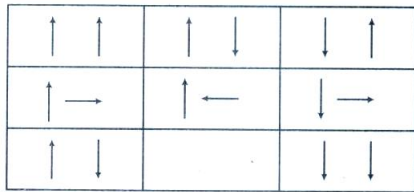
Ans.(b)

Sol. According to the given information,

Person	Shirt's Colour	Pant's Colour
A	Orange	Yellow
B	Black	Orange
C	Blue	Green

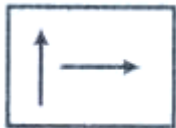
Clearly, C wore pant and shirt of Green and Blue colour, respectively.

76.



Which one of the following figures fits into blank part of the above matrix? [2018]

(a)



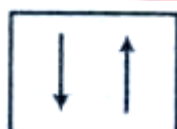
(b)



(c)



(d)



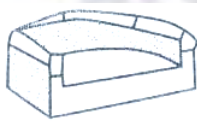
Ans.(b)

Sol. In each column the element on the left hand side remains the same while the element on the right rotates 90° clockwise in each step.

77. Choose the picture that would go in the empty box so that the two bottom pictures are related in the same way as the top two are related. [2018]



(1)



(2)



(3)



(4)

(a) 1 (b) 2 (c) 3 (d) 4

Ans.(b)

Sol. T-shirt and shoes both items are worn on the body. Similarly, rack and sofa are kept inside the house.

78. Decide which of the given conclusions logically follow from the given statement(s).

Statements Some codes are secrets.
All secrets are puzzles.

Conclusions

I. All secrets being codes is a possibility.

II. Atleast some puzzles are codes.

(a) Only conclusion I is true

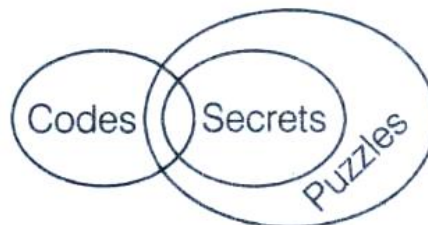
(b) Only conclusion II is true

(c) Either conclusion I or II is true

(d) Both conclusions I and II are true

Ans.(d)

Sol.



Conclusion

I. (✓) II. (✓)

Hence, both conclusions I and II are true.

79. Decide which of the given conclusions logically follow from the given statement(s).

Statements Some metals are alloys.

No metal is a stone.

Conclusions

I. No alloy is stone.

II. Atleast some alloys are metals.

[2018]

(a) Neither conclusion I nor II true

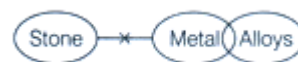
(b) Only conclusion II is true

(c) Only conclusion I is true

(d) Both conclusions I and II are true

Ans.(b)

Sol.

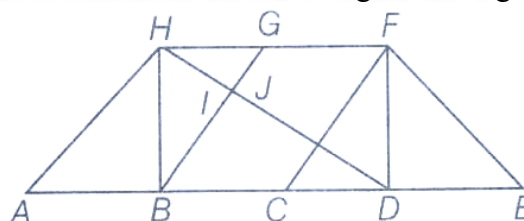


Conclusion

I. (x) II. (✓)

Hence, only conclusion II is true.

80. Find the number of triangles in the given picture.

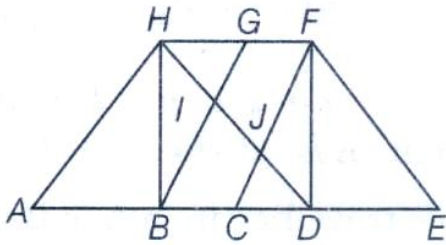


[2018]

(a) 10 (b) 12 (c) 14 (d) 16

Ans.(c)

Sol.



Total number of triangles = 14

($\square ABH, \square HIB, \square HGI, \square FJD, \square CJD, \square FDE, \square HFJ, \square DIB, \square HFD, \square DBH, \square AHD, \square CFE, \square CFD$ and $\square BHG$)

81. Decide which of the given conclusions logically follows from the given statement(s).

Statements All mangoes are golden in colour. No golden coloured things are cheap.

Conclusions

(I) All mangoes are cheap.

(II) Golden-coloured mangoes are not cheap. [2018]

(a) Only conclusion I follows

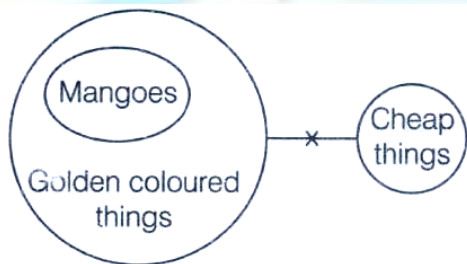
(b) Only conclusion II follows

(c) Either I or II follows

(d) Neither I nor II follows

Ans. (b)

Sol.



Conclusion

I. (x) II. (✓)

Hence, only conclusion II is true.

82. By selling an article, what is the profit percent gained?

I. 5% discount is given on list price.

II. If discount is not given, 20% profit is gained.

III. The cost price of the articles is ₹ 5,000 . [2018]

(a) I and II

(b) II and III

(c) I and III

(d) All of T. II and III

Ans.(d)

Sol. All the statements are necessary to find the profit percentage. Let the marked price of the item be ₹ x .

From statement III,

Cost price of article = ₹ 5000

From statement II,

If discount is not given, profit percentage = 20%

$$\therefore \frac{x - 5000}{5000} \times 100 = 20$$

$\therefore x = \text{marked price} = ₹ 6000$

Now, from statement I,

Discount = 5%

$$\therefore \text{Selling price} = \frac{(95)}{100} \times 6000$$

= ₹ 5700

$$\text{Hence, profit percentage} = \frac{5700 - 5000}{5000} \times 100 = 14\%$$

83. Decide which of the given conclusions logically follows from the given statement(s).

Statements All politicians are honest. All honest are fair.

Conclusions

(I) Some honest are politicians

(II) No honest is politicians

(III) Some fair are politicians

(IV) All fair are politicians

[2018]

(a) None follows

(b) I and IV follow

(c) I and II follow

(d) I and III follow

Ans.(d)

Sol.



Conclusions

I. (✓)

II. (x)

III. (✓)

IV. (x)

Hence, only conclusions I and III follow.

84. Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?

[2018]

(a) 0

(b) 2

(c) 4

(d) 8

Ans.(d)

Sol. Students who watched film A = 13

Students who watched film B = 16

Students who watched film C = 19

But total number of students = 40

Hence, students who watched all the films

$$= (13 + 16 + 19) - 40 = 8$$

85. Two bus tickets from city A to B and three tickets from city A to C cost ₹ 77, but three tickets from city A to B and two tickets from city A to C cost ₹ 73.

What are the fares for cities B and C from A ? [2018]

- (a) ₹ 4 , ₹ 23 (b) ₹ 13 , ₹ 17
(c) ₹ 15 , ₹ 14 (d) ₹ 17 , ₹ 13

Ans.(b)

Sol. Let the fare of 1 ticket from city A to B be ₹ x and the fare of 1 ticket from city A to C be ₹ y According to the question,

$$2x + 3y = 77 \quad \dots(i)$$

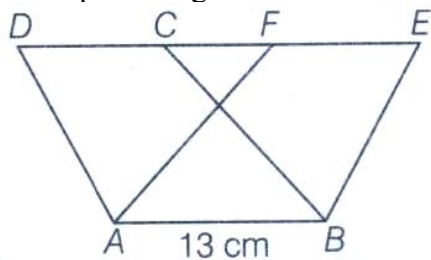
$$3x + 2y = 73 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = 13, y = 17$$

$\therefore$ The fares for cities B and C from A are ₹ 13 and ₹ 17, respectively.

86. In the figure given below, if the area of parallelogram $ABCD$ is 208 cm^2 , what is the height of the parallelogram $ABEF$? [2018]



- (a) 15 cm (b) 15.5 cm
(c) 16 cm (d) 16.5 cm

Ans.(c)

Sol. Area of parallelogram $ABCD = 208 \text{ cm}^2$

$\therefore$ Area of parallelogram = Base $\times$ Height

Let height of parallelogram $ABCD$ be h .

$$\therefore h \times 13 = 208$$

$$\therefore h = 16 \text{ cm}$$

Now, height of parallelogram $ABEF$ = Height of parallelogram $ABCD$

$\therefore$ Required height = 16 cm

87. If the first and the third letters in the group DISTRIBUTION are interchanged and also the second and the fourth letter, the fifth and the seventh and so on, then which of the following would be the seventh letter from the left? [2018]

- (a) U (b) R (c) B (d) T

Ans.(b)

Sol. Original word = DISTRIBUTION

New word = STDIBURIONTI

Hence, seventh letter from left = R

88. In the below series, you will be looking at the letter pattern, diagram pattern or number pattern. Fill in the blank in the end of the series.

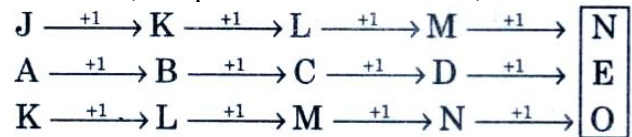
JAK, KBL, LCM, MDN, ____ .

[2018]

- (a) OEP (b) NEO (c) MEN (d) PFQ

Ans.(b)

Sol. Here, the pattern of the series is,



89. A caterpillar crawls up a pole of 75 inches high, starting from the ground. Each day, it crawls up 5 inches and each night it slides down 4 inches. When will it reach the top of the pole? [2018]

- (a) End of 70 days (b) End of 71 days
(c) End of 72 days (d) End of 73 days

Ans.(b)

Sol. A caterpillar crawls up 5 inches each day and slides down 4 inches each night.

$\therefore$ In 1 day it crawls up 1 inch.

Hence, in 70 days it crawls up 70 inches.

At the end of 71 days it will crawl up 5 inches again.

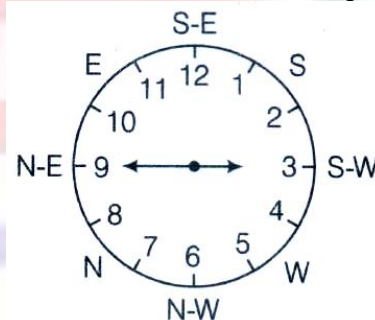
Hence, at the end of 71 days it will reach the top of the pole.

90. The time on the watch is quarter to three. If the minute hand points of North-East, in which direction does the hour hand point? [2018]

- (a) North-West (b) South-West
(c) South-East (d) North-East

Ans.(b)

Sol. Time on the watch is quarter to three, i.e. 2:45



Clearly, the hour hand points South-West direction.

Computer Awareness

91. What is the minimal form of Karnaugh map shown below? (Assume that x denotes a don't care term)

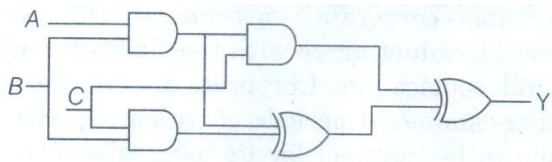
[2018]

	ab	00	01	11	10
cd	00	1	X	X	1
	01	X			1
	11				
	10	1			X

- (a) $\bar{b} \bar{d}$ (b) $\bar{b} \bar{d} + \bar{b} \bar{c}$
(c) $\bar{b} \bar{d} + \bar{a} \bar{d} \bar{c}$ (d) $\bar{b} \bar{d} + \bar{b} \bar{c} + \bar{c} \bar{d}$

Ans.(d)

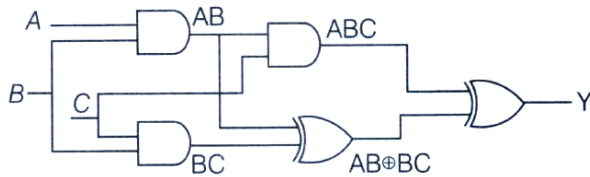
Sol.



- (a) $A + B + C$ (b) $A(B + C)$
 (c) $B(C + A)$ (d) $C(A + B)$

Ans.(c)

Sol.



$$Y = ABC \oplus (AB \oplus BC)$$

So, $Y = B(C + A)$

98. Which one of the following expressions does NOT represent exclusive NOR of x and y ? [2018]

- (a) $xy + \bar{x}\bar{y}$ (b) $x \oplus \bar{y}$ (c) $\bar{x} \oplus y$ (d) $\bar{x} \oplus \bar{y}$

Ans. (d)

Sol. The exclusive NOR and exclusive OR of x and y is given by the expressions

$$x \boxdot y = xy + x'y' \text{ (exclusive NOR or XNOR)}$$

$$x \oplus y = x'y + xy' \text{ (exclusive OR or XOR)}$$

So, option (A) is automatically true.

Option (B) is $x \oplus y'$ which evaluates to $x'y' + x(y')$.

$$= x'y' + xy \text{ which is equivalent to } x \boxdot y.$$

Option (C) is $x' \oplus y$ which evaluates to

$$(x')'y + x'y' = xy + x'y' \text{ which is equivalent to } x \boxdot y$$

Option (D) is $x' \oplus y'$ which evaluates to

$$(x')'y' + x'(y')' = xy' + x'y \text{ which is not equivalent to } x \boxdot y.$$

99. What type of errors are not detected by assemblers? [2018]

- (a) Syntax error (b) Run time error
 (c) Logical error (d) All of these

Ans.(c)

Sol. Logical error.

100. Consider the equation $(43)_x = (y3)_8$, where x and y are unknown. The number of possible solutions is [2018]

- (a) 4 (b) 5 (c) 6 (d) 7

Ans.(b)

$$\text{Sol. } (43)_x = (y3)_8$$

Since, a number in base K can only have digits from 0 to $(K-1)$, we can conclude that

$$x \geq 5 \text{ and } y \leq 7$$

Now, the original equation, when converted to decimal base gives:

$$4x^1 + 3x^0 = y(8^1) + 3(8^0)$$

$$\Rightarrow 4x + 3 = 8y + 3$$

$$\Rightarrow x = 2y$$

So, we have the following constraints

$$x \geq 5$$

$$y \leq 7$$

$x = 2y$, x and y are integers.

The set of values of (x, y) that satisfy these constraints are:

$$(6, 3), (8, 4), (10, 5), (12, 6), (14, 7)$$

So, the number of possible solution is 5.

General English

101. Find the synonym that is most nearly similar in meaning to the word: DEBACLE. [2018]

- (a) Catastrophe (b) Dandy
 (c) Corker (d) Opulence

Ans.(a)

Sol. 'Debacle' means 'a sudden and ignominious failure' and 'catastrophe' also means 'something very unsuccessful'. So, option (a) is the most suitable answers.

102. Choose the phrasal verb the means "To spend time doing unimportant things-instead of doing necessary things". [2018]

- (a) fake out (b) faff about
 (c) fade out (d) fall apart

Ans.(b)

Sol. 'Faff about' means to spend time doing unimportant things instead of doing necessary things.

103. Which of the following is close to the meaning of the word, IMPOLITIC? [2018]

- (a) Impolite (b) Intolerant
 (c) Incongruous (d) Injudicious

Ans.(d)

Sol. Impolitic means 'unwise' and injudicious also means the same. Hence, option (d) is the suitable answer.

104. Choose an option to replace the phrase given in bold. The bank has hired a consultant who will look into any issues which arise during the merger. [2018]

- (a) is looking over (b) will be looked over
 (c) will look out (d) no correction required

Ans.(d)

Sol. No correction required. The given sentence is correct.

105. The sentences given below when properly sequenced, form a coherent paragraph. Each sentence is labeled with a letter. Choose the most logical order of sentences from the given choices to construct a coherent paragraph.

P : It refers to those times when there is no war or fight among nations.

Q : Many people argue that war is a necessary evil and the only process of solving international disputes.

R : Peace means complete freedom from disturbance.

S : Once the world starts believing that wars are unnecessary, they decide to abolish war completely.

T : But in reality, war happens only because people are not ready to settle disputes in a peaceful manner. [2018]

- (a) TSRQP (b) RTQPS
(c) SPTQR (d) QTRPS

Ans.(d)

Sol. The correct sequence is QTRPS.

Directions (Q. Nos. 106 to 108) The following passage and answer the questions based on it.

I do not wish to cast aspersions on the corporate takeover. On the contrary, it is a key facilitator of creative destruction and doubtless the most effective remaining means by which shareholder voices can mold a corporation. But while change in management is often necessary, you cannot effectively run a corporation with differing authoritative voices espousing opposing corporate goals. It has to be one or the other. If the board is riven with conflicting interests, corporate governance will suffer. If directors cannot agree with the CEO's strategy, they should replace him. Corporate dissonance, of course, is unavoidable in periods of transition. But is not a value to be pursued for its own sake. A cacophony produces only red ink.

106. What according to the author facilitates creative destruction? [2018]

- (a) Corporatism (b) Modernism
(c) Terrorism (d) Cosmopolitanism

Ans.(a)

Sol. Corporatism facilitates creative destruction.

107. Which of the following words used in the passage reinforces the idea contained in the word, 'DISSONANCE'? [2018]

- (a) Aspersions (b) Cacophony
(c) Strategy (d) Transition

Ans.(b)

Sol. 'Cacophony' reinforces the idea, contained in dissonance as dissonance means lack of agreement or harmony and cacophony also means the same.

108. What according to the author makes corporate governance suffer? [2018]

- (a) Conflicting strategies of executives
(b) Conflicting interests of customers
(c) Conflicting interests of statesmen
(d) Conflicting interests of the Board of Directors

Ans.(d)

Sol. Conflicting interests of the Board of Directors make corporate governance suffer.

109. Fill in the blank choosing the correct word.

The vote will have to be laid _____ until next week.

[2018]

- (a) on (b) in (c) over (d) from

Ans.(c)

Sol. 'Over' is the suitable answer.

110. Fill in the blank choosing the correct question tag.

He has done his duty, _____ .

[2018]

- (a) Shouldn't he? (b) Hasn't he?
(c) Won't he? (d) Has he?

Ans.(b)

Sol. 'Hasn't he' is the suitable answer.

111. Choose one of the options that is most nearly same as meaning of the word Epitome. [2018]

- (a) Final verdict (b) Climax
(c) Essence (d) Tombstone

Ans.(c)

Sol. Epitome means a person or thing that is a perfect example of a particular quality or type and essence also means the intrinsic nature of something. Therefore, option (c) is the most suitable answer.

112. Which collocation goes with the word AWARE? [2018]

- (a) rightly (b) fully (c) nearly (d) exactly

Ans.(b)

Sol. Fully

113. Change the speech : "If you don't keep quite I shall shoot you", he said to her in a calm voice. [2018]

- (a) He warned her to shoot if she didn't keep quite calmly
(b) He said calmly that I shall shoot you if you don't be quite
(c) He warned her calmly that he would shoot her if she didn't keep quite
(d) Calmly he warned her that be quite or else he will have to shoot her

Ans.(c)

Sol. He warned her calmly that he would shoot her if she didn't kept quiet.

114. Choose the correct spelling for the word given below. **[2018]**

- (a) Ceiling (b) Cealing (c) Ceiling (d) Ceeling

Ans.(c)

Sol. 'Ceiling' is the correct spelling.

115. Select the pair of words, which are related in the same way as the capitalised words are related to each other.

BUTTERFLY : FREEDOM [2018]

- (a) Frog : Tadpole
(b) Self reliant : Buoyant
(c) Alga: Lichens
(d) Chicken : Rooster

Ans.(b)

Sol. Self reliant: Buoyant are related in same way as BUTTERFLY : FREEDOM.

116. Which of the following is the closest in meaning to the word, CLOWN? **[2018]**

- (a) Idiot (b) Dunce (c) Don (d) Jester

Ans.(d)

Sol. 'Clown' means a comic entertainer' and 'jester' means 'a professional joker'. Hence, option (d) is the closest in meaning to Clown.

117. Identify the underlined part which has an error. He could.'t cope up with the hot climate of the place any longer. **[2018]**

- (a) couldn't (b) cope up (c) climate (d) any longer

Ans.(b)

Sol. 'Cope up' should be replaced by 'cope' to make the sentence grammatically correct.

118. The _____ pittance the widow receives from the government cannot keep her from poverty. **[2018]**

- (a) Meagre (b) Indulgent
(c) Meticulous (d) Magnanimous

Ans.(a)

Sol. Meagre is the suitable answer.

119. The other boys or Henry _____ to blame. **[2018]**

- (a) is (b) are (c) were (d) will

Ans.(a)

Sol. 'Is' is the suitable answer because a singular subject 'Henry' is used after the word 'or' so the verb should also be singular.

120. In the question, the word 'Echo' is usual in four different ways, numbered A to D . **[2018]**

Choose the option in which the usage of the word is inappropriate.

- (a) The crowd cheered to the echo in the foot ball match
(b) I started to feel nostalgic as I stood in the old

neighbourhood that echoed with my childhood

(c) A loud thunder echo hit the tallest building

(d) My thoughts echoed with the sounds of spring

Ans.(c)

Sol. A loud thunder echo hit the tallest building. 'Echo' is used inappropriately in this sentence because it is used to represent the reflection of sound which cannot hit the building.