NIMCET-2021 SOLVED PAPER

Mathematics

1. If $|k|=5$ and $0^\circ \le \theta \le 360^\circ$, the number of distinct solutions of $3\cos\theta+4\sin\theta=k$ is **[2021]**

(a) 0 (b) 1 (c) 2 (d) infinite

Ans.(c)

Sol. Given, $|k|=5$ where $k=3\cos\theta+4\sin\theta$

$$\Rightarrow |3\cos\theta+4\sin\theta|=5 \Rightarrow \left|5\left(\frac{3}{5}\cos\theta+\frac{4}{5}\sin\theta\right)\right|=5$$

$$\cos\alpha=\frac{3}{5}\Rightarrow \sin\alpha=\frac{4}{5}$$

Let

$$\therefore \left|5\left(\cos\theta\cdot\cos\alpha+\sin\theta\cdot\sin\alpha\right)\right|=5$$

$$\Rightarrow |\,5(\cos(\theta-\alpha)|=5\Rightarrow 5|\cos(\theta-\alpha)|=5$$

$$\Rightarrow \cos(\theta-\alpha)=\pm1\Rightarrow\theta-\alpha=0^\circ,180^\circ$$

$$\Rightarrow \theta=\alpha,\theta=\alpha+180^\circ$$

$\Rightarrow$ It has two solutions in it.

2. The area of the region bounded by the X-axis and the curves defined by $y=\tan x,-\frac{\pi}{3}\le x\le\frac{\pi}{3}$ and $y=\cot x,\frac{\pi}{6}\le x\le\frac{3\pi}{2}$ is **[2021]**

(a) $-\frac{1}{2}\log 2$ (b) $\frac{1}{2}\log 2$

(c) $\log 2$ (d) None of these

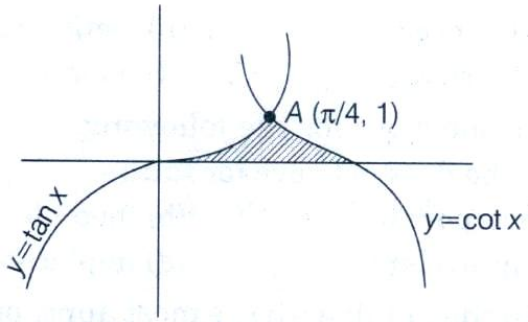
Ans.(c)

Sol. Given curves are $\tan x,-\frac{\pi}{3}\le x\le\frac{\pi}{3}$

and $y=\cot x,\frac{\pi}{6}\le x\le\frac{3\pi}{2}\ldots.(i)$

On solving Eqs. (i) and (ii),

$$\tan x=\cot x\Rightarrow \tan^2 x=1=\tan^2\left(\frac{\pi}{4}\right)$$



$$\Rightarrow x=\frac{\pi}{4}$$

$$\therefore \text{ Required area }=\int_0^{\pi/4}\tan x dx+\int_{\pi/4}^{\pi/2}\cot x dx$$

$$=\left[(\log\sec x)\right]_0^{\pi/4}+\left[(\log\sin x)\right]_{\pi/4}^{\pi/2}$$

$$=\left(\log\sqrt{2}-\log 1\right)+\left(\log 1-\log\left(\frac{1}{\sqrt{2}}\right)\right)$$

$$=2\log\sqrt{2}=\log 2\ldots.(i)$$

3. If $\log\left(1-x+x^2\right)=a_1x+a_2x^2+a_3x^3+\ldots$. Then, $a_3+a_6+a_9+\ldots$ is equal to **[2021]**

(a) $\log 2$ (b) $\frac{2}{3}\log 2$ (c) $\frac{1}{3}\log 2$ (d) $2\log 2$

Ans. (b)

Sol. If $\log\left(1-x+x^2\right)=a_1x+a_2x^2+a_3x^3+\ldots\ (i)$

To find $a_3+a_6+a_9+\ldots$

In Eq. (i), put $x=1$

$$log(1)=a_1+a_2+a_3+a_4+\ldots(ii)$$

In Eq. (i), put $x=\omega$

$$log\left(1-\omega+\omega^2\right)=a_1\omega+a_2\omega^2+a_3\omega^3+\ldots(iii)$$

In Eq. (i), put $x=\omega^2$

$$log\left(1+\omega-\omega^2\right)=a_1\omega^2+a_2\omega+a_3+\ldots(iv)$$

where, $\omega=\frac{-1+\sqrt{3}i}{2}$ and $\omega^2=\frac{-1-\sqrt{3}i}{2}$

$\left[\because 1+\omega+\omega^2=0 \text{ and } \omega^3=1\right]$

On adding Eqs. (ii), (iii) and (iv),

$$\log 1+\log\left(1-\omega+\omega^2\right)+\log\left(1+\omega-\omega^2\right)$$

$$=a_1\left(1+\omega+\omega^2\right)+a_2\left(1+\omega+\omega^2\right)+3a_3+3a_6+\ldots$$

$$\Rightarrow 0+\log(-2\omega)+\log\left(-2\omega^2\right)=a_1(0)+a_2(0)$$

$$+3\left(a_3+a_6+a_9+\ldots\right)$$

$$=\frac{\log\left(4\omega^3\right)}{3}=a_3+a_6+a_9+\ldots=a_3+a_6+a_9+\ldots=\frac{2}{3}\log 2$$

4. $\lim\limits_{x\to\infty}\left(\frac{x+7}{x+2}\right)^{x+5}$ is equal to **[2021]**

(a) e^5 (b) e^{-5} (c) e^2 (d) e^{-2}

Ans.(a)

Sol. We have,

$$\lim_{x\to\infty}\left(\frac{x+7}{x+2}\right)^{x+5}=\lim_{x\to\infty}\left(1+\frac{5}{x+2}\right)^{x+5}\ [\ 1^{\infty}\text{ form }]$$

$$=e^{\lim_{x\to\infty}\left[\frac{5}{x+2}(x+5)\right]}=e^{\lim_{x\to\infty}\frac{5x\left(1+\frac{5}{x}\right)}{x\left(1+\frac{2}{x}\right)}}=e^{\lim_{x\to\infty}\left(\frac{1+\frac{5}{x}}{1+\frac{2}{x}}\right)}=e^{5\left(\frac{1+0}{1+0}\right)}=e^{5}\ldots(i)$$

5. The number of common tangents to the circle $x^2+y^2=4$ and $x^2+y^2-6x-8y=24$ is **[2021]**

(a) 0 (b) 1 (c) 3 (d) 4

Ans. (b)

Sol. Given circles are $x^2+y^2=4$

Centre $C_1=(0,0)$ and radius $(r_1)=2$

and $x^2+y^2-6x-8y=24$

$\Rightarrow x^2-6x+9+y^2-8y+16=24+9+16$

$\Rightarrow (x-3)^2+(y-4)^2=(7)^2$

$\therefore$ Center $(C_2)=(3,4)$ and radius $(r_2)=7$

$\because \quad C_1C_2=\sqrt{(3-0)^2+(4-0)^2}=5$

and $\quad r_1+r_2=2+7=9$

$\because \quad C_1C_2<r_1+r_2$ but $C_1C_2=|r_1-r_2|$

Thus, the given circles touch each other internally. Hence, number of common tangent is only one.

6. In a $\triangle ABC$, if $\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}=k$, then k is always **[2021]**

(a) >2 (b) ≥ 1 (c) $=2$ (d) $=1$

Ans. (b)

Sol. Given that $\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}=k$

Let $\tan\frac{A}{2}=x,\tan\frac{B}{2}=y,\tan\frac{C}{2}=z$

$\therefore\ x^2+y^2+z^2-xy-yz-zx$

$=\frac{1}{2}\left[(x-y)^2+(y-z)^2+(z-x)^2\right]\geq 0$

$\Rightarrow\ x^2+y^2+z^2-xy-yz-zx\geq 0$

$\Rightarrow\ \left(x^2+y^2+z^2\right)\geq xy+yz+zx$

$\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\geq\tan\frac{A}{2}\cdot\tan\frac{B}{2}$

$+\tan\frac{B}{2}\cdot\tan\frac{C}{2}+\tan\frac{C}{2}\cdot\tan\frac{A}{2}$(i)

$\because A+B+C=180^\circ$

$\therefore\ \frac{A}{2}+\frac{B}{2}+\frac{C}{2}=90^\circ$

$\frac{A}{2}+\frac{B}{2}=90^\circ-\frac{C}{2}\Rightarrow\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(90^\circ-\frac{C}{2}\right)$

$\Rightarrow\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\cdot\tan\frac{B}{2}}=\cot\frac{C}{2}$

$\Rightarrow\tan\frac{A}{2}\cdot\tan\frac{C}{2}+\tan\frac{B}{2}\cdot\tan\frac{C}{2}=1-\tan\frac{A}{2}\cdot\tan\frac{B}{2}$

$\Rightarrow\tan\frac{A}{2}\cdot\tan\frac{B}{2}+\tan\frac{B}{2}\cdot\tan\frac{C}{2}+\tan\frac{C}{2}\cdot\tan\frac{A}{2}=1$

From Eq. (i),

$\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\geq 1$ or $k\geq 1$

7. Let $\mathbf{a}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}},\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{c}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ are three vectors. Then, a vector in the plane of $\mathbf{a}$ and $\mathbf{c}$ whose projection on $\mathbf{b}$ is of magnitude $\frac{1}{\sqrt{6}}$ is **[2021]**

(a) $3\hat{\mathbf{i}}-2\hat{\mathbf{j}}$ (b) $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}$

(c) $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}$ (d) $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Ans. (b)

Sol. Given $\mathbf{a}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}},\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{c}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

Let $\mathbf{d}$ be the vector in the plane of $\mathbf{a}$ and $\mathbf{c}$

$\therefore\ \mathbf{d}=\mathbf{a}+6\mathbf{c}=\left(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}\right)+\lambda\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)$

$\mathbf{d}=(2+\lambda)\hat{\mathbf{i}}+(1+\lambda)\hat{\mathbf{j}}+(2-2\lambda)\hat{\mathbf{k}}$

Now, projection of $\mathbf{d}$ on $\mathbf{b}=\frac{\mathbf{d}\cdot\mathbf{b}}{|\mathbf{b}|}$

$\frac{1}{\sqrt{6}}=\frac{(2+\lambda)-(1+\lambda)+2(2-2\lambda)}{\sqrt{1^2+(-1)^2+2^2}}\Rightarrow\frac{1}{\sqrt{6}}=\frac{5-4\lambda}{\sqrt{6}}$

$\therefore\ 5-4\lambda=1\Rightarrow\lambda=1$

$\therefore\ \mathbf{d}=(2+1)\hat{\mathbf{i}}+(1+1)\hat{\mathbf{j}}+(2-2)\hat{\mathbf{k}}\Rightarrow\mathbf{d}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}$

8. For what value of p, the polynomial $x^4-3x^3+2px^2-6$ is exactly divisible by $(x-1)$? **[2021]**

(a) 2 (b) 4 (c) 6 (d) 8

Ans. (b)

Sol. Given polynomial

$p(x)=x^4-3x^3+2px^2-6$

$\because\ p(x)$ is divisible by $(x-1)$

$\Rightarrow \quad p(1)=0 \Rightarrow 1^4-3(1)^3+2p(1)^2-6=0$

$\Rightarrow \quad 8=2p \Rightarrow p=4$

9. If $F(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$, then $F(\theta)F(\alpha)$ is equal to **[2021]**

(a) $F(\theta\alpha)$ (b) $F\left(\frac{\theta}{\alpha}\right)$

(c) $F(\theta+\alpha)$ (d) $F(\theta-\alpha)$

Ans. (c)

Sol.

$$F(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \therefore F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\therefore F(\theta)\cdot F(\alpha)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos\theta\cdot\cos\alpha-\sin\theta\sin\alpha & -(\cos\theta\sin\alpha+\sin\theta\cdot\cos\alpha) & 0\\ \sin\theta\cdot\cos\alpha+\cos\theta\cdot\sin\alpha & \cos\theta\cdot\cos\alpha-\sin\theta\sin\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(\theta+\alpha) & -\sin(\theta+\alpha) & 0\\ \sin(\theta+\alpha) & \cos(\theta+\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\cos(A+B)=\cos A\cos B-\sin A\sin B\sin(A+B)$

$=\sin A\cos B+\cos A\sin B]$

[Using

$=F(\theta+\alpha)$

10. If $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{4!(n-4)!}$ are in the ratio $2:1$, then the value of n is **[2021]**

(a) 0 (b) 2 (c) 4 (d) 5

Ans. (d)

Sol. Given that

$$\frac{n!}{2!(n-2)!}:\frac{n!}{4!(n-4)!}=2:1$$

$$\Rightarrow \frac{n!}{\frac{2!(n-2)(n-3)(n-4)!}{4\cdot3\cdot2!(n-4)!}}=\frac{2}{1}\Rightarrow\frac{12}{n^2-5n+6}=2$$

$$\Rightarrow n^2-5n=0\Rightarrow n=5, n\neq0$$

11. The locus of the point of intersection of tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which meet right angles is **[2021]**

(a) a circle (b) a parabola

(c) an ellipse (d) a hyperbola

Ans. (a)

Sol. Given that,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.....(i)$$

Ellipse .

Equation of tangents on ellipse(i)

$$y=mx\pm\sqrt{(a^2m^2+b^2)}$$

$$\Rightarrow(y-mx)^2=a^2m^2+b^2$$

$$\Rightarrow y^2+m^2x^2-2mxy=a^2m^2+b^2$$

$$\Rightarrow m^2(x^2-a^2)-2mxy+y^2-b^2=0$$

which are perpendicular.

$$\therefore \quad m_1\cdot m_2=1\Rightarrow\frac{y^2-b^2}{x^2-a^2}=-1$$

$\Rightarrow \quad x^2+y^2=a^2+b^2$ which is an equation of circle.

12. If the position vector of A and B relative to O be $\hat{\mathbf{i}}-4\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ respectively, then the median through O of $\triangle ABC$ is **[2021]**

(a) $-2\hat{\mathbf{i}}+2\hat{\mathbf{k}}$ (b) $-\hat{\mathbf{j}}+\hat{\mathbf{k}}$

(c) $-\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ (d) $-\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$

Ans. (b)

Sol. Given that $\mathbf{OA}=\hat{\mathbf{i}}-4\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\mathbf{OB}=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$

Since, the median will passes through the mid-point of AB.

$$\text{Hence, median}=\frac{\mathbf{OA}+\mathbf{OB}}{2}$$

$$=\frac{\left(\hat{\mathbf{i}}-4\hat{\mathbf{j}}+3\hat{\mathbf{k}}\right)+\left(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)}{2}=-\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

13. The general value of θ, satisfying the equation, $\sin\theta=-\frac{1}{2},\tan\theta=\frac{1}{\sqrt{3}}$ is **[2021]**

(a) $n\pi+\frac{\pi}{6}, n\in I$ (b) $n\pi+(-1)^n\left(\frac{7\pi}{6}\right), n\in I$

(c) $2n\pi\pm\frac{7\pi}{6}, n\in I$ (d) $2n\pi+\frac{11\pi}{6}, n\in I$

Ans. (c)

Sol. $\sin\theta=\frac{-1}{2},\tan\theta=\frac{1}{\sqrt{3}}$

Since, $\sin\theta$ is negative and $\tan\theta$ is positive. $\Rightarrow \theta$ lies in third quadrant.

$$\therefore \quad \sin\theta = \frac{-1}{2} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$$

$$\text{and} \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan\left(\pi + \frac{\pi}{6}\right) = \tan\frac{7\pi}{6}$$

$$\theta = 2n\pi \pm \left(\frac{7\pi}{6}\right), n \in I$$

14. The area of the triangle formed by the vertices whose position vectors are $3\hat{\mathbf{i}}+\hat{\mathbf{j}}, 5\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}, \hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ is **[2021]**

(a) $\sqrt{21}$ sq. units (b) $\sqrt{23}$ sq. units
(c) $\sqrt{33}$ sq. units (d) $\sqrt{29}$ sq. units

Ans. (d)

Sol. Let $\mathbf{OA} = 3\hat{\mathbf{i}}+\hat{\mathbf{j}}$

$$\mathbf{OB} = 5\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}} \Rightarrow \mathbf{OC} = \hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$$

$$\therefore \quad \mathbf{AB} = \mathbf{OB} - \mathbf{OA} = 2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

$$\text{and} \quad \mathbf{AC} = \mathbf{OC} - \mathbf{OA} = -\hat{\mathbf{i}}-3\hat{\mathbf{j}}+3\hat{\mathbf{k}}$$

$\therefore$ Area of triangle $= \frac{1}{2}|\mathbf{AB}\times\mathbf{AC}|.....(i)$

$$AB \times AC = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & 1 \\ -2 & -3 & 3 \end{vmatrix} = \hat{\mathbf{i}}(6) - \hat{\mathbf{j}}(8) + \hat{\mathbf{k}}(-4) = 6\hat{\mathbf{i}}8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

$$= \frac{1}{2}\times\sqrt{6^2+(-8)^2+(-4)^2} = \sqrt{29}$$

$\therefore$ Area of triangle sq. units

15. The standard deviation of 20 number is 30 . If each of the numbers is increased by 4 , then the new standard deviation will be **[2021]**

(a) 24 (b) 34 (c) 30 (d) 20

Ans. (c)

Sol. Let the numbers are x_i, where $1 \le i \le 20$

$$\therefore \text{Mean}\left(\bar{x}\right) = \frac{\sum_{i=1}^{20} x_i}{20} \quad \text{Then,} \quad \sigma(x) = \sqrt{\frac{\left(x_i - \bar{x}\right)^2}{n}} = 30$$

[given]

Let the new numbers are y_i, where $y_i = x_i + 4$

Then, $\bar{y} = \frac{\sum_{i=1}^{20}(x_i+4)}{20} \Rightarrow \bar{y} = \bar{x} + 4$

$$\therefore \quad y_i - \bar{y} = (x_i + 4) - \left(\bar{x} + 4\right)$$

$$\therefore \quad y_i - \bar{y} = x_i - \bar{x}$$

$$\therefore \quad \sigma(y) = \sqrt{\frac{\left(y_i - \bar{y}\right)^2}{n}} = \sqrt{\frac{\left(x_i - \bar{x}\right)^2}{n}} = 30$$

16. The function $f(x) = \frac{x}{1+x\tan x}, 0 \le x \le \frac{\pi}{2}$ is maximum when **[2021]**

(a) $x = \sec x$ (b) $x = \tan x$
(c) $x = \cos x$ (d) None of the above

Ans. (c)

Sol. $f(x) = \frac{x}{1+x\tan x}, 0 \le x \le \frac{\pi}{2}$

$$\therefore f'(x) = \frac{(1+x\tan x)\cdot 1 - x\left(x\sec^2 x + \tan x\right)}{(1+x\tan x)^2} = \frac{1-x^2\sec^2 x}{(1+x\tan x)^2}$$

For maxima / minima

$$f'(x) = 0 \Rightarrow 1 - x^2\sec^2 x = 0 \Rightarrow x^2 = \cos^2 x \Rightarrow x = \cos x$$

$\because f''(x) < 0$ at $x = \cos x$

$\Rightarrow f(x)$ is maximum when $x = \cos x$

17. If $f: R \to R$ is defined by **[2021]**

$$f(x) = \begin{cases} \frac{x+2}{x^2+3x+2}, & \text{if } x \in R - \{-1,-2\} \\ -1 & , \text{ if } x = -2 \\ 0 & , \text{ if } x = -1 \end{cases}$$

Then, $f(x)$ is continuous on the set

(a) R (b) $R - \{-2\}$
(c) $R - \{-1\}$ (d) $R - \{-1,-2\}$

Ans. (c)

Sol. $f: R \to R$

$$f(x) = \begin{cases} \frac{x+2}{x^2+3x+2}, & \text{if } \quad x \in R - \{-1-2\} \\ -1, & \text{if} \quad x = -2 \\ 0, & \text{if} \quad x = -1 \end{cases}$$

$\because f(x)$ is continuous for every value of R except $\{-1,-2\}$.

Now, we have to check continuity at $x = -2$ and $x = -1$

$\therefore \quad \lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{x+2}{x^2+3x+2}$

$$\lim_{x\to -2} \frac{(x+2)}{(x+1)(x+2)} = \frac{1}{-2+1} = -1 = f(-2)$$

$\Rightarrow f(x)$ is continuous at $x=-2$

Now at $x=-1$

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{(x+2)}{(x+1)(x+2)} = \lim_{x\to -1} \frac{1}{x+1} = \infty$$

Limit does not exist.

$\Rightarrow f(x)$ is not continuous at $x=-1$

18. A polygon has 44 diagonals, the number of sides are **[2021]**

(a) 9 (b) 10 (c) 11 (d) 12

Ans.(c)

Sol. $\because$ Number of diagonals in n-sided polygon is $\left({}^nC_2 - n\right)$.

$\therefore \quad {}^nC_2 - n = 44 \Rightarrow \frac{n(n-1)}{2} - n = 44$

$\Rightarrow \quad n(n-3) = 88 \Rightarrow n^2 - 3n - 88 = 0$

$\Rightarrow \quad (n-11)(n+8) = 0 \Rightarrow n = 11, n \neq -8$ (never be negative)

19. The probability of occurrence of two events E and F are 0.25 and 0.50 , respectively. The probability of their simultaneous occurrence is 0.14 . The probability that neither E nor F occur is **[2021]**

(a) 0.61 (b) 0.11 (c) 0.39 (d) 0.89

Ans. (c)

Sol. Given that

$$P(E) = 0.25, P(F) = 0.50 \Rightarrow P(E \cap F) = 0.14$$

To find $P(E \cup F)'$

$$\because P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$= 0.25 + 0.50 - 0.14 = 0.61$$
$$\therefore P(E \cup F)' = 1 - P(E \cup F) = 1 - 0.61 = 0.39$$

20. If $y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^3}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, then $\frac{dy}{dx}$ is **[2021]**

(a) $-\frac{1}{1+x^2}$ (b) $\frac{3}{1+x^2}$

(c) $\frac{3}{\sqrt{1+x^2}}$ (d) $\frac{1}{\sqrt{1+x^2}}$

Ans.(b)

Sol. $y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$

Let $x = \tan\theta$

$$\therefore y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta) = 3\theta$$

$$\Rightarrow y = 3\tan^{-1}x \left[\theta = \tan^{-1}x\right]$$

$$\therefore \frac{dy}{dx} = 3\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{3}{1+x^2}$$

21. The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is equal to **[2021]**

(a) 5 (b) 3 (c) 4 (d) 6

Ans. (c)

Sol. $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$$= (\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ)$$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = \frac{2}{\frac{(\sqrt{5}-1)}{4}} - \frac{2}{\frac{(\sqrt{5}+1)}{4}} = \frac{16}{5-1} = 4$$

22. In a triangle, if the sum of two sides is x and their product is y such that $(x+z)(x-z) = y$, where z is the third side of the triangle, then triangle is **[2021]**

(a) equilateral (b) right angled

(c) isosceles (d) obtuse angled

Ans. (d)

Sol. Let sides of triangles be a, b and c.

$\therefore b + c = x, bc = y, a = z$ (Given)

$y = (x-z)(x+z)$

and

$$\Rightarrow bc = (b+c+a)(b+c-a)$$

$$\Rightarrow b^2 + c^2 - a^2 = -bc \Rightarrow \frac{b^2+c^2-a^2}{2bc} = \frac{-1}{2}$$

$$\Rightarrow \cos A = \frac{-1}{2} = \cos 120^\circ \Rightarrow A = 120^\circ$$

Hence, the triangle will be obtuse angled triangle.

23. If $H_1, H_2, \ldots, H_n$ are n harmonic means between a and $b(\neq a)$, then $\frac{H_1 + a}{H_1 - a} + \frac{H_n + b}{H_n - b} =$ **[2021]**

(a) $2n$ (b) $n+1$ (c) $n-1$ (d) $2n+1$

Ans. (a)

Sol. Given $H_1, H_2 \ldots, H_n$ are Harmonic means between a and $b, (a \neq b)$.

$\Rightarrow \frac{1}{H_1}, \frac{1}{H_2} \cdot \frac{1}{H_3}, \ldots, \frac{1}{H_n}$ are Arithmetic mean between $\frac{1}{a}$ and $\frac{1}{b}$.

$$(d)=\frac{1}{H_1}-\frac{1}{a}=\frac{1}{b}-\frac{1}{H_n}$$

$\therefore$ Common difference

$$\Rightarrow \frac{1}{a}=\frac{dH_1}{a-H_1}\Rightarrow a=\frac{a-H_1}{dH_1}$$

$$b=\frac{b-H_n}{dH_n}$$

Also,

Now, $$\frac{H_1+a}{H_1-a}+\frac{H_n+b}{H_n-b}=\frac{1}{d}\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{H_n}-\frac{1}{H_1}\right)$$

$$=(n+1)+(n-1)\left[\because \frac{1}{b}-\frac{1}{a}=d(n+1)\right]$$

$$=2n$$

24. If $y=\sin^{-1}\left(\frac{x^2+1}{\sqrt{1+3x^2+x^4}}\right)(x>0)$, then $\frac{dy}{dx}=$ **[2021]**

(a) $\frac{x^2-1}{x^4+3x^2+1}$ (b) $\frac{x^2+1}{x^4+3x^2+1}$

(c) $\frac{x^2-1}{x^4-3x^2+1}$ (d) $\frac{x^2+1}{x^4-3x^2+1}$

Ans. (a)

Sol. $$y=\sin^{-1}\left(\frac{x^2+1}{\sqrt{1+3x^2+x^4}}\right)$$

$$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1-\frac{x^4+1+2x^2}{1+3x^2+x^4}}}\frac{d}{dx}\left(\frac{x^2+1}{\sqrt{1+3x^2+x^4}}\right)=\frac{\sqrt{1+3x^2+x^4}}{x}$$

$$\frac{\left\{\sqrt{1+3x^2+x^4}\,(2x)-\left(x^2-1\right)\frac{1}{2\sqrt{1+3x^2+x^4}}\left(6x+4x^3\right)\right\}}{\left(1+3x^2+x^4\right)}$$

$$=\frac{\sqrt{1+3x^2+x^4}}{x\left(1+3x^2+x^4\right)}\left\{\frac{2x\left(1+3x^2+x^4\right)-\left(x^2+1\right)\left(3x+2x^3\right)}{\sqrt{1+3x^2+x^4}}\right\}$$

$$\frac{dy}{dx}=\frac{x^2-1}{1+3x^2+x^4}$$

$$=\frac{1}{x\left(1+3x^2+x^4\right)}\left\{2x+6x^3+2x^5-3x^3-2x^5-3x-2x^3\right\}$$

$$=\frac{1}{x\left(1+3x^2+x^4\right)}\left\{4x^3-x\right\}$$

25. If $32\tan^8\theta=2\cos^2\alpha-3\cos\alpha$ and $3\cos2\theta=1$, then the general value of α for **[2021]**

(a) $n\pi\pm\frac{\pi}{3}$ (b) $2n\pi\pm\frac{2\pi}{3}$

(c) $2n\pi\pm\frac{\pi}{3}$ (d) $n\pi\pm\frac{2\pi}{3}$

Ans. (b)

Sol. Given that

$$32\tan^8\theta = 2\cos^2\alpha-3\cos\alpha, 3\cos2\theta=1$$

$$\therefore \quad 3\cos2\theta=1\Rightarrow\cos2\theta=1/3$$

$$\Rightarrow \quad 2\cos^2\theta-1=\frac{1}{3}\Rightarrow 2\cos^2\theta=\frac{2}{3}\Rightarrow\cos\theta=\sqrt{\frac{2}{3}}$$

$$\therefore \quad \tan^2\theta=\sec^2\theta-1\Rightarrow\frac{3}{2}-1=\frac{1}{2}$$

Now, $32\tan^8\theta=2\cos^2\alpha-3\cos\alpha$

$$32\left(\frac{1}{2}\right)^4=2\cos^2\alpha-3\cos\alpha$$

$$\Rightarrow 2\cos^2\alpha-3\cos\alpha-2=0$$

$$\Rightarrow 2\cos^2\alpha-4\cos\alpha+\cos\alpha-2=0$$

$$\Rightarrow 2\cos\alpha(\cos\alpha-2)+1(\cos\alpha-2)=0$$

$$\Rightarrow (\cos\alpha-2)(2\cos\alpha+1)=0$$

$\therefore \cos\alpha=\frac{-1}{2}$ or $\cos\alpha=2$ which is not possible.

$$\cos\alpha=\cos\frac{2\pi}{3}\ \therefore\alpha=2n\pi\pm\frac{2\pi}{3}$$

26. In a $\triangle ABC, a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$, then the sides of the triangle are in **[2021]**

(a) AP (b) GP (c) HP (d) None of the above

Ans. (a)

Sol. Given that

$$a\cos^2\frac{C}{2}+C\cos^2\frac{A}{2}=\frac{3b}{2}$$

$$\Rightarrow \quad 2a\cos^2\frac{C}{2}+2c\cos^2\frac{A}{2}=3b$$

$$\Rightarrow \quad a(1+\cos C)+c(1+\cos A)=3b$$

$$\Rightarrow \quad a+c+(a\cos C+c\cos A)=3b$$

$$\Rightarrow \quad a+c+b=3b\Rightarrow a+c=2b\Rightarrow a,b,c \text{ are in A P.}$$

27. If θ is acute angle between the pair of lines $x^7-7xy+12y^2=0$, then $\frac{2\cos\theta+3\sin\theta}{4\sin\theta+5\cos\theta}=$ **[2021]**

(a) $\frac{29}{69}$ (b) $\frac{61}{39}$ (c) $\frac{39}{61}$ (d) $\frac{69}{29}$

Ans. (a)

Sol. $x^2-7xy+12y^2=0$

On comparing with $ax^2+2hxy+by^2=0$

$$h=\frac{-7}{2}, a=1, b=12$$

$$\therefore \tan\theta=\frac{2\sqrt{h^2-ab}}{a+b} \Rightarrow \tan\theta=\frac{2\sqrt{\frac{49}{4}-12}}{1+12}=\frac{1}{13}$$

$$\therefore \frac{2\cos\theta+3\sin\theta}{4\sin\theta+5\cos\theta}=\frac{2+3\tan\theta}{4\tan\theta+5}=\frac{2+3\left(\frac{1}{13}\right)}{4\left(\frac{1}{13}\right)+5}=\frac{29}{69}$$

28. The four geometric means between 2 and 64 are **[2021]**

(a) $\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{32}$ (b) $4,8,16,32$

(c) $4\sqrt{2},8,16\sqrt{2},32$ (d) None of the above

Ans. (b)

Sol. Let four geometric mean between 2 and 64 are G_1,G_2,G_3,G_4.

$\therefore 2,G_1,G_2,G_3,G_4,64$ from GP.

First term $(a)=2$ and sixth term $=64$

$ar^5=64$, where r is common ratio

$2\cdot r^5=64\Rightarrow r^5=32\Rightarrow r=2$

$\therefore G_1=ar=2\times 2=4$

$G_2=ar^2=2(2)^2=8$

$G_3=ar^3=2(2)^3=16$

$G_4=ar^4=2(2)^4=32$

29. The lines $px+qy=1$ and $qx+py=1$ are respectively the sides AB, AC of the $\triangle ABC$ and the base BC is bisected at (p,q). Equation of the median of the triangle through the vertex A is **[2021]**

(a) $(2pq-1)(qx+py-1)-(p^2+q^2-1)(px+qy-1)=0$

(b) $(2pq-1)(px+qy-1)+(p^2+q^2-1)(qx+py-1)=0$

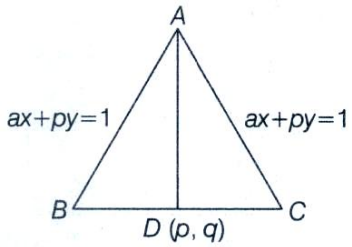
(c) $(2pq-1)(px+qy-1)-(p^2+q^2-1)(qx+py-1)=0$

(d) $(2pq-1)(qx+py-1)+(p^2+q^2-1)(px+qy-1)=0$

Ans. (c)

Sol. $px+qy=1$

$qx+py=1\ldots(ii)$



On solving Eqs. (i) and (ii),

$$px+qy=qx+py\Rightarrow x(p-q)+y(q-p)=0\Rightarrow x=y$$

From Eq. (i);

$$px+qx=1\Rightarrow x=\frac{1}{p+q}=y$$

$$\therefore \quad A=\left(\frac{1}{p+q},\frac{1}{p+q}\right)$$

$\therefore$ Equation of AD,

$$y-q=\frac{\frac{1}{p+q}-q}{\frac{1}{p+q}-p}(x-p)$$

$$(y-q)=\left(\frac{1-p^2-pq}{p+q}\right)=\left(\frac{1-pq-q^2}{p+q}\right)(x-p)$$

$$\Rightarrow y-yp^2-pqy-q+qp^2+pq^2$$
$$=x-xq^2-xpq-p+p^2q+pq^2$$
$$\Rightarrow pqx+q^2x-p^2y-pqy+y-x+p-q=0$$
$$\Rightarrow (p+q)qx-py(p+q)+y-x+(p-q)=0$$

On multiplying $py(p-q)$, we get

$$(p^2-q^2)qx-(p^2-q^2)py+y(p-q)$$
$$-x(p-q)+(p-q)^2=0$$
$$\Rightarrow (2pq-1)(px+qy-1)=(p^2+q^2-1)(qx+py-1)$$
$$\Rightarrow (2pq-1)(px+qy-1)-(p^2+q^2-1)(qx+py-1)=0$$

30. If the vectors $a\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, \hat{\mathbf{i}}+b\hat{\mathbf{j}}+\hat{\mathbf{k}}, \hat{\mathbf{i}}+\hat{\mathbf{j}}+c\hat{\mathbf{k}}\ (a,b,c\neq 1)$ are coplanar, then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$ **[2021]**

(a) 0 (b) 1 (c) 2 (d) 3

Ans. (b)

Sol. Since $\left(a\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right),\left(\hat{\mathbf{i}}+b\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$ and $\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+c\hat{\mathbf{k}}\right)$ are coplanar.

$$\therefore \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & d \end{vmatrix}=0$$

Applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$,

$$\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix}=0$$

$$\Rightarrow (1-a)(1-b)(1-c)\begin{vmatrix} \frac{a}{1-a} & \frac{1}{1-b} & \frac{1}{1-c} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{a}{1-a}+\frac{1}{1-a}+\frac{1}{1-c}=0 \Rightarrow \frac{a}{1-a}+1+\frac{1}{1-b}+\frac{1}{1-c}=1$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

31. Let $\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\mathbf{b}=2\hat{\mathbf{i}}-\hat{\mathbf{k}}$. Then, the point of intersection of the lines $\mathbf{r}\times\mathbf{a}=\mathbf{b}\times\mathbf{a}$ and $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ is **[2021]**

(a) $-\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$

(c) $\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ (d) $3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$

Ans. (d)

Sol. $\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}, \mathbf{b}=2\hat{\mathbf{i}}-\hat{\mathbf{k}}$

$$\times a=b\times a \Rightarrow (r-b)\times a=0 \Rightarrow r=b+ta...(i)$$

and $$\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b} \Rightarrow (\mathbf{r}-\mathbf{a})\times\mathbf{b}=0 \Rightarrow \mathbf{r}=\mathbf{a}+s\mathbf{b}...(ii)$$

where s,t are scalars.

Eqs. (i) and (ii) are same if $s=t=1$

$\therefore$ Point of intersection is $\mathbf{r}=\mathbf{a}+\mathbf{b}$

$$\Rightarrow \mathbf{r}=(\hat{\mathbf{i}}+\hat{\mathbf{j}})+(2\hat{\mathbf{i}}-\hat{\mathbf{k}})=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

32. If the system of equations $3x-y+4z=3, x+2y-3z=-2, 6x+5y+\lambda x=-3$ has at least one solution, then λ is equal to **[2021]**

(a) -5 (b) 3 (c) 5 (d) 6

Ans. (a)

Sol. Given system of equations

$$3x-y+4z=3$$
$$x+2y-3z=-2$$
$$6x+5y+\lambda z=-3$$
$$\Delta=\begin{vmatrix} 6 & 5 & \lambda \\ 1 & 2 & -3 \\ 3 & -1 & 4 \end{vmatrix}$$

$$\Delta=30-65-7\lambda=-35-7\lambda$$

$\therefore$ System of equations has infinite solution if $\Delta=0$

$$\Rightarrow -35-7\lambda=0 \Rightarrow -\lambda=5 \Rightarrow \lambda=-5$$

33. If a variable takes value $0,1,2,\ldots,50$ with frequencies $1, {}^{50}C_1, {}^{50}C_2,\ldots, {}^{50}C_{50}$, then the AM is **[2021]**

(a) 50 (b) 25 (c) $\frac{2^{50}}{50}$ (d) 51

Ans. (b)

Sol. Obtained means is given by

$$\bar{x}=\frac{0\cdot 1+1\cdot {}^{50}C_1+2^{50}C_2+\ldots+50^{50}C_{50}}{1+{}^{50}C_1+{}^{50}C_2+\ldots+{}^{50}C_5}$$

$$=\frac{\sum_{r=0}^{50} r\cdot {}^{50}C_r}{\sum_{r=0}^{50} {}^{50}C_r}=\frac{\sum_{r=0}^{50} r\cdot \frac{50^{49}}{r}C_{r-1}}{\sum_{r=0}^{50} {}^{50}C_r}=\frac{50\cdot 2^{49}}{2^{50}}=\frac{50}{2}$$

$$\bar{X}=25$$

34. If $A=\{1,2,3,4\}$ and $B\{3,4,5\}$, then the number of elements in $(A\cup B)\times(A\cap B)\times(A\Delta B)$ **[2021]**

(a) 18 (b) 20 (c) 24 (d) 30

Ans. (d)

Sol. $A=\{1,2,3,4\}$ and $B=\{3,4,5\}$

$\therefore\ A\cup B=\{1,2,3,4,5\}$ and $A\cap B=\{3,4\}$

$$A\Delta B=(A-B)\cup(B-A)=\{1,2\}\cup\{5\}=\{1,2,5\}$$

$$\therefore n\left((A\cup B)\times(A\cap B)\times(A\Delta B)\right)=5\times 2\times 3=30$$

35. If n is an integer between 0 to 21, then find a value of n for which the value of $n!(21-n)!$ is minimum. **[2021]**

(a) 9 (b) 10 (c) 12 (d) 21

Ans. (b)

Sol. $$n!(21-n)!=21!\cdot\frac{n!(21-n)!}{21!}=21!\frac{1}{{}^{21}C_n}$$

For minimum valúe of $n!(21-n)!, {}^{21}C_n$ is maximum.

$${}^{21}C_n={}^{21}C_{\frac{21-1}{2}}={}^{21}C_{10}$$

Maximum value of $=\frac{21!}{{}^{21}C_{10}}=\frac{21!}{21!}\times 10!11!$

$\therefore$ Minimum value

Hence, $n=10$

36. Suppose $A_1, A_2,\ldots, A_{30}$ are 30 sets each with five elements and $B_1, B_2, B_3,\ldots, B_n$ are n sets (each with three elements) such that $\cup_{i=1}^{30} A_i=\cup_{j=1}^{n} B_j=S$ and each element of S belong to exactly ten of the A_i's and exactly 9 of the B_j's. Then, n is equal to **[2021]**

(a) 15 (b) 45 (c) 75 (d) 90

Ans. (b)

Sol. Given that A_i's are thirty sets with five element each.

$$\text{So}, \sum_{i=1}^{30} n(A_i)=5\times 30=150...(i)$$

If there are m distinct elements in S and each element of S belongs to exactly 10 of the A_i 's, we have

$$\sum_{i=1}^{30} n(A_i) = 10\ m \ldots (ii)$$

From Eqs. (i) and (ii),

$$10m = 150 \Rightarrow m = 15$$

Similarly $\sum_{j=1}^{30} n(B_j) = 3n$ and $\sum_{j=1}^{30} n(B_j) = 9m$

$$\therefore\ 3n = 9m \Rightarrow n = \frac{9m}{3} = 3m = 3\times 15 \Rightarrow n = 45$$

37. The eccentric angle of the extremities of latusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are given by **[2021]**

(a) $\tan^{-1}\left(\pm\frac{ae}{b}\right)$ (b) $\tan^{-1}\left(\pm\frac{be}{e}\right)$

(c) $\tan^{-1}\left(\pm\frac{b}{ae}\right)$ (d) $\tan^{-1}\left(\pm\frac{a}{be}\right)$

Ans. (c)

Sol. Given that, equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Coordinate of any point on ellipse with eccentric angle θ are ($a\cos\theta, b\sin\theta$).

The coordinates of the end point of the latusrectum

$$= \left(ae, \pm\frac{b^2}{a}\right)$$

$\therefore a\cos\theta = ae$ and $b\sin\theta = \pm\frac{b^2}{a}$

$$\Rightarrow tan\theta = \pm\frac{b}{ae} \Rightarrow \theta = \tan^{-1}\left(\pm\frac{b}{ae}\right)$$

38. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3$, then the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is **[2021]**

(a) $3x^2 - 25x = 3 = 0$ (b) $3x^2 + 5x + 3 = 0$

(c) $3x^2 - 5x + 3 = 0$ (d) $3x^2 - 19x + 3 = 0$

Ans. (d)

Sol. Given that $\alpha^2 = 5\alpha - 3 \ldots (i)$ and $\beta = 5\beta - 3 (ii)$

If roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

$\therefore$ Product of roots and sum of root (s)

$$= \frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$$

$$= \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{5(\alpha+\beta)-6}{\alpha\beta} \ldots (iii)$$

$\because$ Root always satisfies the equation. From Eq. (i), we can observe the equation whose roots are α and β is

$$x^2 = 5x - 3 \Rightarrow x^2 - 5x + 3 = 0$$

$$\therefore\ \alpha + \beta = 5 \text{ and } \alpha\beta = 3$$

From Eqs. (iii),

$$S = \frac{5(5)-6}{3} = \frac{19}{3}$$

Required equation,

$$x^2 - \left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 0 \Rightarrow x^2 - \left(\frac{19}{3}\right)x + 1 = 0$$

$$\Rightarrow \quad 3x^2 - 19x + 3 = 0$$

39. The probability that a man who is x years old will die in a year is p. Then, amongst n persons $A_1, A_2, \ldots A_n$ each x year old now, the probability that A_1 will die in one year is **[2021]**

(a) $\frac{1}{n^2}$ (b) $1-(1-p)^n$

(c) $\frac{1}{n^2}\left[1-(1-p)^n\right]$ (d) $\frac{1}{n}\left[1-(1-p)^n\right]$

Ans. (d)

Sol. Given probability that a man who is x years old will die in a year $= p$

p (no one among $A_1, A_2, A_3, \ldots, A_n$ dies within a year)

$$= (1-p)(1-p)\ldots n \text{ times}$$

$$= (1-p)^n$$

p (atleast one among $A_1, A_2, \ldots, A_n$ dies within a year)

$$= 1-(1-p)^n$$

p (A_1 dies within a year and is first to die)

$$= \frac{1}{n}\left[1-(1-p)^n\right]$$

40. Angle between $\mathbf{a}$ and $\mathbf{b}$ is 120°. If $|\mathbf{b}| = 2|\mathbf{a}|$ and the vectors $\mathbf{a}+x\mathbf{b}, \mathbf{a}-\mathbf{b}$ are at right angle, then x is equal to **[2021]**

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{2}{3}$ (d) $\frac{2}{5}$

Ans. (d)

Sol. Given that angle between $\mathbf{a}$ and $\mathbf{b}$ is 120 . $\because \mathbf{a}+x\mathbf{b}, \mathbf{a}-\mathbf{b}$ are right angled.

$$\Rightarrow (\mathbf{a}+x\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = 0$$

$$\Rightarrow |\mathbf{a}|^2 - (x-1)|\mathbf{a}|^2 - 4x|\mathbf{a}|^2 = 0 \quad \Rightarrow \{\because |\mathbf{b}| = 2|\mathbf{a}|\}$$

$$\Rightarrow (2-5x)|\mathbf{a}|^2 = 0 \Rightarrow x = \frac{2}{5} \quad [\because |a| \neq 0]$$

41. If a number x is selected at random from natural numbers $1,2,\ldots,100$, then the probability for $x+\frac{100}{x}>29$ is **[2021]**

(a) $\frac{37}{50}$ (b) $\frac{39}{50}$ (c) $\frac{41}{50}$ (d) $\frac{43}{50}$

Ans. (b)

Sol. Given condition $x+\frac{100}{x}>29$, where x is a natural number. Possible values of x satisfying the above condition are $1,2,3,26,27,28,\ldots..,100$.

Since,

$$x+\frac{100}{x}>29$$

$$\Rightarrow$$

$$x^2-29x+100>0$$

$$(x-4)(x-25)>0$$

$$\therefore x\in\{1,2,3,26,27,\ldots,100\}=E \quad \text{(say)}$$

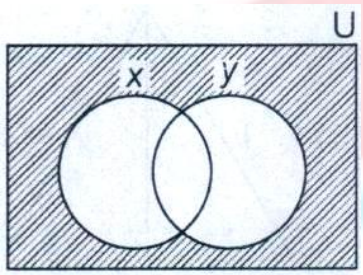
$$\therefore \text{ Probability } =\frac{n(E)}{100}=\frac{78}{100}=\frac{39}{50}$$

42. If X and Y are two sets, then $X\cap Y'\cap(X\cup Y)'$ is **[2021]**

(a) X' (b) Y' (c) ϕ (d) None of these

Ans. (c)

Sol. $X\cap Y'\cap(X\cup Y)'$



$$=X\cap Y'\cap(X'\cap Y')=X\cap((Y'\cap X')\cap(Y'\cap Y'))$$

$$=X\cap(Y'\cap X')=(X\cap Y')\cap(X\cap X')$$

$$=(X\cap Y')\cap\phi=\phi$$

43. $\int e^x(\sinh x+\cosh x)\,dx$ is equal to **[2021]**

(a) $e^x\operatorname{sech}x+C$ (b) $e^x\cosh x+C$

(c) $\sinh 2x+C$ (d) $\cosh 2x+C$

Ans. (b)

Sol. $I=\int e^x(\sinh x+\cosh x)\,dx$

$$\left\{\because \frac{d}{dx}(\sinh x)=\cosh(x) \text{ and } \frac{d}{dx}\cosh x=\sinh x\right.$$

$$\left.\text{and } \int e^x(f(x)+f'(x))\,dx=e^x f(x)+C\right\}$$

$$I=e^x\cosh x+C$$

44. If $\mathbf{e}_1=(1,1,1)$ and $\mathbf{e}_2=(1,1,-1)$ and $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $\mathbf{e}_1=2\mathbf{a}+\mathbf{b}$ and $\mathbf{e}_2=\mathbf{a}+2\mathbf{b}$, then angle between $\mathbf{a}$ and $\mathbf{b}$ is **[2021]**

(a) $\cos^{-1}\left(-\frac{7}{11}\right)$ (b) $\cos^{-1}\left(\frac{7}{11}\right)$

(c) $\cos^{-1}\left(\frac{7}{9}\right)$ (d) $\cos^{-1}\left(\frac{6\sqrt{2}}{11}\right)$

Ans. (a)

Sol. $\mathbf{e}_1=(1,1,1),\mathbf{e}_2=(1,1,-1)$

$$\mathbf{e}_1=2\mathbf{a}+\mathbf{b} \text{ and } \mathbf{e}_2=\mathbf{a}+2\mathbf{b}$$

$$\Rightarrow 2\mathbf{a}+\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}} \quad \text{and} \quad \mathbf{a}+2\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

On solving these equations,

$$\mathbf{a}=\frac{1}{3}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}) \text{ and } \mathbf{b}=\frac{1}{3}(\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}})$$

$$\therefore \quad \cos\theta=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\Rightarrow\cos\theta=\frac{-7}{11}$$

$$\therefore \quad \theta=\cos^{-1}\left(\frac{-7}{11}\right)$$

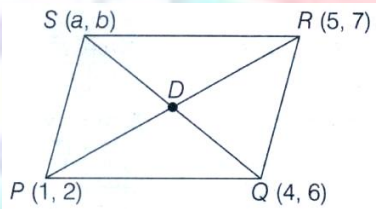
45. If $P(1,2),Q(4,6),R(5,7)$ and $S(a,b)$ are the vertices of a parallelogram $PQRS$, then **[2021]**

(a) $a=2,b=3$ (b) $a=3,b=4$

(c) $a=2,b=4$ (d) $a=3,b=5$

Ans. (a)

Sol. Since we know that diagonals of parallelogram bisect each other.



$$\therefore D=\left(\frac{a+4}{2},\frac{b+6}{2}\right)=\left(\frac{1+5}{2},\frac{2+7}{2}\right)$$

$$\Rightarrow \frac{a+4}{2}=\frac{6}{2} \text{ and } \frac{b+6}{2}=\frac{9}{2}$$

$$\therefore a=2 \text{ and } b=3$$

46. If $a\cos\theta+b\sin\theta=2$ and $a\sin\theta-b\cos\theta=3$, then a^2+b^2 is equal to **[2021]**

(a) 6 (b) 5 (c) 13 (d) 10

Ans. (c)

Sol. $a\cos\theta+b\sin\theta=2\ldots(i)$

$$a\sin\theta-b\cos\theta=3\ldots(ii)$$

On squaring Eqs. (i) and (ii) and adding after that $(a\cos\theta+b\sin\theta)^2+(a\sin\theta+b\cos\theta)^2=2^2+3^2$

$\Rightarrow a^2\left(\cos^2\theta+\sin^2\theta\right)+b^2\left(\sin^2\theta+\cos^2\theta\right)=4+9$

$\therefore a^2+b^2=13$

47. The three thrown of three dice, the probability of throwing triplets not more than twice is **[2021]**

(a) $1-\frac{1}{6^2}$ (b) $1-\frac{1}{6^3}$

(c) $1-\frac{1}{36^2}$ (d) $1-\frac{1}{36^3}$

Ans. (d)

Sol. Since three dice are thrown.

$\therefore \quad n(S)=216$

Triplets are $\{(1,1,1),(2,2,2),(3,3,3),(4,4,4),(5,5,5),(6,6,6)\}$

Number of triplets $=6$

$\therefore$ Probability of throwing triplets not more than twice

$$=1-\left(\frac{6}{216}\right)^3=1-\left(\frac{1}{36}\right)^3=1-\frac{1}{36^3}$$

48. $\int 3^{3^{3x}}\cdot 3^{3^x}\cdot 3^x dx$ is equal to **[2021]**

(a) $\frac{3^{3^{3x}}\cdot 3^x}{(\log 3)^3}+C$ (b) $\frac{3^x}{(\log 3)^3}+C$

(c) $\frac{3^{3^x}}{(\log 3)^3}+C$ (d) $\frac{3^{3^{3x}}}{(\log 3)^3}+C$

Ans. (d)

Sol. $I=\int 3^{3^{3x}}\cdot 3^{3^x}\cdot 3^x dx$

Let $3^{3^{3x}}=t$

$$\Rightarrow \frac{d}{dx}\left(3^{3^{3x}}\right)=\frac{dt}{dx}\Rightarrow 3^{3^{3x}}\cdot 3^{3^x}\cdot 3^x(\log 3)^3=\frac{dt}{dx}$$

$$I=\frac{1}{(\log 3)^3}\int dt\Rightarrow I=\frac{t}{(\log 3)^3}+C=\frac{3^{3^{3x}}}{(\log 3)^3}+C$$

49. There are 50 questions in a paper. Find the number of ways in which a student can attempt one or more questions. **[2021]**

(a) $2^{50}-1$ (b) $2^{50}+1$ (c) $2^{50}-2$ (d) $2^{50}+2$

Ans. (a)

Sol. The number of ways in which a student can attempt one or more questions

$={}^{50}C_1+{}^{50}C_2+{}^{50}C_3+\ldots.+{}^{50}C_{50}$

$=2^{50}-1\left[\because {}^nC_0+{}^nC_1\ldots+{}^nC_n=2^n\right]$

50. Consider the following frequency distribution table. **[2021]**

Class Interval	10 –20	20 –30	30 –40	40 –50	50 –60	60 –70	70 –80
Freq-ency	180	f_1	34	180	136	f_2	50

If the total frequency is 686 and the median is 42.6 , then the value of f_1 and f_2 are

(a) 81, 25 (b) 82,24

(c) 83,23 (d) 84,22

Ans. (b)

Sol.

Class internal	Frequency	Cumulative frequeney
10-20	180	180
20-30	f_1	$180+f_1$
30-40	34	$214+f_1$
40-50	180	$3.94+f_1$
50-60	136	$530+f_1$
60-70	f_2	$530+f_1+f_2$
70-80	50	$580+f_1+f_2$

Given that total, frequency $=686$

$\therefore 580+f_1+f_2=686\Rightarrow f_1+f_2=106$

$\because$ Median=426, so it will lies in 40-50 class interval.

$\because$ Modal class $=40-50$

$\therefore$ Lower limit $(l)=40, h=10$

$f=180, cf=214+f_1$

$\because$ Median $=l+\frac{\left(\frac{N}{2}-cf\right)}{f}\times h$

$$\Rightarrow 42.6=40+\frac{\frac{686}{2}-(214+f_1)}{180}\times 10$$

$\Rightarrow 2.6\times 18=343-214-f_1\Rightarrow 46.8=129-f_1$

$\therefore f_1=82.2\approx 82$ and $f_2=106-82=24$

Analytical Ability & Logical Reasoning

51. Choose the word which has the same relationship among the given three words: **[2021]**

Yellow, Blue, Red

(a) Black (b) Orange (c) White (d) Paint

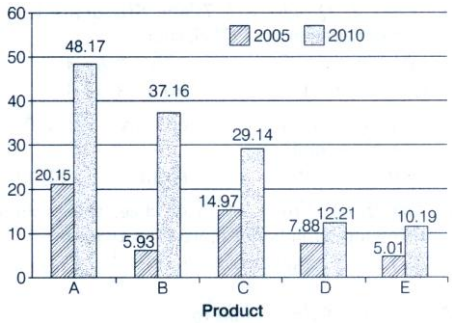
Ans. (b)

Sol. VIBGYOR stands for Violet, Indigo, Blue, Green, Yellow, Orange and Red.

So, Yellow, Blue, Red and Orange are colours in VIBGYOR.

52. A company produces five different products. The sales of these five products (in lakh number of packs) during 2005 and 2010 are shown in the following bar graph. Question is based on this graph.

Sales (in lakh number of packs) of five different products of a company during 2005-2010.



The sales of Product A in 2010 was by that percent more than the sales of Product B in 2010? (rounded off to the nearest integer)

[2021]

(a) 33% (b) 31% (c) 28% (d) 22%

Ans. (b)

Sol. Required percentage $= \dfrac{48.17-37.16}{37.16}\times 100$

$= \dfrac{1101}{37.16} = 29.62\% \approx 31\%$

53. Choose the pair of number that comes next:

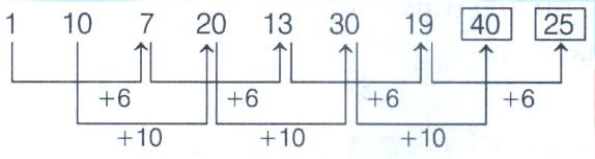
[2021]

1 10 7 20 13 30 19

(a) 25 22 (b) 26 24 (c) 26 23 (d) 25 23

Ans. (*)

Sol. Pattern of the series as follows



54. Read the information given below and answer the following question:

(i) In a family of six persons A, B, C, D, E and F, there are two married couples.

(ii) D is grandmother of A and mother of B .

(iii) C is wife of B and mother of F .

(iv) F is the granddaughter of E .

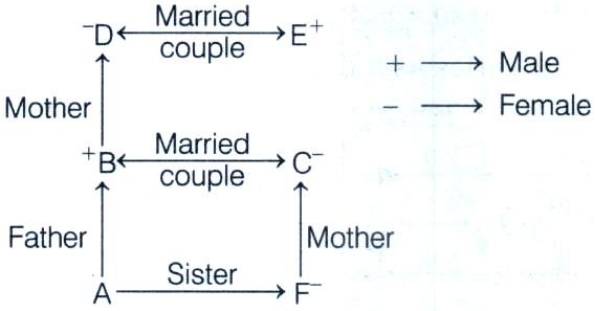
[2021]

Who among the following is one of the couples?

(a) CD (b) DE (c) EB (d) None of these

Ans. (b)

Sol. According to the given information.



Hence, DE is one of the couples.

55. If $Z = 52$ and $ACT = 48$, then BAT will be equal to

[2021]

(a) 46 (b) 39 (c) 44 (d) 41

Ans. (a)

Sol. As, $Z = 52$

$\Rightarrow 26\times 2 = 52$

and $ACT = 48$

$\Rightarrow (1+3+20)\times 2 = 24\times 2 = 48$

Similarly, BAT $\Rightarrow (2+1+20)\times 2 = 23\times 2 = 46$

Directions (Q. Nos. 56 and 57) Read the information given below and answer the following question.

(i) In a family of six persons A, B, C, D, E and F there are two married couples.

(ii) D is grandmother of A and mother of B .

(iii) C is wife of B and mother of F .

(iv) F is the granddaughter of E .

56. Which of the following is true? **[2021]**

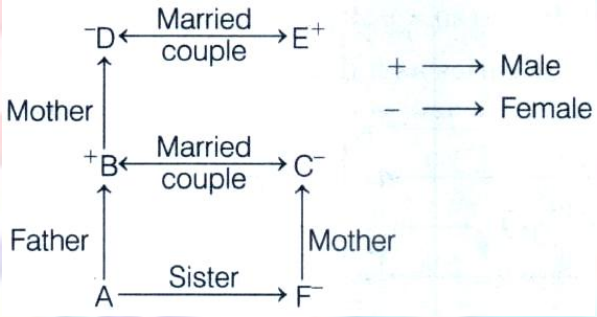
(a) A is brother of F . (b) A is sister of F .

(c) D has two grandsons. (d) None of these

Ans. (d)

Sol. None of these statements are true.

(Q. Nos. 56 and 57) According to the given information,



57. What is C to A ? **[2021]**

(a) Daughter (b) Grandmother

(c) Mother (d) Cannot be determined

Ans. (c)

Sol. C is mother of A .

58. Find the matching term for Music: Guitar: Performer.

[2021]

(a)Dance: Tune: Instrument

(b)Food : Recipe : Cook

(c)Patient : Medicine : doctor

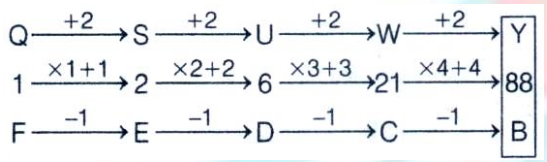
(d) Trick : Rope : Acrobat

Ans. (d)

Sol. 'Trick: Rope: Acrobat' is similar to 'Music: Guitar: Performer'.

59. If $9\times3+8=24, 10\times2+7=35$ and $80\times40+3=6$, then find the value of $12\times4+3=$? **[2021]**
(a) 7 (b) 9 (c) 12 (d) 16
Ans. (b)
Sol. As $9\times3+8=24$
$\Rightarrow (9\times8)\div3=72\div3=24; 10\times2+7=35$
$\Rightarrow (10\times7)\div2=70\div2=35$ and $80\times40+3=6$
$\Rightarrow (80\times3)\div40=240\div40=6$
Similarly, $12\times4+3=$?
$\Rightarrow (12\times3)\div4=36\div4=\boxed{9}$

60. Choose the missing term out of the given alternatives: Q1F, S2E, U6D, W21C, ? **[2021]**
(a) Y66B (b) Y44B (c) Y88B (d) Z88C
Ans. (c)
Sol. Pattern of series as follows



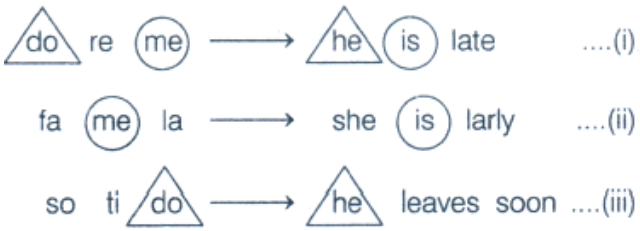
61. Study the following table carefully and answer the following question. **[2021]**

Subjects (Full Marks 50)	40 and above	30 And Above	20 and above	10 and above	0 And Above
Physics	9	32	80	92	100
Chemistry	4	21	66	81	100
Aggregate	7	27	73	87	100

If it is known that atleast 23 students were eligible for a Symposium on Chemistry the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range:
(a) $30-40$ (b) $20-30$
(c) Below 20 (d) Cannot be determined
Ans. (b)
Sol. The minimum qualifying marks in Chemistry for eligibility to symposium would lie in the range 20-30.

62. In a certain code language, "do re me" means "he is late"; "fa me la" means "she is early" and "so ti do" means "he leaves soon". Which word in the language means 'late'? **[2021]**
(a) la (b) do (c) me (d) re
Ans. (d)

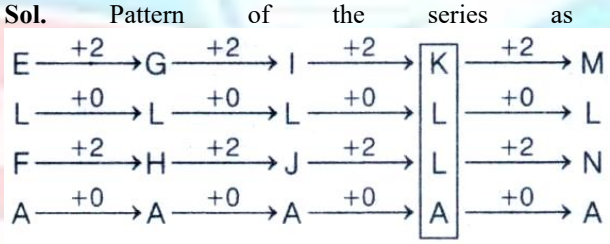
Sol. According to the question,



From Eqs. (i), (ii) and (iii),
late = re

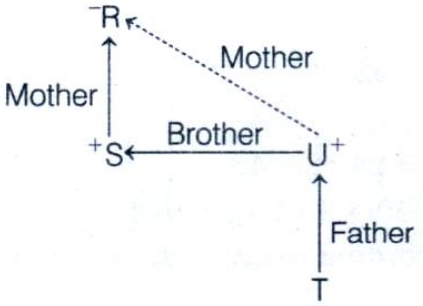
63. Find the missing term $7:56::9:$?. **[2021]**
(a) 63 (b) 81 (c) 90 (d) 99
Ans. (c)
Sol. As $7:56$
$\Rightarrow 7:7\times(7+1)=7:7\times8=7:56$
Similarly, 9 : ?
$\Rightarrow 9:9\times(9+1)=9:9\times10=9:90$

64. In this series, looking at the letter pattern fill the blank in the middle of the series:
ELFA, GLHA, ILJA, ——— ,MLNA **[2021]**
(a) OLPA (b) KLMA (c) LLMA (d) KLLA
Ans. (d)
Sol. Pattern of the series as follows



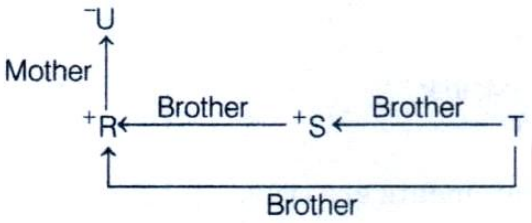
65. Read the following information carefully and then answer the following question: **[2021]**
(i) $P\psi Q$ means P is mother of Q
(ii) $P\in Q$ means P is sister of Q
(iii) $P\$Q$ means P is father of Q
(iv) $P\#Q$ means P is brother of Q

Which of the following means R is brother of T ?
(a) $R\ \psi\ S\#U\$T$
(b) $U\psi R\#\ S\#\ T$
(c) $U\psi R\in S\psi T$
(d) $T\#\ S\$Q\in R$
Ans. (b)
Sol. From option
(a),



$\because R$ is grandmother of T .[incorrect]

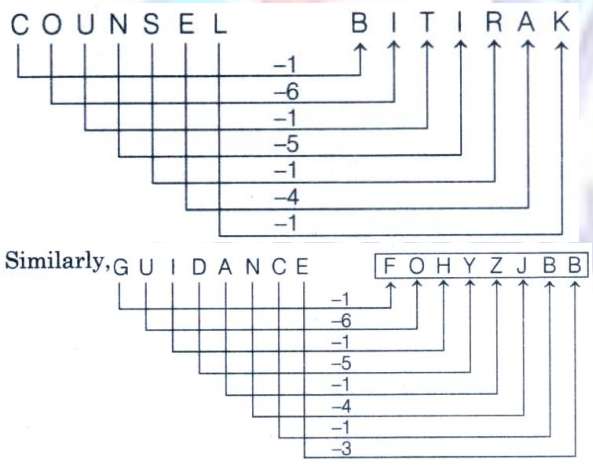
From option (b),



So, option (b) is correct.

66. Choose the pair that best represents a similar relationship to the one expressed in the original pair of words:
WAITRESS : RESTAURANT **[2021]**
(a) doctor : diagnosis
(b)actor:role
(c) driver : truck
(d) teacher : school
Ans. (d)
Sol. As, WAITRESS works in RESTAURANT.
Similarly, Teacher works in School.

67. COUNSEL is to BITIRAK, so also GUIDANCE is to ——— ?
[2021]
(a) EOHYZKBB (b) FOHYZJBB
(c) FPHZZKAB (d) HOHYBJBA
Ans. (b)
Sol. As,



68. Choose the number pair or group that is different from others?
[2021]
(a) $15:46$ (b) $12:37$ (c) $9:28$ (d) $8:33$
Ans. (d)
Sol. $15:46 \Rightarrow 15\times3+1=46, 12:37 \Rightarrow 12\times3+1=37$,

$9:28 \Rightarrow 9\times3+1=28$ and $\boxed{8:33} \Rightarrow 8\times3+1=25\neq33$

69. If it is Saturday on January 1, 2000, then January 1, 2001 would have been **[2021]**
(a) Monday (b) Sunday
(c) Tuesday (d) Friday
Ans. (a)
Sol. $\because$ Year 2000 is a leap year.
So, if it is Saturday on January 1,2000, then January 1, 2021 would have been (Saturday $+2=$ Monday).

70. Radlia is twice as old as Rita was 2 yr ago. If difference between their ages is 2 yr , how old is Radha today?
[2021]
(a) 6 (b) 8 (c) 10 (d) 12
Ans. (b)
Sol. 2 yr ago, Rita's age $=x$ (let) and Radha's present age $=2x$
According to the question,
Difference between their ages $=2\text{yr}$
$\Rightarrow 2x-(x+2)=2 \Rightarrow 2x-x-2=2 \Rightarrow x=4$
Hence, Radha's present age $=4\times2=8\text{yr}$

71. How many times in a day are the hands of a clock pointing opposite to each other? **[2021]**
(a) 4 (b) 20 (c) 22 (d) 14
Ans. (c)
Sol. 22 times in a day are the hands of a clock pointing opposite to each other.

Direction (Q. 72 and 73) Study the following table carefully and answer the following question.

Subjects (Full – Marks 50)	40 and abo-ve	30 and abo-ve	20 and abo-ve	10 and abo-ve	0 and abo-ve
Physics	9	32	80	92	100
Chemistry	4	21	66	81	100
Aggregate	7	27	73	87	100

72. If atleast 60% marks in Physics are required for pursuing higher studies in Physics. How many students will be eligible to pursue higher studies in Physics?
[2021]
(a) 27 (b) 32 (c) 34 (d) 41
Ans. (b)
Sol. Required marks for pursuing studies in Physics
$$=\frac{50\times60}{100}=30$$
Hence, 32 students will be eligible to pursue higher studies in Physics.

73. The number of students scoring less than 40% marks in aggregate is **[2021]**
(a) 13 (b) 19 (c) 27 (d) 20
Ans. (c)
Sol. 40% of $50=50\times\frac{40}{100}=20$

Hence, number of students scoring less than 40%. Marks in aggregate $=100$ - Number of students scoring 20 and above marks in aggregate

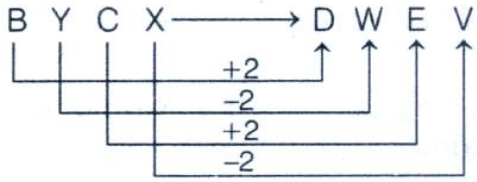
$=100-73=27$

74. Choose the missing term: BYCX : DWEV : : FUGT : ? **[2021]**

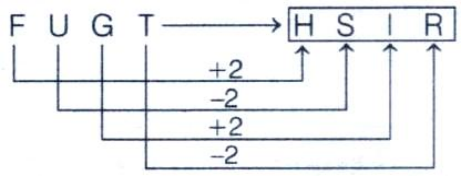
(a) EHIJ (b) GHIJ (c) HSIR (d) SRHS

Ans. (c)

Sol. as,



Similarly



75. Read the information given below and answer the following question: **[2021]**

(i) In a family of six persons A,B,C,D,E and F, there are two married couples.

(ii) D is grandmother of A and mother of B.

(iii) C is wife of B and mother of F

.

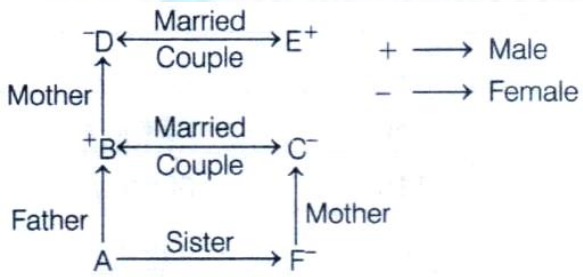
(iv) F is the granddaughter of E .

How many male members are there in the family?

(a) Three
(b)Two
(c) Cannot be determined
(d) None of these

Ans. (c)

Sol. According to the given information, Gender of A cannnot define.



$\therefore$ Number of male members are there in this family cannot be determined.

76. Read the following information carefully and then answer the following question: **[2021]**

(i) $P\Psi Q$ means P is mother of Q

(ii) $P\in Q$ means P is sister of Q

(iii) $P\$Q$ means P is father of Q

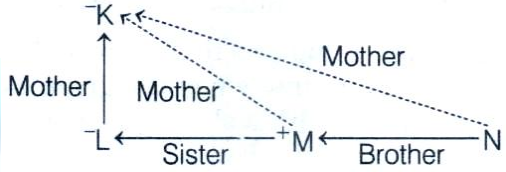
(iv) $P\#Q$ means P is brother of Q

If $K\Psi L\in M\#N$, then how K is related with N ?

(a) Mother (b) Aunt
(c) Aunt (d) Grandmother

Ans. (a)

Sol. According to the given information,



Hence, K is the mother of N .

77. Choose the correct option for the remainder when $X=1!+2!+3!+\ldots+100$! is divided by 24 . **[2021]**

(a) 9 (b) 11 (c) 152 (d) 13

Ans. (a)

Sol. Given,

$$X=1!+2!+3!+\ldots+100!$$

Now,

$$\frac{X}{24}=\left(\frac{1!+2!+3!+\ldots+100!}{24}\right)$$

$$\Rightarrow \frac{X}{24}=\left(\frac{1+2\times1+3\times2\times1+\ldots+100\times99\times\ldots\times1}{24}\right)$$

$$\Rightarrow \frac{X}{24}=\left(\frac{1+2+6+24+120+\ldots+100\times99\times\ldots\times1}{24}\right)$$

$$\Rightarrow \frac{X}{24}=\frac{9}{24}$$

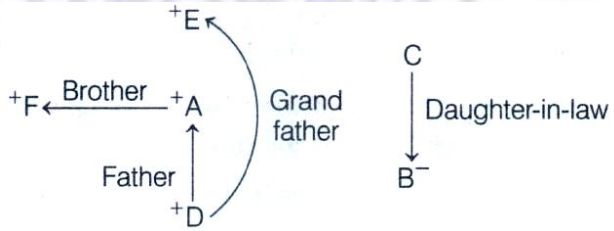
$\therefore$ Required remainder $=9$

78. There are six members in a family. A is the father of D,E is the grandfather of $D.B$ is the daughter-in-law of C. F is the uncle of D. What is the relationship of C with F ? **[2021]**

(a) Sister (b) Mother-in-law
(c) Nephew (d) Data inadequate

Ans. (d)

Sol. According to the question,



Hence, relationship of C with F cannot defines. Hence, data inadequate.

79. Read the following information carefully and then answer the following question: **[2021]**

(i) $P\Psi Q$ means P is mother of Q

(ii) $P \in Q$ means P is sister of Q

(iii) P$Q means P is father of Q

(iv) $P \# Q$ means P is brother of Q

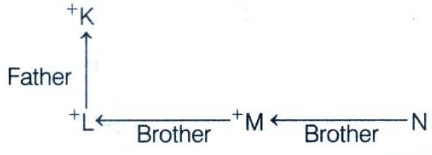
Which of the following means N is definitely daughter of K ?

(a) K $ L#M# N (b) $M\Psi K\$N \in L$

(c) K$M# L∈ N (d) L$K$ N#M

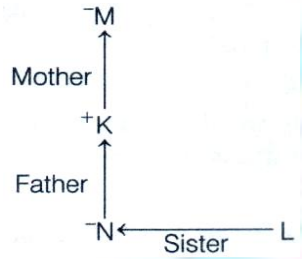
Ans. (b)

Sol. From option (a),



∵ Gender of N cannot defines. (Incorrect)

From option (b),



Hence, N is definitely daughter of K . (correct)

80. Choose the word that is a necessary part of the underlined word: **[2021]**

Harvest

(a) autumn (b) crop

(c) stockpile (d) tractor

Ans. (b)

Sol. Crop is a necessary part of Harvest.

81. Study the following table carefully and answer the following question. **[2021]**

Subjects (Full Marks 50)	40 and abo-ve	30 And Abo-ve	20 And Abo-ve	10 And Abo-ve	0 And Abo -ve
Phys-ics	9	32	80	92	100
Chem-istry	4	21	66	81	100
Aggre-gate	7	27	73	87	100

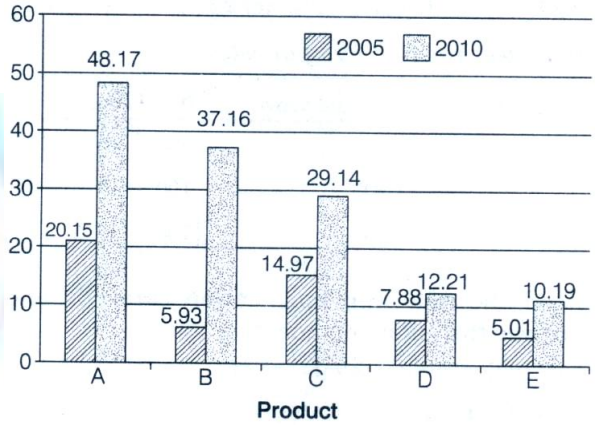
What is the difference between the number of students passed with 30 as cut - off marks in Chemistry and those passed with 30 as cut - off marks in aggregate?

(a) 3 (b) 4 (c) 5 (d) 6

Ans. (d)

Sol. Required difference = Number of students scoring 30 and above marks in Chemistry - Number of students scoring 30 and above marks in aggregate $= 27 - 21 = 6$

82. A company produces five different products. The sales of these five products (in lakh number of packs) during 2005 and 2010 are shown in the following bar graph. Question is based on this graph. Sales (in lakh number of packs) of five different products of a company during 2005-2010.



During the period 2005-2010. the minimum rate of increase in sales is in the case of **[2021]**

(a) Product A (b) Product B

(c) Product D (d) Product E

Ans. (c)

Sol. Rate of increase for Product

$$A = \frac{48.17 - 20.15}{20.15} \times 100\% = 139.05\%$$

Rate of increase for Product

$$B = \frac{37.16 - 5.93}{5.93} \times 100\% = 526.64\%$$

Rate of increase for Product

$$D = \frac{12.21 - 7.88}{7.88} \times 100\% = 54.94\%$$

Rate of increase for Product

$$E = \frac{10.19 - 5.01}{5.01} \times 100\% = 103.39\%$$

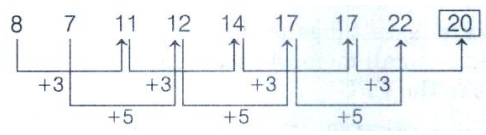
Hence, during the period 2005-2010, the minimum rate of increase in sales is in the case of product D.

83. Insert the missing number $8, 7, 11, 12, 14, 17, 17, 22$, ? **[2021]**

(a) 27 (b) 20 (c) 22 (d) 24

Ans. (b)

Sol. Pattern of the series as follows



84. Choose the pair that best represents a similar relationship to the one expressed in the original pair of words: **[2021]**

PULSATE : THROB

(a) walk : run (b) tired : sleep

(c) examine : scrutinize (d) ballet : dancer
Ans.(c)
Sol. As, Pulsate and Throb are synonyms.
Similarly, Examine and Scrutinize are synonyms.

85. In a class of 50 students, Raghu's rank is twice that of Paul. There are 10 students who have ranked worse than that of Raghu. Paul's rank in the class is **[2021]**
(a) 5^{th} (b) 10 th (c) 15 th (d) 2π
Ans. (d)
Sol. Total number of students in the class $=50$

Let Paul's rank $=x$

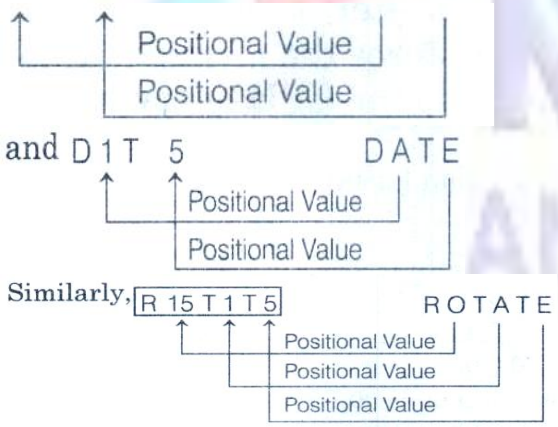
So, Raghu's rank $=2x$

$\because 10$ students who have ranked worse than that of Raghu.
$\therefore$ Raghu's rank $=40$

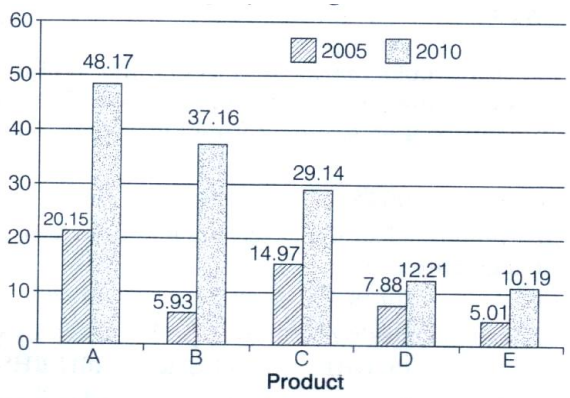
So, Paul's rank $=\frac{40}{2}=20$ th

86. Choose the odd one: **[2021]**
(a) Zebra (b) Rhinoceros
(c) Hyena (d) Giraffe
Ans. (c)
Sol. Except Hyena, all others are vegetarian.

87. If 'A3T15R' stands for 'ACTOR' and 'DIT5' stands for DATE. How will you code 'ROTATE'? **[2021]**
(a) R16TIT5 (b) R15TIT5
(c) R15TITl6 (d) R 15 CIT 7
Ans. (b)
Sol. As, A3T $15R$ ACTOR



88. A company produces five different products. The sales of these five products (in lakh number of packs) during 2005 and 2010 are shown in the following bar graph. Question is based on this graph. Sales (in lakh number of packs) of five different products of a company during 2005-2010.



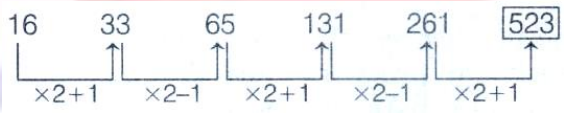
The sales have increased by nearly 55% from 2005 to 2010 in case of **[2021]**
(a) Product A (b) Product B
(c) Product C (d) Product D
Ans. (d)
Sol. Rate of increase for product

$$C=\frac{29.14-14.97}{14.97}\times 100=94.65\%$$

and from answer (82). It is clear, sales have increased by nearly 55% from 2005 to 2010 in case of product D.

89. Which word does NOT belong with the others? **[2021]**
(a) wing (b) fin (c) beak (d) rudder
Ans. (c)
Sol. Except beak, all others are parts of an airplane.

90. Insert the missing number: $16, 33, 65, 131, 261$, ? **[2021]**
(a) 523 (b) 521 (c) 613 (d) 721
Ans. (a)
Sol. Pattern of the series as follows



Computer Awareness

91. The process when processor fetch or decode another instruction during the execution of current instruction is called **[2021]**
(a) Supercomputing (b) Pipelining
(c) Cloud computing (d) Grid computing
Ans. (b)
Sol. Pipe-lining is the process of improving the performance of the system by processing different instructions at the same time, with only one instruction performing one specific operation.

92. Which of the following is used by ALU to store the intermediate results? **[2021]**
(a) Stack (b) Heap
(c) Registers (d) Accumulators
Ans. (d)
Sol. Accumulators are used by ALU (Arithmetic Logic Unit) to store the intermediate results. The ALU is the computational center of the CPU. It performs all mathematical and logical operations.

93. One TeraByte $(TB)=$ ——— GB and

One ExaByte $(EB) =$ ——— GB **[2021]**

(a) 2^{10} GB, 2^{16} GB

(b) 2^{10} GB, 2^{20} GB

(c) 2^{10} GB, 2^{24} GB

(d) 2^{10} GB, 2^{30} GB

Ans. (d)

Sol. 1 GigaByte

$(GB) = 1024 = 10^{10} MB$

1 TeraByte (TB) $= 2^{10}$ GB

1 PetaByte $(PB) = 2^{10}$ TB

1 ExaByte $(EB) = 2^{10}$ PB

$= 2^{10} \times 2^{10}$ TB

$= 2^{10} \times 2^{10} \times 2^{10}$ GB $= 2^{30}$ GB

94. The cache memory is more effective because of **[2021]**

(a) Memory localization (b) Locality of reference

(c) Memory size (d) None of the mentioned

Ans. (b)

Sol. The cache memory is more effective because of locality of reference. This means that the cache depends on the location in the memory that is referenced often.

95. Which of the following is the fastest means of memory access for CPU? **[2021]**

(a) Registers (b) Cache

(c) Main memory (d) Stack

Ans. (a)

Sol. Registers are fastest means of access for CPU. Registers are the small memory locations which are present closet to the CPU.

96. The number $(2217)_8$ is equivalent to **[2021]**

(a) $(608)_{16}$ (b) $(028\ F)_{16}$

(c) $(048\ F)_{16}$ (d) $(2297)_{10}$

Ans. (c)

Sol. Binary equivalent of $(2217)_8$ is

$\underset{010}{2} \quad \underset{010}{2} \quad \underset{001}{1} \quad \underset{111}{7}$

$= (010010001111)_2$

Hexademical equivalent of $(010010001111)_2$ is

$\underset{4}{0100} \quad \underset{8}{1000} \quad \underset{15 \rightarrow F}{1111} = (48\ F)_{16}$

So, $(2217)_8 = (048\ F)_{16}$

97. To fetch data from the secondary memory which one of the following register is used? **[2021]**

(a) MAR (b) PC (c) IR (d) MBR

Ans. (a)

Sol. To fetch data from the secondary memory, MAR (Memory Address Register) is used. MAR is a CPU register that either stores the memory address from which data will be fetched to the CPU or the address to which data will be sent and stored.

98. The binary multiplication $00*11$ will give **[2021]**

(a) 11 (b) 00 (c) 01 (d) 10

Ans. (b)

Sol. Binary multiplication for $00*11$

$$\begin{array}{r} oo \\ \times 11 \\ \hline oo \\ 00\times \\ \hline ooo \end{array}$$

So, it will give output as 00 .

99. Consider a computer system with speed of 10^6 instructions per second. A program P , having $2n^2$ steps is run on this system, where n is the input size. If $n = 10000$, what is the execution time for P ?

[2021]

(a) 1.2 s (b) 20 s (c) 100 s (d) 200 s

Ans. (d)

Sol. Number of steps $= 2n^2$

Speed of computer $= 10^6$ per second

For $n = 10000 (10^4)$

$\because$ Time $= \dfrac{\text{Number of steps}}{\text{Speed of computer}}$

$= \dfrac{2n^2}{10^6} = \dfrac{2 \times (10^4)^2}{10^6} = \dfrac{2 \times 10^8}{10^6}$

$= 2 \times 10^2 = 200 \text{sec}$

100. To access the I/O devices, the status flag is continuously checked in **[2021]**

(a) Program controlled I/O

(b) Memory mapped I/O

(c) I/O mapped

(d) None of these

Ans. (a)

Sol. To access the I/O devices, the status flag is continuously checked in program controlled I/O. In this method, the processor constantly checks the status flags and when it finds that the flag is set it performs the appropriate operation.

General English

Directions (Q. Nos. 101-102) Read the following passage carefully and answer the question that follow:
It is said with truth that the function of a university is to prepare the young to take their place in human society. It must provide its

members with the knowledge and skill necessary to make them efficient citizens. But is the whole duty of man exhausted by the acquisition of knowledge and professional training? Is a university only an institution for higher learning, a factory which churns out clerks and technicians able to run the machinery of the State? Mere knowledge which gratifies curiosity is different from culture which refines personality. Culture is n remembering a mass of serious details about the dates of birth of the great heroes of the world or the interesting names of the fastest ships which cross the Atlantic or entertaining odds and ends gathered from the latest who's who. A well known institution of this country has for its motto sa vidya yavimuchyate: that is, knowledge which is designed for salvation, for the development of the soul, is the best. Such an idea is not merely an Indian idiosyncrasy. Plato said long ago that the culture of soul is "the first and fairest thing that the best of men can ever have. According to Goethe, the object of education is to form tastes and not simply to communicate knowledge. A man's culture is not judged by the amount of tabulated information which he has at his command, but by the quality of mind which he brings to bear on the facts of life. Education is not cramming the mind with a host of technical details, putting sight, as it were, into blind eyes.

The eye of the soul is never blind, only its gaze may be turned to the false and the fleeting. Too often the vision may be dragged downwards by the "leaden weights" of pride and prejudice, of passion and desire. The function of the teacher is not to add to the "leaden weights" but remove them and liberate the soul from the encumbrance so that it may follow its native impulse to soar upwards. The student at a university does not merely learn something, but becomes something by being exposed, in the most elastic period of his life, to transforming influences, such as the onstant clash of mind with mind, the interchange of ideas, che testing of opinions, and the growth of knowledge of human nature.

101. Mere knowledge and culture may be distinguished from each other in that: **[2021]**

(a) the former widens the mental horizon, the latter enlarges the hearts

(b) the former gratifies curiosity, the latter refines personality

(c) the former is concerned with facts, the latter with fiction

(d) the former adds to power, the latter adds to prestige

Ans. (b)

Sol. Mere knowledge and culture may be distinguished from each other in that the former gratifies curiosity, the latter refines personality.

102. What is the function of education according to the ancient Indian philosophers? **[2021]**

(a) Education is the development mind

(b) Education brings about salvation and development of the soul

(c) Education is the cultivation of culture

(d) Education aims at the inculcation of generosity

Ans. (b)

Sol. According to the Ancient Indian Philosophers, Education brings about salvation and development of the soul.

103. Choose the most suitable synonym of the given word: SCINTILLATING **[2021]**

(a) touching (b) nagging

(c) glittering (d) warning

Ans. (c)

Sol. Scintillating means sparkling or shinning brightly. So, glittering is the most suitable synonym of the given word. Glittering also means shining with a shimmering or sparkling light.

104. Choose the most suitable synonym of the given word: BLAND **[2021]**

(a) pleasant (b) harsh

(c) irritating (d) tasteless

Ans. (d)

Sol. Bland means unseasoned, mild-tasting or insipid (of food or drink). So, 'tasteless' is the most suitable synonym of the given word.

Directions (Q. Nos. 105-108) Read the following passage carefully and answer the question that follow:

It is said with truth that the function of a university is to prepare the young to take their place in human society. It must provide its members with the knowledge and skill necessary to make them efficient citizens. But is the whole duty of man exhausted by the acquisition of knowledge and professional training? Is a university only an institution for higher learning, a factory which churns out clerks and technicians able to run the machinery of the State? Mere knowledge which gratifies curiosity is different from culture which refines personality. Culture is not remembering a mass of serious details about the dates of birth of the great heroes of the world or the interesting names of the fastest ships which cross the Atlantic or entertaining odds and ends gathered from the latest who's who. A well known institution of this country has for its motto sa vidya yavimuchyate: that is, knowledge which is designed for salvation, for the development of the soul, is the best. Such an idea is not merely an Indian idiosyncrasy. Plato said long ago that the culture of soul is "the first and fairest thing that the best of men can ever have. According to Goethe, the object of education is to form tastes and not simply to communicate knowledge. A man's culture is not judged by the amount of tabulated information which he has at his command, but by the quality of mind which he brings to bear on the facts of life. Education is not cramming the mind with a host of technical details, putting sight, as it were, into blind eyes. The eye of the soul is never blind, only its gaze may be turned to the false and the fleeting. Too often the vision may be dragged downwards by the "leaden weights" of pride and prejudice, of passion and desire. The function of the teacher is not to add to the "leaden weights" but remove them and liberate the soul from the encumbrance so that it may follow its native impulse to soar upwards. The student at a university does not merely learn something, but becomes something by being exposed, in the most elastic period of his life, to transforming influences, such as the constant clash of mind with mind, the interchange of ideas, the testing of opinions, and the growth of knowledge of human nature.

105. What is meant by "leaden weights"? **[2021]**

(a)Weights made of leaden

(b) Cold scientific knowledge

(c) Hindering factors, of pride and prejudice, passion, and desire, in the liberation of the soul

(d) Social and family responsibilities

Ans. (c)

Sol. 'Leaden weights' means hindering factors, of pride and prejudice, passion, and desire, in the liberation of the soul.

106. The function of university is **[2021]**

(a) to enable the young to gather facts about the world

(b) to learn to do his job

(c) to prepare the young to take their place in society

(d) to enable them to learn to talk with others

Ans. (c)

Sol. The function of university is to prepare the young to take their place in society.

107. According to the passage, the function of the teacher is **[2021]**
(a) to remove leaden weights of pride and prejudice, passion, and desire to liberate the soul
(b) to instill facts into the minds of the students
(c) to teach humanity
(d) to foster brotherhood
Ans. (a)
Sol. According to the passage, the function of the teacher is to remove leaden weights of pride and prejudice, passion and desire to liberate the soul.

108. What are the hindering factors in the liberation and development of the soul? **[2021]**
(a) Power and wealth

(b) Passion and desire, pride and prejudice

(c) Money and influence

(d) Greed and envy
Ans. (b)
Sol. Passion and desire, pride and prejudice are the hindering factors in the liberation and development of the soul.

109. Fill in the blank with the most appropriate word given: I cannot ..... it to you right now; tomorrow we will discuss about it. **[2021]**
(a) demand (b) disturb
(c) explain (d) expect
Ans. (c)
Sol. According to the given context, 'explain' is the most appropriate word for the given blank.

110. Choose the most suitable synonym of the given word? RECUPERATE **[2021]**
(a) recapture (b) reclaim

(c) recover (d) recur
Ans. (c)
Sol. Recuperate means recover from illness or exertion. Hence, 'recover' is its correct synonym.

111. Fill in the blank with the most appropriate word: The ——— with which he is able to wield the paint brush is really remarkable? **[2021]**
(a) ease (b) practice
(c) sweep (d) gait
Ans. (c)
Sol. As per the given context, 'ease' is the most suitable word for the given blank.

Directions (Q. Nos. 112-113) Read the following passage carefully and answer the questions that follow:
It is said with truth that the function of a university is to prepare the young to take their place in human society. It must provide its members with the knowledge and skill necessary to make them efficient citizens. But is the whole duty of man exhausted by the acquisition of knowledge and professional training? Is a university only an institution for higher learning, a factory which churns out clerks and technicians able to run the machinery of the State? Mere knowledge which gratifies curiosity is different from culture which refines personality. Culture is not remembering a mass of serious details about the dates of birth of the great heroes of the world or the interesting names of the fastest ships which cross the Atlantic or entertaining odds and ends gathered from the latest who's who. A well known institution of this country has for its motto sa vidya yavimuchyate: that is, knowledge which is designed for salvation, for the development of the soul, is the best. Such an idea is not merely an Indian idiosyncrasy. Plato said long ago that the culture of soul is "the first and fairest thing that the best of men can ever have. According to Goethe, the object of education is to form tastes and not simply to communicate knowledge. A man's culture is not judged by the amount of tabulated information which he has at his command, but by the quality of mind which he brings to bear on the facts of life. Education is not cramming the mind with a host of technical details, putting sight, as it were, into blind eyes. The eye of the soul is never blind, only its gaze may be turned to the false and the fleeting. Too often the vision may be dragged downwards by the "leaden weights" of pride and prejudice, of passion and desire. The function of the teacher is not to add to the "leaden weights" but remove them and liberate the soul from the encumbrance so that it may follow its native impulse to soar upwards. The student at a university does not merely learn something, but becomes something by being exposed, in the most elastic period of his life, to transforming influences, such as the constant clash of mind with mind, the interchange of ideas, the testing of opinions, and the growth of knowledge of human nature.
112. What is the object of education according to Goethe'? **[2021]**
(a) It teaches social manners

(b) It teaches courtesy

(c) It communicates knowledge

(d) It forms taste
Ans. (d)
Sol. According to Geothe, the object of education is to forms taste.

113.What is the man's culture judged by? **[2021]**
(a) By the quality of mind which be brings to bear on the facts of life
(b) By man's social skill

(c) By the variety of books he reads

(d) By money and influence
Ans. (a)
Sol. The man's culture judged by the quality of mind which brings to bear on the facts of life.

114. Choose the most suitable antonym of given word: ERUDITE **[2021]**
(a) scholarly (b) unfamiliar
(c) illiterate (d) arrogant
Ans. (c)
Sol. Erudite means having or showing great knowledge or learning. So, among the given, illiterate is the most suitable antonym of the given word.

115. Choose one word for the following **[2021]**
An uncivilized/primitive person.
(a) barbarian (b) ascetic
(c) bourgeois (d) altruist
Ans. (a)
Sol. Barbarian is a person who considered extremely primitive and uncivilised.

116. Fill in the blank with the most appropriate word. The car driver was arrested for rash driving and his license was ——— by the

police. **[2021]**
(a) impounded (b) flown

(c) penalized (d) banned
Ans. (a)
Sol. Impounded is the correct word for the given blank. Impounded means seize and take legal custody of something (especially a vehicle, goods or documents).

117. Fill in the blank with the most appropriate word: The State Transportation Corporation has ——— a loss of 5 crore INR this year. **[2021]**
(a) derived (b) incurred

(c) performed (d) formulated
Ans. (b)
Sol. 'Incurred' is the most suitable word for the given blank. Incurred means to experience something unwelcome or unpleasant.

118. Choose the most suitable antonym of given word. NEGLIGENCE **[2021]**