

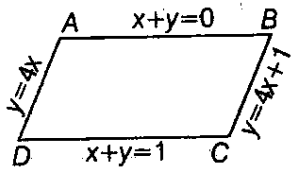
MATHEMATICS

1. Area of the parallelogram formed by the lines $y = 4x$, $y = 4x + 1$, $x + y = 0$ and $x + y = 1$ is [2022]

- (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) 5 (d) 10

Ans. (a)

Sol. Given, equation of lines are $x + y = 0$, $y = 4x + 1$, $x + y = 1$ and $y = 4x$.



Here, $m_1 = 4, m_2 = -1$

and $c_1 = 0, c_2 = 1, d_1 = 0, d_2 = 1$

We know that

$$\therefore \text{Area of parallelogram} = \left| \frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2} \right|$$

$$= \left| \frac{(0 - 1)(0 - 1)}{4 - (-1)} \right| = \frac{1}{5}$$

2. The function $f(x) = \begin{cases} (1+2x)^{\frac{1}{x}}, & x \neq 0 \\ e^2, & x = 0 \end{cases}$ [2022]

- (a) differentiable at $x = 0$
 (b) continuous at $x = 0$
 (c) discontinuous at $x = 0$
 (d) not differentiable at $x = 0$

Ans. (b)

Sol. We have, $f(x) = \begin{cases} (1+2x)^{\frac{1}{x}}, & x \neq 0 \\ e^2, & x = 0 \end{cases}$

At, $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} (1+2x)^{\frac{1}{x}}$$

This is 1^∞ form.

$$\text{Since, } f(0) = e^{\lim_{x \rightarrow 0} (1+2x-1) \times \frac{1}{x}}$$

$$= e^{\lim_{x \rightarrow 0} 2} = e^2$$

$$\text{and RHL} = \lim_{x \rightarrow 0^+} f(x) = e^2$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0) = e^2$$

Hence, $f(x)$ is continuous at $x = 0$.

3. If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is [2022]

- (a) $\frac{x+y}{xy}$ (b) xy (c) $\frac{x}{y}$ (d) $\frac{y}{x}$

Ans. (d)

Sol. Given, $x^m y^n = (x+y)^{m+n}$

Taking $\log_e$ both sides, we get

$$m \log x + n \log y = (m+n) \log(x+y)$$

On differentiating both sides w.r.t. x , we get

$$\frac{m}{x} + \frac{n}{y} \frac{dy}{dx} = \frac{m+n}{(x+y)} \left[1 + \frac{dy}{dx} \right]$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{n}{y} - \frac{m+n}{x+y} \right] = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{nx + ny - my - ny}{y(x+y)} \right] = \frac{mx + nx - mx - my}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \left[\frac{nx - my}{nx - my} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

4. A straight line through the point $(4, 5)$ is such that its intercept between the axes is bisected at A , then its equation is [2022]

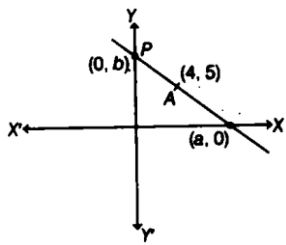
- (a) $3x + 4y = 20$ (b) $3x - 4y + 7 = 0$
 (c) $5x - 4y = 40$ (d) $5x + 4y = 40$

Ans. (d)

Sol. Let A be the mid-point of PQ .

Where $P(0, b)$, $A(4, 5)$ and $Q(a, 0)$

$$\frac{a+0}{2} = 4 \Rightarrow a = 8$$



$$\text{and } \frac{0+b}{2} = 5 \Rightarrow b = 10$$

Therefore, equation of line,

$$\frac{x}{8} + \frac{y}{10} = 1$$

$$\Rightarrow 10x + 8y = 80$$

$$\Rightarrow 5x + 4y = 40$$

5. If $\mathbf{a} = \lambda \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + \lambda \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $[\mathbf{abc}] = 7$, then the values of λ are [2022]

(a) 2, -6 (b) 4, -2

(c) 4, -2 (d) -4, 2

Ans. (a)

Sol. Given, $\mathbf{a} = \lambda \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$$\mathbf{b} = \hat{\mathbf{i}} + \lambda \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

$$\mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$$\text{and } [\mathbf{abc}] = 7$$

$$\text{Here, } [\mathbf{abc}] = 7$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 7$$

$$(\lambda \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) \cdot \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & \lambda & -2 \\ 1 & 1 & 1 \end{vmatrix} = 7$$

$$\Rightarrow (\lambda \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) [\hat{\mathbf{i}}(\lambda + 2) - \hat{\mathbf{j}}(1 + 2) + \hat{\mathbf{k}}(1 - \lambda)] = 7$$

$$\Rightarrow \lambda(\lambda + 2) - 3 - 2(1 - \lambda) = 7$$

$$\Rightarrow \lambda^2 + 2\lambda - 3 - 2 + 2\lambda = 7$$

$$\Rightarrow \lambda^2 + 4\lambda - 5 - 7 = 0$$

$$\Rightarrow \lambda^2 + 4\lambda - 12 = 0$$

$$\Rightarrow \lambda^2 + 6\lambda - 2\lambda - 12 = 0$$

$$(\lambda + 6)(\lambda - 2) = 0$$

$$\therefore \lambda + 6 = 0 \text{ or } \lambda - 2 = 0$$

$$\lambda = -6, 2$$

6. The value of $3^{3-\log_3 5}$ is [2022]

(a) $\frac{5}{27}$ (b) $\frac{27}{5}$ (c) $\frac{9}{5}$ (d) $\frac{5}{9}$

Ans. (b)

Sol. Let $y = 3^{3-\log_3 5}$

Taking $\log_3$ both sides, we get

$$\log_3 y = \log_3 3^{3-\log_3 5}$$

$$\Rightarrow \log_3 y = 3 - \log_3 5$$

$$\Rightarrow \log_3 y + \log_3 5 = 3$$

$$\Rightarrow \log_3 (5y) = 3$$

$$\Rightarrow 5y = (3)^3$$

$$\Rightarrow 5y = 27$$

$$\therefore y = \frac{27}{5}$$

7. If $a_1, a_2, \dots, a_n$ be arithmetic progression with common difference, d , then the sum $\sin d (\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \dots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n)$ is equal to [2022]

(a) $\cot a_1 - \cot a_n$ (b) $\sin a_1 - \sin a_n$

(c) $\operatorname{cosec} a_1 - \operatorname{cosec} a_n$ (d) $a_1 - a_n$

Ans. (a)

Sol. Given, $a_1, a_2, a_3 \dots a_n$ be in AP with d as common difference, then

$$d = a_2 - a_1 = a_3 - a_2 \dots = a_n - a_{n-1}$$

$$\sin d [\operatorname{cosec} a_1 \cdot \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \dots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n]$$

$$= \frac{\sin d}{\sin a_1 \sin a_2} + \frac{\sin d}{\sin a_2 \sin a_3} + \dots + \frac{\sin d}{\sin a_{n-1} \sin a_n}$$

$$= \frac{\sin(a_2 - a_1)}{\sin a_1 \cdot \sin a_2} + \frac{\sin(a_3 - a_2)}{\sin a_2 \sin a_3} + \dots + \frac{\sin(a_n - a_{n-1})}{\sin a_{n-1} \sin a_n}$$

$$= \frac{\sin a_2 \cos a_1 - \sin a_1 \cos a_2}{\sin a_1 \cdot \sin a_2} + \frac{\sin a_3 \cdot \cos a_2 - \cos a_3 \sin a_2}{\sin a_2 \cdot \sin a_3}$$

$$+ \dots + \frac{\sin a_n \cos a_{n-1} - \cos a_n \sin a_{n-1}}{\sin a_{n-1} \sin a_n}$$

$$= \cot a_1 - \cot a_2 + \cot a_2 - \cot a_3 + \dots + \cot a_{n-1} - \cot a_n$$

$$= \cot a_1 - \cot a_n$$

8. The value of $\cot \left(\operatorname{cosec}^{-1} \frac{5}{3} + \tan^{-1} \frac{2}{3} \right)$ is [2022]

(a) $\frac{6}{17}$ (b) $\frac{3}{17}$ (c) $\frac{4}{17}$ (d) $\frac{5}{17}$

Ans. (a)

Sol. We have,

$$\text{Let } \cot \left[\operatorname{cosec}^{-1} \left(\frac{5}{3} \right) + \tan^{-1} \left(\frac{2}{3} \right) \right]$$

$$\Rightarrow \operatorname{cosec}^{-1} \left(\frac{5}{3} \right) = \theta, \tan^{-1} \left(\frac{2}{3} \right) = \phi$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{5}{3}, \tan \phi = \frac{2}{3}$$

$$\Rightarrow \cot \theta = \frac{4}{3}, \cot \phi = \frac{3}{2}$$

$$\text{Now, } \cot(\theta + \phi) = \frac{\cot \theta \cot \phi - 1}{\cot \theta + \cot \phi}$$

$$= \frac{\left(\frac{4}{3} \times \frac{3}{2} \right) - 1}{\frac{4}{3} + \frac{3}{2}} = \frac{6(2-1)}{9+8} = \frac{6}{17}$$

9. If $a_1, a_2, \dots, a_n$ are any real number and n is any positive integer, then **[2022]**

$$(a) n \sum_{i=1}^n a_i^2 < \left(\sum_{i=1}^n a_i \right)^2 \quad (b) n \sum_{i=1}^n a_i^2 \geq \left(\sum_{i=1}^n a_i \right)^2$$

$$(c) \sum_{i=1}^n a_i^2 \geq \left(\sum_{i=1}^n a_i \right)^2 \quad (d) \text{None of these}$$

Ans.(b)

Sol. Since, $a_1, a_2, \dots, a_n$ are positive real numbers.

Using the n th power mean equality,

$$\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n} \geq \left(\frac{a_1 + a_2 + \dots + a_n}{n} \right)^2$$

$$\Rightarrow \frac{n^2}{n} \left(\sum_{i=1}^n a_i^2 \right) \geq \left(\sum_{i=1}^n a_i \right)^2$$

$$\Rightarrow n \left(\sum_{i=1}^n a_i^2 \right) \geq \left(\sum_{i=1}^n a_i \right)^2$$

10. If $\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \phi$, then $9x^2 - 12xy \cos \phi + 4y^2$

is equal to **[2022]**

$$(a) -36 \sin^2 \phi \quad (b) 36 \sin^2 \phi$$

$$(c) 36 \cos^2 \phi \quad (d) 36$$

Ans.(b)

$$\text{Sol. Here, } \cos^{-1} \left(\frac{x}{2} \right) + \cos^{-1} \left(\frac{y}{3} \right) = \phi$$

Taking $\cos$ on both sides, we get

$$\Rightarrow \cos \left(\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} \right) = \cos \phi$$

$$\Rightarrow \cos \left\{ \cos^{-1} \left(\frac{x}{2} \right) \right\} \cos \left\{ \cos^{-1} \left(\frac{y}{3} \right) \right\}$$

$$= \sin \left\{ \cos^{-1} \left(\frac{x}{2} \right) \right\} \sin \left\{ \cos^{-1} \left(\frac{y}{3} \right) \right\} = \cos \phi$$

$$\Rightarrow \frac{xy}{6} - \sin \cos^{-1} \left(\frac{x}{2} \right) \sin \cos^{-1} \left(\frac{y}{3} \right) = \cos \phi$$

We know that, $\cos^{-1} x = \sin^{-1} \sqrt{1-x^2}$

$$\Rightarrow \frac{xy}{6} - \sin \left\{ \sin^{-1} \sqrt{1-\frac{x^2}{4}} \right\} \sin \left\{ \sin^{-1} \sqrt{1-\frac{y^2}{9}} \right\} = \cos \phi$$

$$\Rightarrow \frac{xy}{6} - \sqrt{1-\frac{x^2}{4}} \sqrt{1-\frac{y^2}{9}} = \cos \phi$$

$$\Rightarrow \frac{xy}{6} - \frac{\sqrt{4-x^2}}{2} \frac{\sqrt{9-y^2}}{3} = \cos \phi$$

$$\Rightarrow xy - \sqrt{4-x^2} \sqrt{9-y^2} = 6 \cos \phi$$

$$\Rightarrow -\sqrt{4-x^2} \sqrt{9-y^2} = 6 \cos \phi - xy$$

On squaring both sides, we get

$$\Rightarrow (4-x^2)(9-y^2) = 36 \cos^2 \phi + x^2 y^2 - 12xy \cos \phi$$

$$\Rightarrow 36 - 4y^2 - 9x^2 + x^2 y^2 = 36 \cos^2 \phi + x^2 y^2 - 12xy \cos \phi$$

$$\Rightarrow 4y^2 + 9x^2 - 12xy \cos \phi = -36 \cos^2 \phi + 36$$

$$\Rightarrow 4y^2 + 9x^2 - 12xy \cos \phi = 36(1 - \cos^2 \phi) = 36 \sin^2 \phi$$

$$\therefore 4y^2 + 9x^2 - 12xy \cos \phi = 36 \sin^2 \phi$$

11. If the roots of the quadratic equation

$x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$, respectively, then the value of $2 + p - q$ is **[2022]**

$$(a) 3 \quad (b) 0 \quad (c) 1 \quad (d) 2$$

Ans.(c)

Sol. We have, $x^2 + px + q = 0$

Sum of roots $= -p$

and product of roots $= q$

According to the question,

$$\tan 30^\circ + \tan 15^\circ = -p$$

$$\tan 30^\circ \cdot \tan 15^\circ = q$$

$$\therefore \tan(30^\circ + 15^\circ) = \frac{(\tan 30^\circ + \tan 15^\circ)}{1 - \tan 15^\circ \cdot \tan 30^\circ}$$

$$\Rightarrow \tan 45^\circ = \frac{-p}{1-q}$$

$$\Rightarrow 1 - q = -p \quad [\because \tan 45^\circ = 1]$$

$$\Rightarrow p - q + 1 = 0$$

$$\Rightarrow p - q + 1 + 1 = 0 + 1$$

$$\Rightarrow p - q + 2 = 1$$

12. If $\operatorname{cosec} \theta - \cot \theta = 2$. Find $\operatorname{cosec} \theta$ is **[2022]**

$$(a) \frac{5}{3} \quad (b) \frac{3}{5} \quad (c) \frac{4}{5} \quad (d) \frac{5}{4}$$

Ans.(d)

Sol. Given, $\operatorname{cosec} \theta - \cot \theta = 2$... (i)

$$\operatorname{cosec} \theta + \cot \theta = \frac{1}{2} \dots (ii)$$

$$[\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1]$$

Adding Eq. (i) into Eq. (ii), we get

$$2 \operatorname{cosec} \theta = 2 + \frac{1}{2}$$

$$\Rightarrow 2 \operatorname{cosec} \theta = \frac{5}{2}$$

$$\therefore \operatorname{cosec} \theta = \frac{5}{4}$$

13. Solution of the equation [2022]

$$\tan^{-1} \sqrt{x^2 + x} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2} \text{ are}$$

- (a) 0, 1 (b) 1, -1 (c) 0, -1 (d) 0, -2

Ans. (c)

$$\text{Sol. Here, } \tan^{-1} \sqrt{x^2 + x} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

From the given expression,

$$-1 \leq \sqrt{x^2 + x + 1} \leq 1 \text{ and } x^2 + x \geq 0$$

$$\Rightarrow 0 \leq x^2 + x + 1 \leq 1 \text{ and } x^2 + x + 1 \geq 1$$

$$\Rightarrow x^2 + x + 1 = 1$$

$$\Rightarrow x^2 + x = 0$$

$$\Rightarrow x(x+1) = 0$$

Hence, there will be two solutions.

One at $x = -1$ and another at $x = 0$

14. If $a < b$, then $\int_a^b (|x-a| + |x-b|) dx$, is equal to

[2022]

(a) $\frac{(b-a)^2}{2}$ (b) $\frac{(b^2 - a^2)}{2}$

(c) $\frac{(b^3 - a^3)}{2}$ (d) $(b-a)^2$

Ans. (d)

$$\text{Sol. Here, } \int_a^b (|x-a| + |x-b|) dx$$

$$= \int_a^b |x-a| dx + \int_a^b |x-b| dx$$

$$\text{So, } 0 < a < b, 0 < a \text{ and } a < b$$

$$= \int_a^b |x-a| dx + \int_b^a |x-b| dx$$

$$= \int_a^b x dx - \int_a^b a dx + \int_b^a x dx - \int_b^a b dx$$

$$= \left[\frac{x^2}{2} \right]_a^b - \left[ax \right]_a^b + \left[\frac{x^2}{2} \right]_b^a - \left[bx \right]_b^a$$

$$\begin{aligned} &= \frac{b^2 - a^2}{2} - [ab - a^2] + \frac{a^2}{2} - \frac{b^2}{2} - [ba - b^2] \\ &= \frac{b^2}{2} - \frac{a^2}{2} - ab + a^2 + \frac{a^2}{2} - \frac{b^2}{2} - ba + b^2 \\ &= -ab + a^2 - ba + b^2 \\ &= b^2 + a^2 - 2ab = (b-a)^2 \end{aligned}$$

15. Let $\mathbf{a} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\mathbf{b}$ be another vector such that $\mathbf{a} \cdot \mathbf{b} = 14$ and $\mathbf{a} \times \mathbf{b} = 3\hat{i} + \hat{j} - 8\hat{k}$ the vector $\mathbf{b} =$ [2022]

(a) $5\hat{i} + \hat{j} + 2\hat{k}$ (b) $5\hat{i} - \hat{j} - 2\hat{k}$

(c) $5\hat{i} + \hat{j} - 2\hat{k}$ (d) $3\hat{i} + \hat{j} + 4\hat{k}$

Ans. (a)

$$\text{Sol. Given, } \mathbf{a} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\text{Let } \mathbf{b} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\therefore \mathbf{a} \cdot \mathbf{b} = 14$$

$$\Rightarrow (2\hat{i} + 2\hat{j} + \hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 14$$

$$\Rightarrow 2x + 2y + z = 14 \quad (i)$$

$$\text{Now, } \mathbf{a} \times \mathbf{b} = (3\hat{i} + \hat{j} - 8\hat{k})$$

$$(2\hat{i} + 2\hat{j} + \hat{k}) \times (x\hat{i} + y\hat{j} + z\hat{k}) = 3\hat{i} + \hat{j} - 8\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ x & y & z \end{vmatrix} = 3\hat{i} + \hat{j} - 8\hat{k}$$

$$\Rightarrow \hat{i}(2z - y) - \hat{j}(2z - x) + \hat{k}(2y - 2x) = 3\hat{i} + \hat{j} - 8\hat{k}$$

On comparing with coefficients, we get

$$2z - y = 3 \dots (ii)$$

$$2z - x = -1 \dots (iii)$$

$$y - x = -4 \dots (iv)$$

Solving Eqs. (i), (ii), (iii), (iv), we get

$$x = 5, y = 1, z = 2$$

$$\Rightarrow \mathbf{b} = 5\hat{i} + \hat{j} + 2\hat{k}$$

16. Let a be the distance between lines $-2x + y = 2$ and $2x - y = 2$, and b be the distance between the lines $4x - 3y = 5$ and $6y - 8x = 1$, then [2022]

(a) $40b = 11\sqrt{5}a$ (b) $40\sqrt{2}a = 11b$

(c) $11\sqrt{2}b = 40a$ (d) $11\sqrt{2}a = 40b$

Ans. (a)

Sol. Given, equation of lines $-2x + y = 2$ and $2x - y = 2$

Since, slopes are the same, then lines are parallel. Similarly, we can find that the lines $4x - 3y = 5$ and $6y - 8x = 1$ are also parallel. Since, the slopes are the same.

We know that, the distance between two parallel lines,

$$\therefore d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$$

$$\text{So, } a = \frac{|2 - (-2)|}{\sqrt{(2)^2 + (1)^2}} = \frac{4}{\sqrt{5}}$$

$$b = \frac{\left|-\frac{5}{3} - \frac{1}{6}\right|}{\sqrt{(4/3)^2 + (1)^2}} = \frac{\left|-\frac{10-1}{6}\right|}{\left(\frac{\sqrt{16+9}}{3}\right)}$$

$$b = \frac{11}{2 \times 5} = \frac{11}{10}$$

$$\Rightarrow \frac{a}{b} = \frac{4 \times 10}{\sqrt{5} \times 11}$$

$$\Rightarrow 40b = 11\sqrt{5}a$$

17. The area enclosed within the curve $|x| + |y| = 2$ is [2022]

- (a) 16 sq units (b) 24 sq units
(c) 32 sq units (d) 8 sq units

Ans. (d)

Sol. Given curve, $|x| + |y| = 2$

Four cases arise from the given equation,

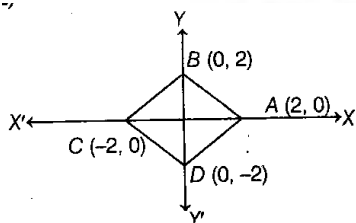
$$x + y = 2, \text{ when } x > 0 \text{ and } y > 0$$

$$-x + y = 2, \text{ when } x < 0 \text{ and } y > 0$$

$$-x - y = 2, \text{ when both } x, y < 0$$

$$x - y = 2 \text{ when } x > 0 \text{ and } y < 0$$

If you plot them you will get coordinates as $(2, 0)(0, 2)(-2, 0)(0, -2)$



A square of side equal to $2\sqrt{2}$ units.

Thus, area enclosed = Area of square

$$(2\sqrt{2})^2 = 8 \text{ sq unit}$$

18. In a harmonic progression, p th term is q and q th term is p . Then, pq th term is [2022]

- (a) 0 (b) 1 (c) pq (d) $pq(p+q)$

Ans. (b)

Sol. Let a be first term and d be common difference of an AP

$$t_p = q \Rightarrow \frac{1}{a + (p-1)d} = q$$

$$\Rightarrow a + (p-1)d = \frac{1}{q} \dots (i)$$

$$\text{Similarly, } t_q = p \Rightarrow \frac{1}{a + (q-1)d} = p$$

$$a + (q-1)d = \frac{1}{p} \dots (ii)$$

Subtracting Eq. (ii) from Eq. (i), we get

$$(p-q)d = \frac{1}{q} - \frac{1}{p}$$

$$\Rightarrow (p-q)d = \frac{p-q}{pq}$$

$$\Rightarrow d = \frac{1}{pq}$$

On putting value of d in Eq. (i), we get

$$a = \frac{1}{pq}$$

$$\text{Now, } t_{pq} = \frac{1}{a + (pq-1)d}$$

$$= \frac{1}{\frac{1}{pq} + (pq-1)\frac{1}{pq}}$$

$$t_{pq} = \frac{pq}{1 + pq - 1} = 1$$

19. Suppose that the temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant, walking on the plate, traverses a circle of radius 5 centred at the origin. What is the highest temperature encountered by the ant? [2022]

- (a) 125 (b) 120 (c) 0 (d) 25

Ans. (a)

Sol. Here, $T(x, y) = 4x^2 - 4xy + y^2$

and

$$g(x, y) = x^2 + y^2 - 25$$

Apply Lagrange multiplier. To apply the method we must define the Lagrange function.

$$F(x, y, z, \lambda) = 4x^2 - 4xy + y^2 - \lambda(x^2 + y^2 - 25)$$

To find the critical points of the Lagrange function all its first order partial derivatives must be zero.

$$F_x = 8x - 4y - 2x\lambda$$

$$F_y = 2y - 4x - 2y\lambda$$

$$F_\lambda = -(x^2 + y^2 - 25)$$

Apply the critical points conditions.

$$8x - 4y - 2x\lambda = 0, 2y - 4x - 2y\lambda = 0$$

and

$$-(x^2 + y^2 - 25) = 0$$

Solve for λ the first two equation

$$\lambda = \frac{8x - 4y}{2x}, \lambda = \frac{2y - 4x}{2y}$$

$$(4x - 2y)y = (y - 2x)x$$

$$(2x - y)(x + 2y) = 0$$

$$\Rightarrow y = 2x \text{ or } x = -2y$$

Substituting $y = 2x$ in equation $-(x^2 + y^2 - 25) = 0$

We get, $x = \pm\sqrt{5}$ and $y = \pm 2\sqrt{5}$

Hence, $(\sqrt{5}, 2\sqrt{5}), (-\sqrt{5}, -2\sqrt{5})$ is critical point of the function.

And substituting $x = -2y$ in equation

$$(-x^2 + y^2 + 25) = 0, \text{ we get,}$$

$$y = \pm\sqrt{5} \Rightarrow x = \pm 2\sqrt{5}$$

So, $(2\sqrt{5}, -\sqrt{5}), (-2\sqrt{5}, \sqrt{5})$ is critical point of the function.

$$\text{Now, } T(x, y) = 4x^2 - 4xy + y^2$$

$$T(\sqrt{5}, 2\sqrt{5}) = 0$$

$$T(-\sqrt{5}, -2\sqrt{5}) = 0$$

$$T(2\sqrt{5}, -\sqrt{5}) = 125$$

$$T(-2\sqrt{5}, \sqrt{5}) = 125$$

Hence, the highest temperature encountered by the ant is 125 units and lowest temperature is 0 units.

20. If $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1, (a > b)$ and $x^2 - y^2 = c^2$ cut at

right angles, then

[2022]

$$(a) a^2 + b^2 = 2c^2 \quad (b) b^2 - a^2 = 2c^2$$

$$(c) a^2 - b^2 = 2c^2 \quad (d) a^2 - b^2 = c^2$$

Ans. (c)

Sol. We can easily determine the coordinates of the intersection point of the curves.

The co-ordinate are

$$(x, y) = \left(\sqrt{\frac{a^2(b^2 + c^2)}{a^2 + b^2}}, \sqrt{\frac{(a^2 - c^2)b^2}{a^2 + b^2}} \right)$$

Since, the curve intersect at right angles, then their slopes at that point will be perpendicular.

That is, the product of slopes will be -1

Given,

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

On differentiating w.r.t x , we get:

$$\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right)_1 = \frac{-2x}{a^2} \times \frac{b^2}{2y} = \frac{-b^2x}{a^2y} \quad (i)$$

$$x^2 - y^2 = c^2$$

Similarly, on differentiating w.r.t to x , we get

$$2x - 2y \frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right)_2 = \frac{x}{y} \quad \dots(ii)$$

According to the question,

$$\Rightarrow \left(\frac{dy}{dx}\right)_1 \times \left(\frac{dy}{dx}\right)_2 = -1$$

$$\Rightarrow \frac{-b^2}{a^2} \times \frac{x}{y} \times \frac{x}{y} = -1$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{a^2}{b^2} \quad \dots(iii)$$

Substituting the points, we get

$$\Rightarrow \frac{b^2}{a^2} = \frac{b^2(a^2 - c^2)}{a^2(b^2 + c^2)} \Rightarrow b^2 + c^2 = a^2 - c^2$$

$$\therefore a^2 - b^2 = 2c^2$$

21. If $(\hat{\mathbf{a}} \times \hat{\mathbf{b}}) \times \hat{\mathbf{c}} = \hat{\mathbf{a}} \times (\hat{\mathbf{b}} \times \hat{\mathbf{c}})$, then [2022]

- (a) $\mathbf{a}$ and $\mathbf{c}$ are collinear
- (b) $\mathbf{a}$ and $\mathbf{c}$ are perpendicular
- (c) $\mathbf{a}$ and $\mathbf{c}$ are collinear
- (d) $\mathbf{a}$ and $\mathbf{c}$ are perpendicular

Ans. (c)

Sol. We know that,

$$\hat{\mathbf{a}} \times (\hat{\mathbf{b}} \times \hat{\mathbf{c}}) = (\hat{\mathbf{a}} \cdot \hat{\mathbf{c}}) \hat{\mathbf{b}} - (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}) \hat{\mathbf{c}}$$

$$\text{Here, } (\hat{\mathbf{a}} \times \hat{\mathbf{b}}) \times \hat{\mathbf{c}} = \hat{\mathbf{a}} \times (\hat{\mathbf{b}} \times \hat{\mathbf{c}})$$

$$\Rightarrow -\hat{c}(\hat{a} \times \hat{b}) = (\hat{a} \cdot \hat{c})\hat{b} - (\hat{a} \cdot \hat{b})\hat{c}$$

$$\Rightarrow (\hat{a} \cdot \hat{c})\hat{b} - (\hat{a} \cdot \hat{b})\hat{c} + \hat{c}(\hat{a} \times \hat{b}) = 0$$

$$\Rightarrow (\hat{a} \cdot \hat{c})\hat{b} - (\hat{a} \cdot \hat{b})\hat{c} + (\hat{c} \cdot \hat{b})\hat{a} - (\hat{c} \cdot \hat{a})\hat{b} = 0$$

$$\Rightarrow (\hat{c} \cdot \hat{b})\hat{a} = (\hat{a} \cdot \hat{b})\hat{c}$$

$$\Rightarrow \lambda \hat{a} = \mu \hat{c}$$

Hence, $\hat{a}$ and $\hat{c}$ are collinear.

22. Angle of elevation of the top of the tower from 3 points (collinear) A, B and C on a road leading to the foot of tower are $30^\circ, 45^\circ$ and 60° , respectively. The ratio of AB and BC is [2022]

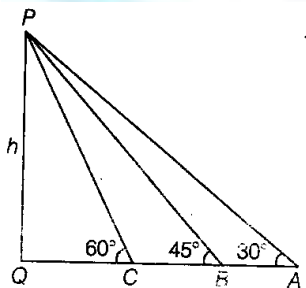
- (a) $\sqrt{3}:1$ (b) $\sqrt{3}:2$ (c) $1:2$ (d) $2:\sqrt{3}$

Ans. (a)

Sol. Here, A, B and C are collinear point.

In $\square PQA$,

$$\tan 30^\circ = \frac{PQ}{AQ} \Rightarrow AQ = \sqrt{3}h \dots (i)$$



$$\text{In } \square PQB, \tan 45^\circ = \frac{PQ}{BQ} \Rightarrow BQ = h \dots (ii)$$

$$\text{In } \square PQC, \tan 60^\circ = \frac{PQ}{CQ} \Rightarrow CQ = \frac{h}{\sqrt{3}} \dots (iii)$$

Now,

$$\frac{AB}{BC} = \frac{AQ - BQ}{BQ - CQ} = \frac{\sqrt{3}h - h}{h - \frac{h}{\sqrt{3}}}$$

$$\frac{AB}{BC} = \frac{\sqrt{3} - 1}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{1}$$

$$\therefore AB:BC = \sqrt{3}:1$$

23. Let a, b and c be distinct non-negative numbers. If

the vector $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane then c is equal to [2022]

(a) the arithmetic means of a and b

(b) the geometric mean of a and b

(c) the harmonic mean of a and b

(d) equal to zero

Ans. (b)

Sol. Given, vector $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar.

$$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$\Rightarrow -1(ab - c^2) - 1(ac - ac) = 0 \Rightarrow c^2 = ab$$

$$\therefore c = \sqrt{ab}$$

Hence, a, b and c are in GP

24. There are two sets A and B with $|A| = m$ and $|B| = n$. If $|P(A)| - |P(B)| = 112$, then choose the wrong option (where $|A|$ denotes the cardinality of A , and $P(A)$ denotes the power set of A) [2022]

(a) $m + n = 11$

(b) $2n - m = 1$

(c) $2m - n = 1$

(d) $3n - m = 5$

Ans. (c)

Sol. Given, $|P(A)| - |P(B)| = 112$

Where $P(A)$ and $P(B)$ denote the power set of A and B , respectively.

$$\therefore 2^m - 2^n = 112$$

$$\Rightarrow 2^n (2^{m-n} - 1) = 112$$

$$\Rightarrow 2^n (2^{m-n} - 1) = 2^4 \times 7$$

$$\Rightarrow 2^n (2^{m-n} - 1) = 2^4 (2^3 - 1)$$

$$\Rightarrow n = 4 \text{ and } m - n = 3 \Rightarrow m = 7$$

$$\therefore 2m - n = 2 \times 7 - 4 = 10$$

Hence, option (c) is wrong.

25. The mean of 25 observations was found to be 38. It was later discovered that 23 and 38 were misread as 25 and 36, then the mean is [2022]

(a) 32

(b) 36

(c) 38

(d) 12

Ans. (c)

Sol. Mean is the sum of the data values divided by the total number of values.

Given that, mean of 25 observations is 38.

Let the values be $X_1, X_2, \dots, X_{25}$

Given that, two values are 25 and 36.

Let $X_{24} = 25$ and $X_{25} = 36$

$$\text{Therefore, } \frac{X_1 + X_2 + \dots + X_{23} + 25 + 36}{25} = 38$$

$$\Rightarrow X_1 + X_2 + \dots + X_{23} = (38 \times 25) - (61)$$

$$\Rightarrow X_1 + X_2 + \dots + X_{23} = 950 - 61$$

$$\Rightarrow X_1 + X_2 + \dots + X_{23} = 889$$

Given that, 23 and 38 are missed as 25 and 36.

Therefore, the correct mean

$$= \frac{X_1 + X_2 + \dots + X_{23} + 23 + 38}{25}$$

$$\Rightarrow \text{Correct mean} = \frac{889 + 23 + 38}{25} = \frac{950}{25} = 38$$

26. There are two circles in xy -plane whose equations are $x^2 + y^2 - 2y = 0$ and $x^2 + y^2 - 2y - 3 = 0$. A point (x, y) is chosen at random inside the larger circle.

Then, the probability that the point has been taken from smaller circle is [2022]

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Ans. (d)

Sol. Given, equation

$$x^2 + y^2 - 2y = 0 \quad \dots(i)$$

$$x^2 + y^2 - 2y - 3 = 0 \quad \dots(ii)$$

and $x^2 + y^2 - 2y = 0$ [from Eq.(i)]

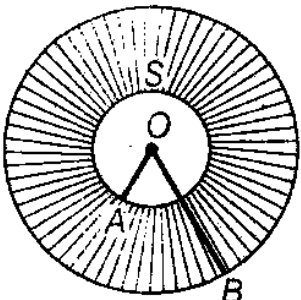
$$(x-0)^2 + y^2 - 2y + 1^2 - 1^2 = 0$$

$$\Rightarrow x^2 + (y-1)^2 = 1^2$$

$$\text{Now } x^2 + y^2 - 2y - 3 = 0$$

$$(x-0)^2 + (y-1)^2 = (2)^2$$

$$\text{Total region} = \pi(2)^2 = 4\pi$$



$$\text{Smaller region} = \pi(1)^2 = \pi$$

$$\therefore \text{Probability} = \frac{\pi}{4\pi} = \frac{1}{4}$$

27. The value of $\int \frac{(x^2-1)dx}{x^3\sqrt{2x^4-2x^2+1}}$ is [2022]

(a) $2\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c$ (b) $2\sqrt{2+\frac{2}{x^2}+\frac{1}{x^4}}+c$

(c) $\frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c$ (d) None of these

Ans. (c)

$$\text{Sol. Let } I = \int \frac{(x^2-1)}{x^3\sqrt{2x^4-2x^2+1}} dx$$

$$I = \frac{x^2-1}{x^5\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}} dx$$

$$I = \frac{\frac{1}{x^3}-\frac{1}{x^5}}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}} dx$$

$$\text{Let } 2-\frac{2}{x^2}+\frac{1}{x^4} = t$$

$$\left(0+\frac{4}{x^3}-\frac{4}{x^5}\right) dx = dt$$

$$\left(\frac{1}{x^3}-\frac{1}{x^5}\right) dx = \frac{dt}{4}$$

Now, substituting above value

$$\Rightarrow \int \frac{dt}{4\sqrt{t}} = \frac{1}{4} \int t^{-1/2} dt \Rightarrow \frac{1}{4} \frac{t^{-1/2+1}}{-1/2+1} + c$$

$$= \frac{1}{4} \times \frac{t^{1/2}}{1} \times 2 + c = \frac{1}{2} t^{1/2} + c$$

$$= \frac{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}{2} + c$$

$$= \frac{\sqrt{2x^4-2x^2+1}}{2x^2} + c$$

$$= \frac{1}{2} \sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}} + c$$

28. If $0 < P(A) < 1$ and $0 < P(B) < 1$, and

$P(A \cap B) = P(A) \cdot P(B)$, then [2022]

(a) $P(B|A) = P(B) - P(A)$

(b) $P(A^c - B^c) = P(A^c) - P(B^c)$

(c) $P(A \cup B)^c = P(A^c)P(B^c)$

(d) $P(A|B) = P(A) - P(B)$

Ans. (c)

Sol. Given, $P(A \cap B) = P(A) \cdot P(B)$

For option (a),

$$P(B|A) = \left(\frac{P(B \cap A)}{P(A)} \right) = \left(\frac{P(B) \cdot P(A)}{P(A)} \right)$$

$$P \cdot (B|A) = P(B)$$

It is not correct.

For option (b),

$$P(A^c - B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$$

Option (b) is not correct.

For option (c),

$$\begin{aligned} P(A \cup B)^c &= (1 - P(A \cup B)) \\ &= (1 - P(A) - P(B) + P(A)P(B)) \\ &= (1 - P(A))(1 - P(B)) \\ &= P(A^c)P(B^c) \end{aligned}$$

Hence, option (c) is correct answer.

29. Which of the following is not true? [2022]

- (a) $\lim_{x \rightarrow \infty} \frac{x}{e^x} = 0$ (b) $\lim_{x \rightarrow 0^+} \frac{1}{xe^x} = 0$
 (c) $\lim_{x \rightarrow 0^+} \frac{\sin x}{1 + 2x} = 0$ (d) $\lim_{x \rightarrow 0^+} \frac{\cos x}{1 + 2x} = 0$

Ans. (d)

Sol. For option (a) $\lim_{x \rightarrow \infty} \frac{x}{e^x}$

Let $x = \frac{1}{h}, x \rightarrow \infty, h \rightarrow 0$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{1}{e^{1/h}} = 0$$

For option (b), $\lim_{x \rightarrow 0^+} \frac{e^{1/x}}{1/x} = 0$

For option (c), $\lim_{x \rightarrow 0^+} \frac{\sin x}{1 + 2x} = \frac{\sin 0}{1 + 2 \times 0} = 0$

For option (d), $\lim_{x \rightarrow 0^+} \frac{\cos x}{1 + 2x} = \frac{1}{1 + 0} = 1$

Hence, option (d) is not true.

30. A four digit number is formed using the digits 1, 2, 3, 4 and 5 without repetition. The probability that it is divisible by 3 is [2022]

- (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$

Ans. (c)

Sol. A number divisible by 3 formed using the digits 1, 2, 3, 4 and 5 would not contain 4 only formed by 1, 2, 3 and 5.

$\therefore$ Four digit number formed by 1, 2, 3 and 5 which divisible by 3 = $4! = 24$

Total number of 4 digit numbers that can be formed using the digits, 1, 2, 3, 4 and 5 without repetition = $5! = 120$

$\therefore$ Required probability = $\frac{24}{120} = \frac{1}{5}$

31. The correct expression for $\cos^{-1}(-x)$ is [2022]

- (a) $\frac{\pi}{2} - \cos^{-1}x$ (b) $\pi - \cos^{-1}x$
 (c) $\pi + \cos^{-1}x$ (d) $\frac{\pi}{2} + \cos^{-1}x$

Ans. (b)

Sol. Here, $\cos^{-1}(-x)$

Let $y = \cos^{-1}(-x)$

$$\Rightarrow \cos y = -x$$

$$\Rightarrow -\cos y = x$$

As we know,

$$\Rightarrow \cos(\pi - y) = x$$

$$\Rightarrow \pi - y = \cos^{-1}x$$

$$\Rightarrow \pi - \cos^{-1}x = y$$

Hence, $y = \cos^{-1}(-x) = \pi - \cos^{-1}x$

32. If α and β are the roots of $x^2 - x - 1 = 0$, and $A_n = \alpha^n + \beta^n$, then arithmetic mean of A_{n-1} and A_n is [2022]

- (a) $2A_n - 1$ (b) $\frac{1}{2}A_{n+1}$
 (c) $2A_n - 2$ (d) None of these

Ans. (b)

Sol. Let α, β be the root of $x^2 - x - 1 = 0$

Then, $\alpha + \beta = 1, \alpha \cdot \beta = -1$

$$\text{or } \frac{1}{\alpha} + \frac{1}{\beta} = -1 \quad \dots (i)$$

Now, $A_n = \alpha^n + \beta^n, A_{n+1} = \alpha^{n+1} + \beta^{n+1}$

$$A_{n-1} = \alpha^{n-1} + \beta^{n-1}$$

The arithmetic mean of A_{n-1} and A_n is

$$\begin{aligned} \frac{A_{n-1} + A_n}{2} &= \frac{\alpha^{n-1} + \beta^{n-1} + \alpha^n + \beta^n}{2} \\ &= \frac{\alpha^n \left(1 + \frac{1}{\alpha}\right) + \beta^n \left(1 + \frac{1}{\beta}\right)}{2} \\ &= \frac{\alpha^n \left(\frac{-1}{\beta}\right) + \beta^n \left(\frac{-1}{\alpha}\right)}{2} \\ &= -\frac{(\alpha^{n+1} + \beta^{n+1})}{2\alpha \cdot \beta} \\ &= \frac{-(\alpha^{n+1} + \beta^{n+1})}{2(-1)} \\ &= \frac{\alpha^{n+1} + \beta^{n+1}}{2} = \frac{1}{2} \times A_{n+1} \end{aligned}$$

33. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2+x & 1 \\ 1 & 1 & 2+y \end{vmatrix}$, $x \neq 0, y \neq 0$, then D is

[2022]

- (a) divisible by x and y
 (b) divisible by x but not by y
 (c) divisible by $(x+1)$ and $(y+1)$
 (d) divisible by $(1+x)$ but not $(1+y)$

Ans.(c)

Sol. Here, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2+x & 1 \\ 1 & 1 & 2+y \end{vmatrix}$

$$C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3$$

$$D = \begin{vmatrix} 0 & 0 & 1 \\ -x-1 & 1+x & 1 \\ 0 & -1-y & 2+y \end{vmatrix}$$

$$= (-x-1)(-1-y)$$

$$= -(1+x) \times -(1+y)$$

$$= (1+x)(1+y)$$

Hence, D is divisible by $(x+1)$ and $(y+1)$.

34. In a $\triangle ABC$, if the tangent of half the difference of two angles is equal to one third of the tangent of the sum of the angles, then the ratio of the sides opposite to the angle is [2022]

- (a) 2:1 (b) 1:2 (c) 3:1 (d) 1:1

Ans.(a)

Sol. Given, $\tan\left(\frac{(A-B)}{2}\right) = \frac{1}{3} \tan\left(\frac{(A+B)}{2}\right)$

Since, $A+B+C = \pi$

$$\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\Rightarrow \frac{1}{3} \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$= \frac{1}{3} \cot(C/2)$$

Now $\frac{a-b}{a+b} \cot(C/2) = \frac{1}{3} \cot(C/2)$

$$\Rightarrow \frac{a-b}{a+b} = \frac{1}{3} \quad [\text{by Napier Analog y}]$$

$$\Rightarrow 3a - 3b = a + b$$

$$\Rightarrow 2a = 4b \text{ or } a = 2b$$

$$\Rightarrow \frac{a}{b} = \frac{2}{1}$$

$$\therefore a:b = 2:1$$

35. If the volume of a parallelopiped whose adjacent edges are

$\mathbf{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\mathbf{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}$, $\mathbf{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then α is equal to [2022]

- (a) 1 (b) $\frac{5}{2}$ (c) $\frac{9}{2}$ (d) 0

Ans.(c)

Sol. The sides of the parallelopiped are

$$\mathbf{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\mathbf{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}$$

$$\mathbf{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$$

$$\therefore \text{Volume} = [\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = \begin{vmatrix} 2 & 3 & 4 \\ 1 & \alpha & 2 \\ 1 & 2 & \alpha \end{vmatrix} = 15$$

$$\Rightarrow 2(a^2 - 4) + 3(2 - a) + 4(2 - a) = 15$$

$$\Rightarrow 2a^2 - 8 + 6 - 3a + 8 - 4a = 15$$

$$\Rightarrow 2a^2 - 7a - 9 = 0$$

$$\Rightarrow 2a^2 - 9a + 2a - 9 = 0$$

$$\Rightarrow a(2a - 9) + (2a - 9) = 0$$

$$\Rightarrow (2a - 9)(a + 1) = 0$$

$$2a - 9 = 0 \text{ or } a + 1 = 0$$

$$\therefore a = 9/2 \text{ or } a = -1$$

36. The first three moments of a distribution about 2 are 1, 16, 40, respectively. Then, mean and variance of the distribution are [2022]

- (a) (2, 16) (b) (2, 15) (c) (3, 15) (d) (1, 16)

Ans.(c)

Sol. Given, $\mu_1 = \frac{(\sum_{i=1}^n x_i - 2)}{n} = 1 \quad \dots(i)$

$$\mu_2 = \frac{\sum_{i=1}^n (x_i - 2)^2}{n} = 16 \dots(ii)$$

$$\mu_3 = \frac{\sum_{i=1}^n (x_i - 2)^3}{n} = 40 \quad \dots(iii)$$

From (i), $\frac{\sum x_i}{n} - 2 = 1$

$$\therefore \text{Mean} = 1+2=3 \left[\because \text{Mean} = \frac{\sum x_i}{n} \right]$$

$$\text{Again } \mu_2 = \frac{\sum_{i=1}^n (x_i - 2)^2}{n} = 16$$

$$\Rightarrow \frac{\sum_{i=1}^n x_i^2 - 4 \sum_{i=1}^n x_i + 4n}{n} = 16$$

$$\Rightarrow \frac{1}{n} \sum_{i=1}^n x_i^2 - 4\bar{x} + 4 = 16$$

$$\Rightarrow \frac{1}{n} \sum_{i=1}^n x_i^2 = 24$$

Hence, variance of x

$$\frac{1}{n} \sum_{i=1}^n x_i^2 - (\bar{x})^2 = (24) - (3)^2 = 15$$

37. Inverse of the function $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is [2022]

(a) $\log_{10}(2-x)$ (b) $\frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$

(c) $\frac{1}{2} \log_{10}(2x-1)$ (d) $\frac{1}{4} \log_{10} \left(\frac{2x}{2-x} \right)$

Ans.(b)

Sol. Let $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} = y$

$$\frac{10^x - 10^{-x}}{10^x + 10^{-x}} = y$$

$$\Rightarrow \frac{10^{2x} - 1}{10^{2x} + 1} = y$$

$$\Rightarrow 10^{2x} = \frac{1+y}{1-y} \text{ (By componendo and dividendo)}$$

$$\Rightarrow x = \frac{1}{2} \log_{10} \left(\frac{1+y}{1-y} \right)$$

$$\Rightarrow f^{-1}(y) = \frac{1}{2} \log_{10} \left(\frac{1+y}{1-y} \right)$$

$$\therefore f^{-1}(x) = \frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$$

38. Coordinate of focus of the parabola

$$4y^2 + 12x - 20y + 67 = 0 \text{ is}$$

[2022]

(a) $\left(-\frac{5}{4}, \frac{17}{4}\right)$ (b) $\left(-\frac{17}{2}, \frac{5}{4}\right)$

(c) $\left(-\frac{17}{4}, \frac{5}{2}\right)$ (d) $\left(-\frac{5}{2}, \frac{17}{4}\right)$

Ans.(c)

Sol. Given, parabola $4y^2 + 12x - 20y + 67 = 0$

$$4y^2 - 20y = -12x - 67$$

$$\Rightarrow \left(y - \frac{5}{2}\right)^2 - \frac{25}{4} = -3x - \frac{67}{4}$$

$$\Rightarrow \left(y - \frac{5}{2}\right)^2 = -3x - \frac{42}{4}$$

$$\Rightarrow \left(y - \frac{5}{2}\right)^2 = -3\left(x + \frac{7}{2}\right)$$

$$\therefore h = -\frac{7}{2}, k = \frac{5}{2}, a = \frac{1}{3}$$

$$\text{Vertex } A(h, k) = \left[-\frac{7}{2}, \frac{5}{2}\right]$$

$$\text{Focus is } \left(h + \frac{1}{4a}, k\right) = \left(-\frac{7}{2} - \frac{3}{4}, \frac{5}{2}\right) = \left(-\frac{17}{4}, \frac{5}{2}\right)$$

39. Which term of the series $\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{4}, \frac{1}{\sqrt{5}}, \dots$ is $\frac{\sqrt{5}}{13}$?

[2022]

(a) 12 (b) 11 (c) 10 (d) 9

Ans.(b)

Sol. We have, $t_1 = \frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{\sqrt{9}}$

$$t_2 = \frac{\sqrt{5}}{4} = \frac{\sqrt{5}}{\sqrt{16}}$$

$$t_3 = \frac{\sqrt{5}}{5} = \frac{\sqrt{5}}{\sqrt{25}} = \frac{1}{\sqrt{5}}$$

$$t_r = \frac{\sqrt{5}}{r+2}$$

$$\text{So, } \begin{matrix} r+2 & = 13 \\ r & = 13-2 = 11 \end{matrix}$$

40. A particle is at rest at the origin. It moves along the X -axis with an acceleration $x - x^2$, where x is the distance of the particle at time t . The particle next comes to rest after it has covered a distance. [2022]

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2

Ans.(c)

Sol. Given, acceleration $a = x - x^2$

$$v = 0, \text{ initial velocity } (u) = 0$$

We know that

$$a = \frac{dv}{dt} = \frac{d^2v}{dt^2} = \frac{d}{dt} \left(\frac{dv}{dt} \right)$$

$$\Rightarrow \frac{dv}{dt} = x - x^2$$

$$\Rightarrow \frac{dv}{dx} \cdot \frac{dx}{dt} = x - x^2$$

$$\Rightarrow \frac{dv}{dx} \cdot v = x - x^2$$

$$\Rightarrow v dv = (x - x^2) dx$$

Integrate on both sides, we get

$$\int_0^v v \cdot dv = \int_0^x (x - x^2) dx$$

$$\left[\frac{v^2}{2} \right]_0^v = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^x$$

$$\frac{v^2}{2} = \frac{x^2}{2} - \frac{x^3}{3}$$

$$\frac{v^2}{2} = \frac{3x^2 - 2x^3}{6}$$

According to the question, final velocity = 0

$$\therefore 0 = \frac{3x^2 - 2x^3}{6}$$

$$\Rightarrow x^2(3 - 2x) = 0$$

$$x = 0 \text{ or } x = \frac{3}{2}$$

Hence, particle next comes after it has covered a distance is $\frac{3}{2}$.

41. The function $f(x) = \log(x + \sqrt{x^2 + 1})$ is [2022]

- (a) an even function
- (b) an odd function
- (c) a periodic function
- (d) neither an even nor an odd function

Ans.(b)

Sol. Given that, $f(x) = \log(x + \sqrt{x^2 + 1})$

Now,

$$f(-x) = \log(-x + \sqrt{x^2 + 1})$$

$$f(x) + f(-x) = \log(x + \sqrt{x^2 + 1}) + \log(\sqrt{x^2 + 1} - x)$$

$$f(x) + f(-x) = \log((x^2 + 1) - x^2) = \log 1 = 0$$

Hence, $f(x)$ is an odd function.

42. The domain of the function $f(x) = \frac{\cos^{-1}x}{[x]}$ is

[2022]

- (a) $[-1, 0) \cup \{1\}$
- (b) $[-1, 1]$
- (c) $[-1, 1)$
- (d) None of these

Ans.(a)

Sol. Given, $f(x) = \frac{\cos^{-1}x}{[x]}$

The domain of $f(x)$ is equivalent to the range of value of x for which $f(x)$ exists.

Let $f(x) = \theta$, we get $x = \cos \theta$

The range of $\cos \theta$ is $[-1, 1]$

The domain of $\cos^{-1}x$ is $[-1, 1]$

Now, $[x] \neq 0$

So, domain of $[x] = R - (0, 1)$

Hence, the domain of $f(x)$ is $[-1, 0) \cup \{1\}$

43. A survey is done among a population of 200 people who like either tea or coffee. It is found that 60% of the population like tea and 72% of the population like coffee. Let x be the number of people who like both tea and coffee. Let $m \leq x \leq n$, then choose then correct option. [2022]

- (a) $n - m = 56$
- (b) $n - m = 28$
- (c) $n - m = 32$
- (d) $n + m = 92$

Ans.(a)

Sol. Let x be the number of people who like tea and coffee, T denote the set of people who like tea and C denote the set of people who like coffee.

$$n(C \cup T) = 200, n(T) = \frac{60}{100} \times 200 = 120$$

$$n(C) = \frac{72}{100} \times 200 = 144$$

and $n(T \cap C) = x$

We know that,

$$n(T \cap C) \leq n(T)$$

$$n(T) + n(C) = 120 + 144 = 264$$

$$\therefore n(C \cup T) = n(C) + n(T) - n(T \cap C)$$

$$n(T \cap C) = 264 - 200 = 64$$

$$\therefore 64 \leq x \leq 120$$

Here, $m = 64$ and $n = 120$

$$\therefore n - m = 120 - 64 = 56$$

44. The solutions of the equation $4\cos^2x + 6\sin^2x = 5$ are [2022]

$$(a) x = n\pi \pm \frac{\pi}{4}$$

$$(b) x = n\pi \pm \frac{\pi}{3}$$

$$(c) x = n\pi \pm \frac{\pi}{2}$$

$$(d) x = n\pi \pm \frac{2\pi}{3}$$

Ans.(a)

Sol. Consider given the trigonometry equation,

$$4\cos^2 x + 6\sin^2 x = 5$$

$$\Rightarrow 4(1 - \sin^2 x) + 6\sin^2 x = 5$$

$$\Rightarrow 2\sin^2 x + 4 = 5$$

$$\Rightarrow 2\sin^2 x = 1$$

$$\Rightarrow \sin x = \frac{1}{\sqrt{2}} \text{ or } \sin x = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow x = \frac{\pi}{4} \text{ or } \frac{-\pi}{4}$$

$$x = n\pi + (-1)^n \frac{\pi}{4} \text{ or } -$$

$$x = n\pi - (-1)^n \frac{\pi}{4}$$

$$\text{So, } x = n\pi \pm (-1)^n \frac{\pi}{4}$$

$$x = n\pi \pm \frac{\pi}{4} \quad [n \text{ is even number}].$$

45. $f(x) = x + |x|$ is continuous for **[2022]**

$$(a) x \in (-\infty, \infty)$$

$$(b) x \in (-\infty, \infty) - \{0\}$$

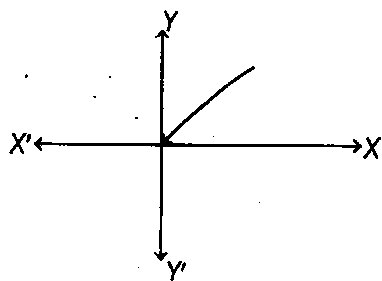
$$(c) \text{ only } x > 0$$

$$(d) \text{ no value of } x$$

Ans.(a)

Sol. Given, $f(x) = x + |x|$

$$\therefore f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



It is clear from the graph $f(x)$ is continuous for every value of x .

46. The 10 th and 50 th percentiles of the observations 32, 49, 23, 29 and 118, respectively are **[2022]**

$$(a) 21, 32 \quad (b) 23, 32 \quad (c) 23, 33 \quad (d) 22, 31$$

Ans.(b)

Sol. Arrange in increasing order

23, 29, 32, 49, 118

$\therefore P_i = \frac{i(n+1)}{100}$, where n is number of term.

$$P_{50} = \frac{50(5+1)}{100} = 3 \text{ th term}$$

Hence, 50 th percentile of the observation is 32.

$$\text{Now, } P_{10} = \frac{10(5+1)}{100} = \frac{6}{10} = 1 \text{ st (Approx term)}$$

Hence, 10th percentile of the observation is 23.

47. For $a \in R$ (the set of all real numbers), $a \neq -1$,

$$\lim_{n \rightarrow \infty} \frac{(1^a + 2^a + \dots + n^a)}{(n+1)^{a-1} [(na+1) + (na+2) + \dots + (na+n)]} = \frac{1}{60}.$$

Then, one of the values of a is **[2022]**

$$(a) 5 \quad (b) 8 \quad (c) -\frac{15}{2} \quad (d) -\frac{17}{2}$$

Ans.(d)

Sol. Given,

$$\lim_{n \rightarrow \infty} \frac{(1^a + 2^a + \dots + n^a)}{(n+1)^{a-1} [(na+1) + (na+2) + \dots + (na+n)]} = \frac{1}{60}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{\sum_{r=1}^n r^a}{(n+1)^{a-1} \left[n^2 a + \frac{n(n+1)}{2} \right]}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{2 \sum_{r=1}^n \left(\frac{r}{n} \right)^a}{\left(1 + \frac{1}{n} \right)^{a-1} (2na + n + 1)}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \left[2 \sum_{r=1}^n \left(\frac{r}{n} \right)^a \times \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{1}{n} \right)^{a-1} \left(2a + 1 + \frac{1}{n} \right)} \right]$$

$$\Rightarrow 2 \int_0^1 x^a dx \times \frac{1}{1(2a+1)} = \frac{2}{(2a+1)(a+1)}$$

$$\therefore \frac{2}{(2a+1)(a+1)} = \frac{1}{60}$$

$$\Rightarrow 120 = (2a+1)(a+1)$$

$$\Rightarrow 120 = 2a^2 + 3a + 1$$

$$\Rightarrow 2a^2 + 3a - 119 = 0$$

$$\Rightarrow (2a+17)(a-7) = 0$$

$$\Rightarrow a = \frac{-17}{2}, 7$$

48. The eccentricity of an ellipse, with its centre at the origin is $\frac{1}{3}$. If one of the directrix is $x = 9$, then the equation of ellipse is [2022]

- (a) $9x^2 + 8y^2 = 72$ (b) $8x^2 + 9y^2 = 72$
(c) $8x^2 + 7y^2 = 56$ (d) $7x^2 + 8y^2 = 56$

Ans.(b)

Sol. Let the equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here, $a > b$ because the directrix is parallel to y -axis. $b^2 = a^2(1 - e^2)$

$$\text{Given, } e = \frac{1}{3}$$

$$\Rightarrow b^2 = a^2 \left(1 - \frac{1}{9}\right) \Rightarrow \frac{8a^2}{9}$$

$$\text{But } ae = \frac{1}{9}$$

$$\Rightarrow a \times \frac{1}{3} = \frac{1}{9}$$

$$\Rightarrow a = 3$$

Putting $a = 3$, we get $b = 2\sqrt{2}$

Required ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

$$\therefore 8x^2 + 9y^2 = 72$$

49. If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ and the

hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ are coincide, then the value

of b^2 is : [2022]

- (a) 25 (b) 16 (c) 64 (d) 49

Ans.(b)

Sol. Given hyperbola, $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$\Rightarrow \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$$

$$a^2 = \frac{144}{25}, b^2 = \frac{81}{25}$$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{81}{144}} = \frac{15}{12}$$

$\therefore$ Foci of hyperbola are $(\pm ae, 0)$ i.e., $(\pm 3, 0)$

Now, given ellipse is $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$

$$\Rightarrow a^2 = 25$$

Assume eccentricity of this ellipse is e then its foci are $(\pm ae, 0), (\pm 5e, 0)$

Given, foci of given hyperbola and ellipse are coincide.

$$\Rightarrow 5e = 3 \Rightarrow e = \frac{3}{5}$$

For ellipse, using eccentricity relationship

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{9}{25} = 1 - \frac{b^2}{25}$$

$$\Rightarrow 9 = 25 - b^2$$

$$\Rightarrow b^2 = 16$$

50. If the angle of elevation of the top of a hill from each of the vertices A, B and C of a horizontal is α , then the height of the hill is [2022]

(a) $\frac{1}{2} b \tan \alpha \sec B$ (b) $\frac{1}{2} b \tan \alpha \operatorname{cosec} A$

(c) $\frac{1}{2} c \tan \alpha \sin C$ (d) $\frac{1}{2} a \tan \alpha \operatorname{cosec} A$

Ans.(d)

Sol. Let $R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C}$

$$\text{So, } \tan \alpha = \frac{h}{R}$$

$$\Rightarrow R \tan \alpha = h$$

$$\Rightarrow h = \tan \alpha \cdot \frac{a}{2 \sin A}$$

$$\Rightarrow h = \frac{1}{2} a \tan \alpha \operatorname{cosec} A$$

Analytical Ability & Logical Reasoning

51. Select the pair of words, which are related in the same way as the capitalised word are related to each other. [2022]

Frugal : Extravagant

- (a) Predecessor: Precursor
(b) Criticise: Advocate
(c) Teacher: Philanthropist
(d) Hermit: Philosopher

Ans.(b)

Sol. As, Frugal and Extravagant are antonyms of each other. Similarly, Criticise and Advocate are antonyms of each other.

52. Fill in the blank: HEC, JGE, LIG, NKI, ____ .

[2022]

- (a) PMK (b) HGF (c) KMP (d) ONM

Ans.(a)

Sol. Pattern of the series is as follows,

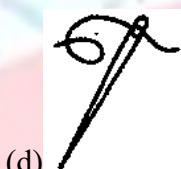
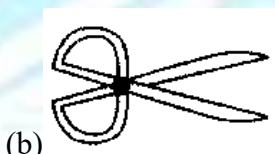
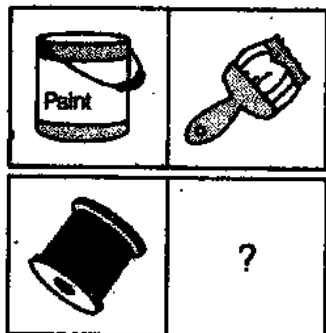
$$H \xrightarrow{+2} J \xrightarrow{+2} L \xrightarrow{+2} N \xrightarrow{+2} P$$

$$E \xrightarrow{+2} G \xrightarrow{+2} I \xrightarrow{+2} K \xrightarrow{+2} M$$

$$C \xrightarrow{+2} E \xrightarrow{+2} G \xrightarrow{+2} I \xrightarrow{+2} K$$

$\therefore ? = PMK$

53. Replace the question mark with an appropriate image to complete the analogous pair. [2022]



Ans.(d)

Sol. As, a can of paint is to be used by paint brush. Similarly, a spool of thread is to be used by sewing needle. Both show the tool needed to perform a task.

54. Deepak, Rahul, Manoj and Vinod are brothers. Who is the heaviest?

I. Rahul is heavier than Deepak and Vinod, but lighter than Manoj.
II. Deepak is lighter than Rahul and Manoj, but heavier than Vinod. [2022]

- (a) Data in both the statements together are not sufficient
(b) Statement II alone is sufficient, but Statement I alone is not sufficient
(c) Either I or II is sufficient
(d) Statement I alone is sufficient, but Statement II alone is not sufficient

Ans.(d)

Sol. From Statement I, Manoj > Rahul > Deepak and Vinod From Statement II, Rahul and Manoj > Deepak > Vinod Hence, Statement I alone is sufficient, but Statement II alone is not sufficient.

55. Select the related word from the given alternatives.

MELT: LIQUID :: FREEZE : ? [2022]

- (a) PUSH (b) CONDENSE
(c) ICE (d) SOLID

Ans.(d)

Sol. As, on melting, liquid is formed. Similarly on freezing solid is formed.

56. Identify the fifth number in the series: 122, 144, 166, 188, ? [2022]

- (a) 210 (b) 310 (c) 345 (d) 234

Ans.(a)

Sol. Pattern of the series is as follows,

$$\begin{array}{ccccccc} 122 & & 144 & & 166 & & 188 & & \boxed{210} \\ & \swarrow & \uparrow & \swarrow & \uparrow & \swarrow & \uparrow & \swarrow & \uparrow \\ & +22 & & +22 & & +22 & & +22 & \end{array}$$

$\therefore ? = 210$

57. Today is Wednesday. What would be the day after 61 days? [2022]

- (a) Sunday (b) Monday
(c) Tuesday (d) Saturday

Ans.(b)

Sol. Given, today is Wednesday. Everyday repeats after 7 days, hence Wednesday will repeat after every 7 days. Now, $61 \text{ days} \div 7 = 8 \text{ weeks } 5 \text{ odd days}$. So, the 5th day after Wednesday is Monday. Hence, Monday is the correct answer.

58. Fill in the blank: JAK, KBL, LCM, MDN, _____ [2022]

- (a) NEO (b) OEP (c) MEN (d) PEQ

Ans.(a)

Sol. Pattern of the series is as follows,

$$\begin{array}{ccccccc} J & \xrightarrow{+1} & K & \xrightarrow{+1} & L & \xrightarrow{+1} & M & \xrightarrow{+1} & \boxed{N} \\ A & \xrightarrow{+1} & B & \xrightarrow{+1} & C & \xrightarrow{+1} & D & \xrightarrow{+1} & E \\ K & \xrightarrow{+1} & L & \xrightarrow{+1} & M & \xrightarrow{+1} & N & \xrightarrow{+1} & \boxed{O} \end{array}$$

$\therefore ? = NEO$

59. DNN, FQQ, HTT, _____ LZZ [2022]

- (a) JWW (b) IWW (c) JVV (d) JXX

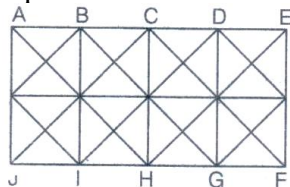
Ans.(a)

Sol. Pattern of the series is as follows;

$$\begin{array}{ccccccc} D & \xrightarrow{+2} & F & \xrightarrow{+2} & H & \xrightarrow{+2} & \boxed{J} & \xrightarrow{+2} & L \\ N & \xrightarrow{+3} & Q & \xrightarrow{+3} & T & \xrightarrow{+3} & \boxed{W} & \xrightarrow{+3} & Z \\ N & \xrightarrow{+3} & Q & \xrightarrow{+3} & T & \xrightarrow{+3} & \boxed{W} & \xrightarrow{+3} & Z \end{array}$$

$\therefore ? = JWW$

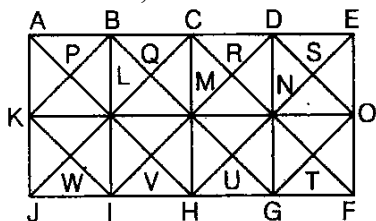
60. In the following figure, find the total number of squares. [2022]



- (a) 24 (b) 20 (c) 36 (d) 18

Ans.(a)

Sol. Given,



Hence, total number of squares = 24 (

□ABKL, □BCML, □CDMN, □DENO, □KLJI, □LMHI, □MNGH, □NOFG, □BQLP, □KPLW, □LWIV, □LQMV, □QCMR, □MRNU, □MVUH, □RDSN, □NSOT, □NTUG, □BKMI, □CLNH, □DMOG, □ACJH, □BDGI and □CEFH)

61. Find the synonym that is most nearly similar in meaning to the word: DEBACLE [2022]

- (a) Catastrophe (b) Dandy
(c) Opulence (d) Corker

Ans.(a)

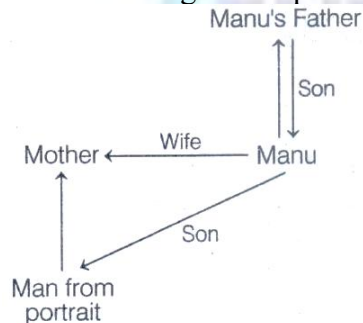
Sol. 'Catastrophe' is most nearly similar in meaning to the word 'DEBACLE'.

62. Looking at the portrait of a man, Manu said, "Her mother is the wife of my father's son and I have no brother and sister." Whose portrait was Manu looking at? [2022]

- (a) His son (b) His daughter
(c) His nephew (d) His father

Ans.(a)

Sol. According to the question,



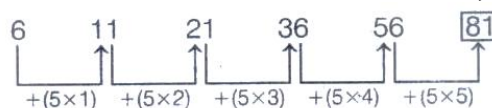
Hence, Manu looking at his son's portrait.

63. Identify the sixth number in the series: 6, 11, 21, 36, 56, ? [2022]

- (a) 82 (b) 21 (c) 81 (d) 52

Ans.(c)

Sol. Pattern of the series is as follows,



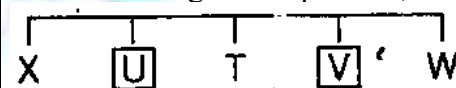
∴ ? = 81

64. U, V, W, X and T are sitting on a bench. T is sitting next to U, V is sitting next to W, W is not sitting with X who is on the left end of the bench. V is in the second position from the right. T is to the right of U and X. T and V are sitting together. In which position T is sitting? [2022]

- (a) Between U and V (b) Between X and W
(c) Between V and X (d) Between U and W

Ans.(a)

Sol. According to the question,



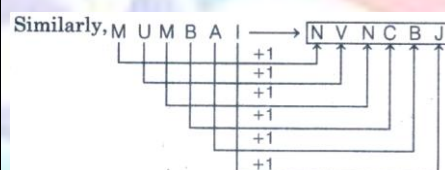
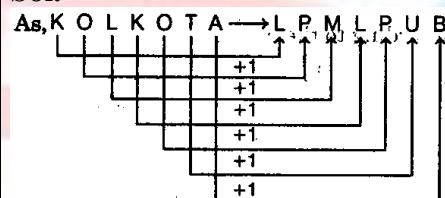
Hence, T is sitting between U and V.

65. If in a certain language, KOLKOTA is coded as LPMLPUB, how is MUMBAI coded in that code? [2022]

- (a) NVNBJCJ (b) OVNBHH
(c) NUNBCH (d) NVNCBJ

Ans.(d)

Sol.



66. Select the pair of words, which are related in the same way as the capitalised words are related to each other.

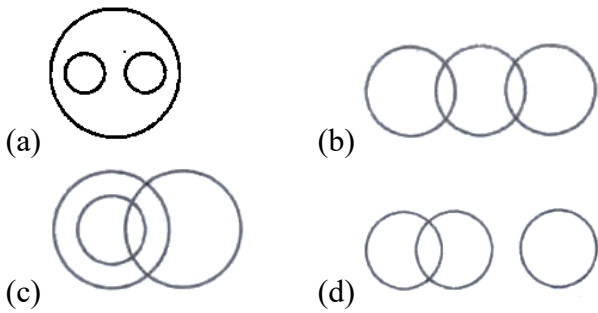
BUTTERFLY : FREEDOM [2022]

- (a) Frog : Water (b) Self-reliant : Buoyant
(c) Horse : Speed (d) Chicken : Rooster

Ans.(b)

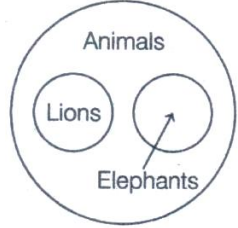
Sol. As, Butterfly and Freedom are inter-related. Similarly, Self-reliant and Buoyant are inter-related.

67. Which of the following diagrams correctly represents lions, elephants and animals? [2022]



Ans.(a)

Sol. According to the question,



68. Choose the word opposite in meaning to the given word: MITIGATE [2022]

- (a) Abate (b) Tranquelize
(c) Intensify (d) Alleviate

Ans.(d)

Sol. 'Alleviate' is opposite in meaning to the given word 'MITIGATE'.

69. Which of the following is to odd one from the given alternatives? [2022]

- (a) Diving (b) Driving
(c) Swimming (d) Sailing

Ans.(b)

Sol. Except 'Driving', all other are related to water, while 'Driving' is related to road.

Directions : (Q. Nos. 70-72) In each question below are given two statements followed by two conclusions numbered I and II. You have to take the given two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusion and then decide which of the following given conclusions logically follows from the two given statements, disregarding commonly known facts.

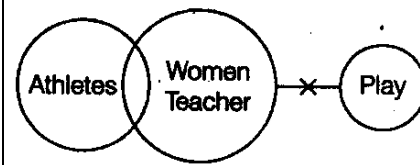
70. Statements No women teacher play. Some women teachers are athletes. [2022]

Conclusions

- I. Male athletes can play.
II. Some athletes can play.
(a) Neither I nor II follows
(b) Only Conclusion I follows
(c) Either I or II follows
(d) Only Conclusion II follows

Ans.(a)

Sol. According to the given statements,



I. (x)

II. (x)

Hence, neither Conclusion I nor II follows.

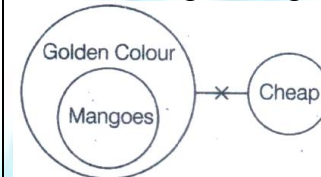
71. Statements All mangoes are golden in colour. No golden coloured things are cheap.

Conclusions

- I. All mangoes are cheap.
II. Golden-coloured mangoes are not chap. [2022]
(a) Neither I or II follows
(b) Only Conclusion II follows
(c) Only Conclusion I follows
(d) Either I or II follow

Ans.(b)

Sol. According to the given statements,



I. (x)

II. (✓)

Hence, only Conclusion II follows.

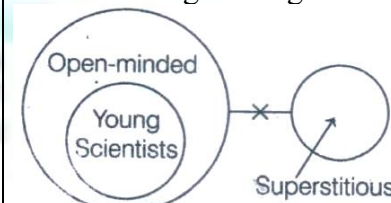
72. Statements All young scientists are open-minded. No open-minded men are superstitious.

Conclusions

- I. No scientist is superstitious
II. No young people are superstitious. [2022]
(a) Either I or II follows
(b) Neither I or II follows
(c) Only Conclusion II follows
(d) Only Conclusion I follows

Ans.(b)

Sol. According to the given statements,



I. (x)

II. (x)

Hence, neither Conclusion I nor II follows.

73. Six books are labelled A, B, C, D, E and F are placed side-by-side. Books B, C, E and F have green covers while others have yellow covers. Book A, B and D are new while the rest are old volumes. Book A, B and C are law reports while the rest are medical extracts. Which two books are old medical extracts and have green covers? [2022]

- (a) B and C (b) E and F
(c) C and F (d) C and E

Ans.(b)

Sol. According to the question,

Qualities/Bodies	Green cover	Yellow cover	New volume	Old volume	Law reports	Medical extracts
A	×	√	√	×	√	×
B	√	×	√	×	√	×
C	√	×	×	√	√	×
D	×	√	√	×	×	√
E	√	√	√	√	×	√
F	√	√	√	√	×	√

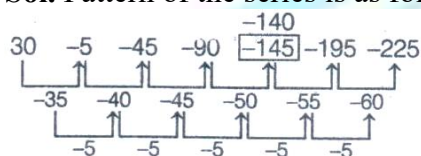
Hence, E and F are two old medical extracts and have green covers.

74. Find out the wrong number in the following series.
30, -5, -45, -90, -145, -195, -255. [2022]

- (a) -145 (b) -255 (c) -5 (d) -195

Ans.(a)

Sol. Pattern of the series is as follows,



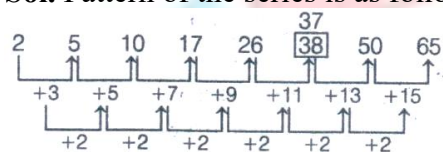
∴ Wrong number = -145

75. Find out the wrong number in the following series.
2, 5, 10, 17, 26, 38, 50, 65. [2022]

- (a) 26 (b) 30 (c) 65 (d) 50

Ans.(b)

Sol. Pattern of the series is as follows,

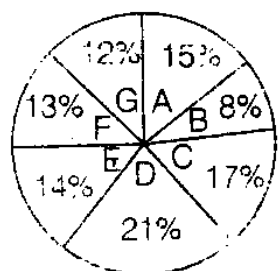


∴ Wrong number = 38

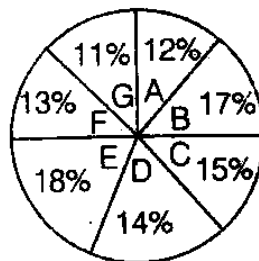
Directions (Q. Nos. 76-78) The following questions are based on the pie-charts given below:

Percentage-wise distribution of students studying in Arts and Commerce in seven different institutions, different institutions - A, B, C, D, E, F and G.

Total number of students studying Arts = 3800



Total number of students studying commerce = 4200



76. What is the total number of students studying Arts in Institutes A and G together? [2022]

- (a) 1206 (b) 1226 (c) 1026 (d) 1126

Ans.(c)

Sol. Total number of students studying Arts in Institutes A and G together

$$= 3800 \times \frac{15}{100} + 3800 \times \frac{12}{100}$$

$$= \frac{3800}{100} \times 27$$

$$= 38 \times 27 = 1026$$

77. How many students from Institute B study Arts and Commerce? [2022]

- (a) 1108 (b) 1180 (c) 1018 (d) 1208

Ans.(c)

Sol. Total number of students from Institute B study Arts and Commerce

$$= 3800 \times \frac{8}{100} + 4200 \times \frac{17}{100}$$

$$= 38 \times 8 + 42 \times 17$$

$$= 304 + 714 = 1018$$

78. The ratio of the number of students studying Arts to that studying Commerce in institute E is [2022]

- (a) 27:14 (b) 19:28
(c) 19:27 (d) 19:16

Ans.(c)

Sol. Required ratio

$$= \frac{\text{Number of students studying Arts in Institute E}}{\text{Number of students studying Commerce in Institute E}}$$

$$= \frac{3800 \times \frac{14}{100}}{4200 \times \frac{18}{100}}$$

$$= \frac{38 \times 14}{42 \times 18} = \frac{19}{27} = 19:27$$

79. Statement Many shops in the local market have extended their shops and occupied most part of the footpath in front of their shops. [2022]

Cause of Action

I. The civic authority should immediately activate a task force to clear all the footpaths encroached by the shop owners.

II. The civic authority should charge hefty penalty to the shop owners for occupying the footpath.

III. The civic authority should setup a monitoring system so that encroachments do not recur in future.

- (a) None follows (b) II and III follow
(c) I and II follow (d) All I, II and III follow

Ans.(d)

Sol. According to the given statement, all of three cause of actions are follows.

80. Statement There is a significant increase in the number of patients affected by some disease in a city.

Cause of Action

I. Municipal corporation of the city should take immediate action.

II. This problem should be raised in the UNESCO.

III. Hospitals in the city should be equipped properly for the treatment of the patients. **[2022]**

- (a) Only III follows (b) All follow
(c) I and II follow (d) I and III follow

Ans.(d)

Sol. According to the given statements, only I and III follows.

Directions : (Q. Nos. 81-85) Read the following information carefully and answer the questions. Five dramas A, B, C, D and E have to be staged in 6h where 1 h needs to be given per drama.

1. A break of 1 h has to be taken in 3rd or 4th hour.

2. Drama show cannot be started with A and cannot end in C.

3. D has to follow B immediately with no break in between.

4. A cannot be done immediately after D.

5. A has to precede E immediately with no break in between.

81. Which hour is a break hour? **[2022]**

- (a) 5th (b) 2nd (c) 3rd (d) 4th

Ans.(c)

Sol. Solutions (Q. Nos. 81-85) According to the given information

Hours	Dramas
1 st	B
2 nd	D
3 rd	Break
4 th	C
5 th	A
6 th	E

3rd hour is a break hour.

82. Which is the drama to be staged first? **[2022]**

- (a) A (b) D (c) B (d) None of these

Ans.(c)

Sol. Drama B is to be staged first.

83. Which is the drama staged immediately after the break? **[2022]**

- (a) A (b) D (c) B (d) None of these

Ans.(d)

Sol. Drama C is to be staged immediately after the break.

84. Which drama is staged immediately after D ? **[2022]**

- (a) None of these (b) E (c) B (d) C

Ans.(a)

Sol. Break hour happens immediately after D.

85. Which drama is staged immediately after E ? **[2022]**

- (a) C (b) A (c) E (d) None of these

Ans.(d)

Sol. Drama E is to be staged last.

86. A person's present age two-fifth of the age of his mother. After 8 yr, he will be one-half of the age of his mother. What is the present age of the mother? **[2022]**

- (a) 40 yr (b) 60 yr (c) 30 yr (d) 50 yr

Ans.(a)

Sol. Let age of mother be x yr.

$$\text{Then, age of person} = \frac{2x}{5}$$

According to the question, after 8 yr

$$\frac{1}{2} \times (x+8) = \frac{2x}{5} + 8$$

$$\Rightarrow x+8 = 2 \left(\frac{2x+40}{5} \right)$$

$$\Rightarrow 5x+40 = 4x+80 \Rightarrow x = 40 \text{ yr}$$

Hence, present age of Mother = 40yr

87. Running at the same constant rate, 6 identical machines can produce a total of 270 bottles per minute. At this rate, how many bottles could 10 such machines produce in 4 min ? **[2022]**

- (a) 2700 (b) 648 (c) 10800 (d) 1800

Ans.(d)

Sol. From, $\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$

Here, $M_1 = 6, M_2 = 10, H_1 = 1, H_2 = 4$

$$W_1 = 270 \text{ and } W_2 = ?$$

Now, $\frac{6 \times 1}{270} = \frac{10 \times 4}{W_2}$
 $\Rightarrow W_2 = \frac{40 \times 270}{.6}$
 $\therefore W_2 = 1800$

88. At what time, in minutes, between 3 o'clock and o'clock both the needles will coincide each other?

[2022]

- (a) $5\frac{1}{11}$ (b) $12\frac{4}{11}$
 (c) $13\frac{4}{11}$ (d) $16\frac{4}{11}$

Ans.(d)

Sol. Here, $n = 3$ and $(n + 1) = 4$

According to the formula,

The two hands will coincide at $\frac{60n}{11}$ min past n

or $\frac{60 \times 3}{11}$ min past 3 or $16\frac{4}{11}$ min past 3 .

89. The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 , respectively is [2022]

- (a) 127 (b) 123 (c) 235 (d) 305

Ans.(a)

Sol. Required number = HCF of $(1657 - 6) = 1651$ and $(2037 - 5) = 2032$

By division method,

1651)2032(1

1651

381)1651(4

1524

127) 381 (3

381

X = 127

90. 1. A is the brother of B .

2. C is the father of A .

3. D is brother of E .

4. E is the daughter of B .

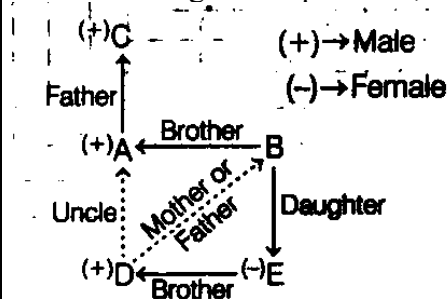
[2022]

Then, the uncle of D is

- (a) B (b) C (c) E (d) A

Ans.(d)

Sol. According to the questions,



Hence, A is (maternal or paternal) uncle of D .

Computer Awareness

91. The Boolean expression $AB + AB' + A'C + AC$ is unaffected by the value of the Boolean variable [2022]

- (a) C (b) A (c) B (d) None of these

Ans.(c)

Sol. Let $F(A, B, C) = AB + AB' + A'C + AC$

$$F(A, B, C) = A(B + B') + C(A' + A)$$

$$= A(1) + C(1) = A + C$$

Since, the minimised Boolean expression does not contain Boolean variable B . Therefore, $F(A, B, C)$ is unaffected by the value of the Boolean variable B .

92. Suppose the largest n bit number requires, ' d ' digits in decimal representation. Which of the following relations between ' n ' and ' d ' approximately correct? [2022]

- (a) $d = 2^n$ (b) $n = 2^d$
 (c) $d < n \log_{10} 2$ (d) $d > n \log_{10} 2$

Ans.(d)

Sol. n bits binary number requires d -decimal digits

So, $10^d > 2^n$

Take log on both side

$$\log_{10} 10^d > \log_{10} 2^n$$

$$d > n \log_{10} 2$$

93. FFFF will be the last memory location in a memory is size [2022]

- (a) 32 K (b) 16 K (c) 64 K (d) 1 K

Ans.(c)

Sol. FFFF will be the last memory location in a memory size 64 K.

The equivalent addressing ranges are 0 to 65535 in decimal and 0 to FFFF in hex.

Hexadecimal (also base 16) is a positional numeral system with a radix or base of 16 . It uses sixteen

distinct symbols, most often the symbols 0-9 to represent values zero to nine and $A-F$ to represent values ten to fifteen.

Bytes	Hex	Address range
16	10	$0-F$
256	100	$0-FF$
1 K	400	$0-3FF$
4 K	1000	$0-FFF^6$
64 K	10000	$0-FFFF$

94. Which of the following is equivalent to the Boolean expression $(X+Y) \cdot (X+\bar{Y}) \cdot (\bar{X}+Y)$ [2022]

- (a) XY (b) $\bar{Y}X$ (c) $\bar{X}Y$ (d) $\bar{X}\bar{Y}$

Ans.(a)

Sol. $(x+y)(x+y')(x'+y)$

$$\begin{aligned}
 &= [(x+y)(x+y')](x'+y) \\
 &= [xx + xy' + xy + yy'](x'+y) \\
 &= [x + xy' + xy + yy'](x'+y) \\
 &= [x + xy' + xy + 0](x'+y) \\
 &= [x + x(y' + y)](x'+y) \\
 &= [x + x(1)](x'+y) \\
 &= [x + x](x'+y) \\
 &= x(x'+y) \\
 &= xx' + xy \\
 &= 0 + xy \\
 &= xy
 \end{aligned}$$

95. 'Floating point representation' is used to represent [2022]

- (a) Integers (b) Real numbers
(c) Boolean values (d) Whole numbers

Ans.(b)

Sol. Floating point representation is used to represent real numbers. This representation does not reserve a specific number of bits for the integer part or the fractional part.

The floating number representation of a number has two part : the first part represents a signed fixed point number called Mantissa and the second part of designates the position of the decimal point and is called the exponent.

96. If a processor clock is rated as 2500 million cycles per second, then its clock period is [2022]

- (a) 2.50×10^{-10} sec (b) 4.00×10^{-10} sec
(c) 1.00×10^{-10} sec (d) None of these

Ans.(b)

Sol. We know that, frequency is defined as the number of cycles in one second.

Number of cycle in 1sec = 2500 million

Frequency = 2500MHz

We know that time period is the inverse of frequency and is defined as the time taken by one cycle.

$$\Rightarrow T = 1 / F$$

$$\begin{aligned}
 &= \frac{1}{2500 \times 10^6} \\
 &= \frac{100}{2500 \times 10^6 \times 100} = \frac{100}{25 \times 10^{10}} \\
 &= \frac{100}{25} \times 10^{-10} = 4 \times 10^{-10} \text{ sec}
 \end{aligned}$$

97. The minimum number of NAND gates required for implementing of Boolean expression, [2022]

$AB + A\bar{B}C + A\bar{B}\bar{C}$ is

- (a) 1 (b) 0 (c) 2 (d) 3

Ans.(b)

Sol. Let $F(A, B, C) = AB + AB'C + AB'C'$

$$\begin{aligned}
 &= AB + AB'(C + C') \\
 &= AB + AB'(1) \\
 &= AB + AB'
 \end{aligned}$$

$$= A(B + B') = A(1) = A$$

So, there is no required of NAND gate.

98. The maximum and minimum value represented in signed 16 bit 2's complement representations are [2022]

- (a) - 16384 and 16383 (b) 0 and 32767
(c) - 32768 and 32767 (d) 0 and 65535

Ans.(c)

Sol. The maximum and minimum value represented in signed 16 bit 2's complement representations are - 32768 and 32767 .

99. Write the simplified form of the Boolean

expression $(A+C)(A+AD)+AC+C$ [2022]

- (a) $A+C'$ (b) $A+C$
(c) $A'+C$ (d) $A+D$

Ans.(b)

Sol. $(A+C)(A+AD)+AC+C$

$$\begin{aligned}
 &= (A + C) \cdot A(1 + D) + AC + C \\
 &= (A + C) \cdot A + AC + C \\
 &= A \cdot A + AC + AC + C \\
 &= A + AC + AC + C \\
 &= A + AC + C = A(1 + C) + C = A + C
 \end{aligned}$$

100. If a signal passing through a gate is inhibited by sending a low into one of the inputs, and the output is high, the gate is a(n) [2022]

- (a) OR (b) NOR (c) NAND (d) AND

Ans.(c)

Sol.In NAND gate, output is high if any of the input is low. The truth table for the NAND gate is

A	B	Output
1	1	0
1	0	1
0	1	1
0	0	1

General English

101. 'Bite and bullet' means [2022]

- (a) to stop a conflict
(b) to become mad
(c) to accept something that is difficult or unpleasant
(d) to analyse your faults

Ans.(c)

Sol. 'Bite and Bullet' means to force yourself to perform an unpleasant or difficult action or to be brave in a difficult situation. Hence, option (c) is its correct answer.

102. What can you call a person who leads an unconventional style or living? [2022]

- (a) Cynic (b) Bohemian
(c) Altruist (d) Agnostic

Ans.(b)

Sol. Bohemian is the practice of an unconventional lifestyle, oft a in the company of like-minded people and with permanent ties. Hence, option (b) is its correct answer.

103. Fill in the blanks with the correct option.
Technical writing demands _____ use of language.

- (a) poetic (b) factual (c) figurative (d) dramatic [2022]

Ans.(b)

Sol. Here, 'factual' is the correct word for the given blank.

Something that is factual is concerned with facts or contains facts, rather than giving theories or personal interpretations. Hence, option (b) is its correct answer.

104. The county cleared this path and paved it with packed gravel, so they have a peaceful place to hike and bike. Which of the following alternatives to the underlined portion would NOT be acceptable? [2022]

- (a) Path, paving (b) Path and then paved
(c) Path before paving (d) Path paved

Ans.(d)

Sol. 'Path paved' would not be acceptable to the given underlined portion.

105. Select the correct form of verb/subject verb agreement.

The principal, along with his assistants, _____ the meeting. [2022]

- (a) are attending (b) attend
(c) is attending (d) attending

Ans.(c)

Sol.When two nouns joined together by along with, the verb is governed by the former noun. So, here, "The principal" is singular subject so, singular verb 'is attending' should be used.

106. Select the correct articles.

He is _____ M.A. with PhD and teaches in _____ university. [2022]

- (a) the, the (b) an, the
(c) a, an (d) a, the

Ans.(b)

Sol. In first blank, article 'an' should be used because article 'an' is used before the word start with vowel sound. Hence, option (b) is its correct answer.

107. Fill in the blank with the most appropriate option.

Kedar _____ this project for a month and now he is about to join a new project. [2022]

- (a) guides (b) guided
(c) has been guiding (d) guiding

Ans.(c)

Sol. The given sentence belongs to Present Perfect Continuous tense, so 'has been guiding' is the correct alternative for the given blank.

Comprehension Type Questions

Directions : Read the following passage carefully and answer the questions:

You might think you've experienced VR, and you might have been pretty impressed. Particularly, if

you're a gamer, there are some great experiences to be had out there (or rather, in there) today. But over the next few years, in VR

as in all fields of technology, we're going to see things that make what is cutting-edge today look like space invaders. And although the games will be amazing, the effects of this transformation will be far broader, touching on our work, education, and social lives.

Today's most popular VR applications involve taking total control of user's sense (sight and hearing, particularly) to create a totally immersive experience that places the user in a fully virtual environment that feels pretty realistic. Climb up something high and look down, and you're likely to get a sense of vertigo. If you see an object moving quickly towards your head, you'll feel an urge to duck out of the way.

Very soon, VR creators will extend this sensory hijacking to our other facilities—for example, touch and smell to deepen that sense of immersion. At the same time, the devices we use to visit these virtual worlds will become cheaper and lighter, removing the friction that can currently be a barrier.

I believe Extended Reality (XR)—a term that covers Virtual Reality (VR), Augmented Reality (AR), and Mixed reality (MR)—will be one of the most transformative tech trends of the next five years. It will be enabled and augmented by other tech trends, including super-fast networking, that will let us experience VR as a cloud service just like we currently consume music and movies. And Artificial Intelligence (AI) will provide us with more personalised virtual worlds to explore, even giving us realistic virtual characters to share our experiences with.

108. The passage states all the following about VR applications except. **[2022]**

- (a) Future AI will allow us to share our experiences with realistic virtual characters.
- (b) Vertigo is a major feature of all AI applications.
- (c) VR applications create a virtual environment that feels pretty realistic.
- (d) VR applications take control of the user's senses.

Ans.(b)

Sol. The passage states all the following about VR applications except that Vertigo is a major feature of all AI applications.

109. 'Duck out of something' means **[2022]**

- (a) to avoid doing something
- (c) to fall down
- (b) to hit something hard
- (d) to meet with an accident

Ans.(a)

Sol. 'Duck out of something' means to avoid doing something that you have to do or have promised to do. Hence, option (a) is its correct answer.

110. Select an antonym for the word 'argument' from the options given below. **[2022]**

- (a) Aggrandize
- (b) Curtail
- (c) Inflate
- (d) Reinforce

Ans.(b)

Sol. 'Curtail' is an correct antonym for the word 'Argument'.

Argument means a reason or reasons why you support or oppose an idea or suggestion, or the process of explaining these reasons.

Curtail means something is to slow it down, restrictions on it or stop it entirely.

111. Read the following sentence to find if there is any error in any part. **[2022]**

If i were him/i would teach/him a lesson/no error

- (a) him a lesson
- (b) no error
- (c) if i were him
- (d) i would teach

Ans.(b)

Sol. The given sentence has no error.

112. Fill in the blanks with the correct preposition: Jugdish is waiting for me _____ the campus. **[2022]**

- (a) on
- (b) at
- (c) in
- (d) out

Ans.(a)

Sol. Preposition 'on' is suitable for the given blank.

113. Select the word which means the same as the following.

To read something carefully **[2022]**

- (a) Presume
- (b) Perverse
- (c) Peruse
- (d) Persiflage

Ans.(c)

Sol. Peruse means read (something), typically in a thorough or careful way.

114. Select the one which best expresses the same sentence in indirect/direct speech: **[2022]**

He said, "I am glad to be here this evening."

- (a) He says he was glad to be here his evening.
- (b) He said he was glad to be here this evening.
- (c) He said that he was glad to be there that evening.
- (d) He asked he is glad to be here this evening.

Ans.(c)

Sol. The sentence given in option (c) He said that he was glad to be there that evening', is the correct indirect speech of the given sentence.

115. Choose the antonym. **[2022]**

BOLD

- (a) Nervous
- (b) Timid
- (c) Fearful
- (d) Coy

Ans.(b)

Sol. 'Timid' is the antonym of 'Bold'.

116. Which of the phrases given below should replace the phrase printed in bold to make the sentence grammatically correct. **[2022]**

He is addicted to smoke.

- (a) addicted with smoking (b) addict of smoke
(c) used to smoke (d) addicted to smoking

Ans.(d)

Sol. 'Addicted to smoking' should replace the phrase given in bold to make the sentence grammatically correct.

117. Choose the correct alternative with the correct choice given below each statement. **[2022]**

You are to conform _____ the rules of the institute.

- (a) with (b) to (c) on (d) of

Ans.(b)

Sol. Preposition 'to' is suitable in the given blank to make the sentence grammatically correct.

118. Choose the synonym. **[2022]**

LIBERAL

- (a) Generous (b) Affectionate
(c) Sober (d) Reactionary

Ans.(a)

Sol. Liberal means someone who believes in personal freedom and that society should change gradually so that money, property and power are shared more fairly. So, 'Generous' would be its correct synonym, as it also means willing to give money, help, kindness, etc.

119. Identify the word that is similar in meaning to the underlined word.

Raghu made adulatory remarks about the waiter who served the food. **[2022]**

- (a) Complimentary (b) Ironic
(c) Derogatory (d) Slandorous

Ans.(a)

Sol. The given underlined word 'Adulatory' means showing great admiration or praise for someone, especially when it is more than is deserved. So, 'Complimentary' would be its correct similar meaning word.

120. Fill in the blank. **[2022]**

Neither Peter nor I _____ responsible for this blunder.

- (a) am (b) are (c) is (d) were

Ans.(a)

Sol. 'Am' is suitable for the given blank.