

Mathematics

1. A circle touches the X -axis and also touches the circle with centre $(0,3)$ and radius 2. The locus of the centre of the circle is [2023]

- (a) a circle (b) an ellipse
(c) a parabola (d) a hyperbola

Ans. (c)

Sol. Let $C_1(h, k)$ be the centre of the circle.

Circle touches the X -axis, then its radius is $r_1 = k$.

Also, circle touches the circle with centre $C_2(0,3)$ and radius $r_2 = 2$.

$$\begin{aligned} \therefore |C_1C_2| &= r_1 + r_2 \\ \Rightarrow \sqrt{(h-0)^2 + (k-3)^2} &= |k+2| \\ \Rightarrow h^2 - 10k + 5 &= 0 \end{aligned}$$

Change h to x and k to $y \Rightarrow x^2 - 10y + 5 = 0$

It is a parabola.

2. A computer producing factory has only two plants T_1 and T_2 . Plant T_1 produces 20% and plant T_2 produces 80% of total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P (computer turns out to be defective given that it is produced in plant T_1) = 10P (computers turns out to be defective given that it is produced in plant T_2), where $P(E)$ denotes the probability of an event E . A computer produced in the factory is randomly selected and it does not turn out to be defective. Then, the probability that it is produced in plant T_2 is [2023]

- (a) $\frac{36}{73}$ (b) $\frac{47}{79}$ (c) $\frac{78}{93}$ (d) $\frac{75}{83}$

Ans. (c)

Sol. Let $x = P$ (computer turns out to be defective, given that it is produced in plant T_2).

$$\Rightarrow x = P\left(\frac{D}{T_2}\right) \dots (i)$$

where, D = Defective computer.

$\therefore P$ (computer turns out to be defective given that it is produced in plant T_1) = $10x$

$$\text{i.e., } P\left(\frac{D}{T_1}\right) = 10x \dots (ii)$$

$$\text{Also, } P(T_1) = \frac{20}{100}$$

$$\text{and } P(T_2) = \frac{80}{100}$$

$$\text{Given, } P(\text{defective computer}) = \frac{7}{100}$$

$$\text{i.e., } P(D) = \frac{7}{100}$$

Using law of total probability,

$$P(D) = P(T_1) \cdot P\left(\frac{D}{T_1}\right) + P(T_2) \cdot P\left(\frac{D}{T_2}\right)$$

$$\therefore \frac{7}{100} = \left(\frac{20}{100}\right) \cdot 10x + \left(\frac{80}{100}\right) \cdot x$$

$$\Rightarrow 7 = (280)x \Rightarrow x = \frac{1}{40} \dots (iii)$$

$$\therefore P\left(\frac{D}{T_2}\right) = \frac{1}{40} \text{ and } P\left(\frac{D}{T_1}\right) = \frac{10}{40}$$

$$\Rightarrow P\left(\frac{\bar{D}}{T_2}\right) = 1 - \frac{1}{40} = \frac{39}{40}$$

$$\text{and } P\left(\frac{\bar{D}}{T_1}\right) = \frac{30}{40} \dots (iv)$$

Using Baye's theorem,

$$\begin{aligned} P\left(\frac{T_2}{\bar{D}}\right) &= \frac{P(T_2) \cdot P\left(\frac{\bar{D}}{T_2}\right)}{P(T_1) \cdot P\left(\frac{\bar{D}}{T_1}\right) + P(T_2) \cdot P\left(\frac{\bar{D}}{T_2}\right)} \\ &= \frac{\frac{80}{100} \cdot \frac{39}{40}}{\frac{20}{100} \cdot \frac{30}{40} + \frac{80}{100} \cdot \frac{39}{40}} = \frac{78}{93} \end{aligned}$$

3. The mean of 5 observation is 5 and their variance is 124. If three of the observations are 1, 2 and 6; then the

mean deviation from the mean of the data is [2023]

- (a) 2.5 (b) 2.6 (c) 2.8 (d) 2.4

Ans. (c)

Sol. Let the two numbers be x and y .

Given, variance $\sigma^2 = 124$

Mean, $\bar{x} = 5$ and $n = 5$

$$\frac{1+2+6+x+y}{5} = 5$$

$$x+y=16$$

So, mean deviation

$$\begin{aligned} & |1-5| + |2-5| + |6-5| \\ & + |x-5| + |y-5| \\ & = \frac{\quad}{5} \end{aligned}$$

Now, we consider $x, y > 5$

Mean deviation

$$\begin{aligned} & = \frac{4+3+1+(x+y-10)}{5} \\ & = \frac{8+16-10}{5} = \frac{14}{5} = 2.8 \end{aligned}$$

4. The perimeter of a $\triangle ABC$ is 6 times the arithmetic mean of the sines of its angles. If the side a is 1, then the angle A is [2023]

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

Ans. (a)

Sol. Let the side of the triangle are a, b, c . It is given that the perimeter of a $\triangle ABC$ is 6 times the arithmetic mean of the sines of its angles.

$$\therefore a+b+c = 6 \left(\frac{\sin A + \sin B + \sin C}{3} \right)$$

$$a+b+c = 2(\sin A + \sin B + \sin C) \dots (i)$$

From the law of sine,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\Rightarrow a = k \sin A, b = k \sin B$$

and $c = k \sin C$

$$\therefore a+b+c = k(\sin A + \sin B + \sin C) \dots (ii)$$

Hence, $k = 2$

$$\Rightarrow a = 2 \sin A \Rightarrow 1 = 2 \sin A$$

$$\Rightarrow \sin A = \frac{1}{2} \Rightarrow A = \frac{\pi}{6}$$

5. In an examination of nine papers, a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is [2023]

- (a) 255 (b) 256 (c) 128 (d) 9×8 !

Ans. (b)

Sol. The candidate is unsuccessful if he fails in 9 or 8 or 7 or 6 or 5 papers.

$\therefore$ The number of ways to be unsuccessful

$$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$$

$$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$$

$$= \frac{1}{2} ({}^9C_0 + {}^9C_1 + \dots + {}^9C_9)$$

$$= \frac{1}{2} 2^9 = 256$$

6. For a group of 100 candidates, the mean and standard deviation of scores were found to be 40 and 15 respectively. Later on, it was found that the scores 25 and 35 were misread as 52 and 53 respectively.

Then the corrected mean and standard deviation corresponding to the corrected figures are [2023]

- (a) 39.9, 14.97 (b) 39.5, 14
(c) 39.55, 14.97 (d) 40.19, 15.1

Ans. (c)

Sol. $\bar{x} = 40$

$$= \frac{\sum x_i}{100}$$

$$\Rightarrow \sum x_i = 4000$$

$$\sum x_i = 4000 - (52 + 53) + (25 + 35)$$

$$= 3955$$

$$\Rightarrow \text{Correct } \bar{x} = 39.55$$

As from given options only (c) option is matched.

7. Consider the following frequency distribution table

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	180	f_1	34	180	136	f_2	50

If the total frequency is 685 and median is 42.6, then the values of f_1 and f_2 are [2023]

- (a) 80, 25 (b) 83, 22 (c) 79, 26 (d) 82, 23

Ans. (d)

Sol.

Class	Frequency	Cumulative Frequency
10-20	180	180
20-30	f_1	$180 + f_1$
30-40	34	$214 + f_1$
40-50	180	$394 + f_1$
50-60	136	$530 + f_1$
60-70	f_2	$530 + f_1 + f_2$
70-80	50	$580 + f_1 + f_2$

Total frequency = 685

$$580 + f_1 + f_2 = 685$$

$$f_1 + f_2 = 105$$

Median = 42.6 (given) lies in (40-50) interval

Class = 40-50

$$\text{Median} = l + \frac{\frac{N}{2} - C \cdot f}{f} \times h$$

$$l = 40, h = 10$$

$$C \cdot f = 214 + f_1$$

$$42.6 = 40 + \frac{\frac{685}{2} - (214 + f_1)}{180} \times 10$$

$$2.6 \times 18 = 342.5 - 214 - f_1$$

$$f_1 = 817 \cong 82$$

$$f_2 = 105 - 82 = 23$$

8. If $f(x) = \lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{\log_e 9(1 - \cos x)}$ is a real number then

$$\lim_{x \rightarrow 0} f(x) = \quad [2023]$$

- (a) 2 (b) 3 (c) $\log_e 2$ (d) $\log_e 3$

Ans. (c)

$$\text{Sol. } \lim_{x \rightarrow 0} \frac{(3^x - 1)(2^x - 1)}{2 \log_e 3 \left(2 \sin^2 \frac{x}{2} \right)}$$

$$\text{Using } \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

$$\begin{aligned} &= \frac{1}{4 \log_e 3} \lim_{x \rightarrow 0} \frac{(3^x - 1)(2^x - 1)}{x \cdot x} \cdot \frac{4 \left(\frac{x^2}{4} \right)}{\sin^2 \frac{x}{2}} \\ &= \frac{1}{\log_e 3} \log_e 3 \log_e 2 = \log_e 2 \end{aligned}$$

9. The sum of infinite terms of decreasing GP is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ in the interval $[-2, 3]$ and difference between the first two terms is $f'(0)$. Then, the common ratio of the GP is

[2023]

- (a) $-\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

Ans. (c)

$$\text{Sol. } f(x) = x^3 + 3x - 9, x \in [-2, 3]$$

Differentiate with respect to x

$$f'(x) = 3x^2 + 3$$

Hence, $f(x)$ is strictly increasing function so its greatest value will be at $x = 3$

$$f(3) = 3^3 + 3 \times 3 - 9 = 27$$

$$\frac{a}{1-r} = 27$$

$$\Rightarrow a = 27 - 27r$$

$$\Rightarrow a + 27r = 27 \dots (i)$$

$$f'(0) = 3$$

Also given, $a - ar = f'(0)$

$$\Rightarrow 1 - r = \frac{3}{a} \dots (ii)$$

From Eqs. (i) and (ii), we get

$$a + 27 \left(1 - \frac{3}{a} \right) = 27$$

$$a + 27 - \frac{81}{a} = 27$$

$$\Rightarrow a^2 = 81 \Rightarrow a = \pm 9$$

$\therefore$ GP is decreasing

$$\therefore a = 9$$

$$\text{Now, } \frac{9}{1-r} = 27$$

$$\begin{aligned} \Rightarrow \frac{1}{3} &= 1 - r \Rightarrow r = 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

10. The value of $\int_{-\pi/3}^{\pi/3} \frac{x \sin x}{\cos^2 x} dx$ is [2023]

- (a) $\frac{1}{3}(4\pi + 1)$ (b) $\frac{4\pi}{3} - 2 \log \tan \frac{5\pi}{12}$
(c) $\frac{4\pi}{3} + \log \tan \frac{5\pi}{12}$ (d) $\frac{4\pi}{3} - \log \tan \frac{5\pi}{3}$

Ans. (b)

$$\text{Sol. } I = 2 \int_0^{\pi/3} x \tan x \sec x dx$$

Using integration by parts

$$\begin{aligned} I &= 2[x \sec x]_0^{\pi/3} - 2 \int_0^{\pi/3} \sec x dx \\ &= 2 \left[\frac{\pi}{3} \times 2 \right] - 2 [\ln |\sec x + \tan x|]_0^{\pi/3} \\ &= \frac{4\pi}{3} - 2 \left[\ln |2 + \sqrt{3}| \right] \\ &= \frac{4\pi}{3} - 2 \ln \tan \frac{5\pi}{12} \end{aligned}$$

11. The equation of the tangent at any point of curve $x = a \cos 2t, y = 2\sqrt{2}a \sin t$ with m as its slope is

[2023]

- (a) $y = mx + a\left(m - \frac{1}{m}\right)$ (b) $y = mx - a\left(m + \frac{1}{m}\right)$
 (c) $y = mx + a\left(a + \frac{1}{a}\right)$ (d) $y = amx + a\left(m - \frac{1}{m}\right)$

Ans. (b)

Sol. Equation of tangent, with slope m ,

$$x = a \cos 2t$$

$$y = 2\sqrt{2}a \sin t$$

$$\frac{dx}{dt} = -2a \sin 2t$$

$$\frac{dy}{dt} = 2\sqrt{2}a \cos t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\sqrt{2}a \cos t}{-2a \sin 2t}$$

$$= \frac{\sqrt{2} \cos t}{-2 \sin t \cos t} = \frac{-1}{\sqrt{2} \sin t}$$

$$= m \text{ (given)}$$

$$\Rightarrow \sin t = \frac{-1}{\sqrt{2}m}$$

Then, $x = a \cos 2t$

$$y = 2\sqrt{2}a \sin t$$

$$= a(1 - 2 \sin^2 t)$$

$$= 2\sqrt{2}a \times \left(\frac{-1}{\sqrt{2}m}\right)^2$$

$$= a \left(1 - 2 \times \left(\frac{-1}{\sqrt{2}m}\right)^2\right)$$

$$y = \frac{-2a}{m}$$

$$x = a \left(1 - \frac{1}{m^2}\right)$$

Then, equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y + \frac{2a}{m} = m \left(x - a \left(1 - \frac{1}{m^2}\right)\right)$$

$$y + \frac{2a}{m} = mx - am + \frac{a}{m}$$

$$y = mx - a \left(m + \frac{1}{m}\right)$$

12. If $\prod_{i=1}^n \tan(\alpha_i) = 1 \forall \alpha_i \in \left[0, \frac{\pi}{2}\right]$ where $i = 1, 2, 3, \dots, n$

. Then, maximum value of $\prod_{i=1}^n \sin \alpha_i$ [2023]

- (a) $\frac{1}{2^n}$ (b) $\frac{1}{2^{n/2}}$ (c) 1 (d) None of these

Ans. (b)

Sol. $\sin \alpha_1 \sin \alpha_2 \sin \alpha_3 \dots \sin \alpha_n$

$$= \cos \alpha_1 \cos \alpha_2 \dots \cos \alpha_n$$

Multiplying both sides by $\sin \alpha_1 \sin \alpha_2 \sin \alpha_3 \dots \sin \alpha_n$

$$\Rightarrow \sin^2 \alpha_1 \sin^2 \alpha_2 \sin^2 \alpha_3 \dots \sin^2 \alpha_n$$

$$= (\sin \alpha_1 \cos \alpha_1)(\sin \alpha_2 \cos \alpha_2)$$

$$\dots (\sin \alpha_n \cos \alpha_n)$$

$$\Rightarrow \sin^2 \alpha_1 \sin^2 \alpha_2 \sin^2 \alpha_3 \dots \sin^2 \alpha_n$$

$$= \frac{1}{2^n} (\sin 2\alpha_1)(\sin 2\alpha_2) \dots (\sin 2\alpha_n)$$

As, we know maximum value of $\sin \theta$ is 1.

$$\Rightarrow \sin^2 \alpha_1 \sin^2 \alpha_2 \sin^2 \alpha_3 \dots$$

$$\sin^2 \alpha_n \leq \frac{1}{2^n}$$

$$\Rightarrow \sin \alpha_1 \sin \alpha_2 \sin \alpha_3 \dots \sin \alpha_n \leq \frac{1}{2^{n/2}}$$

Hence, maximum value of

$$\prod_{i=1}^n \sin \alpha_i = \frac{1}{2^{n/2}}$$

13. A speaks truth in 60% and B speaks the truth in 50% cases. In what percentage of cases they are likely to contradict each other while narrating some incident is [2023]

- (a) 1/2 (b) 1/4 (c) 2/3 (d) 1/3

Ans. (a)

Sol. Probability A speaks truth

$$P(AT) = \frac{60}{100} = \frac{3}{5}$$

$\Rightarrow$ Probability A speaks lie

$$P(AL) = \frac{40}{100} = \frac{2}{5}$$

Probability B speaks truth

$$P(BT) = \frac{50}{100} = \frac{1}{2}$$

$\Rightarrow$ Probability B speaks lie

$$P(BL) = \frac{50}{100} = \frac{1}{2}$$

Probability that they contradict each other stating the same fact

$$= P(AT \cap BL) + P(AL \cap BT)$$

$$= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{1}{2} = \frac{1}{2}$$

14. If $\mathbf{a}$ and $\mathbf{b}$ are vector in space, given by $\mathbf{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$

and $\mathbf{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of

$$(2\mathbf{a} + \mathbf{b}) \cdot [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} - 2\mathbf{b})] \text{ is } \quad [2023]$$

(a) 3 (b) 4 (c) 5 (d) 6

Ans.(c)

$$\text{Sol. } (c) - (2\mathbf{a} + \mathbf{b}) \cdot [(\mathbf{a} - 2\mathbf{b}) \times (\mathbf{a} \times \mathbf{b})]$$

$$= -(2\mathbf{a} + \mathbf{b}) \cdot [\{(\mathbf{a} - 2\mathbf{b}) \cdot \mathbf{b}\} \mathbf{a} - \{(\mathbf{a} - 2\mathbf{b}) \cdot \mathbf{a}\} \mathbf{b}]$$

$$= (2\mathbf{a} + \mathbf{b}) [\{(\mathbf{a} - 2\mathbf{b}) \cdot \mathbf{a}\} \mathbf{b} - \{(\mathbf{a} - 2\mathbf{b}) \cdot \mathbf{b}\} \mathbf{a}]$$

$$= (2\mathbf{a} + \mathbf{b}) [\{|\mathbf{a}|^2 - 2\mathbf{b} \cdot \mathbf{a}\} \mathbf{b} - \{\mathbf{a} \cdot \mathbf{b} - 2|\mathbf{b}|^2\} \mathbf{a}]$$

$$\text{Now, } \mathbf{a} \cdot \mathbf{b} = \left(\frac{\hat{i} - 2\hat{j}}{\sqrt{5}} \right) \cdot \left(\frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}} \right)$$

$$= 2 - 2 = 0$$

$$|\mathbf{a}| = 1, |\mathbf{b}| = 1$$

$$= (2\mathbf{a} + \mathbf{b}) \cdot [\{1 - 0\} \mathbf{b} - \{0 - 2\} \mathbf{a}]$$

$$= (2\mathbf{a} + \mathbf{b}) \cdot [\mathbf{b} + 2\mathbf{a}]$$

$$= (2\mathbf{a} + \mathbf{b}) \cdot [\mathbf{b} + 2\mathbf{a}]$$

$$= 2\mathbf{a} \cdot \mathbf{b} + 4|\mathbf{a}|^2 + |\mathbf{b}|^2 + 4\mathbf{a} \cdot \mathbf{b}$$

$$= 0 + 4 + 1 + 0 = 5$$

15. Let $\mathbf{A} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\mathbf{B} = \hat{i} + \hat{j}$. If $\mathbf{C}$ is a vector such that $|\mathbf{C} - \mathbf{A}| = 3$ and the angle between $\mathbf{A} \times \mathbf{B}$ and

$\mathbf{C}$ is 30° , then $[(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}] = 3$. Then, the value of

$\mathbf{A} \cdot \mathbf{C}$ is equal to [2023]

(a) $25/8$ (b) 2 (c) 5 (d) $1/8$

Ans. (b)

$$\text{Sol. } \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(2) - \hat{j}(2) + \hat{k}(1)$$

$$\mathbf{A} \times \mathbf{B} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$|\mathbf{C} - \mathbf{A}| = 3$$

$$|\mathbf{C}|^2 + |\mathbf{A}|^2 - 2\mathbf{C} \cdot \mathbf{A} = 9 \quad \dots (i)$$

$$|\mathbf{C}|^2 - 2\mathbf{C} \cdot \mathbf{A} = 0$$

$$\text{Now, } |(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}| = |\mathbf{A} \times \mathbf{B}| |\mathbf{C}| \sin \theta$$

$$= \sqrt{4 + 4 + 1} \times |\mathbf{C}| \times \sin 30^\circ = 3$$

$$\text{Given, } \theta = 30^\circ \Rightarrow |\mathbf{C}| = 2$$

Put this value in Eq. (i), we get

$$\mathbf{A} \cdot \mathbf{C} = 2$$

16. Let A and B be two sets, $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X , then find relation between A and B . [2023]

(a) $A = B$ (b) $A \cup B = X$ (c) $B = X$ (d)

$A = X$

Ans. (a)

Sol. Let A and B be two sets such that

$A \cap X = B \cap X = \phi$ and

$A \cup X = B \cup X$ for some set X .

For $A = B$

$$A = A \cap (A \cup X)$$

$$= A \cap (B \cup X) [A \cup X = B \cup X]$$

$$= (A \cap B) \cup (A \cap X)$$

[Distributive law]

$$= (A \cap B) \cup \phi [\because A \cap X = \phi]$$

$$= A \cap B \dots (i)$$

$$\text{Now, } B = B \cap (B \cup X)$$

$$= B \cap (A \cup X) [\because A \cup X = B \cup X]$$

$$= (B \cap A) \cup (B \cap X) \text{ [Distributive law]}$$

$$= (B \cap A) \cup \phi [\because B \cap X = \phi]$$

$$= B \cap A = A \cap B \dots (ii)$$

Hence, from Eqs. (i) and (ii), we get

$$A = B$$

17. If a, b, c, d are in HP and arithmetic mean of ab, bc, cd is 9, then which of the following number is the value of ad ? [2023]

(a) 3 (b) 9 (c) 12 (d) 4

Ans. (b)

$$\text{Sol. } \frac{ab + bc + cd}{3} = 9$$

$$\Rightarrow ab + bc + cd = 27$$

$$a, b, c \text{ are in HP} \Rightarrow b = \frac{2ac}{a+c}$$

$$\Rightarrow a+c = \frac{2ac}{b} \dots (i)$$

$$b, c, d \text{ are in HP} \Rightarrow c = \frac{2bd}{b+d}$$

$$\Rightarrow b+d = \frac{2bd}{c} \dots (ii)$$

Multiply Eqs. (i) and (ii),

$$(a+c)(b+d) = \frac{2ac}{b} \frac{2bd}{c} = 4ad$$

$$(ab+ad+bc+dc) = 4ad$$

$$\Rightarrow 3ad = ab+bc+cd = 27$$

$$\Rightarrow ad = 9$$

18. Find foci of the equation $x^2 + 2x - 4y^2 + 8y - 7 = 0$ [2023]

$$(a) (\sqrt{5} \pm 1, 1) \quad (b) (-1 \pm \sqrt{5}, 1)$$

$$(c) (-1, \sqrt{5} \pm 1) \quad (d) (1, -1 \pm \sqrt{5})$$

Ans. (b)

$$\text{Sol. } (x^2 + 2x + 1) - 4(y^2 - 2y + 1) = 7 + 1 - 4$$

$$\Rightarrow (x+1)^2 - 4(y-1)^2 = 4$$

$$\Rightarrow \frac{(x+1)^2}{4} - \frac{(y-1)^2}{1} = 1$$

Hence, centre is $(-1, 1)$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow 1 = 4(e^2 - 1)$$

$$\Rightarrow e = \frac{\sqrt{5}}{2} \Rightarrow ae = \sqrt{5}$$

Foci are at a distance ae from centre. Hence, foci will be $(-1 + \sqrt{5}, 1)$ and $(-1 - \sqrt{5}, 1)$

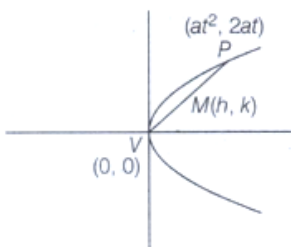
19. The locus of the mid-point of all chords of the parabola $y^2 = 4x$ which are through its vertex is [2023]

$$(a) y^2 = 8x \quad (b) y^2 = 2x$$

$$(c) y^2 + 4y^2 = 16 \quad (d) x^2 = 2y$$

Ans. (b)

Sol.



Let $M(h, k)$ be the mid-point of VP .

$$h = \frac{0+2at^2}{2} \Rightarrow at^2 = 2h \dots (i)$$

$$k = \frac{0+2at}{2} \Rightarrow at = k \Rightarrow t = \frac{k}{a}$$

Put value of t in Eq. (i), we get

$$k^2 = 2ah$$

Replace $h \rightarrow x$ and $k \rightarrow y$

$$y^2 = 2ax$$

20. If $\mathbf{a} = \hat{i} - \hat{k}$, $\mathbf{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and

$\mathbf{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$, then $[\mathbf{abc}]$ depends on [2023]

(a) neither x nor y

(b) only x

(c) only y

(d) both x and y

Ans. (a)

$$\text{Sol. } \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$$= \{(1+x-y) - (x-x^2) - 0 - (x^2-y)\}$$

$$= 1+x-y+x-x^2-x^2+y=1$$

Depends neither x nor y .

21. If $\mathbf{a}, \mathbf{b}$ are unit vectors such that $2\mathbf{a} + \mathbf{b} = 3$, then which of the following statement is true? [2023]

(a) $\mathbf{a}$ is parallel to $\mathbf{b}$

(b) $\mathbf{a}$ is perpendicular to $\mathbf{b}$

(c) $\mathbf{a}$ is perpendicular to $2\mathbf{a} + \mathbf{b}$

(d) $\mathbf{b}$ is parallel to $2\mathbf{a} + \mathbf{b}$

Ans. (a)

Sol. Fundamentally this question is wrong because sum of two vectors can not be equal to scalar. But if we have solve this equation.

$$|2\mathbf{a} + \mathbf{b}|^2 = 9$$

$$\Rightarrow 4|\mathbf{a}|^2 + |\mathbf{b}|^2 + 4\mathbf{a} \cdot \mathbf{b} = 9$$

$$\Rightarrow |\mathbf{a} \cdot \mathbf{b}| = 1$$

$$\Rightarrow |\mathbf{a}| \cdot |\mathbf{b}| \cos \theta = 1$$

$$\Rightarrow \cos \theta = 1$$

$$\theta = 0$$

22. $\int f(x) dx = g(x)$, then $\int x^5 f(x^3) dx$ [2023]

$$(a) \frac{1}{3} x^3 g(x^3) - 3 \int x^4 g(x^3) dx + c$$

(b) $\frac{1}{3}x^3g(x^3) - \int x^2g(x^3)dx + c$

(c) $\frac{1}{3}x^3g(x^3) - \int x^3g(x^3)dx + c$

(d) None of the above

Ans. (b)

Sol. $I = \int x^5 f(x^3) dx$

$= \int x^2 x^3 f(x^3) dx$

Let $x^3 = t \Rightarrow 3x^2 dx = dt$

$I = \frac{1}{3} \int t f(t) dt$

Using integration by parts.

$I = \frac{1}{3} \left[t \int f(t) dt - \int \left(\frac{dt}{dt} \int f(t) dt \right) dt \right]$

$= \frac{1}{3} \left[t g(t) - \int g(t) dt \right]$

$= \frac{1}{3} t g(t) - \frac{1}{3} \int g(t) dt$

As $x^3 = t$ and $dt = 3x^2 dx$

$= \frac{1}{3} x^3 g(x^3) - \frac{3}{3} \int x^2 g(x^3) dx$

$= \frac{1}{3} x^3 g(x^3) - \int x^2 g(x^3) dx + c$

23. $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^2}{x^2 - k^2}$, then find k . **[2023]**

(a) $8/3$ (b) $4/3$ (c) $2/3$ (d) 1

Ans. (a)

Sol. Using L' Hospital rule,

$\lim_{x \rightarrow 1} \frac{4x^3}{1} = \lim_{x \rightarrow k} \frac{3x^2}{2x}$

$\Rightarrow 4 = \frac{3}{2} k$

$k = \frac{8}{3}$

24. The graph of function $f(x) = \log_e(x^3 + \sqrt{x^6 + 1})$ is symmetric about **[2023]**

(a) X -axis (b) Y -axis (c) origin (d) $y = x$

Ans. (c)

Sol. $f(x) = \log(x^3 + \sqrt{x^6 + 1})$

$f(-x) = \log[(-x)^3 + \sqrt{(-x)^6 + 1}]$

$f(-x) = \log[\sqrt{x^6 + 1} - x^3]$

$= \log \left[\frac{(\sqrt{x^6 + 1} - x^3)(\sqrt{x^6 + 1} + x^3)}{\sqrt{x^6 + 1} + x^3} \right]$

$= \log \left(\frac{1}{x^3 + \sqrt{x^6 + 1}} \right)$

$= \log(x^3 + \sqrt{x^6 + 1})^{-1}$

$= -\log(x^3 + \sqrt{x^6 + 1}) = -f(x)$

Odd function

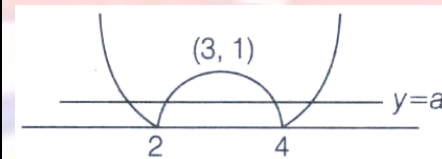
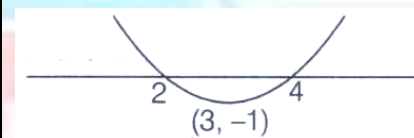
We should know that odd functions are symmetrical about origin.

25. If the equation $|x^2 - 6x + 8| + a$ has four real solution, then find the value of a ? **[2023]**

(a) $a \in 0$ (b) $a = 1$ (c) $a \in (0, 1)$ (d) $a \in [1, 2]$

Ans. (c)

Sol. Let $y = x^2 - 6x + 8$



Hence, for 4 solutions a must lie between $(0, 1)$.

26. Largest value of $\cos^2 \theta - 6 \sin \theta \cos \theta + 3 \sin^2 \theta + 2$ **[2023]**

(a) 4 (b) 0 (c) $4 + \sqrt{10}$ (d) $4 - \sqrt{10}$

Ans. (c)

Sol. $\cos^2 \theta - 6 \sin \theta \cos \theta + 3 \sin^2 \theta + 2$

$= 2 \sin^2 \theta - 6 \sin \theta \cos \theta + 3$

$= 3 - 3 \sin 2\theta + [1 - \cos 2\theta]$

$= 4 - [3 \sin 2\theta + \cos 2\theta]$

$-\sqrt{10} \leq 3 \sin 2\theta + \cos 2\theta \leq \sqrt{10}$

For maximum value of given expression

$3 \sin 2\theta + \cos 2\theta$

It should be minimum.

Hence, maximum value if $4 + \sqrt{10}$

27. Given two events A and B such that odd in favour A are 2: 1 and odd in favour of $A \cup B$ are 3:1. Consistent with this information the smallest and largest value for the probability of event B are given by [2023]

- (a) $\frac{1}{12} \leq P(B) \leq \frac{3}{4}$ (b) $\frac{1}{3} \leq P(B) \leq \frac{1}{2}$
 (c) $\frac{1}{6} \leq P(B) \leq \frac{1}{3}$ (d) None of these

Ans. (a)

Sol. $P(A) = \frac{2}{3}$

$$P(A \cup B) = \frac{3}{4}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{3}{4} = \frac{2}{3} + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(B) - \frac{1}{12}$$

$$0 \leq P(A \cap B) \leq P(A)$$

$$0 \leq P(B) - \frac{1}{12} \leq \frac{2}{3}$$

$$\frac{1}{12} \leq P(B) \leq \frac{2}{3} + \frac{1}{12}$$

$$\frac{1}{12} \leq P(B) \leq \frac{3}{4}$$

28. If A and B are square matrices such that $B = -A^{-1}BA$, then $(A+B)^2$ is [2023]

- (a) 0 (b) $A^2 + B^2$
 (c) $A^2 + 2AB + B^2$ (d) $A+B$

Ans. (b)

Sol. $B = -A^{-1}BA$

$$AB = -(AA^{-1})BA$$

$$AB = -BA$$

$$(A+B)^2 = A^2 + B^2 + AB + BA$$

$$= A^2 + B^2$$

29. A bag contain different kind of balls in which 5 yellow, 4 black and 3 green balls. If 3 balls are drawn at random, then find the probability that no black ball is chosen. [2023]

- (a) $\frac{14}{55}$ (b) $\frac{1}{66}$ (c) $\frac{2}{9}$ (d) None of these

Ans. (a)

Sol. $5Y + 4B + 3G = 12$ balls

No black ball = 8 balls

$$P(\text{No black ball is selected}) = \frac{{}^8C_3}{{}^{12}C_3}$$

$$= \frac{8 \times 7 \times 6}{12 \times 11 \times 10} = \frac{14}{55}$$

30. Between any two real roots of the equation $e^x \sin x = 1$, then equation $e^x \cos x = -1$ has [2023]

- (a) atleast one root (b) exactly one root
 (c) no root (d) None of these

Ans. (a)

Sol. $e^x \sin x = 1$

$$f(x) = \sin x - e^{-x}$$

Let it has 2 real roots α, β .

$$f'(x) = \cos x + e^{-x} = 0$$

Using Rolle's theorem

There will be atleast one real root of its derivative between α and β atleast one $c \in (\alpha, \beta)$

$$f'(c) = 0$$

$$e^{-c} + \cos c = 0$$

$$1 + e^c \cos c = 0 = g(c)$$

$$g(x) = e^x \cos x + 1$$

$$\alpha < c < \beta$$

Atleast one real root of $g(x)$ between two real roots of $f(x)$.

31. If $f(x)$ is a polynomial of degree 4, $f(n) = n+1$ and $f(0) = 25$, then find $f(5) = ?$ [2023]

- (a) 30 (b) 20 (c) 25 (d) None of these

Ans. (a)

Sol. $f(x) \rightarrow 4$ degree polynomial

Let $f(x) = \lambda(x-1)(x-2)(x-3)$

$$(x-4) + (x+1)$$

$$f(0) = \lambda(-1)(-2)(-3)(-4) + 1 = 25$$

$$\Rightarrow \lambda = 1$$

$$f(5) = 6 + 24\lambda \quad [\text{Put } \lambda = 1]$$

$$f(5) = 30$$

32. The maximum value of $f(x) = (x-1)^2(x+1)^3$ is equal to $\frac{2^p 3^q}{3125}$, then the ordered pair of (p, q) will be [2023]

- (a) (3, 7) (b) (7, 3) (c) (5, 5) (d) (4, 4)

Ans. (b)

Sol. $f(x) = (x-1)^2(x+1)^3$

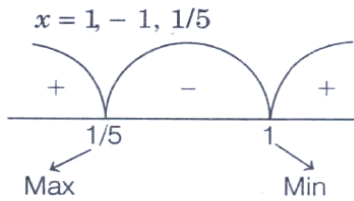
$$f'(x) = 2(x-1)(x+1)^3$$

$$+ 3(x+1)^2(x-1)^2$$

$$= (x-1)(x+1)^2 [2(x+1) + 3(x-1)] = 0$$

$$\Rightarrow (x-1)(x+1)^2 [5x-1] = 0$$

$$x = 1, -1, 1/5$$



$$f\left(\frac{1}{5}\right) = \left(\frac{1}{5}-1\right)^2 \left(\frac{1}{5}+1\right)^3 = \left(-\frac{4}{5}\right)^2 \left(\frac{6}{5}\right)^3$$

$$= \frac{16}{5^2} \times \frac{6^3}{5^3} = \frac{2^7 \times 3^3}{5^5}$$

Hence, $p = 7, q = 3$

33. The coefficient of x^{50} in the expression of $(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \dots + 1001x^{1000}$

[2023]

- (a) $^{1005}C_{50}$ (b) $^{1005}C_{48}$ (c) $^{1002}C_{50}$ (d) $^{1002}C_{51}$

Ans. (c)

Sol. Let $S = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \dots + 1000x^{999}(1+x) + 1001x^{1000} \dots$ (i)

Above is a GP of common ratio

$$r = \frac{x}{1+x}$$

$$\therefore \left[\frac{x}{(1+x)} \right] S = x(1+x)^{999} + 2x^2(1+x)^{998} + \dots$$

$$\dots + 1000 \cdot x^{1000} + \frac{1001x^{1001}}{1+x} \dots$$
 (ii)

Subtracting Eq. (i) from Eq. (ii),

$$\left(1 - \frac{x}{1+x}\right) S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \dots + x^{1000} - \frac{1001x^{1001}}{1+x}$$

Or, $S = (1+x)^{1001} + x(1+x)^{1000} + x^2(1+x)^{999} + \dots + x^{1000}(1+x) - 1001x^{1001}$

$$= \frac{(1+x)^{1001} [1 - (x - (1+x))^{1001}]}{1-x} - 1001x^{1001}$$

Sum of GP $= (1+x)^{1002} \left[1 - \left(\frac{x}{1+x} \right)^{1001} \right] - 1001x^{1001}$

$$= (1+x)^{1002} - x^{1001}(1+x) - 1001x^{1001}$$

$$= (1+x)^{1002} - x^{1002} - 1002x^{1001} \dots$$
 (i)

Now, the coefficients of x^{50} on the R.H.S of Eq. (i) $= {}^{1002}C_{50}$

34. If $x_k = \cos\left(\frac{2\pi k}{n}\right) + i\sin\left(\frac{2\pi k}{n}\right)$ then $\sum_{k=1}^n x_k = ?$ [2023]

- (a) 1 (b) -1 (c) 0 (d) None of these

Ans. (c)

Sol. We should know that $\cos\theta + i\sin\theta = e^{i\theta}$

$$x_k = \cos\left(\frac{2\pi k}{n}\right) + i\sin\left(\frac{2\pi k}{n}\right) = e^{i\frac{2\pi k}{n}}$$

$$\sum_{k=1}^n x_k = \sum_{k=1}^n e^{i\frac{2\pi k}{n}} = e^{i\frac{2\pi}{n}} + e^{i\frac{4\pi}{n}} + e^{i\frac{6\pi}{n}} + \dots + e^{i\frac{2n\pi}{n}}$$

Let $e^{i\frac{2\pi}{n}} = \alpha$

Hence, this series $= \alpha + \alpha^2 + \alpha^3 + \dots + \alpha^n = \frac{\alpha(1-\alpha^n)}{1-\alpha}$

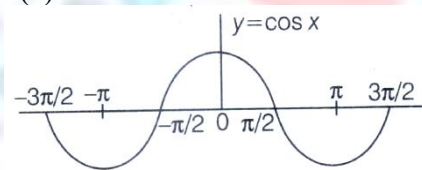
$$= \frac{e^{i\frac{2\pi}{n}} \left(1 - e^{i\frac{2\pi}{n}n} \right)}{1 - e^{i\frac{2\pi}{n}}} = \frac{e^{i\frac{2\pi}{n}} (1 - e^{i2\pi})}{1 - e^{i\frac{2\pi}{n}}} = 0$$

$[\because e^{i2\pi} = \cos 2\pi + i\sin 2\pi = 1]$

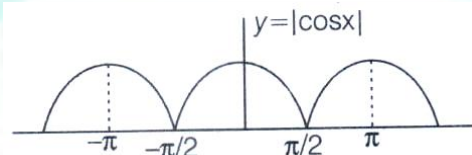
35. Number of points of which $f(x)$ is not differentiable $f(x) = |\cos x| + 3$ in $[-\pi, \pi]$ [2023]

- (a) 2 (b) 3 (c) 4 (d) None of these

Ans. (a)



Sol.



It is not differentiable at two points (We should know the function is not differentiable where there is sharp turns).

$y = |\cos x| + 3$ is not differentiable at $x = -\frac{\pi}{2}, \frac{\pi}{2}$

36. If n_1 and n_2 are the number of real valued solutions $x = |\sin^{-1} x|$ and $x = \sin(x)$ respectively, then the value of $n_2 - n_1$ is

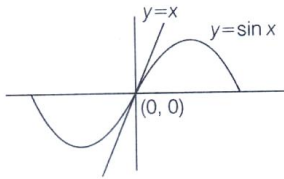
[2023]

- (a) 1 (b) 0 (c) 2 (d) 3

Ans. (b)

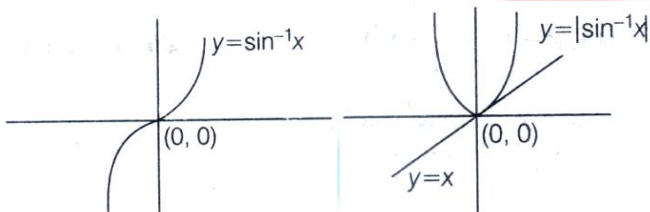
Sol. $x = \sin x$

We should know that $\sin x < x < \tan x$ when $x \in \left(0, \frac{\pi}{2}\right)$



Hence $n_2 = 1$ (It has only one solution that $(0,0)$)

We should know that graph of $f^{-1}(x)$ is mirror image of $f(x)$ with respect to $y=x$ as a mirror.



Number of solutions $n_1 = 1$ (only $(0,0)$ is the solution of on this equation) of Eq. $x = |\sin^{-1} x|$.

Hence, $n_2 - n_1 = 0$.

37. Let a, b, c, d be non zero numbers. If the point of intersection of the line $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistance from the two are then [2023]

- (a) $a + b + c + d = 0$ (b) $ad - bc = 0$
(c) $3bc - 2ad = 0$ (d) $3bc + 2ad = 0$

Ans. (c)

Sol. . If it lies in the fourth quadrant, we get $(x, -x)$

$2ax + c = 0$ and $3bx + d = 0$

$$\frac{c}{2a} = \frac{d}{3b}$$

$$3bc - 2ad = 0$$

38. The negation of $\sim S \vee (\sim R \wedge S)$ is equivalent to

[2023]

- (a) $S \vee (R \vee \sim S)$ (b) $S \wedge \sim R$
(c) $S \wedge R$ (d) $S \wedge (R \wedge \sim S)$

Ans. (c)

Sol. $\sim (\sim S \vee (\sim R \wedge S)) = S \wedge (R \vee \sim S)$

$$= S \cdot (R + \bar{S}) = S \cdot R + S \cdot \bar{S}$$

$$= S \cap R + S \cap \bar{S} = S \cdot R + \phi$$

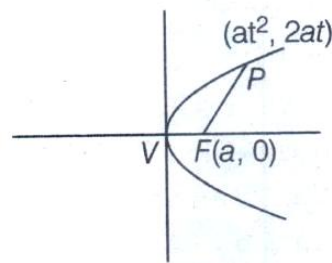
$$= (S \cap R) \cup \phi = S \cap R$$

39. A point P in the first quadrant, lies on $y^2 = 4ax, a > 0$, and keeps a distance of $5a$ units from its focus. Which of the following points lies on the locus of P ? [2023]

- (a) $(1,0)$ (b) $(1,1)$ (c) $(0,2)$ (d) $(2,0)$

Ans. (b)

Sol.



$$FP = 5a \Rightarrow a + at^2 = 5a$$

$$1 + t^2 = 5 \Rightarrow t^2 = 4$$

$$t = \pm 2$$

$$t > 0 \Rightarrow t = 2$$

Hence $P \equiv (4a, 4a)$

From given options only $(1,1)$ satisfy P .

40. If $\int x \sin x \sec^3 x dx = \frac{1}{2} \left[f(x) \sec^2 x + g(x) \left(\frac{\tan x}{x} \right) \right] + c$

then which of the following is true [2023]

- (a) $f(x) - g(x) = 0$ (b) $f(x) \cdot g(x) = 0$
(c) $f(x) + g(x) = 0$ (d) $f(x) + g(x) = 1$

Ans. (c)

Sol. $\int x \sin x \sec^3 x dx = \int x \tan x \sec^2 x dx$.

Using integration by part

$$= \left[x \int \sec^2 x \tan x dx \right] - \int \left(\frac{dx}{dx} \int \sec^2 x \tan x dx \right) dx$$

$$= \left[x \frac{\tan^2 x}{2} \right] - \int \frac{\tan^2 x}{2} dx$$

$$= \frac{1}{2} \left[x \tan^2 x - \int (\sec^2 x - 1) dx \right]$$

$$= \frac{1}{2} \left[x \tan^2 x - \tan x + x \right] + c$$

$$= \frac{1}{2} \left[x(1 + \tan^2 x) - \tan x \right] + c$$

$$= \frac{1}{2} [x \sec^2 x - \tan x] + c$$

$$= \frac{1}{2} \left[f(x) \sec^2 x + g(x) \left(\frac{\tan x}{x} \right) \right] + c$$

Hence, $f(x) = x; g(x) = -x$

Hence, $f(x) + g(x) = 0$

41. $\theta = \cos^{-1} \left(\frac{3}{\sqrt{20}} \right)$ is the angle between

$\mathbf{a} = \mathbf{i} - 2x\mathbf{j} + 3y\mathbf{k}$ and $\mathbf{b} = x\mathbf{i} + \mathbf{j} + y\mathbf{k}$ then possible values at (x, y) that lie on the locus [2023]

(a) (0,1) (b) (1,0) (c) (1,1) (d) (0,0)

Ans. (a)

Sol. $\mathbf{a} = \mathbf{i} - 2x\mathbf{j} + 3y\mathbf{k}$

and $\mathbf{b} = x\mathbf{i} + \mathbf{j} + y\mathbf{k}$

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} \Rightarrow \cos \cos^{-1} \left(\frac{3}{\sqrt{20}} \right) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

$$\Rightarrow \frac{3}{\sqrt{20}} = \frac{x - 2x + 3y^2}{\sqrt{1 + 4x^2 + 9y^2} \sqrt{x^2 + 1 + y^2}}$$

$$\Rightarrow 3\sqrt{1 + 4x^2 + 9y^2} \sqrt{x^2 + 1 + y^2} = \sqrt{20} [3y^2 - x]$$

From given option (0,1) satisfy given equations.

42. Let R be reflexive relation on the finite set a having 10 elements and if m is the number of ordered pair in R , then [2023]

(a) $m \geq 10$ (b) $m = 100$ (c) $m = 10$ (d) $m \leq 10$

Ans. (a)

Sol. Given R has m order pairs.

Since, R is reflexive relation on A , therefore

$$(a, a) \in R \forall a \in A$$

Then, the minimum number of ordered pairs in R is 10.

Therefore, $m \geq 10$

43. If $|x - 6| = |x^2 - 4x| - |x^2 - 5x + 6|$, where x is a real variable. [2023]

(a) $x = (2, 5)$ (b) $x \in [2, 3] \cup [6, \infty)$

(c) $R - [2, 6]$ (d) None of these

Ans. (b)

$$\text{Sol. } |x - 6| = |x^2 - 4x| - |(x - 3)(x - 2)|$$

Its one solution is

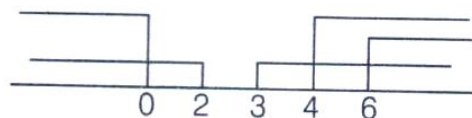
$$x(x - 4) \geq 0$$

$$\text{and } x^2 - 5x + 6 \geq 0$$

$$x \geq 6$$

And $x \leq 0$ or $x \geq 4$

$$x \leq 2 \text{ and } x \geq 3$$

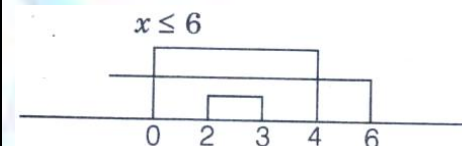


Its intersection $x \geq 6 \Rightarrow x \in [6, \infty)$

Its second solution $x(x - 4) \leq 0$ and $x^2 - 5x + 6 \leq 0$

$$0 \leq x \leq 4 \text{ and } 2 \leq x \leq 3 \text{ and } x < 6$$

And



Its intersection is $2 \leq x \leq 3 \Rightarrow x \in [2, 3]$

Union of both solutions is $x \in [2, 3] \cup [6, \infty)$

44. The range of values of θ in the interval $(0, \pi)$ such that the points $(3, 2)$ and $(\cos \theta, \sin \theta)$ lie on the same sides of the line $x + y - 1 = 0$, is [2023]

(a) $\left(0, \frac{3\pi}{4}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$ (c) $\left(0, \frac{\pi}{3}\right)$ (d) $\left(0, \frac{\pi}{4}\right)$

Ans. (b)

Sol. $L: x + y - 1 = 0$

As $(3, 2)$ and $(\cos \theta, \sin \theta)$ lies on same side of line

$$L: (3, 2) 3 + 2 - 1 = 4 > 0$$

$$\text{So, } L: (\cos \theta, \sin \theta) \equiv \sin \theta + \cos \theta - 1 > 0$$

$$\Rightarrow \sqrt{2} \left[\frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta \right] > 1$$

$$\sin \left(\theta + \frac{\pi}{4} \right) > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4} \Rightarrow 0 < \theta < \frac{\pi}{2}$$

45. Which of the following number is the coefficient of x^{100} in the expansion of $\log_e \left(\frac{1+x}{1+x^2} \right), |x| < 1$?

[2023]

(a) 0.01 (b) 0.02 (c) -0.03 (d) -0.01

Ans. (a)

Sol. $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots - \frac{x^{100}}{100} + \dots$

$$\ln(1+x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \dots - \frac{(x^2)^{50}}{50}$$

Coefficient of x^{100} in $\ln(1+x) - \ln(1+x^2)$

$$= -\frac{1}{100} + \frac{1}{50} = 0.01$$

46. A real valued function f is defined as

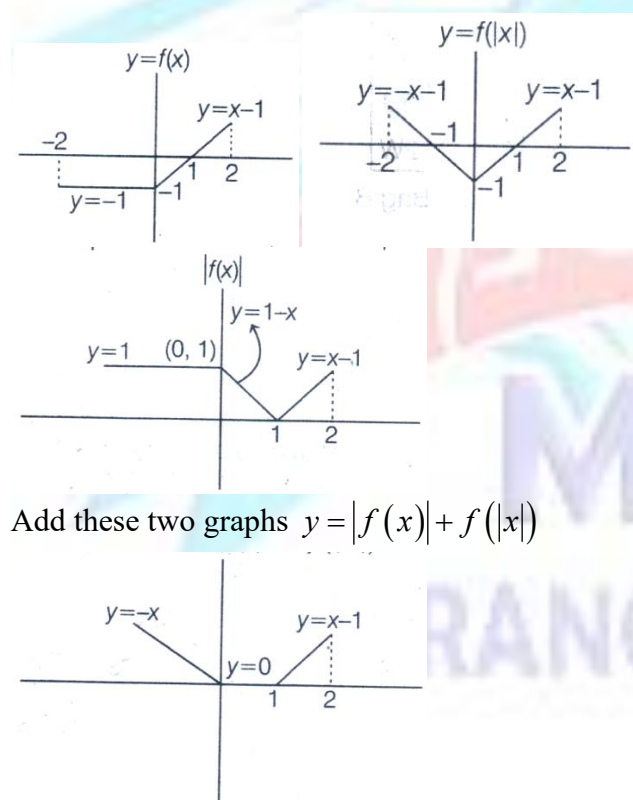
$$f(x) = \begin{cases} -1 & -2 \leq x \leq 0 \\ x-1 & 0 \leq x \leq 2 \end{cases}$$

Which of the following statement is FALSE? [2023]

- (a) $f(|x|) = |x| - 1$, if $0 \leq x \leq 1$
 (b) $|f(x)| = x - 1$, if $1 \leq x \leq 2$
 (c) $f(|x|) + |f(x)| = 1$, if $0 \leq x \leq 1$
 (d) $f(|x|) - |f(x)| = 0$, if $1 \leq x \leq 2$

Ans. (c)

Sol. Graphical solution



From given graphs we can find that statement (c) is false.

47. A line segment AB of length 10 m is passing through the foot of the perpendicular of a pillar, which is standing at right angle to the ground. Top of the pillar subtends angles $\tan^{-1}3$ and $\tan^{-1}2$ at A and B respectively. Which of the following choice represents

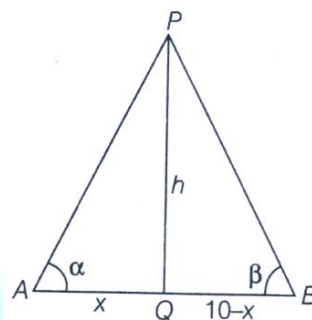
the height of the pillar?

[2023]

- (a) 10 m (b) 8 m (c) 12 m (d) 15 m

Ans. (c)

Sol.



Let

$$\alpha = \tan^{-1}3 \Rightarrow \tan \alpha = 3$$

$$\text{and } \beta = \tan^{-1}2 \Rightarrow \tan \beta = 2$$

In $\triangle APQ$ $\tan \alpha = \frac{h}{x} = 3 \Rightarrow x = \frac{h}{3}$

In $\triangle BPQ$ $\tan \beta = \frac{h}{10-x} = 2 \Rightarrow h = 20 - 2x$
 $\Rightarrow h = 20 - 2h/3 \Rightarrow h = 12 \text{ m}$

48. If a vector having magnitude of 5 units, makes equal angle with each of the three mutually perpendicular axes, then the sum of the magnitude of the projections on each of the axis is [2023]

- (a) $15/3$ unit (b) $5\sqrt{3}$ unit
 (c) $\frac{15\sqrt{3}}{2}$ units (d) None of these

Ans. (b)

Sol. $|v| = 5$

As v makes equal angle with all the three axis.

$$v = x\hat{i} + x\hat{j} + x\hat{k}$$

$$|v| = \sqrt{3}x = 5$$

$$x = \frac{5}{\sqrt{3}}$$

Sum of component on X, Y, Z axis

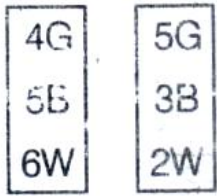
$$x + x + x = 3\left(\frac{5}{\sqrt{3}}\right) = 5\sqrt{3}$$

49. Box A contains 4 green, 5 black and 6 white balls. Box B contains 5 green, 3 black and 2 white balls. A ball is chosen at random from the box A and placed in Box B and then a ball is chosen at random from box B . What is the probability that a black ball was transferred from box A to box B given that the final ball chosen from box B is green? [2023]

- (a) $30/79$ (b) $25/79$
 (c) $24/79$ (d) None of these

Ans. (b)

Sol.



Bag A Bag B

B : Black ball is drawn from bag A

G : Green ball drawn from bag B

$$P\left(\frac{B}{G}\right) = \frac{P(B \cap G)}{P(G)}$$

$$= \frac{\frac{5}{15} \times \frac{5}{11}}{\frac{5}{15} \times \frac{5}{11} + \frac{4}{15} \times \frac{6}{11} + \frac{6}{15} \times \frac{5}{11}} = \frac{25}{79}$$

50. Let $f(x) = \frac{(x^2 - 1)}{(|x| - 1)}$. Then the value of $\lim_{x \rightarrow -1} f(x)$ is

[2023]

- (a) -1 (b) 1 (c) 2 (d) 3

Ans. (c)

Sol. $\lim_{x \rightarrow -1} \frac{x^2 - 1}{(|x| - 1)} = \lim_{x \rightarrow -1} \frac{(x-1)(x+1)}{-(x+1)} = 2$

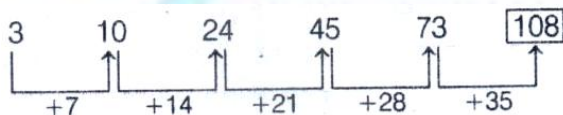
Analytical Ability & Logical Reasoning

51. Complete the series: 3, 10, 24, 45, 73 ... [2023]

- (a) 69 (b) 91 (c) 108 (d) 121

Ans. (c)

Sol. Pattern of the series is as follows,



This series consists of a pattern of addition of multiple of 7.

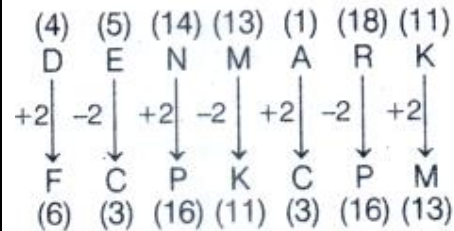
52. In a certain language, DENMARK is coded as FCPKCPM. Find the code of SINGAPORE.

[2023]

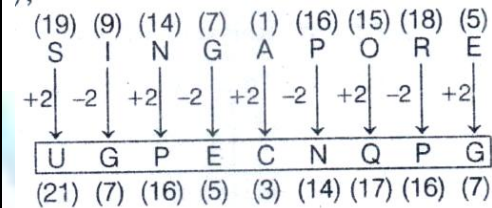
- (a) UGPECNQPG (b) UGPFCNRPG
(c) UGPEDNQTC (d) UGPECNQTC

Ans. (a)

Sol. As,



Similarly,



53. On Monday, Akash ran 4 km less than the distance he ran on Tuesday. Sanjay, who ran the same distance on Monday and Tuesday, ran 5 km more on Tuesday than the distance Akash ran on Monday. Find the difference between the distances covered by Akash and Sanjay over the two days. [2023]

- (a) 5 km (b) 6 km (c) 4 km (d) 9 km

Ans. (b)

Sol. Let Akash ran x km on Tuesday.

So, Akash ran $(x - 4)$ km on Monday.

Distance covered by Sanjay on Monday = Distance covered by Sanjay on Tuesday

$$= x - 4 + 5 = (x + 1) \text{ km}$$

Difference between the distances covered by Akash and Sanjay over two days = $(2x + 2) - (2x - 4)$

$$= 2x + 2 - 2x + 4 = 6 \text{ km}$$

Direction (Q. Nos. 54) Read the statements and conclusions carefully and decide which of the conclusion follows from the statements.

54. Statements

No table is chair.

Not a single chair is stand.

Every stand is statue.

Conclusions

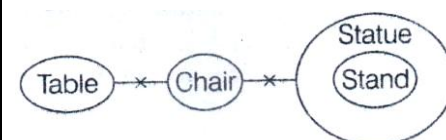
I. Some statue which are stand are table as well

II. Some statue are not chair. [2023]

- (a) If neither Conclusion I nor II follow
(b) If only Conclusion I follows
(c) Only Conclusion II follows
(d) None follows

Ans. (c)

Sol. According to the given statements,



Conclusions I. (X) II. ($\sqrt{\quad}$)
Hence, only Conclusion II follows.

55. A can do a work in 10 days, B can do the same work in 20 days. After working for 4 days B left the job and C started working at the place of B . The remaining work was done by both A and C in 2 days. In how many days C alone can do the work?[2023]

- (a) 15 days (b) 10 days
(c) 12 days (d) 20 days

Ans. (b)

Sol. A can do a work in 10 days.

$$\text{So, work done by } A \text{ in 1 day} = \frac{1}{10}$$

B can do a work in 20 days.

$$\text{So, work done by } A \text{ in 1 day} = \frac{1}{20}$$

Let, C can do a work in x days.

$$\text{So, work done by } C \text{ in 1 day} = \frac{1}{x}$$

According to the question,

$$\begin{aligned} 1 - \left(\frac{1}{10} + \frac{1}{20} \right) \times 4 &= \left(\frac{1}{10} + \frac{1}{x} \right) \times 2 \\ \Rightarrow \frac{8}{20} &= \left(\frac{1}{10} + \frac{1}{x} \right) \times 2 \\ \Rightarrow \frac{8}{40} - \frac{1}{10} &= \frac{1}{x} \Rightarrow \frac{4}{40} = \frac{1}{x} \\ \Rightarrow x &= \frac{40}{4} \Rightarrow x = 10 \end{aligned}$$

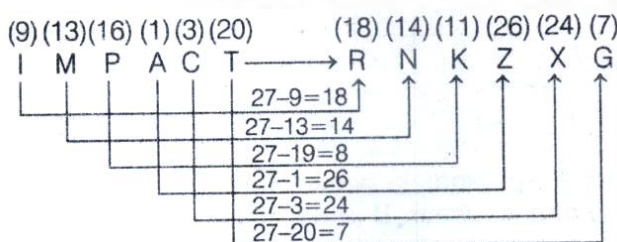
So, C alone can do the work in 10 days.

56. If the word IMPACT is coded as RNKZXG, then which of the following represents the code for the word DESCEND? [2023]

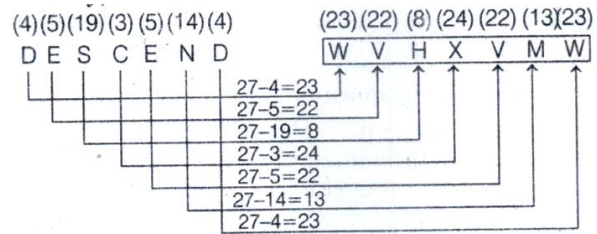
- (a) WVHXVMW (b) MNBLNWM
(c) MFBDFOM (d) MFBDNOM

Ans. (a)

Sol. Here, each letter is coded by its opposite letter.
As,



Similarly,



57. A 30 L mixture of milk and water contains 10% water. How much milk should be added so that the percentage of water in the mixture comes down to 2%? [2023]

- (a) 120 L (b) 80 L (c) 90 L (d) 60 L

Ans. (a)

Sol. In 30 L of mixture,

Water = 10% of 30 L = 3 L

Let milk to be added = x L

$$\begin{aligned} \frac{3}{30+x} \times 100 &= 2 \Rightarrow 60 + 2x = 300 \\ \Rightarrow x &= \frac{300-60}{2} = 120 \text{ L} \end{aligned}$$

58. If yellow is called white, white is called black, black is called green, green is called pink, pink is called blue and blue is called water, what is the colour of sky? [2023]

- (a) Blue (b) Water (c) Black (d) White

Ans. (b)

Sol. We know that, the colour of sky is 'blue' but here 'blue' is called 'water'. So, the colour of sky will be 'water'.

59. A junior school is offering five after-school activities Karate, Handwork, Music, Dance and Gymnastics. Each of the five students - Leena, Megan, Neha, Omar, and Pixie, has subscribed to at least one activity. As per the school rules, anyone who subscribes to Gymnastics must also subscribe to Dance. Karate and Handwork must always be subscribed together. Music and Dance cannot be subscribed together. The following information is available about the students' subscriptions.

- Megan subscribed to four activities.
- Leena subscribed to Gymnastics but not Karate.
- Pixie subscribed to only one activity and is the only one to subscribe to that activity.
- Omar subscribed to three activities.
- Neha subscribed to only one activity.

How many activities are subscribed by exactly two people each? [2023]

- (a) 1 (b) 4 (c) 2 (d) 3

Ans. (d)

Sol. According to the question,

Karate Handwork Music Dance Gymnastic

Megan	✓	✓	×	✓	✓
Leena	×	×	×	✓	✓
Pixie	×	×	✓	×	×
Omar	✓	✓	×	✓	×
Neha	×	×	×	✓	×

It can clearly seen that there are three activities (Karate, Handwork and Gymnastic) subscribed by exactly two people each.

60. If 'E' stands for '+', 'F' stands for '-', 'M' stands for 'x', 'N' stands for '÷', then $19 M 5 E 39 N 3 F 8 = ?$

[2023]

(a) 105 (b) 100 (c) 95 (d) 90

Ans. (b)

Sol Given, $19M5E39N3F8 = ?$

Replacing the proper signs in the given expression, we have

$$19 \times 5 + 39 \div 3 - 8 = ?$$

$$19 \times 5 + 13 - 8 = ?$$

$$\Rightarrow 95 + 13 - 8 = ?$$

[using BODMAS rule]

$$\Rightarrow = 100.$$

61. In a reality show, two judges independently provided marks based on the performance of the participants. If the marks provided by the second judge are given by $y = 1 + x$, where x is the marks provided by the first judge. Then, for a participant [2023]

- (a) Ranks given by both the judges differ by 2
 (b) Rank given by the second judge is more than that of the first judge
 (c) Ranks given by both the judges are same
 (d) Rank given by the first judge is more than that of the second judge

Ans. (b)

Sol. According to the question,

Given that, marks provided by the second judge are given by $y = 1 + x$,

It means the marks given by y is 1 more than x .

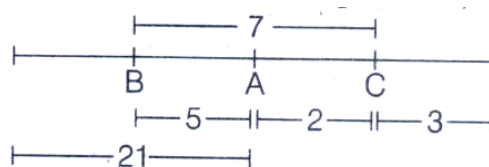
So, ranks given by the second judge is more than that of the first judge.

62. Three persons A, B and C are standing in a queue. There are five persons between A and B and eight persons between B and C . If there are three persons ahead of C and 21 behind A , then what could be the minimum number of persons in the queue? [2023]

(a) 40 (b) 27 (c) 28 (d) 41

Ans. (c)

Sol. According to the question,



There are 5 persons between A and B , 7 persons between B and C , 2 persons between A and C , 3 persons after C , and 21 persons behind A .
 So, total number of persons in a queue
 $= 21 + A + 2 + C + 3 = 28$

63. In the given word "LAVISHLY" if all the consonants replaced with its previous letter and all the vowels replaced with its next letter after that remove all the repeated letter and arranged them in alphabetical order then, which of the following letters is 3rd from the left end? [2023]

(a) G (b) R (c) J (d) U

Ans. (c)

Sol. According to the question, After all the consonants replaced with previous letter and all the vowels replaced with next letter, we get

KBUJRGKX

After removing all repeated letters and arrange in alphabetical order

(Left) B G J R U X (Right) J will be the 3rd letter from the left.

64. A sum of money distributed among four persons P, Q, R, S in the proportion $2:5:4:3$. If Q gets ₹2000 more than S , then what will be the total amount? [2023]

(a) ₹ 14000 (b) ₹ 18000
 (c) ₹ 16000 (d) ₹ 15000

Ans. (a)

Sol. A sum of money is to be distributed among P, Q, R, S in the proportion $2:5:4:3$.

Let the share of P, Q, R and S be ₹ $2x$, ₹ $5x$, ₹ $4x$ and ₹ $3x$ respectively.

According to the question,

$$5x - 3x = 2000$$

$$2x = 2000$$

$$\Rightarrow x = 1000$$

$$P \text{ share} = ₹ 1000 \times 2 = ₹ 2000$$

$$Q \text{ share} = ₹ 1000 \times 5 = ₹ 5000$$

$$R \text{ Share} = ₹ 1000 \times 4 = ₹ 4000$$

$$S \text{ Share} = ₹ 1000 \times 3 = ₹ 3000$$

$$\text{Total amount} = (2000 + 5000 + 4000 + 3000) \\ = ₹ 14000$$

65. Book : Publisher :: Film : ? [2023]

(a) Director (b) Producer
 (c) Writer (d) Editor

Ans. (b)

Sol. As, a publisher publish a book. Similarly, a producer produce a film.

66. Arrange the words given below in a meaningful sequence. [2023]

- (1) Database (2) Analysis
(3) Survey (4) Policy formulation
(5) Interpretation
(a) 2,1,5,3,4 (b) 5,4,3,1,2
(c) 3,1,2,4,5 (d) 3,1,2,5,4

Ans. (d)

Sol. After arranging the words in meaningful sequence, we get

Survey → Database → Analysis → Interpretation
→ Policy formation So, the sequence will be
3,1,2,5,4.

67. In the half-yearly exam only 60% of the students were passed. Out of these (passed in half-yearly) only 70% students are passed in annual exam, out of remaining students (who fail in half-yearly exam) 80% passed in annual exam. What per cent of the students passed the annual exam? [2023]

- (a) 72% (b) 76% (c) 65% (d) 74%

Ans. (d)

Sol. Let the total student be x .

Given that, 60% students passed in half yearly examination.

Then, passed student in half year

$$y = \frac{60}{100} \times x = 0.60x$$

Out of these passed in half-yearly, 70% students passed in annual exam.

Then, passed students in annual examination

$$= \frac{70}{100} \times 0.60x = 0.42x$$

The number of students who did not pass in half-yearly examination

$$= x - 0.60x = 0.40x$$

Given, out of these who didn't pass the half yearly exam, 80% students passed in annual exam.

Then, out of these students who did not pass in half-yearly examination, number of students who passed in

$$\text{annual exam} = \frac{80}{100} \times 0.40x = 0.32x$$

Then, total number of students who passed annual

$$\text{exam} = 0.42x + 0.32x = 0.74x$$

Therefore, percentage of students who passed annual

$$\text{exam} = \frac{0.74x}{x} \times 100 = 74\%$$

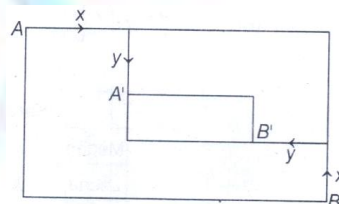
Hence, 74% students passed the annual exam.

68. Two friends A and B were standing at the diagonally opposite corners of a rectangular plot whose perimeter is 100 m. A first walked x m along the length of the plot towards East and then y m towards the South. B walked x m along the breadth towards North and then y m towards West. At the end of their walks, A and B were standing at the diagonally opposite corners of a smaller rectangular plot whose perimeter is 40 m. How much distance did A walk? [2023]

- (a) 15 (b) 25 (c) 40 (d) 50

Ans. (a)

Sol. According to the question,



$$2(A + B) = 100$$

$$\text{and } 2(A' + B') = 40$$

$$A + B = 50$$

$$\text{and } A' + B' = 20$$

$$\text{Now, } A + B = 50$$

$$\Rightarrow (x + y + A' + x + y + B') = 50$$

$$\Rightarrow 2x + 2y + A' + B' = 50$$

$$\Rightarrow 2x + 2y + 20 = 50$$

$$\Rightarrow 2(x + y) = 30$$

$$\therefore x + y = 15$$

Hence, A walked 15 m of distance.

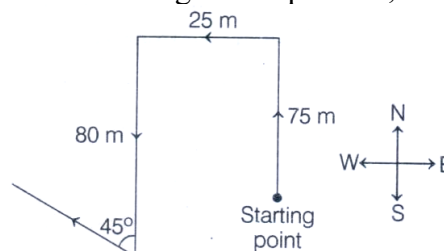
69. Deepa moved a distance of 75 m towards the North. She then turned to the left and after walking for about 25 m, turned left again and walked 80 m.

Finally, she turned to the right at an angle of 45° . In which direction was she moving finally? [2023]

- (a) North-East (b) North-West
(c) South-West (d) South-East

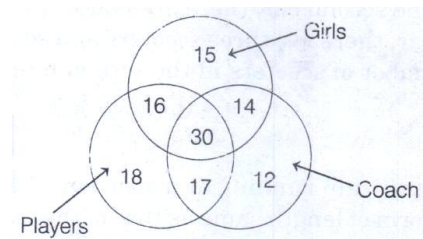
Ans. (c)

Sol. According to the question,



It can be clearly seen that Deepa is moving in South-West direction.

70. Study the diagram and answer the question that follows:



How many players are there who are girls but not a coach? [2023]

- (a) 16 (b) 31 (c) 15 (d) 61

Ans. (a)

Sol. According to the venn diagram, Girls who are plays but not a coach is represented by the number common to only girl's and player's circle, which is 16 .

71. How many meaningful words can be formed with the first, the third, the seventh, and the ninth letters of the word SEPARATION using each letter only once in each word? [2023]

- (a) 4 (b) More than 4 (c) 3 (d) 2

Ans. (b)

Sol. First, third, seventh, and ninth letters of word separation are S, P, T, O.

Possible meaningful words = STOP, SPOT, POST, POTS, OPTS, TOPS

Hence, there are more than 4 meaningful words.

72. Fill the blanks with the most appropriate combination of options.

Further, to augment bond market liquidity, corporates need to be encouraged to _____ exiting bonds under the same International Securities Identification Number, to duly shore up floating _____. [2023]

- (a) affect, negotiate (b) abandon, imply
(c) precaution, abstract (d) reissue, stocks

Ans. (d)

Sol. Reissue, stocks

73. A vessel contains total 95 L mixture of milk and water in the ratio of 15: 4 respectively. P L of mixture taken out from the vessel and 18L water added in remaining mixture, then the new ratio of milk to water becomes 3:2, find the value of P . [2023]

- (a) 57 (b) 19 (c) 27.5 (d) 38

Ans. (d)

Sol. Total milk in mixture initially = $95 \times \frac{15}{19} = 75$ L

Total water in mixture initially

$$= 95 \times \frac{4}{19} = 20 \text{ L}$$

According to the question,

$$\frac{75 - P \times \frac{15}{19}}{20 - P \times \frac{4}{19} + 18} = \frac{3}{2}$$

$$\therefore P = 38$$

Hence, 38 L of mixture was taken out from the vessel.

74. Rakesh says to Mahesh, "I am as old as you were when I was one-third as old as you are." If the sum of their ages is 60 yr, find the present age of Mahesh (in years). [2023]

- (a) 45 (b) 36 (c) 30 (d) 24

Ans. (b)

Sol. Let Rakesh and Mahesh's present age are x yr and y yr, respectively.

$$x + y = 60 \dots (i)$$

Let k years ago,

$$\frac{y}{3} = x - k \dots (ii)$$

$$y - k = x \dots (iii)$$

$$\Rightarrow y - x = k \dots (iii)$$

Putting the value of k from Eq (iii) in Eq (ii),

$$\frac{y}{3} = x - y + x$$

$$\Rightarrow y = 6x - 3y$$

$$\Rightarrow 4y = 6x$$

$$\Rightarrow 2y = 3x \dots (iv)$$

From Eq. (i),

$$2x + 2y = 2 \times 60$$

$$\Rightarrow 2x + 3x = 120 \text{ [form Eq (iv)]}$$

$$\Rightarrow x = \frac{120}{5} = 24 \text{ yr}$$

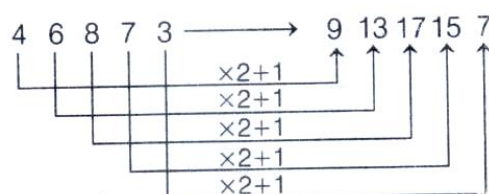
$$y = 36 \text{ [form Eq (i)]}$$

75. For security reasons, a bank manager decided to encrypt the account number in the server. If A/C No. 46873 is coded as 91317157, then 52191 is coded as [2023]

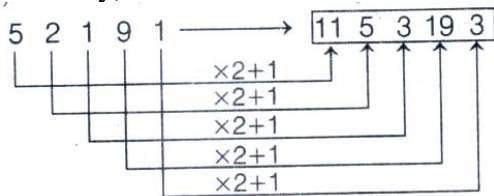
- (a) 4108041 (b) 5219152
(c) 1153193 (d) 1043293

Ans. (c)

Sol. As,



Similarly,



76. If 3 is subtracted from the middle digit of each of the following numbers and then the position of the digits are reversed, which of the following will be last digit of the middle number after they are arranged in descending order? [2023]

589 362 554 371 442

- (a) 3 (b) 2 (c) 4 (d) 1

Ans. (a)

Sol. After subtracting 3 from middle digit of each, we get 559 332 524 341 412

Reversed position $\Rightarrow$ 955 233 425 143 214

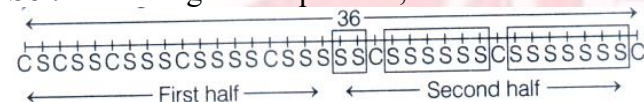
Arranged in descending order $\Rightarrow$ 955 425 233 214 143
Hence, 3 is the last digit of the middle number

77. Thirty-six vehicles are parked in a parking-lot in a single row. After the first car, there is one scooter. After the second car, there are two scooters. After the third car, there are three scooters and so on. Work out the number of scooters in the second half row. [2023]

- (a) 15 (b) 12 (c) 10 (d) 17

Ans. (a)

Sol. According to the question,



Hence, there are 15 scooters in the second half of the row.

78. Rajdhani train running at a speed of 54 km/h crosses a platform of length same as that of the train in 36 sec. If a Duronto train, which is 230 m long crosses the same platform in 25 sec, then find speed of Duronto train (in km/h)? [2023]

- (a) 54 km/h (b) 72 km/h
(c) 84 km/h (d) 90 km/h

Ans. (b)

Sol. Let length of Rajdhani train be x .
Then, the length of platform is also x .

Speed of train = 54 km/h

$$= 54 \times \frac{5}{18} \text{ m/s}$$

$$= 15 \text{ m/s}$$

According to the question,

$$= \frac{(x+x)}{15} = 36$$

$$\Rightarrow 2x = 540$$

$$x = 270 \text{ m}$$

A Duronto train of length 230 m crosses the platform in 25 s.

Let the speed of train be V m/s.

$$\Rightarrow \frac{270+230}{V} = 25$$

$$\Rightarrow V = 20 \text{ m/s}$$

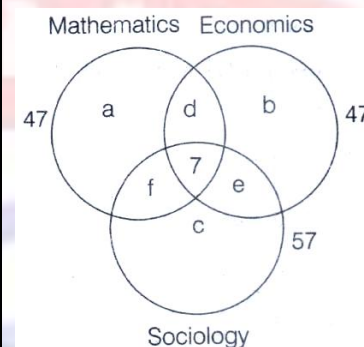
Speed of train = 20 m/s

79. A university is offering elective courses in Mathematics, Economics and Sociology. Each of its 100 undergraduate students has to opt for at least one of these electives. Course enrollment data showed that 47 students enrolled for Mathematics, 47 students enrolled for Economics and 57 students enrolled for Sociology. If 7 students enrolled for all three courses, how many students enrolled for exactly one course? [2023]

- (a) Cannot be determined (b) 58
(c) 56 (d) 60

Ans. (c)

Sol. From the given information,
Here,



$$a + b + c + d + e + f = 100 - 7 = 93 \dots (i)$$

$$a + d + b = 100 - 57 = 43 \dots (ii)$$

$$b + e + c = 100 - 47 = 53 \dots (iii)$$

$$a + f + c = 100 - 47 = 53 \dots (iv)$$

Adding Eqs. (ii), (iii) and (iv), we get

$$2a + 2b + 2c + d + e + f = 149 \dots (v)$$

From Eq. (v) Eq. (i), we get

$$a + b + c = 149 - 93 = 56$$

So, number of students enrolled for exactly one course = $a + b + c = 56$.

80. Hari invested an amount at a certain rate of interest on simple interest and he got 60% more amount after eight years. If he invests ₹ 9600 at the same rate of interest on simple interest, then find the total interest

he would get after four years?

[2023]

- (a) ₹ 2520 (b) ₹ 2260
(c) ₹ 2880 (d) ₹ 2160

Ans. (c)

Sol. Hari gets 60% more amount after 8 yr .

Let principal amount be = ₹ P.

$$\Rightarrow \text{Amount} = ₹ \frac{160}{100} P$$

$$SI = \text{Amount} - \text{Principal}$$

$$= \frac{160}{100} P - P = \frac{60P}{100}$$

$$SI = \frac{P \times R \times T}{100} \Rightarrow \frac{60P}{100} = \frac{P \times R \times 8}{100}$$

$$\Rightarrow R = \frac{60}{8} \Rightarrow R = 7.5$$

He invests ₹ 9600 .

$$SI \text{ after } 4\text{yr} = \frac{9600 \times 7.5 \times 4}{100} \Rightarrow SI = ₹ 2880$$

∴ Simple interest received by the man is ₹2880.

81. Find a number from the given options which best completes the series [2023]

39, 416, 525, 749, 864

- (a) 439 (b) 636 (c) 436 (d) 644

Ans. (b)

Sol. The series follow the following logic,
The last digits of a number are square of the first digit.

As,

$$39 \Rightarrow 3^2 = 9$$

$$416 \Rightarrow 4^2 = 16$$

$$525 \Rightarrow 5^2 = 25$$

Similarly,

This logic is followed in option (b) only.

$$636 \Rightarrow 6^2 = 36$$

$$749 \Rightarrow (7)^2 = 49$$

$$864 \Rightarrow (8)^2 = 64$$

Hence, the correct answer is optc. (b)

82. Ritu purchased 20 dozen of bananas at the rate of ₹ 375 per dozen. She sold each one of them at the rate of ₹ 33 . What was her profit percentage? [2023]

- (a) 12.3% (b) 5.6% (c) 6.5% (d) 10%

Ans. (b)

Sol. CP of 20 dozen bananas = 375×20
= ₹ 7500

$$SP \text{ of } 20 \text{ dozen bananas} = 33 \times 12 \times 20$$

$$= ₹ 7920$$

$$\therefore \text{Profit} = 7920 - 7500$$

$$= ₹ 420$$

$$\therefore \text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$= \frac{420 \times 100}{7500} = 5.6\%$$

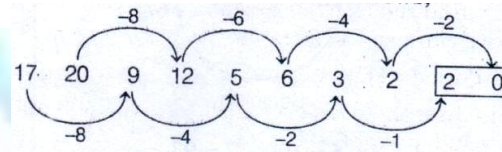
83. Identify the next two numbers in the following sequence: [2023]

17, 20, 9, 12, 5, 6, 3, 2, _____, _____

- (a) 2, 1 (b) 0, 2 (c) 1, 2 (d) 2, 0

Ans. (d)

Sol. The pattern of the series is as follows,



84. Read the statements and conclusions carefully and decide which of the conclusions follow from the statements. [2023]

Statements

Most teachers are boys.

Some boys are students.

Conclusions

I. Some students are boys.

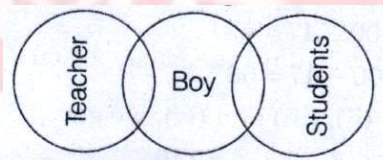
II. Some teachers are students.

(a) Neither I nor II follow (b) Either I or II follow

(c) Only II follows (d) Only I follows

Ans. (d)

Sol. According to the following statements,



Conclusions

I. (✓) II. (×).

Hence, only Conclusion I follows.

85. If $20-10$ means 200 , $8 \div 4$ means 12 , 6×2 means 4 , then $100-10 \times 1000 \div 1000 + 100 \times 10 = ?$

[2023]

- (a) 0 (b) 20 (c) 1090 (d) None of these

Ans. (a)

Sol. Here, $20-10$ means 200

So, '-' means 'x'

$8 \div 4$ means 12

So, ' $\div$ ' means '+'

6×2 means 4

So, 'x' means '-'

Given,

$$100-10 \times 1000 \div 1000 + 100 \times 10 = ?$$

Replacing the proper signs in the expression, we have

$$100 \times 10 - 1000 + 1000 \div 100 - 10 = ?$$

Using BODMAS rule,

$$\Rightarrow 100 \times 10 - 1000 + 10 - 10 = ?$$

$$\Rightarrow 1010 - 1010 = ?$$

$$? = 0$$

86. Which word among the following words can be formed from the letters of "RECOMMENDATION"?

[2023]

- (a) REMINDER (b) COMMUNICATE
(c) MEDICINE (d) MEDIATE

Ans. (d)

Sol. MEDIATE can be formed from the word RECOMMENDATION.

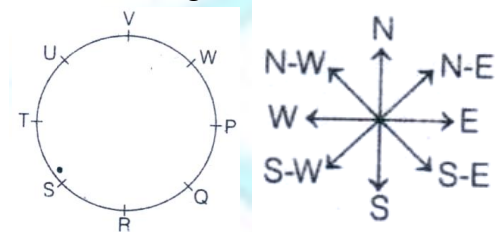
87. P, Q, R, S, T, U, V and W are sitting around a round table in the same order, for group discussion at equal distances. Their positions are clockwise. If V sits in the North, then what will be the position of S?

[2023]

- (a) South-East (b) South-West
(c) South (d) East

Ans. (b)

Sol. According to the question,



It can be clearly seen that S is in South-West direction.

88. Shweta, Yamini, Pooja and Rakhi are sisters. Yamini is older than at least two of her sisters. Pooja is younger than Rakhi, who is not the oldest. Shweta is older than Pooja but younger than Rakhi. Who is the oldest among them?

[2023]

- (a) Pooja (b) Yamini (c) Shweta (d) Rakhi

Ans. (b)

Sol. According to the question,
Yamini > Rakhi > Shweta > Pooja
So, Yamini is the oldest among them.

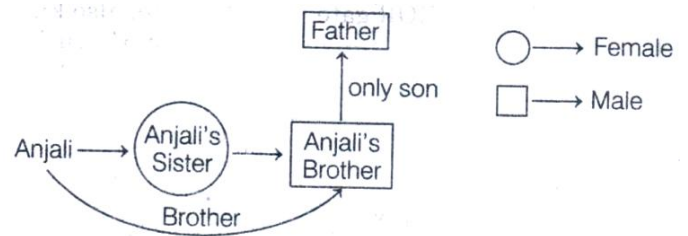
89. Pointing towards a person in the photograph, Anjali said, "He is the only son of the father of my sister's brother." How is that person related to Anjali?

[2023]

- (a) Mother (b) Father
(c) None of these (d) Cousin

Ans. (c)

Sol. According to the question,



So, that person is the "Brother" of Anjali.
Hence, none of these is the correct answer.

90. Hemant deposits 10% of his salary in PF. He saves 30% of the remaining salary. The ratio of his expense on medicine and groceries is 3 : 4 of the remaining salary after saving. If his expense on the medicine was ₹2700, then find his monthly salary.

[2023]

- (a) ₹ 30000 (b) ₹ 20000
(c) ₹ 10000 (d) ₹ 15000

Ans. (c)

Sol. Let the salary of Hemant be ₹ x .
He deposits 10% in PF.

$$\text{Remaining} = x - \frac{10x}{100} = \frac{90x}{100}$$

He saves 30% of remaining

$$\text{Savings of Hemant} = \frac{30}{100} \times \frac{90x}{100} = \frac{27x}{100}$$

$$\text{Expenses of Hemant} = \frac{90x}{100} - \frac{27x}{100}$$

$$= \frac{63x}{100} \quad (i)$$

The ratio of expenses of medicine and groceries is 3 : 4 and the expenses of medicine is ₹2700.

$$\Rightarrow 3 \text{ units} = 2700$$

$$\Rightarrow 1 \text{ unit} = 900$$

$$\Rightarrow 4 \text{ units} = 3600$$

$$\text{Total expenses} = 2700 + 3600$$

$$= 6300 \quad (ii)$$

From Eqs. (i) and (ii)

$$\Rightarrow \frac{63x}{100} = 6300$$

$$\Rightarrow x = 10000$$

∴ Monthly salary of Hema.. is ₹10000.

Computer Awareness

91. The time required for fetching and execution of one machine instruction is

[2023]

- (a) Delay time (b) Seek time
(c) Real time (d) CPU cycle

Ans. (d)

Sol. The time required for fetching and executing one machine instruction is typically measured in terms of CPU cycles. A CPU cycle, also known as a clock cycle, is the basic unit of time used by a processor to execute instructions. Each instruction requires a certain number of CPU cycles to complete, which includes the time for fetching the instruction from memory, decoding it, and executing the corresponding operation.

92. The number of minterms in a n variable truth table is [2023]

- A. n^2 B. $(n-1)^2$ C. 2^n D. 2^{n-1}

(a) D (b) B (c) A (d) C

Ans. (d)

Sol. The number of minterms in a n -variable truth table is 2^n .

93. The maximum and minimum value represented in signed 16-bit 2s complement representation are [2023]

- (a) 0 and 32767 (b) 0 and 65535
(c) -16384 and 16383 (d) -32768 and 32767

Ans. (d)

Sol. In a signed 16-bit 2's complement representation, the most significant bit (MSB) is reserved for the sign of the number. The remaining 15 bits represent the magnitude of the number.

For a signed 16-bit representation, the range of values that can be represented is from -2^{15} to $2^{15}-1$.

$$-2^{15} = -32768$$

$$2^{15}-1 = 32767$$

Therefore, the maximum value represented is +32767, and the minimum value represented is -32768.

94. A bulb in the staircase has two switches, one switch is at the ground floor and the other one is at the first floor. The bulb can be turned ON and also can be turned OFF by any of the switches irrespective of the state of the other switch. The logic of the switching of the bulb resembles [2023]

- (a) XOR Gate (b) AND Gate
(c) OR Gate (d) XNOR Gate

Ans. (a)

Sol. In this scenario, the behaviour of the switches can be represented by an XOR gate. An XOR gate, also known as an Exclusive OR gate, produces an output of "true" (or ON) only when the number of input signals that are true is odd. In this case, the bulb will be ON when either switch is flipped, regardless of the state of the other switch.

95. What is a potential problem of 1's complement representation of numbers? [2023]

- (a) Binary subtractions are not possible
(b) There are two different representations of zero
(c) Multiplication of two numbers cannot be carried out
(d) Binary additions are not possible

Ans. (b)

Sol. In 1's complement representation of numbers, the sign of a number is determined by the leftmost bit (MSB), where 0 represents a positive number and 1 represents a negative number. To obtain the 1's complement of a number, each bit of the original number is inverted.

However, one potential problem with 1's complement representation is that there are two different representations of zero. In regular binary representation, zero is represented as all zeros.

However, in 1's complement representation, zero can be represented as either all zeros or all ones.

96. A wrong sentence related to FAT 32 and NTFS file systems is [2023]

- (a) FAT 32 has lower disk utilisation compared to NTFS
(b) Read and write speeds of NTFS are faster than that of FAT 32
(c) FAT 32 store individual files of size up to 32 GB
(d) NTFS stands for New Technology File System

Ans. (c)

Sol. In general, NTFS (New Technology File System) tends to provide faster read and write speeds compared to FAT32 (File Allocation Table). NTFS incorporates various improvements over FAT32, including better file organisation, enhanced security features, and improved performance.

97. Consider the circuit shown below and find minimum number of NAND gates required to design it.



[2023]

- (a) 4 (b) 6 (c) 3 (d) 5

Ans. (a)

Sol. A NAND gate is a type of logic gate that performs the logical operation called NAND (NOT-AND). It is a combination of an AND gate followed by a NOT gate. The minimum number of NAND gates required to design given circuit is 4.

98. Consider the following minterm expression for $F: F(P, Q, R, S) = \Sigma(0, 2, 5, 7, 8, 10, 13, 15)$. The minterms 2, 7, 8 and 13 are don't care terms. The minimal sum of products form for F is **[2023]**

- (A) $\bar{Q}S + Q\bar{S}$
 (B) $\bar{Q}\bar{S} + QS$
 (C) $\bar{Q}\bar{R}\bar{S} + \bar{Q}R\bar{S} + Q\bar{R}S + QRS$
 (D) $\bar{P}\bar{Q}\bar{S} + \bar{P}QS + PQS + P\bar{Q}\bar{S}$
 (a) A (b) D (c) B (d) C

Ans. (c)

Sol. $F(P, Q, R, S) = \Sigma(0, 2, 5, 7, 8, 10, 13, 15)$

PQ \ RS	00	01	11	10
00	1			x
01		1	x	
11		x	1	
10	x			1

While putting the terms to K-map following things happen,

- 3rd and 4th columns are swapped
- 3rd and 4th rows,
- term 2 is going to (0,3) column instead of (0,2)
- 8 is going to (3,0) instead of (2,0)

Solving, the above K-map. we get $\bar{Q}\bar{S} + QS$

99. The reduced form of the Boolean function $F = xyz + xyz' + x'y z + xy'z$ is **[2023]**

- (a) $xy + yz$ (b) $x + yz + xz$
 (c) $x + y + z$ (d) $xy + yz + xz$

Ans. (d)

Sol. Let's simplify the Boolean expression

$$F = xyz + xyz' + x'y z + xy'z$$

$$xy + yz$$

We can factor out y from the first two terms:

$y(xz + z')$. The expression $xy + yz$ is not equivalent to F .

$$x + yz + xz$$

We can factor out z from the first and last terms:

$xz(y + x' + 1)$. The expression $x + yz + xz$ is not

equivalent to F .

$$x + y + z$$

This expression is not equivalent to F as it does not contain the term $xy'z$.

$$xy + yz + xz$$

We can see that this expression matches the given Boolean function F . It is the reduced form of F .

Therefore, the correct reduced form of $F = xyz + xyz' + x'y z + xy'z$ is $xy + yz + xz$.

100. If a processor clock is rated as 2500 million cycles per second, then its clock period is **[2023]**

- (A) 2.50×10^{-10} sec (B) 4.00×10^{-10} sec
 (C) 1.00×10^{-10} sec (D) 5.00×10^{-10} sec
 (a) D (b) C (c) A (d) B

Ans. (d)

Sol. To find the clock period of a processor given its clock rate, we can use the formula:

$$\text{Clock Period} = 1 / \text{Clock Rate}$$

In this case, the processor clock rate is 2500 million cycles per second, or 2500 MHz.

Converting MHz to Hz :

$$2500 \text{ MHz} = 2500 \times 10^6 \text{ Hz}$$

Now, we can calculate the clock period

$$\text{Clock Period} = 1 / (2500 \times 10^6 \text{ Hz})$$

$$\text{Clock Period} = 0.0000004 \text{ seconds or } 400 \text{ nanoseconds}$$

Therefore, the clock period of the processor is 400 nanoseconds or $4.00 \times 10^{-10} \text{ sec}$.

101. Suppose we have a 10-bit computer that uses 10-bit int (2's complement representation). The number representation of -35 is **[2023]**

- (A) 0000100011 (B) 1100100011
 (C) 1111011101 (D) 1111011101
 (a) B (b) D (c) A (d) C

Ans. (d)

Sol. To represent -35 in a 10-bit computer that uses 10-bit two's complement representation, follow these steps: Represent the positive value of 35 in binary: 0000100011.

Invert all the bits: 1111011100.

Add 1 to the inverted value: 1111011101.

Thus, the 10-bit two's complement representation of -35 would be 1111011101.

102. Consider the following Boolean expression for

$FF(P, Q, R, S) = PQ + \bar{P}QR + \bar{P}Q\bar{R}S$. The minimum sum of products form of F is **[2023]**

- (A) $PQ + QR + QS$ (B) $P + Q + R + S$
 (C) $\bar{P} + \bar{Q} + \bar{R} + \bar{S}$ (D) $\bar{P}R + \bar{P}\bar{R}S + P$
 (a) D (b) A (c) C (d) B

Ans. (b)

Sol. $F(P, Q, R, S) = PQ + \bar{P}QR + \bar{P}Q\bar{R}S$

$$= Q(P + \bar{P}R + \bar{P}\bar{R}S)$$

$$= Q(P + R + \bar{R}S)$$

$$= Q(P + R + S) = PQ + QR + QS$$

103. What is the name of the storage device that compensates the difference in rates of flow of data from one device to another? [2023]

- (a) Cache (b) Buffer
(c) Concentrator (d) RAM

Ans. (b)

Sol. A buffer is a storage device or area that is used to compensate for the difference in rates of flow of data between two devices. It temporarily holds data when there is a difference in data transfer rates or when the receiving device is not ready to process the data. Buffers help smooth out the data transfer process and prevent data loss or disruption.

104. Equivalent of the decimal number $(25.375)_{10}$ in binary form [2023]

- (A) $(11001.011)_2$ (B) $(11101.011)_2$
(C) $(11011.111)_2$ (D) $(11001.101)_2$
(a) A (b) D (c) B (d) C

Ans. (a)

Sol. To convert the decimal number $(25.375)_{10}$ to binary form, we can convert the integer and fractional parts separately.

Integer Part

The integer part of $(25.375)_{10}$ is 25. To convert it to binary, we divide it by 2 repeatedly until the quotient becomes 0, and read the remainders in reverse order.

$$25 \div 2 = 12 \text{ remainder } 1$$

$$12 \div 2 = 6 \text{ remainder } 0$$

$$6 \div 2 = 3 \text{ remainder } 0$$

$$3 \div 2 = 1 \text{ remainder } 1$$

$$1 \div 2 = 0 \text{ remainder } 1$$

Reading the remainders in reverse order, we get the binary representation of the integer part:

$$(25)_{10} = (11001)_2$$

Fractional Part

The fractional part of $(25.375)_{10}$ is 0.375. To convert it to binary, we multiply it by 2 repeatedly until the fractional part becomes 0, and read the integer parts.

$$0.375 \times 2 = 0.75 \rightarrow 0$$

$$0.75 \times 2 = 1.5 \rightarrow 1$$

$$0.5 \times 2 = 1.0 \rightarrow 1$$

Reading the integer parts, we get the binary representation of the fractional part:

$$(0.375)_{10} = (0.011)_2$$

Combining the binary representation of the integer and fractional parts, we get:

$$(25.375)_{10} = (11001.011)_2 \text{ in binary form.}$$

105. Which of the following registers is used to keep track of address of the memory location where the next instruction is located? [2023]

- (a) Program Counter
(b) Memory Data Register
(c) Instruction counters
(d) Memory Address Register

Ans. (a)

Sol. The Program Counter (PC) is a register used in computer architecture to keep track of the address of the memory location where the next instruction to be fetched is located. It is also sometimes referred to as the Instruction Pointer (IP) in certain computer architectures. The PC is automatically incremented after each instruction is fetched, allowing the processor to fetch the next instruction from the sequential memory location.

106. A CPU generates 32-bit virtual addresses. The page size is 4 KB. The processor has a translation look-aside buffer (TLB) which can hold a total of 128 page table entries and is 4-way set associative. The minimum size of the TLB tag is [2023]

- (a) 13 bits (b) 20 bits
(c) 11 bits (d) 15 bits

Ans. (d)

Sol. To determine the minimum size of the TLB tag, we need to consider the number of page table entries the TLB can hold and the page size.

Given:

CPU generates 32-bit virtual addresses.

TLB can hold a total of 128 page table entries.

The TLB is 4-way set associative.

Page size is 4 KB.

To calculate the TLB tag size, we can use the formula:

Tag size = Address size - Index size - Offset size

First, we need to find the index size. Since, the TLB is 4-way set associative and can hold 128 page table entries, the number of sets is $128/4 = 32$. Therefore, the index size is $\log_2(32) = 5$ bits.

Next, we calculate the offset size. The page size is 4 KB, which is equivalent to (2^{12}) bytes. So, the

offset size is $\log_2(2^{12}) = 12$ bits.

Now, we can calculate the tag size:

Tag size = 32 bits - 5 bits - 12 bits = 15 bits

Therefore, the minimum size of the TLB tag is 15 bits.

107. Which of the following is true about Von Neumann architecture? [2023]

- (a) It has separate storage for input/output operations
(b) It has a separate processing unit for data and instructions
(c) It has separate memory for data and instructions

(d) It has a single memory unit for both data and instructions.

Ans.(d)

Sol. It has a single memory unit for both data and instructions.

Von Neumann architecture, named after computer scientist John von Neumann, is a computer architecture design that is widely used in most modern computers.

It is characterised by the following features:

It has a single memory unit: In Von Neumann architecture, both instructions and data are stored in the same memory unit. This means that the program instructions and data are fetched from and stored in the same memory.

It has a Central Processing Unit (CPU): The CPU is responsible for executing instructions and performing calculations. It fetches instructions from memory, decodes them, and executes them.

108. Let $\oplus$ and $\square$ denote the Exclusive -OR and Exclusive -NOR operations respectively. Which one of the following is not correct? [2023]

(A) $\bar{P} \oplus \bar{Q} = P \square Q$

(B) $\bar{P} \oplus Q = P \square Q$

(C) $\bar{P} \oplus \bar{Q} = P \oplus Q$

(D) $\left(P \oplus \bar{P} \right) \oplus Q = \left(P \square \bar{P} \right) \square \bar{Q}$

(a) C (b) D (c) B (d) A

Ans. (b)

Sol. XOR gate is a digital logic gate that gives a true (1 or HIGH) output when the number of true inputs is odd. The XNOR (Exclusive-NOR) gate is a combination XOR gate followed by an inverter. Its output is "true" if the inputs are the same, and "false" if the inputs are different.

109. Suppose we have a 10-bit computer that uses 10-bit floating point computational unit (Float number uses IEEE floating-point arithmetic where a floating point number has 1 sign bit, 5 exponent bits, and 4 fraction bits). The representation for $+\infty$ (plus infinity) is

[2023]

(a) 0111110000

(b) 1111110000

(c) 0000001111

(d) 0111111111

Ans. (a)

Sol. The correct representation for $+\infty$ (plus infinity) in the given 10-bit floating point format using IEEE floating-point arithmetic would be:

0111110000

In this representation, the sign bit is 0 (indicating a positive value), the exponent bits are all 1s (1111),

and the fraction bits are all 0s(0000). This follows the IEEE 754 standard for representing infinity in floating-point arithmetic.

110. How many $32\text{ K} \times 1$ RAM chips are needed to provide a memory capacity of 256 K-bytes? [2023]

(a) 8 (b) 128 (c) 64 (d) 32

Ans. (c)

Sol. Data

RAM chip's capacity = $32\text{ K} \times 1 = 32 \times 1024 \times 1$ bits

Memory Capacity = 256 K bytes = $256 \times 1024 \times 8$ bits

Formula

Number of chips needed

$$= \frac{\text{Memory Capacity}}{\text{RAM Chip's Capacity}}$$

Calculation

Number of chips needed

$$= \frac{256 \times 1024 \times 8}{32 \times 1024 \times 1} = 64 \text{ chips}$$

General English

Directions (Q. Nos. 111 and 112) Read the following passage carefully and answer the questions that follow. Science and religion - the two terms have come to signify a mutual antagonism. The two, it is commonly declared, are poles apart; their spheres of activity and their methods differ widely, so much so that they are considered to be irreconcilable.

On the face of it, science and religion appear to be the two opposite poles of man's consciousness. Science is basically concerned with the material world; its efforts are directed towards unravelling the 'how' of reality while religion is concerned with the 'why' of reality. Science deals with analysing tangible entities into its minutest parts, and then arrives at conclusions about the way in which tangible realities are organised. While science is analytical, religion takes the ultimate reality for granted. Religion follows the metaphysical path; the concept of God is ultimately a matter of faith and it is this faith which is the basis of the religious man's attribution of a design or meaning for the reality. The modes of action are different in science and religion. Science relies on experiment, whereas religion is based on experience. Science benefits mankind by providing material comforts. The frontiers of science do not end in knowledge but are extended to the formation of appliances for actual use. Science, it has been somewhat unfairly charged, cultivates the materialistic thinking. However, it has to be admitted that the mental attitude promoted by religion is entirely different. While the basis of

scientific progress is unbridled curiosity and courageous endeavour, the truly religious spirit cavils at such presumption that man's mind can penetrate the mysteries of the universe. Science promotes fearless inquiry, while an essential ingredient of religion is the humility born of fear of God. Science incorporates a love of experimental knowledge, while religion does not believe in the rational approach.

111. Which of the following statements according to the passage is correct? [2023]

- (a) The religious spirit assumes that human mind can penetrate the mysteries of the universe.
- (b) Science follows the metaphysical path.
- (c) Science believes in the humility born of fear of God.
- (d) Religion believes in ultimate reality.

Ans. (d)

Sol. According to the passage, science is analytical while religion believes in and takes the ultimate reality for granted.

112. Which of the following reasons according to the passage provides material comforts to people in case of science? [2023]

- (a) Tangible proofs of the theories of science
- (b) Materialistic thinking being cultivated by science
- (c) Promotion of fearless inquiry by science
- (d) The subjectivity of science

Ans. (a)

Sol. The passage suggests that tangible proofs of the theories of science provide material comforts to people.

113. Select the most appropriate meaning of the underlined idiom in the given sentence.

Off and on, I take a break from my hectic schedule to refresh myself.

- A. Periodically
- B. Rarely
- C. Seldom
- D. Immediately

(a) B (b) A (c) C (d) D

Ans. (b)

Sol. The idiom 'off and on' means now and then; periodically.

114. I have _____ umbrella, I bought it _____ year ago. [2023]

- (a) a, an
- (b) an, a
- (c) an, the
- (d) the, an

Ans. (b)

Sol. 'An' and 'a' are appropriate to fill the blanks.

115. Synonym for "Nonplussed" is [2023]

- (a) Flummoxed
- (b) Dumbfounded
- (c) Befuddled
- (d) Oriented

Ans. (*)

Sol. 'Nonplussed' means surprised and confused. The words 'flummoxed', 'dumbfounded' and 'befuddled' are its synonyms.

116. Select the most appropriate preposition to fill in the blank.

A baby sitter is someone who looks _____ other people's children. [2023]

- (a) after
- (b) for
- (c) on
- (d) over

Ans. (a)

Sol. 'Look after' means to take care of someone. Hence, 'after' is appropriate to fill the blank.

117. Select the most appropriate preposition to fill in the blank.

We haven't been to Delhi _____ almost five years. [2023]

- (a) to
- (b) since
- (c) from
- (d) for

Ans. (d)

Sol. 'For' is appropriate to fill the blank.

118. Meaning of 'Abrogate' is [2023]

- (a) Abolish
- (b) Absorb
- (c) Abstract
- (d) Ablaze

Ans. (a)

Sol. 'Abrogate' means to repeal or do away with. Hence, 'abolish' is similar in meaning to it.

119. Choose the best option that indicates the change of voice for the sentence given below. [2023]

They sent for a doctor because Pamela had fainted.

- (a) Pamela fainted and a doctor was sent for.
- (b) A doctor was sent for them because Pamela had fainted.
- (c) A doctor was sent for because Pamela had fainted.
- (d) Pamela had sent for a doctor because they had fainted.

Ans. (c)

Sol. The correct passive form of the given sentence is- 'A doctor was sent for because Pamela had fainted'.

120. Antonym for 'Spendthrift' is [2023]

- (a) Profligate
- (b) Extravagant
- (c) Frugal
- (d) Squanderer

Ans. (c)

Sol. 'Spendthrift' means spending money in an extravagant way. Hence, 'frugal' meaning thrifty is its antonym.