NIMCET-2019 SOLVED PAPER

Mathematics

1. For two circles $x^2+y^2=16$ and $x^2+y^2-2y=0$, there is/are **[2019]**

(a) One pair of common tangent
(b) Two pair of common tangents
(c) Three pair of common tangents
(d) No common tangents

Ans. (d)

Sol. The equations of circles are given as

$$x^2+y^2=16$$

$$\Rightarrow (x-0)^2+(y-0)^2=4^2$$

and $x^2+y^2-2y=0$

$$\Rightarrow (x-0)^2+(y-1)^2=1^2$$

$\therefore$ Centres of circles are $C_1(0,0)$ and $C_2(0,1)$ and radius $r_1=4$ and $r_2=1$

Now, $C_1C_2=\sqrt{(0-0)^2+(0-1)^2}=\sqrt{2}$

and $r_1-r_2=4-1=3$

$\therefore$ $C_1C_2<r_1-r_2$

So circle $x^2+y^2-2y=0$ is completely within $x^2+y^2=16$.
Hence, there is no common tangent.

2. Let $f:R\to R$ be defined by

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right), & \text{if } x>0 \\ 0, & x\le 0 \end{cases}$$

Then, **[2019]**

(a) f is neither continuous nor differentiable at $x=0$
(b) f is continuous and differentiable at $x=0$
(c) f is continuous but not differentiable at $x=0$
(d) f is not continuous but differentiable at $x=0$

Ans. (c)

Sol. We observe that

$$\lim_{x\to 0^-} f(x) = 0,\ f(0)=0$$

and

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x\sin\left(\frac{1}{x}\right)$$

$$= \lim_{h\to 0} h\sin\left(\frac{1}{h}\right)=0$$

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)=f(0)$$

So, $f(x)$ is continuous at $x=0$

Again, $\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$

$$=\lim_{x\to 0}\frac{x\sin 1/x}{x}=\lim_{x\to 0}\sin\left(\frac{1}{x}\right)=\sin(\infty)$$

$=$ An oscillating value between -1 and 1

$\therefore f(x)$ is not differentiable at $x=0$

3. A particle P starts from the point $z_0=1+2i$, where $i=\sqrt{-1}$. It moves first horizontally away from the origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i}+\hat{j}$ and, then it moves through an angle $\pi/2$ in an anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by **[2019]**

(a) $6+7i$ (b) $-7+6i$ (c) $7+6i$ (d) $-6+7i$

Ans. (d)

Sol. It is given that CE is in the direction of vector $i+j$ which makes 45° with X-axis. Also, $CE=\sqrt{2}$ unit

$\therefore CD=DE=1$

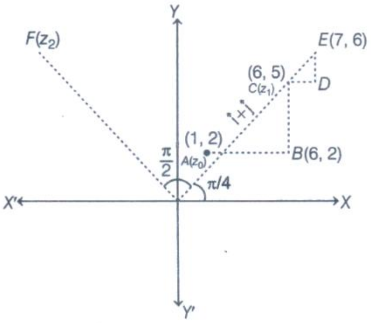
So, the coordinates of E are $(7,6)$.

OE is rotated through an angle $\frac{\pi}{2}$ in anticlockwise direction to reach a point $F(z_2)$

$$\therefore z_2 = \overrightarrow{OE}e^{i\pi/2}$$

$$=(7+6i)i$$

$$=-6+7i$$



4. If $\Delta = a^2-(b-c)^2$, where Δ is the area of the $\triangle ABC$, then $\tan A$ equals **[2019]**

(a) $\frac{15}{16}$ (b) $\frac{8}{15}$ (c) $\frac{8}{17}$ (d) $\frac{1}{2}$

Ans. (b)

Sol. We have

$$\Delta = a^2-(b-c)^2$$
$$=(a-b+c)(a+b-c)$$
$$=(a+b+c-2b)(a+b+c-2c)$$
$$=(2s-2b)(2s-2c)$$
$$=4(s-b)(s-c)$$
$$\therefore \quad \frac{(s-b)(s-c)}{\Delta}=\frac{1}{4}$$
$$\Rightarrow \quad \tan\frac{A}{2}=\frac{1}{4}$$
$$\Rightarrow \quad \tan\frac{A}{2}=\frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$$
$$=\frac{2\times\frac{1}{4}}{1-\frac{1}{16}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}$$

5. Two numbers a and b are chosen at random from a set of first 30 natural numbers, then the probability that a^2-b^2 is divisible by 3 , is **[2019]**

(a) $\frac{47}{87}$ (b) $\frac{15}{87}$ (c) $\frac{12}{87}$ (d) $\frac{9}{87}$

Ans. (a)

Sol. The number of ways of choosing two numbers from the set of first 30 natural numbers is $^{30}C_2=435$

Let us divide given 30 numbers into three groups G_1, G_2 and G_3 as follows :

$G_1: 3,6,9,12,15,18,21,24,27,30$

$G_2: 1,4,7,10,13,16,19,22,25,28$

$G_3: 2,5,8,11,14,17,20,23,26,29$

We have $a^2-b^2=(a-b)(a+b)$

Therefore, a^2-b^2 will be divisible by 3 if either a and b are chose from the same group or one of these is chose from G_2 and other from G_3.

$\therefore$ Favourable elementary events

$$={}^{10}C_2+{}^{10}C_2+{}^{10}C_2+{}^{10}C_1\times{}^{10}C_1$$
$$=3\times{}^{10}C_2+(10)^2$$
$$=\frac{3\times10\times9}{2\times1}+100=235$$

$\therefore$ Required probability $=\frac{235}{435}=\frac{47}{87}$

6. A man takes a step forward with probability 0.4 and backward with probability 0.6 . The probability that at the end of eleven steps, he is one step away from the starting point is **[2019]**

(a) $462(0.34)^2$ (b) $462(0.04)^2$

(c) $462(0.14)^2$ (d) $462(0.24)^2$

Ans. (*)

Sol. We have,

$n=11, p=0.4, q=0.6$

Since, the man is one step away from the initial point, he is either one step forward or one step backward.

$\therefore r=5,6$

$\therefore$ Required probability

$$=P(X=5)+P(X=6)$$
$$={}^{11}C_5(0.4)^5(0.6)^6+{}^{11}C_6(0.4)^6(0.6)^5$$
$$={}^{11}C_5(0.4)^5(0.6)^5[0.6+0.4]$$
$$={}^{11}C_5(0.4)^5(0.6)^5$$
$$=462(0.24)^5$$

7. Let $x_i,\ i=1,2,\ldots,n$ be n observations and $w_i=px_i+k, i=1,2,\ldots,n$ where p and k are constants. If the mean of $x_i's$ is 48 and standard deviation is 12 , whereas the mean of $w_i's$ is 55 and standard deviation is 15 , then the value of p and k should be **[2019]**

(a) $p=1.25, k=-5$ (b) $p=-1.25, k=5$

(c) $p=2.5, k=-5$ (d) $p=25, k=5$

Ans. (a)

Sol. We have,

$$w_i = px_i+k \text{ for } i=1,2,\ldots,n$$
$$\Rightarrow \sum_{i=1}^{n} w_i = p\sum_{i=1}^{n} x_i+nk$$
$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} w_i = p\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)+k$$
$$\Rightarrow 55=48p+k \ldots..(i)$$

Again, $w_i=px_i+k$

$$\Rightarrow \operatorname{var}(w_i)=p^2\operatorname{var}(x)$$
$$\Rightarrow \sigma_w=|p|\sigma_x$$

$\Rightarrow 15 = 12|p|$

$\Rightarrow \boldsymbol{p} = \pm 125$

Now, $p = 125 \Rightarrow k = -5$

and $p = -125 \Rightarrow k = 115$

8. If x, y, z are distinct real numbers and $\begin{vmatrix} x & x^2 & 2+x^3 \\ y & y^2 & 2+y^3 \\ z & z^2 & 2+z^3 \end{vmatrix} = 0$, then $xyz =$ **[2019]**

(a) 1 (b) -1 (c) 2 (d) -2

Ans. (d)

Sol. We have,

$$\Rightarrow \begin{vmatrix} x & x^2 & 2+x^3 \\ y & y^2 & 2+y^3 \\ z & z^2 & 2+z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 2 \\ y & y^2 & 2 \\ z & z^2 & 2 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + (xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + (xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (2+xyz) = 0$$

$xyz + 2 = 0 \; [\because x, y, z \text{ are distinct}]$

$\Rightarrow xyz = -2$

9. Let $f(x)$ be a polynomial satisfying $f(0) = 2, f'(0) = 3$ and $f''(x) = f(x)$. Then, $f(4)$ is equal to **[2019]**

(a) $5\dfrac{(e^8-1)}{2e^4}$ (b) $\dfrac{(5e^8-1)}{2e^4}$ (c) $\dfrac{2e^4}{5e^8-1}$ (d) $\dfrac{2e^4}{5(e^8+1)}$

Ans. (b)

Sol. We have,

Let $f''(x) = f(x)$

$\Rightarrow f(x) = y$

and $f'(x) = \dfrac{dy}{dx}$

$\therefore f''(x) = \dfrac{d^2y}{dx^2}$

$\Rightarrow \dfrac{d^2y}{dx^2} = y$

$\Rightarrow \dfrac{d^2y}{dx^2} - y = 0$

$\Rightarrow y = Ae^x + Be^{-x}$

and $f(x) = Ae^x + Be^{-x}$

Now, $f'(x) = Ae^x - Be^{-x}$

$\Rightarrow f(0) = 2$

and $A + B = 2$

$\Rightarrow f'(0) = 3$

$\therefore A - B = 3$

$\therefore A = \dfrac{5}{2}, B = -\dfrac{1}{2}$

$\therefore f(1) = \dfrac{5}{2}e^x - \dfrac{1}{2}e^{-x} = \dfrac{5e^x - e^{-x}}{2}$

$f(4) = \dfrac{5e^4 - e^{-4}}{2} = \dfrac{5e^8 - 1}{2e^4}$

10. If $a, a_1, a_2, a_3, \ldots, a_{2n-1}, b$ are in AP, $a, b_1, b_2, \ldots, b_{2n-1}, b$ are in GP and $a, c_1, c_2, c_3, \ldots, c_{2n-1}, b$ are in HP, where a, b are positive, then the equation $a_n x^2 - b_n x + c_n = 0$ has its roots **[2019]**

(a) Real and equal

(b) Real and unequal

(c) Imaginary

(d) One real and one imaginary

Ans. (c)

Sol. Clearly, a_n, b_n, c_n are the middle terms of the given AP , GP, HP respectively

So, $a_n =$ AM of a and b

$b_n =$ GM of a and b

$c_n =$ HM of a and b

Since, AM, GM, HM are in GP.

$\therefore b_n^2 = a_n c_n$

Now, given equation is

$a_n x^n - b_n x + c_n = 0$

$\therefore \quad D = b_n^2 - 4a_n c_n$

$= a_n c_n - 4a_n c_n$

$= -3a_n c_n$

$D < 0$

$[\because a_n \cdot c_n > 0 \text{ as a and b are positive}]$

So, given equation has imaginary roots.

11. Solution set of inequality $\log_3 (x+2)(x+4) + \log_{\frac{1}{3}}(x+2) < \dfrac{1}{2}\log_{\sqrt{3}} 7$ is

[2019]

(a) $(-2,-1)$ (b) $(-2,3)$
(c) $(-1,3)$ (d) $(3,\infty)$

Ans. (d)

Sol. We have,

$$\log_3(x+2)(x+4)+\log_{1/3}(x+2)<\frac{1}{2}\log_{\sqrt{3}}7$$

$\Rightarrow \quad \log_3(x+2)(x+4)-\log_3(x+2)<\log_3 7$

$\therefore \quad (x+2)(x+4)>0$ and $(x+2)>0$

Again, $\log_3\dfrac{(x+2)(x+4)}{(x+2)}>\log_3 7, x\neq -2$

$\Rightarrow \quad \log_3(x+4)>\log_3 7$

$\Rightarrow \quad x+4>7$

$\Rightarrow \quad x>3$

$\therefore \quad x\in(3,\infty)$

12. If a,b,c are in GP and $\log a-\log 2b, \log 2b-\log 3c$ and $\log 3c-\log a$ are in AP, then a,b,c are the lengths of the sides of a triangle which is **[2019]**

(a) Acute angle (b) Obtuse angled
(c) Right angles (d) Equilateral

Ans. (b)

Sol. We have,

a,b,c are in GP

$\Rightarrow b^2=ac$

Also, $\log a-\log 2b, \log 2b-\log 3c, \log 3c-\log a$ are in AP.

$\therefore \quad (\log 2b-\log 3c)-(\log a-\log 2b)$

$\Rightarrow \quad =(\log 3c-\log a)-(\log 2b-\log 3c)$

$\Rightarrow \quad 2\log 2b-\log 3c-\log a$

$\Rightarrow \quad 2\log 3c-\log a-\log 2b$

$\Rightarrow \quad 3\log 2b=3\log 3c$

$\Rightarrow \quad \log 2b=\log 3c$

$\Rightarrow \quad 2b=3c$

$\Rightarrow \quad b=\dfrac{3}{2}c$

Now, $b^2=ac$

$\Rightarrow \dfrac{9}{4}c^2=ac$

$\Rightarrow a=\dfrac{9}{4}c$

$\therefore a$ is greatest side.

Now, $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$

$$=\frac{\frac{9}{4}c^2+c^2-\frac{81}{16}c^2}{2\times\frac{3}{2}c^2}<0$$

$\therefore \angle A$ is obtuse angle.

So, $\square ABC$ is obtuse angled triangle.

13. If $x, 2x+2, 3x+3$ are the first three terms of a geometric progression, then 4 th term in the geometric progression is **[2019]**

(a) -13.5 (b) 13.5 (c) -27 (d) 27

Ans. (b)

Sol. We have,

$x, 2x+2, 3x+3$ are in GP.

$\therefore \quad (2x+2)^2=x(3x+3)$

$\Rightarrow \quad 4(x+1)^2=3x(x+1)$

$\Rightarrow \quad 4x+4=3x$

$\Rightarrow \quad x=-4$

$\therefore$ The GP is

$-4,-6,-9,\ldots\ldots$

$\because a_4 \quad =-4\left(\dfrac{3}{2}\right)^3$

$=-4\times\dfrac{27}{8}=-\dfrac{27}{2}=-13.5$

14. If $(1+x-2x^2)^6=1+a_1x+a_2x^2+\ldots+a_{12}x^{12}$, then the value of $a_2+a_4+a_6+\ldots+a_{12}$ is **[2019]**

(a) 29 (b) 30 (c) 31 (d) 32

Ans. (c)

Sol. We have,

$(1+x-2x^2)^6=1+a_1x+a_2x^2+\ldots+a_{12}x^{12}$

Put $x=1$, we get

$(1+1-2)^6=1+a_1+a_2+\ldots+a_{12}$

$\Rightarrow 1+a_1+a_2+\ldots+a_{12}=0$ ...(i)

Put $x=-1$, we get

$(1-1-2)^6=1-a_1+a_2-\ldots+a_{12}$

$\Rightarrow 1-a_1+a_2+\ldots+a_{12}=64$ ...(ii)

Add Eqs. (i) and (ii), we get

$2(1+a_2+a_4+\ldots+a_{12}) \quad =64$

$\Rightarrow a_2+a_4+\ldots+a_{12} \quad =31$

15. If a and b are greatest values of ${}^{2n}C_r$ and ${}^{2n-1}C_r$ respectively, then **[2019]**

(a) $a=2b$ (b) $b=2a$ (c) $a=b$ (d) $a^2=2b^2$

Ans. (a)

Sol. Given, $a={}^{2n}C_n$ and $b={}^{2n-1}C_n$

Now, $\frac{a}{b}=\frac{{}^{2n}C_n}{{}^{2n-1}C_n}$

$\Rightarrow \frac{a}{b}=\frac{2n!}{n!n!}\times\frac{(n-1)!n!}{(2n-1)!}$

$\Rightarrow \frac{a}{b}=\frac{2n!(n-1)!}{n!(2n-1)!}$

$\Rightarrow \frac{a}{b}=\frac{(2n)(2n-1)!(n-1)!}{n(n-1)!(2n-1)!}$

$\Rightarrow \frac{a}{b}=\frac{2n}{n}=\frac{2}{1}$

$\Rightarrow a=2b$

16. Let U and V be two events of a sample space S and $P(A)$ denote the probability of an event A. Which of the following statements is true? **[2019]**

(a) If $P(U)=P(V)$, then $U=V$

(b) If $P(U)=0$, then $U^c=S$

(c) If $U\cap V=\phi$ then U and V are independent

(d) If U and V are independent, then U^c and V^c are independent

Ans. (d)

Sol. If U and V are independent events, then

$P(U\cap V)=P(U)\cdot P(V)$

Now,

$P\left(\bar{U}\cap\bar{V}\right)=P\left(\overline{U\cup V}\right)$

$P\left(\bar{U}\cap\bar{V}\right)=1-P(U\cup V)$

$P\left(\bar{U}\cap\bar{V}\right)=1-\left[P(U)+P(V)-P(U\cap V)\right]$

$=1-P(U)-P(V)+P(U)P(V)$

$=(1-P(U))-P(V)(1-P(U))$

$=(1-P(U))(1-P(V))$

$=P\left(\bar{U}\right)(P\left(\bar{V}\right)$

17. If a man purchase a raffle ticket, he can win a first prize of ₹ 5000 or a second prize of ₹ 2000 with probabilities 0.001 and 0.003 respectively. What should be a fair price to pay for the ticket? **[2019]**

(a) ₹ 11 (b) ₹15 (c) ₹ 2000 (d) None of these

Ans. (a)

Sol. Let the winning price be x and $P(X)$ be its probability.

Then,

x	5000	2000
$P(x)$	0.001	0.003

Now, fair price of the ticket $=\boldsymbol{E(x)}$

$=\Sigma xP(x)$

$=5000\times0.001+2000\times0.003$

$=5+6=11$

$\therefore$ Fair price of the ticket is ₹11.

18. If the mean deviation $1,1+d,1+2d,\ldots,1+100d$ from their mean is 255, then d is equal to **[2019]**

(a) 10.1 (b) 10.2 (c) 10.3 (d) 10.4

Ans. (a)

Sol. $\bar{X}=\frac{1}{101}\left[1+(1+d)+(1+2d)+\ldots-(1+100d)\right]$

$=\frac{1}{101}\times\frac{101}{2}\left[1+(1+100d)\right]$

$=1+50d$

$\therefore$ Mean deviation from mean

$=\frac{1}{101}[|1-(1+50d)|+|1+d-(1+50d)|$

$+\ldots+|1+100d-(1+50d)|]$

$=\frac{2|d|}{101}\{1+2+\ldots+50\}$

$=\frac{2|d|}{101}\times\frac{50\times51}{2}=\frac{2550}{101}|d|$

Now, $\frac{2550}{101}|d|=255$

$\Rightarrow |d|=\frac{255\times101}{2550}$

$\Rightarrow |d|=10.1$

19. If $\sum_{i=1}^{n}x_i=80$ and $\sum_{i=1}^{n}x_i^2=400$, then a possible value of n among the following is **[2019]**

(a) 9 (b) 12 (c) 15 (d) 18

Ans. (d)

Sol. As we know that,

Root mean square $\geq$ Arithmetic mean

$\therefore \sqrt{\frac{\sum_{i=1}^{n}x_i^2}{n}}\geq\frac{\sum_{i=1}^{n}x_i}{n}$

$\Rightarrow \sqrt{\frac{400}{n}}\geq\frac{80}{n}$

$\Rightarrow n\geq16$

Hence, possible value of n is 18.

20. Let S be the set $\{a \in Z^+ : a \le 100\}$. If the equation $\left[\tan^2 x\right] - \tan x - a = 0$ has real roots (where [.] is the greatest integer function), then the number of elements in S is **[2019]**

(a) 10 (b) 8 (c) 9 (d) 0

Ans. (c)

Sol. Given equation

$$\left[\tan^2 x\right] - \tan x - a = 0$$

$\because \left[\tan^2 x\right]$ and ' a ' are integers, so $\tan x$ must be integer,

so $\left[\tan^2 x\right] = \tan^2 x$

Now, $\tan^2 x - \tan x - a = 0$

$\Rightarrow a = \tan x(\tan x - 1)$

$\because a \in z^+$ and $a \le 100$ and $\tan x \in I$

$\therefore$ Possible values of

$$a = 2, 6, 12, 20, 30, 42, 56, 72, 90$$

$\therefore \quad n(S) = 9.$

21. If $\sin^2 x \tan x + \cos^2 x \cot x - 2\sin 2x = 1 + \tan x + \cot x$, $x \in (0, \pi)$, then x is **[2019]**

(a) $\frac{3\pi}{12}, \frac{5\pi}{12}$ (b) $\frac{5\pi}{12}, \frac{7\pi}{12}$

(c) $\frac{7\pi}{12}, \frac{11\pi}{12}$ (d) $\frac{7\pi}{12}, \frac{9\pi}{12}$

Ans. (c)

Sol. We have,

$$\sin^2 x \tan x + \cos^2 x \cot x - \sin 2x = 1 + \tan x + \cot x$$

$$\Rightarrow \frac{\sin^3 x}{\cos x} + \frac{\cos^3 x}{\sin x} - 2\sin x \cos x$$

$$= 1 + \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$\Rightarrow \sin^4 x + \cos^4 x - 2\sin^2 x \cos^2 x$$

$$= \sin x \cos x + \sin^2 x + \cos^2 x$$

$$\Rightarrow \left(\cos^2 x - \sin^2 x\right)^2 = 1 + \sin x \cos x$$

$$\Rightarrow \cos^2 2x = 1 + \frac{1}{2}\sin 2x$$

$$\Rightarrow 1 - \sin^2 2x = 1 + \frac{1}{2}\sin 2x$$

$$\Rightarrow \sin 2x + \frac{1}{2}\sin 2x = 0$$

$$\Rightarrow \sin 2x\left[\sin 2x + \frac{1}{2}\right] = 0$$

$$\Rightarrow \sin 2x = 0 \text{ or } \sin 2x = -\frac{1}{2}$$

But, $\sin 2x \ne 0$

$$\therefore \quad \sin 2x = -\frac{1}{2}$$

$$\Rightarrow \quad 2x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\Rightarrow \quad x = \frac{7\pi}{12}, \frac{11\pi}{12}$$

22. $\vec{a}$ and $\vec{b}$ are non-zero non-collinear vectors such that $|\vec{a}| = 2, \vec{a} \cdot \vec{b} = 1$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/3$. If $\vec{r}$ is any vector satisfying $\vec{r} \cdot \vec{a} = 2, \vec{r} \cdot \vec{b} = 8$, $\left(\vec{r} + 2\vec{a} - 10\vec{b}\right) \cdot \left(\vec{a} \times \vec{b}\right) = 6$ and $\vec{r} + 2\vec{a} - 10\vec{b} = \lambda\left(\vec{a} \times \vec{b}\right)$, then $\lambda =$ **[2019]**

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{4}{\sqrt{3}}$ (d) 3

Ans. (b)

Sol. Given,

$$\left(\vec{r} + 2\vec{a} - 10\vec{b}\right) \cdot \left(\vec{a} \times \vec{b}\right) = 6 \quad ...(i)$$

$$\text{and } \vec{r} + 2\vec{a} - 10\vec{b} = \lambda\left(\vec{a} \times \vec{b}\right) \quad ...(ii)$$

Putting value of $\vec{r} + 2\vec{a} - 10\vec{b}$

$= \lambda\left(\vec{a} \times \vec{b}\right)$ in Eq. (i)

$$\lambda\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{a} \times \vec{b}\right) = 6$$

$$\Rightarrow \lambda(\vec{a} \times \vec{b})^2 = 6$$

$$\Rightarrow \lambda |\vec{a} \times \vec{b}|^2 = 6$$

Now, from Lagrange's identity theorem,

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$\lambda\left[|\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2\right] = 6 \quad ...(iii)$$

Given, $|\vec{a}| = 2, \vec{a} \cdot \vec{b} = 1$ and angle between **a** and **b** is $\frac{\pi}{3}$.

Consider $\vec{a} \cdot \vec{b} = 1$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = 1$$

$$\Rightarrow 2\vec{b}\Big|\cos\frac{\pi}{3} = 1$$

$$\Rightarrow |\vec{b}| = 1$$

Putting all the values in Eq. (iii), we get

$$\lambda\left[(2)^2 (1)^2 - 1^2\right] = 6$$

$$\Rightarrow \quad \lambda[4 - 1] = 6$$

$$\Rightarrow \quad 3\lambda = 6$$

$$\Rightarrow \quad \lambda = 2$$

23. In a chess tournament, n men and 2 women players participated. Each player plays 2 games against every other player. Also, the total number of games played by the men among themselves exceeded by 66 the number of games that the men played against the women. Then, the total number of players in the tournament is **[2019]**

(a) 13 (b) 11 (c) 9 (d) 7

Ans. (a)

Sol. Let there be ' n ' men participated. Then the number of games that the men play between themselves is $2{}^{n}C_2$ and the number of games that the men played with women is $2\cdot(2n)$

$\therefore \quad 2\cdot{}^{n}C_2 - 2\cdot 2n = 66$

$\Rightarrow \quad \dfrac{2n(n-1)}{2} - 4n - 66 = 0$

$\Rightarrow \quad n^2 - 5n - 66 = 0$

$\Rightarrow n^2 - 11n + 6n - 66 = 0$

$\Rightarrow n(n-11) + 6(n-11) = 0$

$\Rightarrow n - 11 = 0$ or $n + 6 = 0$

$\Rightarrow n = 11 [\because n \neq -6]$

$\therefore$ Number of participants $= 11$ men $+2$ women $= 11 + 2 = 13$

24. Suppose $A_1, A_2, A_3, \ldots, A_{30}$ are thirty sets each having 5 elements with no common elements across the sets and $B_1, B_2, \ldots, B_n$ are n sets each with 3 elements with no common elements across the sets. Let $\cup_{i=1}^{30} A_i = \cup_{j=1}^{n} B_j = S$ and each elements of S belongs to exactly 10 of the $A_i's$ and exactly 9 of the $B_j's$. Then, n is equal to **[2019]**

(a) 15 (b) 30 (c) 40 (d) 45

Ans. (d)

Sol. It is given that each A_i has 5 elements.

$\therefore \sum_{i=1}^{30} n(A_i) = 5 \times 30 = 150$

Suppose S has m distinct elements of A_i 's and B_i 's.

Since each element of S belongs to exactly 10 of A_i 's.

$\therefore \quad \sum_{i=1}^{30} n(A_i) = 10m$

$\Rightarrow \quad 10m = 150$

$\Rightarrow \quad m = 15$

Since, each B_j has 3 elements and each element of S belongs to exactly 9 of the B_j 's.

$\therefore \quad \sum_{j=1}^{n} n(B_j) = 3n$ and $\sum_{j=1}^{n} n(B_j) = 9m$

$\Rightarrow \quad 3n = 9m$

$\Rightarrow \quad n = 3m$

$\Rightarrow \quad n = 45$

$[\because m = 15]$

25. Let $f(x) = \begin{cases} \cos[x], & x \geq 0 \\ |x| + a, & x < 0 \end{cases}$, where $[x]$ denotes the greatest integer $\leq x$. If f should be continuous at $x = 0$, then a must be **[2019]**

(a) 0 (b) 1 (c) 2 (d) -1

Ans. (b)

Sol. LHL $= \lim\limits_{x \to 0^-} f(x)$

$= \lim\limits_{x \to 0^-} |x| + a$

$= \lim\limits_{h \to 0} |0 - h| + a$

$= a$

RHL $= \lim\limits_{x \to 0^+} f(x)$

$= \lim\limits_{x \to 0^+} \cos[x]$

$= \lim\limits_{h \to 0} \cos[0 + h]$

$= \cos 0$

$= 1$

Since, $f(x)$ is continuous at $x = 0$

$\therefore$ LHL $=$ RHL

$\Rightarrow a = 1$

26. If x is real, then the minimum value of $\dfrac{x^2 - x + 1}{x^2 + x + 1}$ is **[2019]**

(a) $\dfrac{1}{2}$ (b) 2 (c) 3 (d) $\dfrac{1}{3}$

Ans. (d)

Sol. Let $y = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow \quad x^2 y + xy + y = x^2 - x + 1$

$\Rightarrow \quad x^2(y-1) + x(y+1) + (y-1) = 0$

Since, x is real.

$\therefore D \geq 0$

$\Rightarrow \quad (y+1)^2 - 4(y-1)^2 \geq 0$

$\Rightarrow \quad y^2 + 2y + 1 - 4y^2 + 8y - 4 \geq 0$

$\Rightarrow \quad -3y^2 + 10y - 3 \geq 0$

$\Rightarrow \quad 3y^2 - 10y + 3 \leq 0$

$\Rightarrow \quad 3y^2 - 9y - y + 3 \leq 0$

$\Rightarrow \quad 3y(y-3) - 1(y-3) \leq 0$

$\quad (y-3)(3y-1) \leq 0$

$\therefore y \in \left[\frac{1}{3}, 3\right]$

$\therefore$ Minimum value of y is $\frac{1}{3}$.

27. If $\int \cos x \cos 2x \cos 5x dx = A_1 \sin 2x + A_2 \sin 4x + A_3 \sin 6x + A_4 \sin 8x + c$, then the values of A_1, A_2, A_3, A_4 are **[2019]**

(a) $A_1 = \frac{1}{2}, A_2 = \frac{1}{4}, A_3 = \frac{1}{6}, A_4 = \frac{1}{8}$

(b) $A_1 = \frac{1}{8}, A_2 = \frac{1}{16}, A_3 = \frac{1}{24}, A_4 = \frac{1}{32}$

(c) $A_1 = \frac{1}{6}, A_2 = \frac{1}{12}, A_3 = \frac{1}{18}, A_4 = \frac{1}{24}$

(d) $A_1 = \frac{1}{4}, A_2 = \frac{1}{8}, A_3 = \frac{1}{12}, A_4 = \frac{1}{16}$

Ans. (b)

Sol. Let $I = \int \cos x \cos 2x \cos 5x dx$

$$= \frac{1}{2}\int 2\cos x \cos 2x \cos 5x dx$$

$$= \frac{1}{2}\int (\cos 3x + \cos x)\cos 5x dx$$

$[2\cos A \cos B = \cos(A+B) + \cos(A-B)]$

$$= \frac{1}{4}\int (2\cos 3x \cos 5x + 2\cos x \cos 5x)\, dx$$

$$= \frac{1}{4}\int (\cos 8x + \cos 2x + \cos 6x + \cos 4x)\, dx$$

$$= \frac{1}{4}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2} + \frac{\sin 6x}{6} + \frac{\sin 4x}{4}\right] + c$$

$$= \frac{1}{8}\sin 2x + \frac{1}{16}\sin 4x + \frac{1}{24}\sin 6x + \frac{1}{32}\sin 8x + c$$

$\therefore A_1 = \frac{1}{8}, A_2 = \frac{1}{16}, A_3 = \frac{1}{24}, A_4 = \frac{1}{32}$

28. If $\int_{\log 2}^{x} \frac{1}{\sqrt{e^x - 1}} dx = \frac{\pi}{6}$, then $x =$ **[2019]**

(a) $\log 2$ (b) $2\log 2$ (c) $3\log 2$ (d) $4\log 2$

Ans. (b)

Sol. $\int_{\log 2}^{x} \frac{1}{\sqrt{e^x - 1}} dx = \frac{\pi}{6}$.

let $e^x - 1 = t^2$

$\Rightarrow e^x = t^2 + 1$

$\Rightarrow e^x dx = 2t dt$

$dx = \frac{2t}{t^2+1} dt$

Now, as $x = \log 2$

$\Rightarrow t^2 = e^{\log 2} - 1$

$\Rightarrow t^2 = 1 \Rightarrow t = 1$

and $e^x - 1 = t^2$

$\Rightarrow t = \sqrt{e^x - 1}$

$\therefore \int_1^{\sqrt{e^x-1}} \frac{2t dt}{t(t^2+1)} = \frac{\pi}{6}$

$\Rightarrow 2\int_1^{\sqrt{e^x-1}} \frac{dt}{t^2+1} = \frac{\pi}{6}$

$\Rightarrow 2\left|\tan^{-1} t\right|_1^{\sqrt{e^x-1}} = \frac{\pi}{6}$

$\Rightarrow 2\left[\tan^{-1}\sqrt{e^x - 1} - \tan^{-1}(1)\right] = \frac{\pi}{6}$

$\Rightarrow 2\tan^{-1}\sqrt{e^x - 1} = \frac{\pi}{6} + \frac{\pi}{2}$

$\Rightarrow 2\tan^{-1}\sqrt{e^x - 1} = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1}\sqrt{e^x - 1} = \frac{\pi}{3}$

$\Rightarrow \sqrt{e^x - 1} = \tan\left(\frac{\pi}{3}\right)$

$\Rightarrow e^x - 1 = (\sqrt{3})^2$

$\Rightarrow e^x = 3 + 1 = 4$

$\Rightarrow e^x = 4$

Taking $\log$ on both sides,

$$x \log_e e = \log_e 4$$

$$x = 2\log_e 2$$

29. Equation of the tangent from the point $(3, -1)$ to the ellipse $2x^2 + 9y^2 = 3$ is **[2019]**

(a) $2x - 3y - 3 = 0$ (b) $2x + 3y - 3 = 0$

(c) $2x + y - 3 = 0$ (d) None of these

Ans. (b)

Sol. Given equation of ellipse is

$$2x^2+9y^2=3$$

$$\Rightarrow \frac{x^2}{3}+\frac{y^2}{\frac{1}{3}}=1 \quad \text{(i)}$$

Now, any tangent to the given ellipse is

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

$$\Rightarrow y=mx\pm\sqrt{\frac{3}{2}m^2+\frac{1}{3}} \quad \text{(ii)}$$

Given, tangent to the ellipse passes through $(3,-1)$.

So, putting ($3,-1$) in Eq. (ii), we get

$$-1=3m\pm\sqrt{\frac{3}{2}m^2+\frac{1}{3}}$$

$$\Rightarrow -(3m+1)=\pm\sqrt{\frac{3}{2}m^2+\frac{1}{3}}$$

Squaring both sides, we get

$$(3m+1)^2=\frac{3}{2}m^2+\frac{1}{3}$$

$$\Rightarrow 9m^2+1+6m=\frac{3}{2}m^2+\frac{1}{3}$$

$$\Rightarrow \frac{15}{2}m^2+6m+\frac{2}{3}=0$$

$$\Rightarrow 45m^2+36m+4=0$$

$$\Rightarrow 45m^2+30m+6m+4=0$$

$$\Rightarrow 15m(3m+2)+2(3m+2)=0$$

$$\Rightarrow (15m+2)(3m+2)=0$$

putting $m=-\frac{2}{3}$ in Eq. (ii), we get

$$\Rightarrow y=\frac{-2}{3}x\pm\sqrt{\frac{3}{2}\times\frac{4}{9}+\frac{1}{3}}$$

$$\Rightarrow y=\frac{-2}{3}x\pm 1$$

$$\Rightarrow m=-\frac{2}{15} \text{ and } -\frac{2}{3}$$

Taking positive sign,

$$y=\frac{-2}{3}x+1$$

$$\Rightarrow 3y=-2x+3$$

$$\Rightarrow 3y+2x=3$$

$\therefore$ Equation of tangent from the point $(3,-1)$ to the ellipse $2x^2+9y^2=3$ is $2x+3y-3=0$.

30. The position vectors of the vertices A,B,C of a tetrahedron $ABCD$ are $\hat{i}+\hat{j}+\hat{k},\hat{i}$ and $3\hat{i}$ respectively and the altitude for the vertex D to the opposite face ABC meets the face at E. If the length of ED is 4 and the volume of tetrahedron is $\frac{2\sqrt{2}}{3}$, then the length of DE is **[2019]**

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

Sol. We have,

$A(\hat{i}+\hat{j}+\hat{k}), B(\hat{i})$ and $C(3\hat{i})$

$$\therefore \overrightarrow{AB}=-\hat{j}-\hat{k} \text{ and } \overrightarrow{AC}=2\hat{i}-\hat{j}-\hat{k}$$

$$\therefore \text{ar}(\triangle ABC)=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$$

$$=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & -1 & -1\\ 2 & -1 & -1\end{matrix}\right\|$$

$$=\frac{1}{2}\left[\left|2\hat{j}+2\hat{k}\right|\right]$$

$$=\left|\hat{j}+\hat{k}\right|$$

$$=\sqrt{2}$$

Now volume of tetrahedron

$$=\frac{1}{3}\text{ar}(\triangle ABC)\times \text{ height}$$

$$\Rightarrow \frac{2\sqrt{2}}{3}=\frac{1}{3}\times\sqrt{2}\times \text{ height}$$

$$\Rightarrow \text{ height}=2$$

$\therefore$ Length of $DE=2$.

31. If S and S' are foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, B$ is the end of the minor axis and BSS' is an equilateral triangle, then the eccentricity of the ellipse is **[2019]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

Ans. (a)

Sol. Given that, S and S' are foci of ellipse with coordinates are $(ae,0),(-ae,0)$ respectively & coordinates of end points of minor axis B is $(0,b)$.

Given, BSS' is an equilateral triangle.

$$BS=SS'=2ae$$

$$\Rightarrow \sqrt{a^2e^2+b^2}=2ae$$

$$\Rightarrow a^2e^2+b^2=4a^2e^2$$

$$\Rightarrow b^2=3a^2e^2$$

$$\Rightarrow \frac{b^2}{a^2}=3e^2 \quad ...(i)$$

Now, $e^2 = 1 - \dfrac{b^2}{a^2}$

$\Rightarrow e^2 = 1 - 3e^2$ [By Eq. (i)]

$\Rightarrow e^2 + 3e^2 = 1$

$\Rightarrow 4e^2 = 1$

$\Rightarrow e^2 = \dfrac{1}{4}$

$\Rightarrow e = \dfrac{1}{2}$

32. The equation of the circle passing through the point $(4,6)$ and whose diameters are along $x+2y-5=0$ and $3x-y-1=0$ is **[2019]**

(a) $x^2+y^2-2x-6y-20=0$

(b) $x^2+y^2-6x-2y-20=0$

(c) $x^2+y^2-2x-4y-20=0$

(d) $x^2+y^2-4x-2y-20=0$

Ans. (c)

Sol. Since, the diameter is along $x+2y-5=0$ and $3x-y-1=0$ the point of intersection of these two lines will be the centre of the circle.

Hence, solving these two lines will give $x+2y-5=0$ and $6x-2y-2=0$.

On adding both equations, we get

$\Rightarrow 7x-7=0$

$\Rightarrow 7x=7$

$\Rightarrow x=1$

Putting $x=1$ in $6x-2y-2=0$, we get

$6-2y-2=0$

$\Rightarrow \quad -2y=-4$

$\Rightarrow \quad y=2$

Hence, coordinates of the center is $(1,2)$. The equation of the circle with center as $(1,2)$ is given by

$(x-1)^2+(y-2)^2=r^2$

Since, the circle is passing through $(4,6)$, we have

$(4-1)^2+(6-2)^2=r^2$

$\Rightarrow 3^2+4^2=r^2$

$\Rightarrow r^2=25$

$\Rightarrow r=5$

Hence, the above equation can be written as

$(x-1)^2+(y-2)^2=25$

$\Rightarrow x^2+y^2-2x-4y+1+4=25$

$\Rightarrow x^2+y^2-2x-4y-20=0$

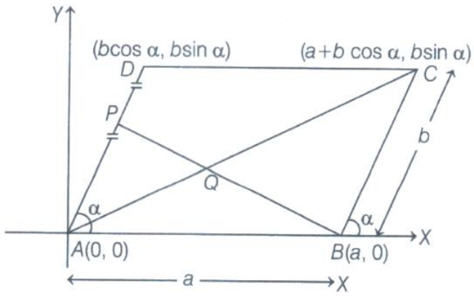
Which is the required equation of circle.

33. In a parallelogram $ABCD, P$ is the mid-point of AD. Also, BP and AC intersect at Q. Then, $AQ:QC=$ **[2019]**

(a) $1:3$ (b) $3:1$ (c) $2:1$ (d) $1:2$

Ans. (d)

Sol.



Since, P is the mid-point of AD.

$\therefore$ Co-ordinates of P are $\left(\dfrac{b\cos\alpha}{2}, \dfrac{b\sin\alpha}{2}\right)$

$\therefore$ Equation of line BP is

$y = \dfrac{b\sin\alpha}{b\cos\alpha - 2a}(x-a)$ ...(i)

Let Q divides AC in ratio $k:1$ Then coordinates of Q are $\left(\dfrac{k(a+b\cos\alpha)}{k+1}, \dfrac{kb\sin\alpha}{k+1}\right)$

Since Q lies on BP

$$\therefore \frac{kb\sin\alpha}{k+1} = \frac{b\sin\alpha}{b\cos\alpha-2a}\left[\frac{k(a+b\cos\alpha)}{k+1}-a\right]$$

$$\Rightarrow \frac{k(b\cos\alpha-2a)}{k+1} = \frac{k(a+b\cos\alpha)-a(k+1)}{k+1}$$

$$\Rightarrow kb\cos\alpha - 2ak = kb\cos\alpha - a$$

$$\Rightarrow -2ak = -a$$

$$\Rightarrow k = \frac{1}{2}$$

$$\therefore \text{ Required ratio } = \frac{1}{2}:1 = 1:2$$

34. The median AD of $\triangle ABC$ is bisected at E and BE is extended to meet the side AC in F. The $AF:AC=$ **[2019]**

(a) $1:3$ (b) $2:1$ (c) $1:2$ (d) $3:1$

Ans. (a)

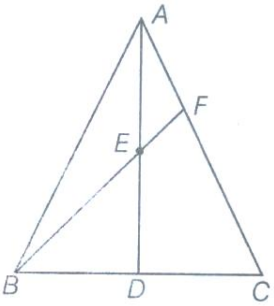
Sol. Let the position vector of A with respect to B is a and that of C with respect to B is c. Then

Position vector of D w.r.t. $B = \dfrac{0+\vec{c}}{2} = \dfrac{\vec{c}}{2}$

Position vector of $E = \dfrac{\vec{a}+\vec{c}/2}{2} = \dfrac{\vec{a}}{2}+\dfrac{\vec{c}}{4}$... (i)

Let $AF:FC=\lambda:1$ and $BE:EF=\mu:1$. Then position vector of $F = \dfrac{\lambda\vec{c}+\vec{a}}{1+\lambda}$

Now, position vector of E



$$E=\frac{\mu\left(\frac{\lambda\vec{c}+\vec{a}}{1+\lambda}\right)+1\times 0}{\mu+1} \quad ...(ii)$$

From Eq. (i) and (ii), we get

$$\Rightarrow \frac{\vec{a}}{2}+\frac{\vec{c}}{4}=\frac{\mu}{(1+\lambda)(1+\mu)}\vec{a}+\frac{\lambda\mu}{(1+\lambda)(1+\mu)}\mathbf{c}$$

$$\Rightarrow \frac{1}{2}=\frac{\mu}{(1+\lambda)(1+\mu)} \text{ and } \frac{1}{4}=\frac{\lambda\mu}{(1+\lambda)(1+\mu)}$$

$$\Rightarrow \lambda=\frac{1}{2}$$

Therefore,

(i) $\frac{AF}{AC}=\frac{AF}{AF+FC}=\frac{\lambda}{1+\lambda}=\frac{1/2}{3/2}=\frac{1}{3}$

35. Let $P(x)$ be a quadratic polynomial such that $P(0)=1$. If $P(x)$ leaves remainder 4 when divided by $x-1$ and it leaves remainder 6 when divided by $x+1$, then **[2019]**

(a) $P(-2)=11$ (b) $P(2)=11$

(c) $P(2)=19$ (d) $P(-2)=19$

Ans. (d)

Sol. Let $P(x)=(x-1)(ax+b)+4$

Now, $P(0)=(0-1)(a\cdot 0+b)+4=1$

$\Rightarrow (-1)b+4=1$

$\Rightarrow b=3$

Also, $6=P(-1)\Rightarrow(-1-1)(-a+b)+4$

$\Rightarrow (-2)(-a+b)+4=6$

$\Rightarrow (-2)(-a+3)+4=6$

$\Rightarrow 2a-6+4=6$

$\Rightarrow 2a=8\Rightarrow a=4$

$\therefore P(x)=(x-1)(4x+3)+4$

So, $P(2)=(2-1)(4\times 2+3)+4$
$=15$

and $P(-2)=(-2-1)(-8+3)+4$
$=15+4=19$

36. The tangent at the point $(2,-2)$ to the curve $x^2y^2-2x=4(1-y)$ does not pass through the point **[2019]**

(a) $(-2,-7)$ (b) $(-4,-9)$ (c) $\left(4,\frac{1}{3}\right)$ (d) $(8,5)$

Ans. (a)

Sol. Given equation of curve is

$x^2y^2-2x=4(1-y)$

Differentiating w.r.t. x,

$$2x^2y\frac{dy}{dx}+2xy^2-2=-4\frac{dy}{dx}$$

At point $(2,-2)$,

$$\Rightarrow 2(2)^2(-2)\frac{dy}{dx}+2(2)(-2)^2-2=-4\frac{dy}{dx}$$

$$\Rightarrow 14=12\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{14}{12}=\frac{7}{6}$$

$\therefore$ Equation of tangent at $(2,-2)$,

$$(y+2)=\frac{7}{6}(x-2)$$

$6y+12=7x-14$

$=7x-6y-26=0$

Now, $(-4,-9),\left(4,\frac{1}{3}\right)$ and $(8,5)$ satisfies the Eq. (i).

But $(-2,-7)$ does not satisfies the Eq. (i).

So, given curve does not pass through $(-2,-7)$.

37. The integral $\int\sqrt{1+2\cot x(\operatorname{cosec}x+\cot x)}\,dx$, $\left(0<x<\frac{\pi}{2}\right)$ (where c is a constant of integration) is equal to **[2019]**

(a) $2\log\left(\sin\frac{x}{2}\right)+c$ (b) $2\log\left(\cos\frac{x}{2}\right)+c$

(c) $4\log\left(\cos\frac{x}{2}\right)+c$ (d) $4\log\left(\sin\frac{x}{2}\right)+c$

Ans. (a)

Sol. Given, $1+2\cot x(\operatorname{cosec}x+\cot x)$

$=1+2\cot x\operatorname{cosec}x+2\cot^2x$

$=(1+\cot^2x)+2\cot x\operatorname{cosec}x+\cot^2x$

$=\operatorname{cosec}^2x+2\cot x\operatorname{cosec}x+\cot^2x$

$=(\operatorname{cosec}x+\cot x)^2$

$\therefore I=\int\sqrt{(\operatorname{cosec}x+\cot x)^2}\,dx$

$$=\int\left(\frac{1}{\sin x}+\frac{\cos x}{\sin x}\right)dx$$

38. If all the words, with or without meaning, are written using the letters of the word QUEEN add are arranged as in English Dictionary, then the position of the word QUEEN is **[2019]**

(a) 47th (b) 44 th (c) 45 th (d) 46 th

Ans. (d)

Sol. Letters of word QUEEN are

E, E, N, Q, U

Words beginning with E$(4!)=$ Rank (1 to 24)

Words beginning with N$(4!/2!)=(25$ to $36)$

Words beginning with QE$(3!)=(37$ to $42)$

Words beginning with QN$(3!/2!)=(43$ to $45)$

QUEEN is the next word and has rank 46^{th}.

39. The curve satisfying the differential equation $ydx-\left(x+3y^2\right)dy=0$ and passing through the point $(1,1)$ also passes through the point _____ **[2019]**

(a) $\left(\frac{1}{4},\frac{1}{2}\right)$ (b) $\left(\frac{1}{4},-\frac{1}{2}\right)$ (c) $\left(-\frac{1}{3},\frac{1}{3}\right)$ (d) $\left(\frac{1}{3},-\frac{1}{3}\right)$

Ans. (c)

Sol. Write the given differential equation as

$$\frac{dx}{dy}+\left(-\frac{1}{y}\right)x=3y \quad ...(i)$$

$\therefore$ I.F $=e^{\int-\frac{1}{y}dy}$

$$=e^{-\ln(y)}=\frac{1}{y}$$

$\therefore$ solution of equation is $x+\frac{1}{y}=\int 3y+\frac{1}{y}dy+c$

$$\frac{x}{y}=3y+c$$

As it passes through $(1,1)$, we get

$\Rightarrow \quad 1=3+c$

$\Rightarrow \quad c=-2$

$\therefore \quad x=3y^2+cy$

$\Rightarrow \quad x=3y^2-2y$

$\Rightarrow \quad x=y(3y-2) \ ...(i)$

So, point $\left(-\frac{1}{3},\frac{1}{3}\right)$ passes through Eq. (i).

40. $\lim_{x\to3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to **[2019]**

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

Ans. (d)

Sol. Given,

$$\lim_{x\to3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$$

$$=\lim_{x\to3}\frac{\left(\sqrt{3x}-3\right)\left(\sqrt{3x}+3\right)\left(\sqrt{2x-4}+\sqrt{2}\right)}{\left(\sqrt{2x-4}-\sqrt{2}\right)\left(\sqrt{2x-4}+\sqrt{2}\right)\left(\sqrt{3x}+3\right)}$$

$$=\lim_{x\to3}\frac{(3x-9)\left(\sqrt{2x-4}+\sqrt{2}\right)}{(2x-6)\left(\sqrt{3x}+3\right)}$$

$$=\frac{3}{2}\lim_{x\to3}\frac{\left(\sqrt{2x-4}+\sqrt{2}\right)}{\left(\sqrt{3x}+3\right)}$$

$$=\frac{3}{2}\times\frac{\sqrt{2\times3-4}+\sqrt{2}}{\sqrt{3\times3}+3}$$

$$=\frac{3}{2}\times\frac{2\sqrt{2}}{6}$$

$$=\frac{1}{\sqrt{2}}$$

41. The sum of infinite terms of a decreasing GP is equal to the greatest value of the function $f(x)=x^3+3x-9$ in the interval $[-2,3]$ and the difference between the first two terms is $f'(0)$. Then, the common ratio of GP is **[2019]**

(a) $-\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

Ans. (c)

Sol. Given, $f(x)=x^3+3x-9$

Differentiating w.r.t. x, we get

$f'(x)=3x^2+3$

For maxima/minima, $f'(x)=0$

$\Rightarrow 3x^2+3=0$

$\Rightarrow x^2=-1($ not possible for $x\in R)$

$\therefore f(-2)=(-2)^3+3(-2)-9$

$=-8-6-9=-23$

and $f(3)=(3)^3+3(3)-9$

$=27+9-9=27$

$\therefore$ Greatest value of function is 27 .

According to the question,

$\Rightarrow a\dfrac{a}{1-r}=27$

$a=27-27r$

$\Rightarrow a+27r=27$ ...(i)

Now, $f'(0)=3$

Also given $a-ar=f'(0)$

$\Rightarrow a(1-r)=3$

$\Rightarrow 1-r=\dfrac{3}{a}$ ...(ii)

From Eqs. (i) and (ii), we get

$$a+27\left(1-\frac{3}{a}\right)=27$$

$$a+27-\frac{81}{a}=27$$

$$\Rightarrow a=81$$

$$\Rightarrow a=\pm 9$$

$\because$ G.P is decreasing

$\therefore \quad a=9$

Now, $\dfrac{9}{1-r}=27$

$$\Rightarrow \frac{1}{3}=1-r$$

$$\Rightarrow r=1-\frac{1}{3}=\frac{2}{3}$$

42. Number of onto (surjective) functions from A to B if $n(A)=6$ and $n(B)=3$, is **[2019]**

(a) 2^6-2 (b) 3^6-3 (c) 340 (d) 540

Ans. (d)

Sol. Number of onto function is given by

$$\sum_{r=1}^{n}(-1)^{n-r}\,{}^{n}C_r(r)^m$$

where $n=3, m=6$

$$\therefore \text{ Onto function}=\sum_{r=1}^{3}(-1)^{3-r}\,{}^{3}C_r\cdot(r)^6$$

$$={}^3C_1(1)+(-1)^3C_2(2)^6+{}^3C_3(3)^6$$

$$=3-192+729=540$$

43. If $|z|<\sqrt{3}-1$, then $|z^2+2z\cos\alpha|$ is **[2019]**

(a) less than 2 (b) $\sqrt{3}+1$

(c) $\sqrt{3}-1$ (d) None of these

Ans. (a)

Sol. We have,

$$|z^2+2z\cos\alpha|\le|z^2|+|2z\cos\alpha|$$

$$|z^2+2z\cos\alpha|\le|z^2|+2|z||\cos\alpha|$$

$$|z^2+2z\cos\alpha|\le|z^2|+2|z|$$

$$|z^2+2z\cos\alpha|<(\sqrt{3}-1)^2+2(\sqrt{3}-1)\quad[\because |z|<\sqrt{3}-1]$$

$$|z^2+2z\cos\alpha|<3+1-2\sqrt{3}+2\sqrt{3}-2$$

$$|z^2+2z\cos\alpha|<2$$

44. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of the computers produced in the factory turns out to be defective. It is known that P (computer turn out to be defective given that it is produced in plant T_1) $=10P$ (computer turns out to be defective given that it is produced in plant T_2). A computer produced in the factory is randomly selected and it does not turn out to be defective. Then, the probability that it is produced in plant T_2 is **[2019]**

(a) $\dfrac{36}{73}$ (b) $\dfrac{47}{79}$ (c) $\dfrac{78}{93}$ (d) $\dfrac{75}{83}$

Ans. (c)

Sol. Let P_1 be the defective computers that are produced from plant T_1 and P_2 be that from plant T_2.

The total percentage of the defective computers produced is 7%.

Now, $P_1=10P$ and $P_2=P$

The computer produced that are defective,

$$\frac{20}{100}\times P_1+\frac{80}{100}\times P_2=\frac{7}{100}$$

$$\Rightarrow 20P_1+80P_2=7$$

$$\Rightarrow 200P+80P=7$$

$$\Rightarrow 280P=7$$

$$\Rightarrow P=\frac{7}{280}=\frac{1}{40}$$

P (Computers that are not defective)

$$=1-\frac{1}{40}=\frac{39}{40}$$

Now, the probability of the defective products is calculated as follows:

$$\frac{20}{100}P_1+\frac{80}{100}P_2=\frac{20}{100}\times\frac{1}{4}+\frac{80}{100}\times\frac{1}{40}$$

$$=\frac{28}{400}=\frac{7}{100}$$

The probability of producing not defectivecomputer is

$$=1-\frac{7}{100}=\frac{93}{100}$$

The probability that plant T_2 produces not defective computers is calculated

$$=\frac{\left(\frac{80}{100}\right)\times\left(\frac{39}{40}\right)}{\left(\frac{93}{100}\right)}=\frac{78}{93}$$

45. If $A>0, B>0$ and $A+B>\frac{\pi}{6}$, then the minimum value of $\tan A+\tan B$ is **[2019]**

(a) $\sqrt{3}-\sqrt{2}$ (b) $4-2\sqrt{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) $2-\sqrt{3}$

Ans. (b)

Sol. We have,

$$A+B=\frac{\pi}{6}$$

$$\Rightarrow \quad B=\frac{\pi}{6}-A$$

Now,

$$\tan A+\tan B=\tan A+\tan\left(\frac{\pi}{6}-A\right)$$

$$=\tan A+\frac{\frac{1}{\sqrt{3}}-\tan A}{1+\frac{\tan A}{\sqrt{3}}}$$

$$=x+\frac{1-\sqrt{3}x}{\sqrt{3}+x} \text{ [where } \tan A=x \text{]}$$

$$=\frac{\sqrt{3}x+x^2+1-\sqrt{3}x}{\sqrt{3}+x}$$

$$=\frac{x^2+1}{x+\sqrt{3}}=f(x) \text{ (say)}$$

$$\therefore f(x)=\frac{x^2+1}{x+\sqrt{3}}$$

$$\Rightarrow f'(x)=\frac{2x\left(x+\sqrt{3}\right)-\left(x^2+1\right)}{(x+\sqrt{3})^2}$$

$$=\frac{x^2+2\sqrt{3}x-1}{(x+\sqrt{3})^2}$$

For minimum, $f'(x)=0$

$$\Rightarrow x^2+2\sqrt{3}x-1=0$$

$$\Rightarrow x=\frac{-2\sqrt{3}\pm\sqrt{12+4}}{2}$$

$$=\frac{-2\sqrt{3}\pm 4}{2}=-\sqrt{3}\pm 2$$

Now, $f''(x)$ at $x=2-\sqrt{3}$ is positive

So, $f(x)$ is minimum at $x=2-\sqrt{3}$.

$$\therefore \quad \tan A=2-\sqrt{3}$$

$$\Rightarrow \quad A=\frac{\pi}{12}$$

$$\therefore \quad B=\frac{\pi}{6}-\frac{\pi}{12}=\frac{\pi}{12}$$

$\therefore$ Minimum value of $\tan A+\tan B$

$$=2\tan A=2\left(2-\sqrt{3}\right)$$

$$=4-2\sqrt{3}$$

46. The mean of 5 observations is 5 and their variance is 12.4. If three of the observations are $1,2,6$, then the mean deviation from the mean of the data is **[2019]**

(a) 2.5 (b) 2.6 (c) 2.8 (d) 2.4

Ans. (c)

Sol. Let the two numbers be x and y.

Given, Variance $\sigma^2=124$

Mean, $\bar{x}=5$ and $n=5$

$$\therefore \frac{1+2+6+x+y}{5}=5$$

$$\Rightarrow x+y=16$$

Now,

$$124=\frac{(1-5)^2+(2-5)^2+(6-5)^2+(x-5)^2+(y-5)^2}{5}$$

$$\Rightarrow \quad 124=\frac{16+9+1+(x-5)^2+(y-5)^2}{5}$$

$$\Rightarrow \quad (x-5)^2+(16-x-5)^2=124\times 5-26$$

$$\Rightarrow \quad (x-5)^2+(11-x)^2=62-26$$

$$\Rightarrow \quad x^2+25-10x+121+x^2-22x=36$$

$$\Rightarrow \quad 2x^2-32x+110=0$$

$$\Rightarrow \quad x^2-16x+55=0$$

$$\Rightarrow \quad x^2-5x-11x+55=0$$

$$\Rightarrow \quad x(x-5)-11(x-5)=0$$

$$\Rightarrow \quad (x-5)=0 \text{ or } x-11=0$$

$$\Rightarrow \quad x=5 \text{ or} 11$$

Hence, $y=11$ or 5

So, mean deviation

$$=\frac{|1-5|+|2-5|+|6-5|+|5-5|+|11-5|}{5}$$

$$=\frac{4+3+1+0+6}{5}=\frac{14}{5}=2.8$$

47. In a beauty contest, half the number of experts voted for Mr. A and two third voted for Mr. B. 10 voted for both and 6 did not for either. How many experts were there in all? **[2019]**

(a) 18 (b) 36 (c) 24 (d) None of these

Ans. (c)

Sol. Let the total number of voters $=x$

According to the question,

$\frac{x}{2}$ voters voted Mr. A

and $\frac{2x}{3}$ voters voted for Mr. B.

$\Rightarrow \frac{x}{2}+\frac{2x}{3}-10=x-6$

$\Rightarrow \frac{x}{2}+\frac{2x}{3}-x=10-6$

$\Rightarrow \frac{3x+4x-6x}{6}=4$

$\Rightarrow \frac{7x-6x}{6}=4$

$\Rightarrow \frac{x}{6}=4$

$\Rightarrow x=24$

Hence, total number of voters $=24$

48. The value of non-zero scalars α and β such that for all vectors $\vec{a}$ and $\vec{b}$, such that

$\left(\vec{b}-\alpha\vec{a}\right)-\left(2\vec{a}+3\vec{b}\right)=0$ **[2019]**

(a) $\alpha=2,\beta=1$ (b) $\alpha=-2,\beta=-3$

(c) $\alpha=1,\beta=3$ (d) $\alpha=-2,\beta=3$

Ans. (a)

Sol. We have,

$(2-\alpha)\vec{a}-(\beta-1)\vec{b}=0$

$\therefore\ 2-\alpha=0$ and $\beta-1=0$

$\Rightarrow\ \alpha=2,\beta=1$

49. A force of 78 grams acts at the point $(2,3,5)$. The direction ratios of the line of action being $2,2,1$. The magnitude of its moment about the line joining the origin to the point $(12,3,4)$ is **[2019]**

(a) 24 (b) 136 (c) 36 (d) 0

Ans. (b)

Sol. Given direction ratios of the line action is $(2,2,1)$.

$\therefore$ Unit vector along the force is $\vec{F}=\frac{2\hat{i}+2\hat{j}+\hat{k}}{3}$.

Now, moment of the force about the point $A(2,3,5)$ is magnitude of moment about the line joining the origin to the point $\vec{B}(12,3,4)=\left(\overrightarrow{AB}\times\vec{F}\right)\cdot\hat{a}$

Now, $\overrightarrow{AB}=(12-2,3-3,4-5)=(10,0,-1)$

Now, $\overrightarrow{AB}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 10 & 0 & -1\\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3}\end{vmatrix}$

$=\frac{1}{3}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 10 & 0 & -1\\ 2 & 2 & 1\end{vmatrix}$

$=\frac{1}{3}\left[\hat{i}(0+2)-\hat{j}(10+2)+\hat{k}(20-0)\right]$

$=\frac{1}{3}\left[2\hat{i}-12\hat{j}+20\hat{k}\right]$

$\therefore$ Required moment

$=78\times\left[\frac{2\hat{i}-12\hat{j}+20\hat{k}}{3}\right]\frac{\left(12\hat{i}+3\hat{j}+4\hat{k}\right)}{13}$

$=2[24-36+80]=2\times 68$

$=136$

(Since, magnitude cannot be negative)

50. Number of real solution of the equation $\sin\left(e^x\right)=5^x+5^{-x}$ is **[2019]**

(a) 0 (b) 1 (c) 2 (d) Infinitely many

Ans. (a)

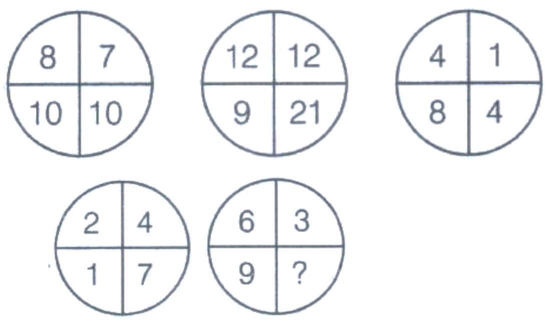
Sol. We know that,

$\sin\left(e^x\right)\le 1$ and $5^x+5^{-x}\ge 2$

It is clear that equation doesn't hold true for any real value of x.

Analytical Ability and Logical Reasoning

51. Which number replaces the question mark in the figure given below? **[2019]**



(a) 11 (b) 6 (c) 3 (d) 21

Ans. (c)

Sol. With the help of lower two circles as a source, the values in corresponding segments of the upper left circle equal the sums of the numbers in the lower two circles. The values in the upper central circle equal the products of the values in the lower two circles and the upper right circle equals the difference between values in the lower two circles.

$\therefore ? = 3$

52. Statement I: Out of total of 200 readers, 100 read Indian Express, 120 read Times of India and 50 read Hindu.

Statement II : Out of a total of 200 readers, 100 read Indian Express, 120 reads Times of India and 50 read neither.

How many people (from the group surveyed) read both Indian Express and Times of India? **[2019]**

(a) The question can be answered with the help of statement II, alone.

(b) Both, statement I and statement II are needed to answer the question.

(c) The question can be answered with the help of statement I alone.

(d) The question cannot be answered even with the help of both the statements.

Ans. (a)

Sol. Let A be the number of people who read Indian express and B be the number of people who reads Time of India then to find

$n(A \cap B) \quad ...(i)$

In statement-1 $n(U) = 200, n(A) = 100, n(B) = 120$

$n(\text{ Hindu }) = 50$

It cannot be determined.

In statement-II

$$n(U) = 200, n(A) = 100, n(B) = 120$$

$$n(\overline{A \cup B}) = 50, n(U) - n(A \cup B) = 50$$

$$\Rightarrow \quad n(A) + n(B) - nA(\cap B) = 150$$

$$\Rightarrow \quad 100 + 120 - n(A \cap B) = 150$$

$$\Rightarrow \quad n(A \cup B) = 70$$

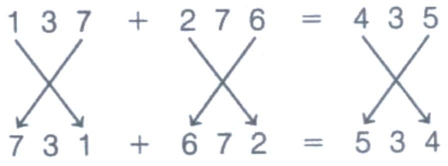
Hence, question can be answered with the help of statement II alone.

53. If $137 + 276 = 435$, how much is $731 + 672 =$? **[2019]**

(a) 534 (b) 1403 (c) 1623 (d) 1531

Ans. (a)

Sol.



54. Study the information carefully and answer the questions given below

If we arrange the alphabets in the word "RATE" in the English alphabetical order, word "AERT" is formed. Then the third alphabet from the left in this word is "R". Similarly, from the word "OPEN" we get-"ENOP" and the third alphabet from the left is " O ". 5 m the

word "CHEF" we get-"CEFH" the third alphabet from the left "F". From the word "TOY" we get-"OTY" and the third alphabet from the left is " Y ". From the word "70P" we qu-"DPT" and the third alphabet from the left is ' T '. If we use all these letters, then a meaningful English word "FORTY" can be formed. Now find which of the following word set DOES NOT give a meaningful word in the similar way. **[2019]**

(a) SAME, ROOM, BEST, AUTO

(b) GOAT, PEST, WATT, ARMY

(c) MALE, FIND, LOST, THAT

(d) JUMP, LIME, DUMB, SOME

Ans. (d)

Sol. From option (a), SAME, ROOM, BEST, AUTO

Now, AEMS, MOQR, BEST, AOTU

Now, meaningful word formed from M, O, S and T is MOST.

From option (b), GOAT, PEST, WATT, ARMY

Now, AGQT, EPST, ATTW, AMRY

Now, meaningful word formed from O, S, T and R is SORT.

From option (c), MALE, FIND, LOST, THAT

Now, AELM, DFIN, LOST, AHTT

Now, meaningful word formed from L, I, S and T is LIST.

From option (d), JUMP, LIME, DUMB, SOME

Now, JMPU, EILM, BDMU, EMOS

Here, no meaningful word can be formed from P, L, M and O .

55. Navjivan Express from Ahmedabad to Chennai leaves Ahmedabad at 6:30 am and travels at 50 kmph towards Baroda situated 100 km away. At 7.00 am Howrah-Ahmedabad Express leaves Baroda towards Ahmedabad and travels at 40 kmph . At 7:30 am Mr. Shah, the traffic controller at Baroda realizes that both trains are running on the same track. How much time does he have to avert a head-on collision between the two trains? **[2019]**

(a) 15 min (b) 20 min (c) 25 min (d) 30 min

Ans. (b)

Sol. Initially, 6:30 AM distance between both trains is 100 km now at 7:00 AM, Navjeevan travels 25 km , so remaining distance between both trains is 75 km . Again at 7:30 AM, Navjeevan travels is 25 km more and also Howrah travel 20 km .
So, final distance between then is
$75-(25+20)\text{km}=30$ km , so time taken to travel 30 km at relative speed
$(50 \text{ km/h}+40 \text{ km/h}) 90 \text{ km/h}$
$=\frac{30}{90} \text{ h}=\frac{1}{3}\times 60=20 \text{ min}$

56. If the points $P(a^2,a)$ lie in the region corresponding to the acute angle between the lines $x=2y$ and $x=4y$ then **[2019]**

(a) $a\in(2,6)$ (b) $a\in(4,6)$
(c) $a\in(2,4)$ (d) $a\in(10,14)$

Ans. (c)
Sol. The joint equation of two lines
$x=2y$ and $x=4y$
i.e. $u\equiv x-2y=0$ and $v\equiv x-4y=0$ is
$u\cdot v=0$
i.e. $(x-2y)\cdot(x-4y)=0$
i.e. $x^2-4xy-2xy+8y^2=0$
$S(x,y)\equiv x^2-6xy+8y^2=0 \quad ...(i)$
If the point $P(a^2,a)$ lies in the interior of the acute angle formed by these lines, then the value of $S(x,y)$ at $P(x=a^2,y=a)$ is negative or is less than zero.
i.e. $S(a^2,a)\equiv(a^2)^2-6(a^2)(a)+8a^2<0$
$\therefore \quad a^2(a^2-6a+8)<0$
$\therefore \quad a^2-6a+8<0$
$\therefore \quad (a-2)(a-4)<0$
$\therefore \quad 2<a<4$
i.e. $a\in(2,4)$

57. Some friends planned to contribute equally to jointly buy a CD player. However, two of them decided to withdraw at the last minute. As a result, each of the others had to shell out one rupee more than what they had planned for. If the price (in ₹) of the CD player is an integer between 1000 and 1100, find the number of friends who actually contributed? **[2019]**

(a) 44 (b) 23 (c) 21 (d) 46

Ans. (a)
Sol. Let the cost of CD be ₹ y
and total number of friends be x.
Now, each of them has to pay ₹ $\frac{y}{x}$.
Now, after withdrawing 2 friends the remaining has to pay $=\frac{y}{x-2}$
Now, according to given condition,
$$\frac{y}{x-2}-\frac{y}{x}=1$$
$$\Rightarrow \quad y=\frac{x(x-2)}{2}$$
Now, given $1000<y<1100$
$\Rightarrow x=46$
So, number of friends who actually contributed is
$46-2=44$

58. Two liquids A and B are in the ratio $5:1$ in container 1 and in the ratio $1:3$ in container 2 . In what ratio should the contents of the two containers be mixed so as to obtain a mixture of A and B in the ratio $1:1$? **[2019]**

(a) $2:3$ (b) $4:3$ (c) $3:2$ (d) $3:4$

Ans. (d)
Sol. Let the required ratio be $x:y$.
Liquid A in x L of mixture in container $1=\frac{5x}{6}$ L
Liquid B in x L of mixture in container $1=\frac{x}{6}$ L
Liquid A in y L of mixture in container $2=\frac{y}{4}$ L
Liquid B in y L of mixture in container $2=\frac{3y}{4}$ L
According to the question,
$$\left[\frac{5x}{6}+\frac{y}{4}\right]:\left[\frac{x}{6}+\frac{3y}{4}\right]=1:1$$
Given, $\dfrac{\frac{5x}{6}+\frac{y}{4}}{\frac{x}{6}+\frac{3y}{4}}=\frac{1}{1}$

$$\Rightarrow \frac{5x}{6}+\frac{y}{4}=\frac{x}{6}+\frac{3y}{4}$$

$$\Rightarrow \frac{5x}{6}-\frac{x}{6}=\frac{3y}{y}-\frac{y}{4}$$

$$\Rightarrow \frac{4x}{6}=\frac{2y}{4}$$

$$\Rightarrow \frac{2x}{3}=\frac{y}{2}$$

$$\Rightarrow \frac{x}{y}=\frac{3}{4}$$

Hence, the required ratio is $3:4$.

59. Each family in a locality has at most adults, and no family has fewer than 3 children. Considering all the families together, there are more adults than boys, more boys than girls, and more girls than families. Then, the minimum possible number of families in the locality is **[2019]**

(a) 4 (b) 3 (c) 2 (d) 5

Ans. (b)

Sol. Let n_f be number of families, n_g be number of girls, n_b be number of boys, n_a be number of adults.

Number of adults more than number of boys, number of boys more than number of girls and number of girls more than number of families.

Hence, we have

$n_a > n_b > n_g > n_f$ ...(i)

We check the above relation (1) with different values for number of families starting with $n_f = 1$

Let $n_f = 1$, then $n_b + n_g = 3$, if $n_g = 2$, then $n_b = 1$ But number of boys is greater than number of girls due to this contradiction, $n_f \neq 1$

Let $n_f = 2$, then $n_b + n_g = 6$, if $n_g = 3$, then $n_b = 3$. But number of boys is greater than number of girls due to this contradiction, $n_f \neq 2$.

Let $n_f = 3$, then $n_b + n_g = 9$, if $n_g = 4$, then $n_b = 5$, number of adults $n_a = 6$

Hence, the relation $n_a > n_b > n_g > n_f$ is satisfied for the above numbers, $6 > 5 > 4 > 3$.

Hence, minimum number of families is 3 .

60. Fresh grapes contain 90% by weight while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 20 kg of fresh grapes? **[2019]**

(a) 2.5 kg (b) 2.4 kg (c) 2 kg (d) 10 kg

Ans. (a)

Sol. 90% of the weight of the fresh grapes is water, so 10% is not water.

$10\% \times 20\text{ kg} = 2\text{ kg}$ of total waterless grapes. Dried grapes contain 20% water. So, 80% of the weight of dried grapes is not water $80\% \times$ dried grapes weight $= 2\text{ kg}$

so, dried grape weight $= \frac{2\text{ kg}}{80\%} = \frac{2}{80} \times 100 = 25\text{ kg}$

Directions (Q. Nos. 61 and 62) for questions 61 and 62 Answer the questions on the basis of the information given below

A, B, C, D, E and F are a group of friends. There are two housewives, one professor, one engineer, one accountant and one lawyer in the group. There are only two married couples in the group. The lawyer is married to D, who is a housewife. No woman in the group is either an engineer or an accountant. C, the accountant, is married to F , who is a professor. A is married to a housewife. E is not a housewife.

61. What is E's profession? **[2019]**

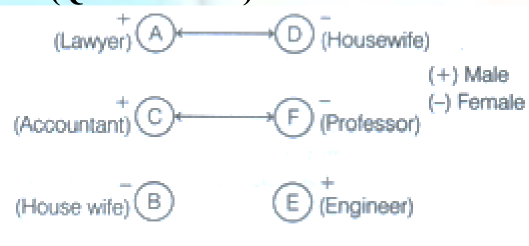
(a) Accountant (b) Lawyer
(c) Professor (d) Engineer

Ans. (d)

Sol. Engineer

According to given question diagram is as following

Sol. (Q. Nos. 61-63)



62. How many members of the group are males? **[2019]**

(a) 2 (b) 3 (c) 4 (d) Cannot be determined

Ans. (b)

Sol. 3

63. Wrong number in the series $7, 8, 18, 57, 228, 1165$, 6996 is **[2019]**

(a) 288 (b) 18 (c) 57 (d) 28

Ans. (a)

Sol. Given series is $7, 8, 18, 57, 228, 1165, 6996$

$$7 \times 1 + 1 = 8$$
$$8 \times 2 + 2 = 18$$
$$18 \times 3 + 3 = 57$$
$$57 \times 4 + 4 = 232 \neq 228$$
$$23 \times 5 + 5 = 1165$$
$$1165 \times 6 + 6 = 6996$$

So, wrong term is 228 .

Directions (Q. Nos. 64 and 65)

Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both statements and

Give the answer (U) if the data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

Give the answer (V) if the data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

Give the answer (W) if the data either in Statement I or in Statement II alone are sufficient to the answer of the question.

Give the answer (X) if the data in both statements I and II together are not sufficient to answer the question.

Give the answer (Y) if the data in both statements I and II together are necessary to answer the question.

64. How much time will leak take to empty the full cistern? **[2019]**

I. The cistern is normally filled in 9 h .

II. It takes one hour more than the usual time to fill the cistern because of a leak in the bottom.

(a) V (b) U (c) X (d) Y

Ans. (d)

Sol. (i) The cistern fill the tank in 9 hours.

(ii) It takes one hour more than the usual time to till the cistern because of a leak in the bottom.

So, far finding the leakage to empty the full cistern, we have required both statements.

65. How will long it take to empty the tank if both the inlet pipe P1 and the outlet pipe P2 are opened simultaneously? **[2019]**

I. $\dot{P}_1$ can fill the tank in 16 min .

II. P_2 can empty the full tank in 8 min .

(a) X (b) U (c) Y (d) V

Ans. (c)

Sol. (i) P_1 fill the tank in 16 minutes

$\therefore P_1$ fill in the tank in 1 minute $=\dfrac{1}{16}$

(ii) P_2 empty the full tank in 8 minutes.

$\therefore P_2$ empty the tank in 1 min $=-\dfrac{1}{8}$

$\therefore P_1$ and P_2 empty the tank in 1 min $=\dfrac{1}{8}-\dfrac{1}{16}=\dfrac{1}{16}$

Hence, both statements are required for finding the answer.

66. How many positive numbers less than 10000 are such that the product of their digits is 210 ? **[2019]**

(a) 36 (b) 42 (c) 48 (d) 54

Ans. (d)

Sol. $210=1\times 2\times 3\times 5\times 7$

$=1\times 6\times 5\times 7$ (only 2×3 makes the single digit 6) so, four digit numbers with combinations of the digits $\{1,6,5,7\}$ and $\{2,3,5,7\}$ and three digit numbers with combinations of digits $\{6,5,7\}$ will have the product of their digits equal to 210.

Number using $1,6,5,7=4!=24$

Number using $2,3,5,7=4!=24$

Number using 6, 5,7=3!=6

$\therefore$ Total numbers $=24+24+6=54$

67. Each of the five people K, L, M, P and Q is of a different weight. It is known that the number of people heavier than P is the same as the number of people lighter than Q. L is the heaviest and K is not the lightest. Who is the lightest? **[2019]**

(a) M (b) L (c) Q (d) P

Ans. (a)

Sol. According to question

$L>P>K>Q>M$

So, least weight or lightest is M .

68. John, Johny and Janardan participated in a race and each won a different medal among Gold, Silver and Bronze, not necessarily in that order. Each person among them gives two replies to any question, one of which is true and the other is false (in any order). When asked about the details of the medals obtained by them, the following were their replies.

John : I won the Gold medal. Johny won the Bronze medal.

Johny : John won the Silver medal. I won the Gold medal.

Janardan : Johny won the Silver medal. I won the Gold medal.

Which among the following is the correct order of the persons who won the Gold medal, the Silver medal and the Bronze medal respectively? **[2019]**

(a) John, Johny, Janardan (b) Janardan, John, Johny

(c) Johny, Janardan, John (d) Janardan, Johny, John

Ans. (b)

Sol. It is given that, one of the replies of a person is true and the other is false (in any order).

So, firstly we assume that John's first statement i.e. I won Gold medal is true and second statement i.e. Johny won the Bronze medal is false. So, John won the Gold medal.

Now, we take Johny's replies.

If John won the Gold medal then Johny's first

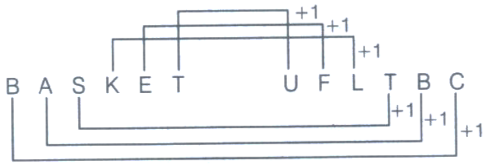
statement i.e. John won the Silver medal becomes false. Also, his second statement which means Johny won the Gold medal becomes false. But, at the same time both the replies can not be false.
Therefore, John's first reply i.e. I won the Gold medal is false. It means his second reply i.e. Johny won the Bronze medal is true.
Now, if Johny won the Bronze medal is true then Johny's second reply i.e. I won the Gold medal becomes false. It means his first reply i.e. John won the Silver medal is true.
Now, if John won the Silver medal is true then Janardan's first reply i.e. Johny won the Silver medal becomes false.
So, Janardan's second reply i.e. I won the Gold medal is true.
Hence, Janardan won Gold medal, John won Silver medal and Johny won Bronze medal.

69. Each of A, B and C is a different digit among 1 to 9 . How many different values of the sum of A, B and C are possible, if $\mathrm{ABA} \times \mathrm{AA} = \mathrm{ACCA}$? **[2019]**
(a) 1 (b) 3 (c) 7 (d) 8
Ans. (d)
Sol. $\mathrm{ABA} \times \mathrm{AA} = \mathrm{ACCA}$
different value for A, B and C

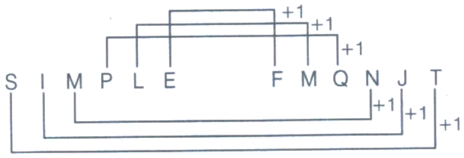
A	B	C	sum
1	1	2	4
1	2	3	6
1	3	4	8
1	4	5	10
1	5	6	12
1	6	7	14
1	7	8	16
1	8	9	18

There are 8 values that satisfies the Eq. (i). Hence, 8 different values of sum A, B and C are possible.

70. In a certain language, if the word 'BASKET' is coded as 'UFLTBC', then how is the word 'SIMPLE' coded in that language? **[2019]**
(a) FMQNJT (b) FMQGNJ
(c) FMQNJH (d) MFNQTJ
Ans. (a)
Sol. As,



Similarly,



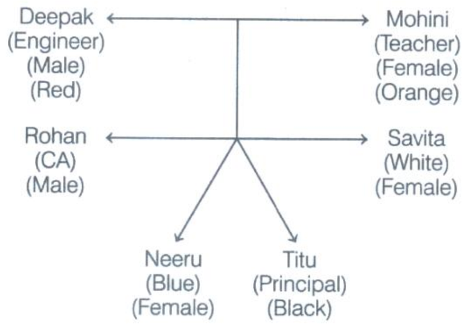
71. A dealer offers sells half of the eggs that he has and another half an egg to Anurag.Then, he sells half of the balance eggs and another half an egg to Deepak. Then, he sells half of the balance eggs and another half an egg to Sivani. In the end he is left with just 7 eggs and he claims that he never broke an egg. How many eggs did he start with? **[2019]**
(a) 65 (b) 63 (c) 67 (d) 69
Ans. (b)
Sol. Let he start with x eggs.
Then the amount of eggs grocer have after selling to Anurag
$$= x - \left[\frac{x}{2} + \frac{1}{2}\right] = \frac{x}{2} - \frac{1}{2}$$
Similarly after selling to Deepak
$$= \frac{x-1}{2} - \left[\frac{x-1}{4} + \frac{1}{2}\right]$$
$$= \frac{x-3}{4}$$
And after selling to Shivani
$$= \frac{x-3}{4} - \left[\frac{x-3}{8} + \frac{1}{2}\right] = \frac{x-7}{8}$$
Since, grocer had left with 7 eggs.
Therefore, $\frac{x-7}{8} = 7$
$\Rightarrow x = 63$
Hence, grocer had initially 63 eggs.

72. There are eight poet, namely, P, Q, R, S, T, U, V and W and H in respect of whom questions are being asked in the examination. The first four are ancient poets and the last four are modern poets. The question on ancient and modern poets is being asked in alternate years. Those who like W also like V , those who like S like R also. The examiner who sets question is not likely to ask question on S because he has written an article on him. But he likes S . Last year a question was asked on U. Considering these facts, on whom the question is most likely to be asked this year? **[2019]**
(a) Q (b) R (c) S (d) V
Ans. (b)

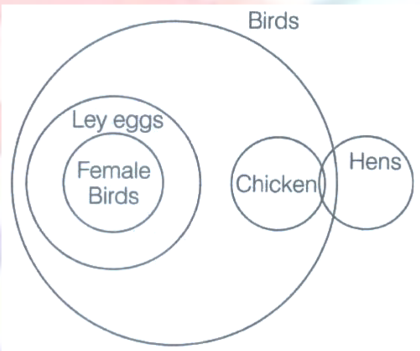
Sol. Given, Ancient poets - P, Q, R, S
Modern poets - T, U, V, W
Last year a question was asked on U i.e. on Modern poet. So, this year the question will be asked on ancient poet because the question on ancient and modern poets is being asked in alternate year.
It is also given, the question on S will not be asked but the examiner likes S , so he also likes R . Hence, the question is most likely to be asked this year on R .

73. A team must be selected from ten probable's-A, B, C, D, E, F, G, H, I and J. Of these, A, C, E and J are forwards, B, G and H are point guards and D, F and I are defenders. The team must have at least one forward,one point guard and one defender. If the team includes J , it must also include F . The team must include E or B , but not both. If the team includes G , it must also include F . The team must include exactly one among C, G and I. C and F cannot be members of the same team. D and H cannot be members of the same team. The team must include both A and D or neither of them. There is no restriction on the number of members in the team. What could be the maximum size of the team that includes G ? **[2019]**
(a) 4 (b) 5 (c) 6 (d) More than 6
Ans. (c)
Sol. The maximum members of the team which includes G
can be 6 i.e. A, D, G, F, E, J. or A, D, G, F, B, J

74. How many 4-digit numbers that can be formed from the digits $2,3,5,6,7$ and 9 , which are divisible by 5 and none of the digits is repeated? **[2019]**
(a) 216 (b) 60 (c) 24 (d) 25
Ans. (b)
Sol. Number of required numbers
$=5\times4\times3\times1$
$=60$

75. In a family of six persons, there are people from three generations. Each person has separate profession and also they like different colours. There are two couples in the family. Rohan is a CA and his wife neither is a
doctor nor likes green colour. Engineer likes red colour and his wife is a teacher. Mohini is mother in law of Savita and she likes orange colour. Deepak is grandfather of Titu and Titu, who is a principal, likes black colour. Nerru is granddaughter of Mohini and she likes blue colour. Nerru's mother likes white colour. Savita is a **[2019]**
(a) Doctor (b) Teacher
(c) Housewife (d) None of these
Ans. (d)
Sol. We have,


Since, no information is given regarding profession of Savita. So, option (d) is correct.

76. 1. All chickens are birds.
2. Some chickens are hens.
3. Female birds lay eggs.
If the above three statements are facts, which of the following statement must also be a fact?
I. All birds lay eggs.
II. Hens are birds.
III. Some chickens are not hens. **[2019]**
(a) II Only
(b) II and III only
(c) I, II and III
(d) None of the statements is known as fact
Ans. (d)
Sol.


None of the statements is known as fact.

77. The number that comes next in the series
$1,2,3,6,11,\ 20,37,68,\ldots$ **[2019]**
(a) 105 (b) 124 (c) 125 (d) 126
Ans. (c)
Sol. The pattern of the series is as follows
$1+2+3=6$
$2+3+6=11$
$3+6+11=20$
$6+11+20=37$
$11+20+37=68$
$20+37+68=125$
Hence, the required number is 125 .

78. Using only $2, 5, 10, 25$ and 50 paise coins, the smallest number of coins required to pay exactly 79 paise, 66 and ₹1.01 to three different persons is **[2019]**

(a) 17 (b) 20 (c) 19 (d) 18

Ans. (d)

Sol. For 1.01 paise,

$1\times50+1\times25+2\times10+3\times2=7$ coins

For 66 paise,

$1\times50+1\times10+3\times2=5$ coins

For 79 paise,

$1\times50+2\times10+1\times5+2\times2=6$ coins

$\therefore$ Total smallest number of coins $=7+5+6=18$ coins

79. What come next in the following sequence

$99, 90, 83, 78, 75, \ldots.$ **[2019]**

(a) 74 (b) 69 (c) 67 (d) 63

Ans. (a)

Sol. The pattern of the series is

=74

80. A dealer offers a cash discount of 20% and still makes a profit of 20%, when he further allows 16 articles to a dozen to a particularly sticky bargainer. How much per cent above the actual price were his was the listed price of the article? **[2019]**

(a) 100% (b) 80% (c) 75% (d) 66%

Ans. (a)

Sol. Let the CP of the article be ₹ x, since he earns a profit of 20%, hence $SP = x + 20\%$ of $x = 1.2x$

It is given that he incurs loss by selling 16 articles at the cost of 12 articles.

$$\therefore \quad \text{loss} = \frac{16-12}{16} = 25\%$$

His selling price $= SP - 25\%$ of SP

$= SP \times 0.75$

Hence, $SP \times 0.75 = 1.2x$

$$\Rightarrow \text{SP} = \frac{12 \times x}{0.75} = 1.6x$$

This SP is arrived after giving a discount of 20% of MP.

Let $MP = y$

$$y - 20\% \text{ of } y = SP$$
$$\Rightarrow 0.80y = 16x$$
$$\Rightarrow y = 2x$$

It means that the articles has been marked 100% above the cost price or marked price was twice of cost price.

Questions (Q. Nos. 81-83) are based on the following instructions,

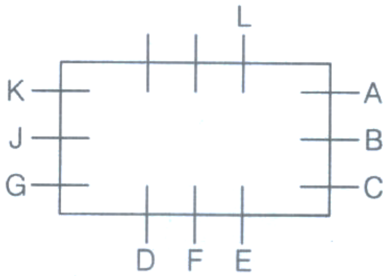
Twelve classmates A, B, C, D, E, F, G, H, I, J, K and L are sitting on a square table with 3 persons on each side. ABC and GJK are sitting on opposite sides. A and L are adjacent to each other. K is sitting diagonally to C .

81. If F is sitting between D and E , who is sitting to the left of K ? **[2019]**

(a) H (b) I (c) H or I (d) None of these

Ans. (c)

Sol.



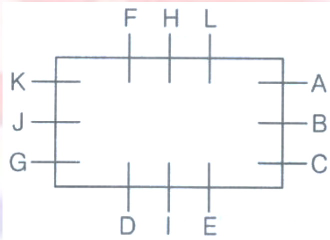
According to the above diagram only H or I can be seated to the left of K.

82. If H is sitting between L and F, then who will be facing H ? **[2019]**

(a) D (b) E (c) G (d) I

Ans. (d)

Sol.



H is sitting between L & F.

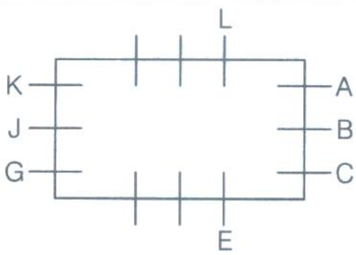
According to above diagram H will face I .

83. If G and E are facing C and L respectively, the neighbours of C are **[2019]**

(a) J and H (b) J and E (c) H and J (d) B and E

Ans. (d)

Sol.



According to above diagram neighbours of C are B and E .

84. The integers 34041 and 32506 , when divided by a 3 -digit integer n, leave the same remainder. What can be the value of n ? **[2019]**

(a) 289 (b) 307 (c) 367 (d) 493

Ans. (b)

Sol. Let the common remainder be x. Then numbers ($34041-x$) and ($32506-x$) would be completely divisible by n.

Hence, the difference of the numbers $(34041-x)$ and $(32506-x)$ will also be divisible by n.

$$34041-x-(32506-x)$$
$$=34041-x-32506+x$$
$$=1535$$

Factors of $1535-1,5,307$

Here, 307 is a three digit factor of 1535 .

So, $n=307$

34041 and 32506 divided by 307 leaves remainder 271.

Hence, the value of n is 307 .

85. The number of solid spheres, each of diameter 3 cm that could be moulded to form a solid metal cylinder of height 54 cm and diameter 4 cm is **[2019]**

(a) 16 (b) 24 (c) 36 (d) 48

Ans. (d)

Sol. Given, diameter of solid sphere $=3$ cm

$\therefore$ Radius of solid sphere $=\frac{3}{2}$ cm

Solid sphere volume $(V_1)=\frac{4}{3}\pi r^3$

$$=\frac{4}{3}\times\frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}$$
$$=\frac{99}{7}\text{ cm}^3$$

Also given, height of the cylinder $=54$ cm

and diameter of the cylinder $=4$ cm

$\therefore$ Radius of the cylinder $=\frac{4}{2}=2$ cm

Volume of the cylinder $(V_2)=\pi R^2 h$

$$=\frac{22}{7}\times2\times2\times54$$
$$=\frac{4752}{7}\text{ cm}^3$$

Hence, the number of spheres $=\frac{V_2}{V_1}$

$$=\frac{\frac{4752}{7}}{\frac{99}{7}}=\frac{4752}{99}=48$$

86. A clock is set right at 5 AM . The clock loses 16 min in 24 h . What will be the true time when the clock indicates 10 PM on 4th day? **[2019]**

(a) 11:15 PM (b) 11:00 PM

(c) 12:00 PM (d) 12:30 PM

Ans. (b)

Sol. Time from 5 am on a day to 10 pm on 4 th day $=89$ h

Now 23 hours 44 min of this clock $=24$ hours of correct clock.

$23\frac{44}{60}=356/15$ h of this clock $=24$ h of correct clock.

89 h of this clock $=\left(24\times\frac{15}{356}\times89\right)$h of correct clock.

$=90$ h of correct clock.

so, the correct time is 11 pm .

87. A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph and passes them completely in 9 sec and 10 sec respectively. The length of the train is **[2019]**

(a) 72 m (b) 54 m (c) 50 m (d) 45 m

Ans. (c)

Sol. Let the length of the train be x m and speed by y m/s.

Speed of the first person $=2$ km/h

$$=2\times\frac{5}{18}=\frac{5}{9}\text{ m/s}$$

Speed of the second person $=4$ km/h

$$=4\times\frac{5}{18}=\frac{10}{9}\text{ m/s}$$

When pass first person, $\frac{x}{y-\frac{5}{9}}=9$

$$\Rightarrow\ 9y-5=x$$
$$\Rightarrow\ 90y-50=10x\ \ ...(i)$$

and when pass second person $\frac{x}{y-\frac{10}{9}}=10$

$$\Rightarrow 90y-100=9x\ \ ...(ii)$$

Subtracting Eqs. (i) and (ii), we get

$90y-50-90y+100=10x-9x$

$\Rightarrow\ x=50$

So, the length of the train $=50$ m

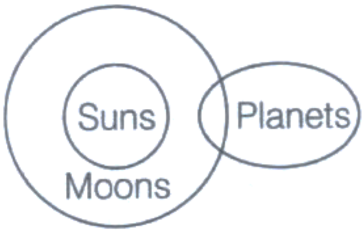
88. Decide which of the give conclusions logically follow from the given students(s)

Statements

All suns are moons.

Some moons are planets

Conclusions
I. All moons are suns.
II. At least some moons are planets. **[2019]**
(a) Either conclusion I or II is true
(b) Neither conclusion I nor II is true
(c) Both conclusion I or II is true
(d) Only conclusion II is true
Ans. (d)
Sol.



Conclusions I-X II- √

89. Ten points are marked on a straight line and eleven points are marked on another straight line. How many triangles can be constructed with vertices from among the above points? **[2019]**
(a) 495 (b) 550 (c) 1045 (d) 2475
Ans. (c)
Sol. Triangle is formed by picking 3 point 2 from 1st line and 1 from 2 md or 1 point from 1st line or 2 from 2 nd .
i.e. ${}^{10}C_2 \times {}^{11}C_1 + {}^{10}C_1 + {}^{11}C_2$

$$= \frac{10!}{2!(10-2)!} \times \frac{11!}{1!(11-1)!} + \frac{10!}{1!(10-1)!} \times \frac{11!}{2!(11-2)!}$$

$$= \frac{10!}{2!8!} \times \frac{11!}{1!10!} + \frac{10!}{1!9!} \times \frac{11!}{2!9!}$$

$$= \frac{10 \times 9}{2 \times 1} \times \frac{11}{1} + \frac{10}{1} \times \frac{11 \times 10}{2 \times 1}$$

$$= 45 \times 11 + 10 \times 55$$

$$= 495 + 550$$

$$= 1045$$

90. The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively, is **[2019]**
(a) 123 (b) 127 (c) 235 (d) 305
Ans. (b)
Sol. Required number

$$= \text{HCF of}(1657-6) \text{ and} (2037-5)$$

$$= \text{HCF of} 1651 \text{ and} 2032 = 127$$

<u>Computer Awareness</u>

91. In IEEE single precision floating point representation, exponent is represented in ____ **[2019]**
(a) 8 bit sign-magnitude representation
(b) 8 bit 2's complement representation
(c) Biases exponent representation with a bias value 127
(d) Biases exponent representation with a bias value 128
Ans. (c)
Sol. In IEEE single precision floating point representation, exponent is represented in Biases exponent representation. The biased exponent is used for representation of negative exponents.
The biased exponent (E) of single precision number can be obtained as

$$E = e + 127$$

92. With 4-bit 2's complement arithmetic, which of the following addition will result in overflow? **[2019]**
(a) $1111+1101$ (b) $0110+0110$
(c) $1101+0101$ (d) $0101+1011$
Ans. (b)
Sol. The rules for detecting overflow in a 2's complement sum are :
(i) If the sum of two positive numbers, yields a negative result; $(+A)+(+B)=-C$
(ii) If the sum of two negative numbers, yields a positive result; $(-A)+(-B)=+C$
(iii) Otherwise, the sum has not overflowed.
From the given options, only option (b) will produce the result in overflow.

93. If we can generate a maximum of 4 Boolean functions using n Boolean variables, what will be minimum value of n ? **[2019]**
(a) 65536 (b) 16 (c) 1 (d) 4
Ans. (c)
Sol. On n Boolean variables, there are 2^{2^n} different Boolean functions.
If Boolean functions are 4 , then

$$2^{2^n} = 4$$

$$2^{2^n} = 2^2$$

On comparing both sides, when base are equal

$$2^n = 2$$

$$2^n = 2^1$$

$$n = 1$$

94. If the 2 's complement representation of a number is $(011010)_2$, what is its equivalent hexadecimal representation? **[2019]**

(a) $(110)_{16}$ (b) $(1\text{ A})_{16}$ (c) $(16)_{16}$ (d) $(26)_{16}$

Ans. (b)

Sol. To convert the binary number into hexadecimal, making the group of 4 bits each from right to left. If the left most group has less than 4 bits, put in the necessary numbers of leading O's on the left.
Now each group will be converted to decimal number.

$$(011010)_2 \rightarrow (?)_{16}$$

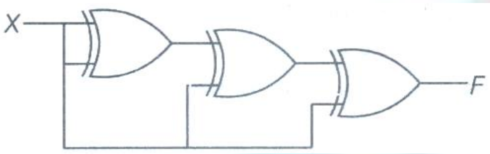
$$(011010)_2 \rightarrow (?)_{16}$$

$$\underline{0001}\ \ \underline{1010}$$

$$1 \quad 10 = \text{A}$$

So,

95. For the circuit shown below, the complement of the output

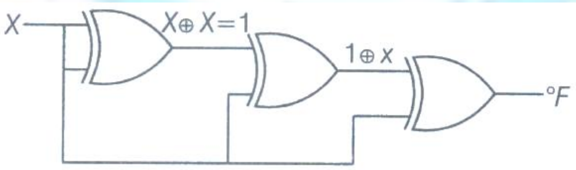


[2019]

(a) 0 (b) X (c) X' (d) 1

Ans. (d)

Sol.



The complement of $(F=0)$ is $(F^*=1)$.

96. If N is a 16 -bit signed integer, then 2 's complement representation of N is $(\text{F87 B})_{16}$. Then, 2's complement representation of 8^* N is **[2019]**

(a) $(\text{C3D8})_{16}$ (b) $(\text{187 B})_{16}$

(c) $(\text{F878})_{16}$ (d) $(\text{987 B})_{16}$

Ans. (a)

Sol. $\text{N} = (\text{F87 B})_{16}$ is 1111100001111011

The most significant bit in the binary representation is 1 , which implies that the number is negative.
To get the value of the number perform the 2 's complement of the number. We get N as -1925 and 8^* N as -15400.
Since, 8^* N is also negative, we need to find 2 's complement of it (-15400).
Binary of $15400 = 0011110000101000$
2's complement $= 1100001111011000 = (\text{C3D8})_{16}$

97. The base (or radix) of the number system such that the following equation holds $312/20 = 13.1$ is **[2019]**

(a) 3 (b) 4 (c) 5 (d) 6

Ans. (c)

Sol. Let x be the base of the number system.
The equation is

$$\frac{3x^2+1x^1+2x^0}{2x^1+0x^0} = 1x^1+3x^0+1x^{-1}$$

$$\frac{3x^2+x+2}{2x} = x+3+\frac{1}{x}$$

$$\frac{3x^2+x+2}{2x} = \frac{x^2+3x+1}{x}$$

$$3x^2+x+2 = 2x^2+6x+2$$

$$3x^2-2x^2+x-6x+2-2=0$$

$$x^2-5x=0$$

$$x(x-5)=0$$

$$x=0 \text{ or } x=5$$

As base of a number system cannot be zero.
So, $x=5$

98. Which of the following represents $(\text{D4})_{16}$? **[2019]**

(a) $(\text{4E})_{16}-(\text{5 B})_{16}$ (b) $(\text{14E})_{16}-(\text{7 A})_{16}$

(c) $(15C)_{16}-(6D)_{16}$ (d) $(\text{1E4})_{16}-(\text{A7})_{16}$

Ans. (b)

Sol. First we convert all hexadecimal to decimal.

$$(\text{D4})_{16} = \text{D}\times16^1+4\times16^0$$

$$=13\times16+4\times1$$

$$=208+4=(212)_{10}$$

This question will be solved by option. So, we will take all options one by one.
Option (a)

$$(\text{4E})_{16} = 4\times16^1+\text{E}\times16^0$$

$$=4\times16+14\times1=(78)_{10}$$

$$(\text{5 B})_{16} = 5\times16^1+\text{B}\times16^0$$

$$=5\times16+11\times1=(91)_{10}$$

$$(\text{4E})_{16}-(\text{5 B})_{16} = (78)_{10}-(91)_{10}$$

$$=(-13)_{10}$$

This is wrong answer.
Option (b)

$$(\text{14E})_{16} = 1\times16^2+4\times16^1+\text{E}\times16^0$$

$$= 1\times256+4\times16+14\times1$$

$$= (334)_{10}$$

$$(\text{7 A})_{16} = 7\times16^1+\text{A}\times16^0$$

$$= 7\times16+10\times1=(122)_{10}$$

$$(\text{14E})_{16}-(\text{7 A})_{16} = (334)_{10}-(122)_{10}$$

$$=(212)_{10}$$

This is right answer.
Option (c)

$(15C)_{16} = 1\times16^2 + 5\times16^1 + C\times16^0$

$= 1\times256 + 5\times16 + 12\times1 = (348)_{10}$

$(6D)_{16} = 6\times16^1 + D\times16^0$

$= 6\times16 + 13\times1 = (109)_{10}$

$(15C)_{16} - (6D)_{16} = (348)_{10} - (109)_{10}$

$= (239)_{10}$

This is wrong answer.
Option (d)

$(1E4)_{16} = 1\times16^2 + E\times16^1 + 4\times16^0$

$= 1\times256 + 14\times16 + 4\times1$

$= (484)_{10}$

$(A7)_{16} = A\times16^1 + 7\times16^0$

$= 10\times16 + 7\times1 = (167)_{10}$

$(1E4)_{16} - (A7)_{16} = (484)_{10} - (167)_{10}$

$= (317)_{10}$

This is wrong answer.

99. How many Boolean expressions can be formed with 3 Boolean variables? **[2019]**
(a) 16 (b) 1024 (c) 32 (d) 256
Ans. (d)
Sol. For the case of n Boolean variables, there are 2^{2^n} Boolean expression can be formed.
If value of Boolean variable is 3 .
Then Boolean expression will be 2^{2^3}
$= 2^8 = 256$

100. In an 8 bit representation of computer system the decimal number 47 has to be subtracted from 38 and the result in binary 2's complement is ____ **[2019]**
(a) 11110111 (b) 10001001
(c) 11111001 (d) 11110001
Ans. (a)
Sol. Binary representation of $47 = 101111$
8 bit representation of $47 = 00101111$
Binary representation of $38 = 100110$
8 bit representation of $38 = 00100110$

$$\begin{array}{r} (47)_{10} - (38)_{10} = 00101111 \\ -00100110 \\ \hline 00001001 \end{array}$$

1's complement = 11110110

$$\begin{array}{r} \text{2's complement} = 11110110 \\ +1 \\ \hline 11110111 \end{array}$$

General English

101. Choose the phrasal verb to replace the explanation in brackets.
"We must (be quick) ____ or we'll be late for school". **[2019]**
(a) act up (b) hurry up
(c) fasten on (d) speed in
Ans. (b)
Sol. According to the given sentence, option (b) 'hurry up' would be the correct alternative to fill the blank. Hurry up means be quick.

102. Anne had to pay everything because as usual,"Peter ____ his wallet at home. **[2019]**
(a) had left (b) was leaving
(c) left (d) leave
Ans. (a)
Sol. The given sentence is in past perfect tense, so 'had left' would be its correct answer.

103. Extreme old age when a man behaves like a child. **[2019]**
(a) Imbecility (b) Senility
(c) Dotage (d) Superannuation
Ans. (c)
Sol. Dotage is the period of life in which a person is old and behaves like a child.

104. Identify the word that means "To try to achieve something is difficult circumstances despite a setbacks". **[2019]**
(a) Persuade (b) Persevere
(c) Picturesque (d) Perspective
Ans. (b)
Sol. 'Persevere' is a word that means try to achieve something in a difficult circumstances despite a setbacks.

105. In the following a part of the sentence is underlined. Four different ways of phrasing the underlined part are indicated below. Choose the best alternative among the four choices given.
When he entered the house, it was in sixes and sevens. **[2019]**
(a) It was six O'clock when he entered the house
(b) Inmates were eulogized when he entered
(c) House was in pandemonium
(d) House was bissful
Ans. (c)
Sol. The given underlined idiom 'at sixes and seven' means in a confused, badly organised or difficult situation. So, 'House was in pandemonium' is its

correct answer. Pandemonium also means wild and noisy disorder or confusion.

106. Fill in the blanks choosing the correct adjective :
The phoenix is a _____ Bird. **[2019]**
(a) Mythical (b) Ethical
(c) Natural (d) Carnivorous
Ans. (a)
Sol. 'Mythical' is the correct word for the given blank.

107. Which of the following is the correct passive of the sentence, "JOHN HAS EATEN THE APPLES??" **[2019]**
(a) The apples are being eaten by John.
(b) The apples are eaten by John.
(c) The apples have been eaten by John.
(d) The apples will be eaten by John.
Ans. (c)
Sol. The given sentence belongs to Present Perfect tense. To change it, in passive voice has/have been $+V_3+$ by should be used. Hence, option (c) 'The apples have been eaten by John' is the correct passive voice of the given sentence.

108. Choose one of the words that is most nearly same as meaning of the given word. Indemnify **[2019]**
(a) Insure (b) Compensate for loss
(c) Assure (d) Sue for damages
Ans. (b)
Sol. Indemnify means compensate someone for harm or loss. So, option (b) is its correct answer.

109. Select a word from the given alternatives which has the same meaning as the underlined word.
He has a propensity for getting into debt. **[2019]**
(a) Tendency (b) Aptitude
(c) Characteristics (d) Quality
Ans. (a)
Sol. 'Propensity' means an inclination or natural tendency to behave in a particular way. So, 'Tendency' would be its correct answer.

110. Select the most suitable synonym from the given choice for the word : "ANTEDILUVIAN". **[2019]**
(a) Recluse (b) Maverick
(c) Archaic (d) Bellicose
Ans. (c)
Sol. 'Antediluvian' means ridiculously old-fashioned. So, 'Archaic' would be its correct synonym. Archaic also means very old or old fashioned.

111. Select the most suitable antonym from the given choices for the word : "SANGFROID". **[2019]**
(a) Equanimity (b) Steadiness
(c) Aplomb (d) Turbulence
Ans. (d)
Sol. 'Sangfroid' means ability to remain calm in a dangerous or difficult situation.
Among the given alternatives, 'Turbulence' is its correct antonym because it means a state of confusion, conflict or violent disorder.

112. The word PIN is used in four different ways. Choose the option in which the usage of the word is incorrect or inappropriate? **[2019]**
(a) She combed her hair backward and secured it with a pin.
(b) Jack managed to grab the thief and pin him against the wall until the police arrived on the scene.
(c) It is imprudent to pin your hopes on someone to help you out of his situation.
(d) You can't pin the blame at anyone without verifying facts.
Ans. (d)
Sol. The usage of the given word is inappropriate in option (d). 'Pin the blame' is incorrect.

113. Pick the most appropriate substitute to the capitalised word in the following sentence
The weapon inspector's report was not expected to provide INCONTROVERTIBLE evidence of weapons of mass destruction. **[2019]**
(a) Conclusive (b) Disputable
(c) Inconvenient (d) Indecisive
Ans. (a)
Sol. 'Incontrovertible' means not able to be denied or disputed. So, conclusive would be its correct answer to substitute the given capitalised word.

114. In the sentence given below, a part of sentence is underlined. Four different ways of phrasing the underlined part are indicated as four choices. Choose the best alternative and mark its corresponding letter as your answer.
A nation is built not be legislation but by the stirrings in the heart of the people. **[2019]**
(a) By legislation and by inspiration
(b) Not by laws but by the excitement of the people
(c) By law and by inciting the people
(d) More by the passions in the hearts of the people than by laws
Ans. (d)
Sol. Option (d) is its correct answer.

115. Select the most suitable synonym for the underlined word in the sentence.
All the members of organisation expressed implacable opposition to the move. **[2019]**
(a) Indignant (b) Adamant
(c) Unified (d) Quixotic

Ans. (b)
Sol. Implacable means not capable of being appeased, significantly changed or mitigated. So, among the given options, 'Adamant' would be its correct synonym. Adamant means not to change or refuse their mind about something.

116. There are two blanks in the sentence given below. From the pairs of word given below the sentences, choose the pair that fills the blank most appropriately.
Private companies supplying 'breakfast cereals' have started .... in agriculture in poorer countries. This has .... the spectre of land grabs and political conflicts. **[2019]**
(a) Spending ... intensified (b) Dealing ... inflated
(c) Ploughing ... increased (d) Investing ... raised
Ans. (d)
Sol. 'Investing' and 'raised' is the correct pair of words that fills the given blanks are make the sentence meaningful.

117. Use the appropriate phrasal verb and complete the sentence given below.
The new system in education is aimed at _____ the differences between rich and poor. **[2019]**
(a) Goof around (b) Evening Out
(c) Glossing Over (d) Give Over
Ans. (b)
Sol. 'Evening out' is the correct phrasal verb to fill the given blank. Evening out means became more balanced or equal with smaller differences and it is also correct according to the given sentence.

Read the following passage and answer questions.
I have, myself, full confidence that if all do their duty, if nothing is neglected, and if the best arrangements are made, as they are being made, we shall prove ourselves once again able to defend our Island home, to ride out the storm, of war, and to outlive the menace of tyranny, if necessary for years, if necessary alone. At any rate, that is what we are going to try to do. That is the resolve of His Majesty's Government-every man of them.
That is the will of the Parliament of our Nation. The British Empire and the French Republic, linked together in their cause and in their need, will defend to the death their native soil, aiding each other like good comrades to the utmost of their strength. Even though large tracts of Europe and many old and famous States have fallen or may fall into the grip of the Gestapo and all the odious apparatus of Nazi rule, we shall not flag or fail.
We shall go on to the end, we shall fight in France, we shall fight on the seas and oceans, we shall fight with growing confidence and growing strength in the air, we shall defend our Island, whatever the cost may be, we shall fight on the beaches, we shall fight on the landing grounds, we shall fight on the landing grounds, we shall fight in the fields an in the streets, we shall fight in the hills; we shall never surrender, and even if, which I do not for a moment believe, this island or a large part of it were subjugated and starving, then the British Fleet, would carry on the struggle, until, in God's good time, the New World, with all its power and might, steps forth to the rescue and the liberation of the old."
118. What does the term ride out the storm mean? **[2019]**
(a) Handle a crisis successfully
(b) Hide from storm
(c) Hide in some place where one cannot be found
(d) Ride on a boat at the time of storm
Ans. (a)
Sol. The term 'ride out the storm' means handle a crisis successfully.

119. What does subjugate mean? **[2019]**
(a) Surrender (b) Compare
(c) Control (d) Abandon
Ans. (c)
Sol. 'Subjugate' means bring under domination or control.

120. "That is the resolve of His Majesty's Government." What is their resolve? **[2019]**
(a) Surrender to the Nazis
(b) Negotiate with the Nazis
(c) Run away from the Nazis
(d) Fight the Nazis
Ans. (d)
Sol. Their resolve is to fight the Nazis.